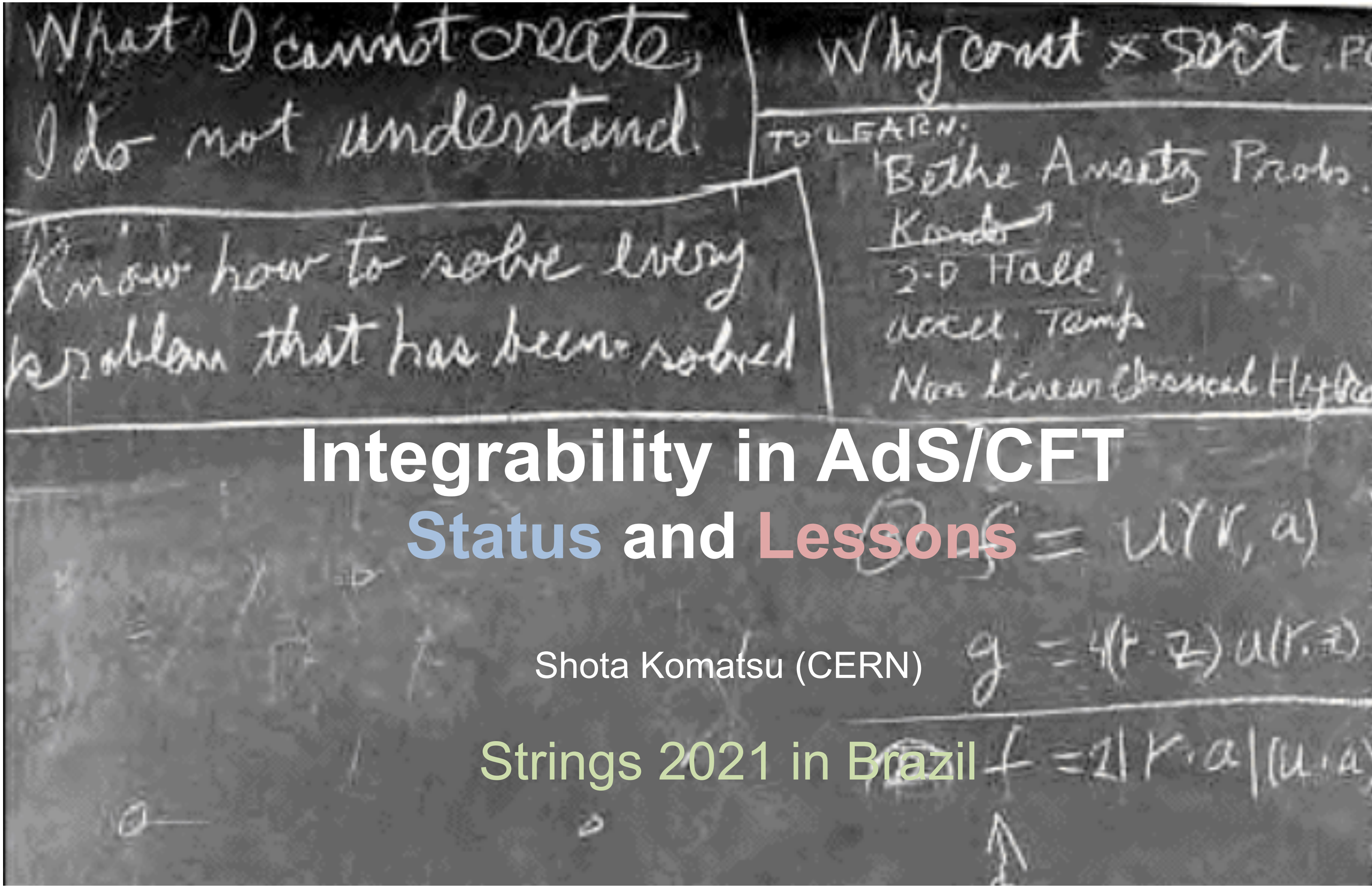
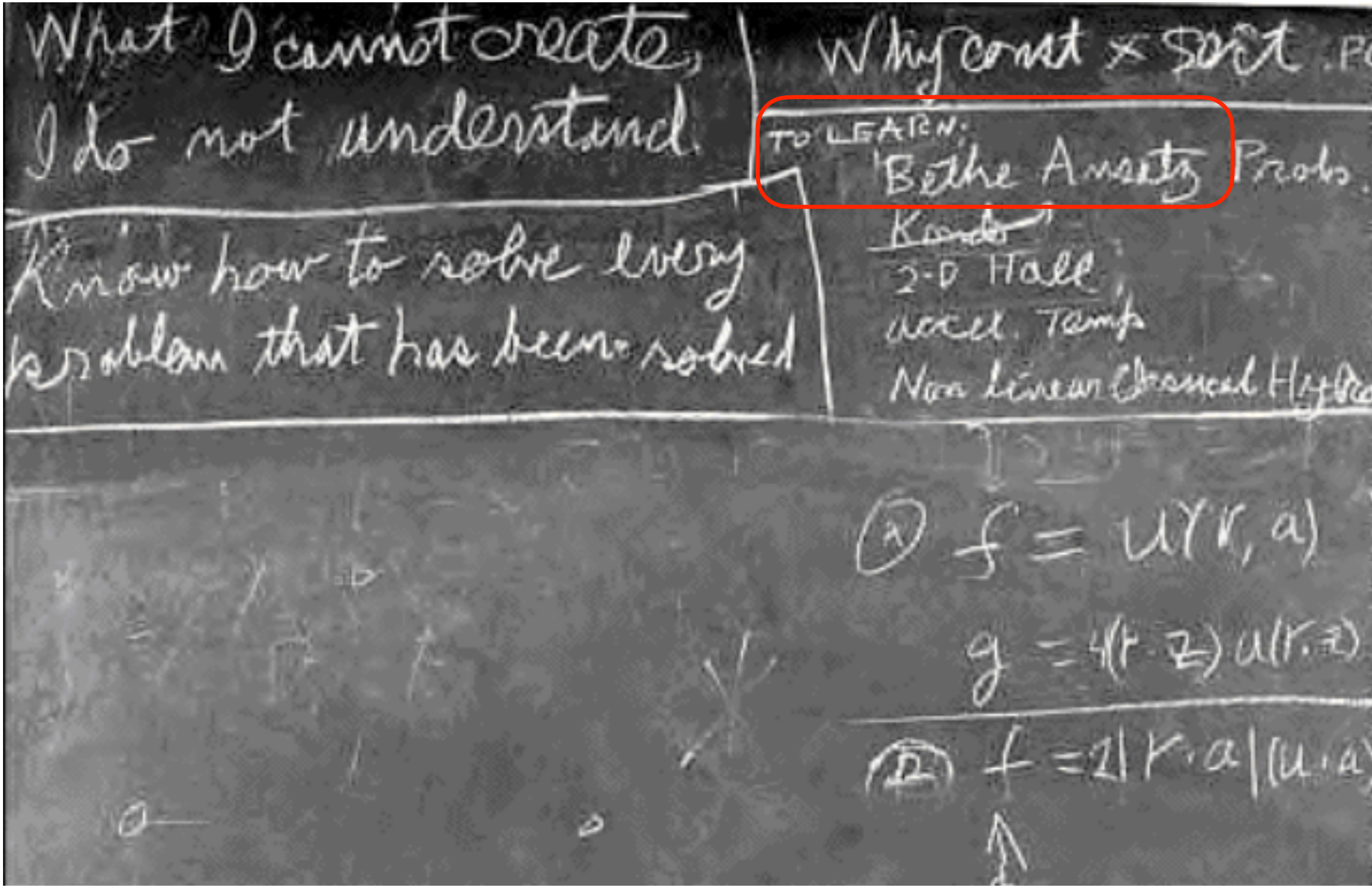


Integrability in AdS/CFT

Status and Lessons

Shota Komatsu (CERN)

Strings 2021 in Brazil



Feynman’s last blackboard taken from 1810.07409 by Dhar, Patel, Wadia, (Trivedi)
cf. Colloquium by Ooguri @ Aspen 2018

Disclaimer

I will focus on N=4 super Yang-Mills in 4d.

- ABJM, AdS_3/CFT_2, deformations, N=2 SCFT…..

2106.08449 by Pomoni, Rabe, Zoubos
Dynamical Yang-Baxter eq.

Goals are

- To give a "feeling" of what the Quantum Spectral Curve is (and explain applications)
- To explain recent developments involving D-branes (e.g. determinant ops)

Things that I will NOT discuss (or will only mention briefly)

- Hexagons for correlation functions, Pentagons for amplitudes / form factor
  talk by Coronado   review by Vieira in Strings 2017, [Sever, Tumanov, Wilhelm]
- Yangian from defects in large N SQFT   Ishtiaque, Moosavian, Zhou, Costello, Gaiotto, Dedushenko, Oh…..
  cf. talk by Dedushenko
- Conformal fishnet theories: non-unitary log CFT obtained by deforming N=4 SYM
  Gurdogan, Kazakov, Caetano, Mamroud, Torrents…..

What do we mean by "N=4 SYM is integrable"?

- $\mathcal{O}_k(x_k)$: single-trace non-BPS op

$$\left\langle \prod_k \mathcal{O}_k(x_k) \right\rangle = \left(\bullet + \frac{1}{N^2}\bullet + \frac{1}{N^4}\bullet + \cdots \right) + e^{-N\bullet}\left(\bullet + \frac{1}{N^2}\bullet + \cdots \right) + \cdots$$

Instantons

$$+ e^{-N^2\star}\left(\star + \frac{1}{N^2}\star + \cdots \right) + \cdots + e^{-e^{N^2\star}} + \cdots$$

black holes, worm holes (single bdy)

doubly nonperturbative

Expectation: we can compute • terms at finite λ but not ★ terms.

- **Heavier** operators:
  Some correlation functions involving $\Delta \sim O(N)$ are computable.
  D-branes
  But operators with $\Delta \sim O(N^2)$ are NOT*.
  Black holes

*See however attempts [de Mello Koch, Kim, Zyl]

- As long as operators do not back-react $\mathrm{AdS}_5 \times S^5$, there's a chance we can compute things using integrability.

Why should we care?

- First example of solvable interacting gauge theory in 4d.
- String theory on RR-flux background.

Alternative approaches: [Berenstein, Leigh], [Cho, Collier, Yin]
[Berkovits, Vafa, Witten], String Field Theory

- Qualitative/quantitative similarities with pure Yang-Mills: e.g. BFKL physics
- Starting point for conformal perturbation/Hamiltonian truncation (at large N).

- Might tell us "how the world sheet theory emerges from gauge theory"

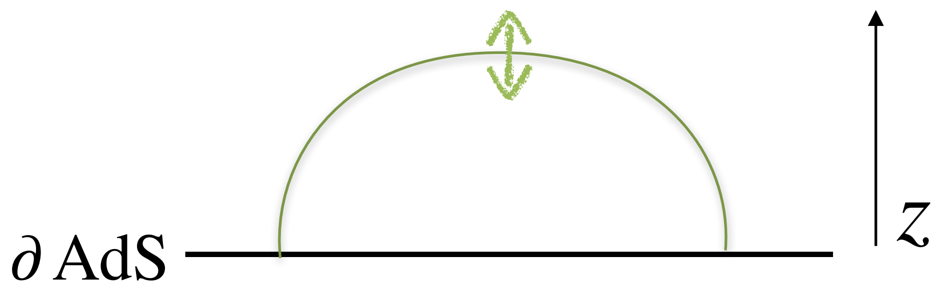
Simpler version of the question: How do bulk/gravity emerge from QFT/QM?

- Hopefully give us hints about the worldsheet dual of pure Yang-Mills?

talk by Gaberdiel

Radial direction from fermion bilinear

- Fluctuations on GKP string (= null Wilson lines)
  [Gubser, Klebanov, Polyakov]



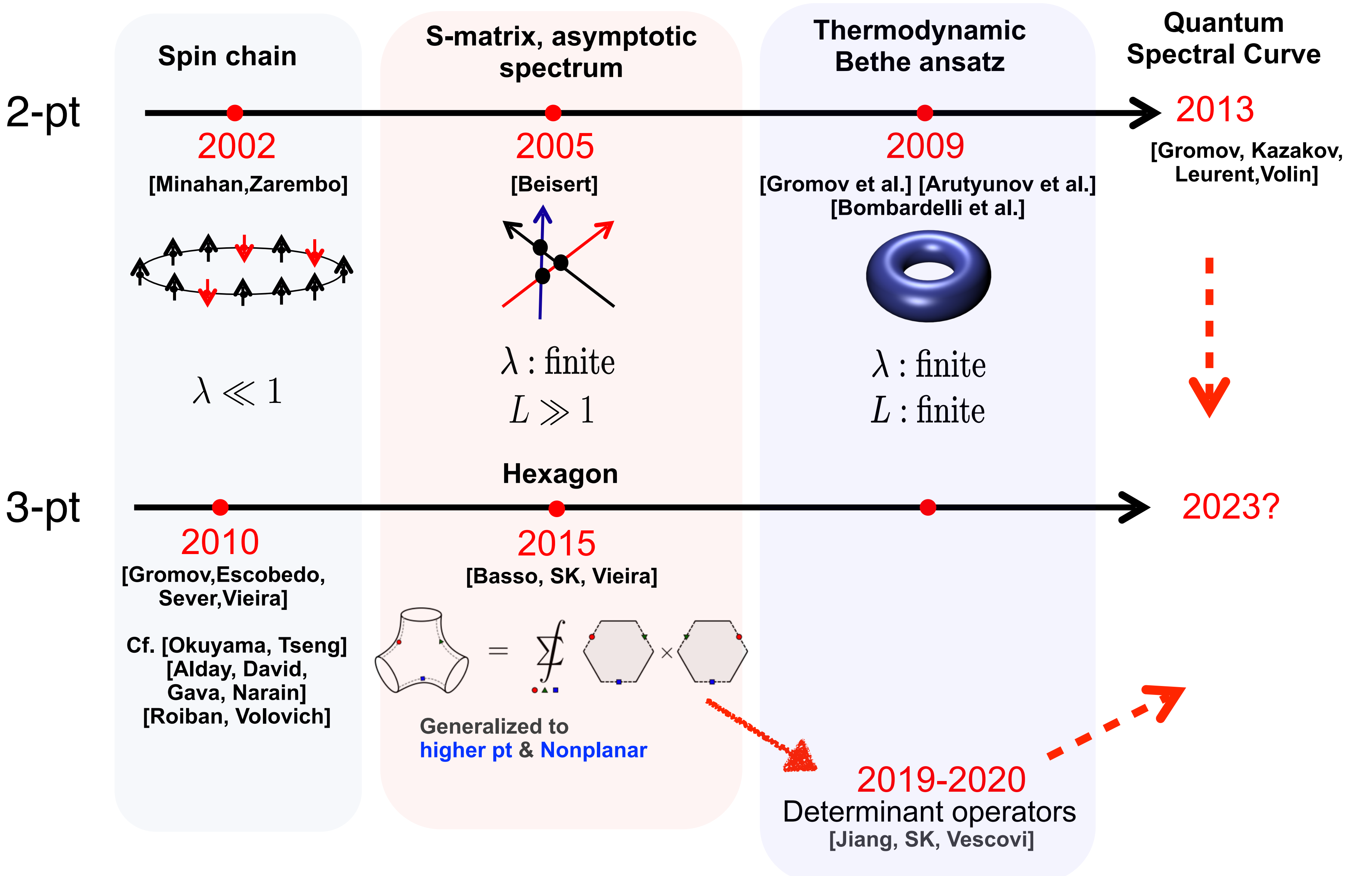
@ Weak coupling (=gauge), No mode corresponding to the radial fluctuation.

- Radial mode = 2 fermion (threshold) bound state
  [Basso], [Basso, Sever, Vieira]

$$\bar{\psi}\psi \longrightarrow z$$

- A similar mechanism for probe branes discussed in [Ferrari et al, '12-'16]

Status: $\mathcal{O} = \mathrm{tr}(\underbrace{XZ\cdots ZX}_{L})$



Main message

N=4 SYM is….

Spin chain

Large N gauge theory / matrix model

So, let’s start with basics….

It all started from here.... [Minahan, Zarembo '02]

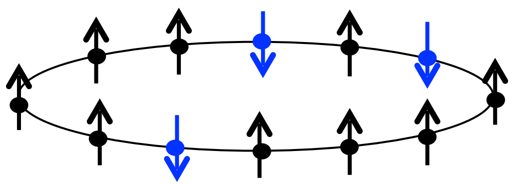
- Minahan and Zarembo found a relation between a 1-loop dilatation operator and a Hamiltonian of spin chain.

N=4 SYM	spin chain
$\mathcal{O} = \mathrm{tr}\,[\, Y Z Y Z Z \cdots]$	$\lvert \downarrow \uparrow \downarrow \uparrow \uparrow \cdots \rangle$
$D_{\text{1-loop}}$	$H_{\text{Heisenberg}} \propto \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$
Δ	Energy

- The spin chain turns out to be solvable by Bethe ansatz.

Bethe ansatz

$$H_{\text{Heisenberg}} \propto \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$



- **Step 1**: Write an ansatz

$$|p_1, p_2, p_3\rangle = \sum_{n_1<n_2<n_3} \Psi_{n_1,n_2,n_3} |\uparrow \cdots \underset{n_1}{\downarrow} \cdots \underset{n_2}{\downarrow} \cdots \underset{n_3}{\downarrow} \cdots \rangle$$

$$\Psi_{n_1,n_2,n_3} = e^{i(p_1 n_1 + p_2 n_2 + p_3 n_3)} + S(p_1,p_2) e^{i(p_2 n_1 + p_1 n_2 + p_3 n_3)} + S(p_1,p_2)S(p_1,p_3) e^{i(p_2 n_1 + p_3 n_2 + p_1 n_3)}$$
$$+ (\text{permutations})$$

- **Step 2**: Impose $H|p_1, p_2, p_3\rangle = E|p_1, p_2, p_3\rangle$

→

1. $S(p_1, p_2) = \dfrac{u_1 - u_2 - i}{u_1 - u_2 + i}$ $\left(e^{ip} \equiv \dfrac{u + i/2}{u - i/2} \right)$ Rapidity variable

2. $e^{ip_j L} \prod_{k \neq j} S(p_j, p_k) = 1$ Bethe equation

3. $E = \sum_j \dfrac{1}{u_j^2 + \frac{1}{4}}$ Energy = sum over energies of magnons

Finite λ

$$\mathcal{O} = \mathrm{tr}(\underbrace{XZ\cdots ZX}_{L})$$

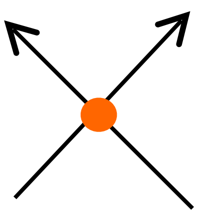
- Assume integrability persists at finite λ: For L>>1, we can write the Bethe equation at finite λ.

$$e^{ip_j L}\prod_{k\neq j} S^{(\lambda)}(p_j, p_k) = 1$$

- $S^{(\lambda)}(p,q)$ can be determined by imposing the **centrally-extended** $\mathrm{SU}(2|2)^2$ symmetry. [Beisert], [Janik]

$$[j, S^{(\lambda)}] = 0\,, \qquad j \in \underline{\mathrm{SU}(2|2)^2} + \boxed{P+K}$$

⊂ superconformal sym.



Central charge = gauge transf = translation

- P, K are field-dependent ("large") gauge transformation

$$Q(\text{boson}) = (\text{fermion}), \qquad Q'(\text{fermion}) = \cdots + \underbrace{g_{\text{YM}}[Z, (\text{boson})]}_{= P}$$

$$\{Q, Q'\} = P$$

- P, K are discrete translations on the spin chain

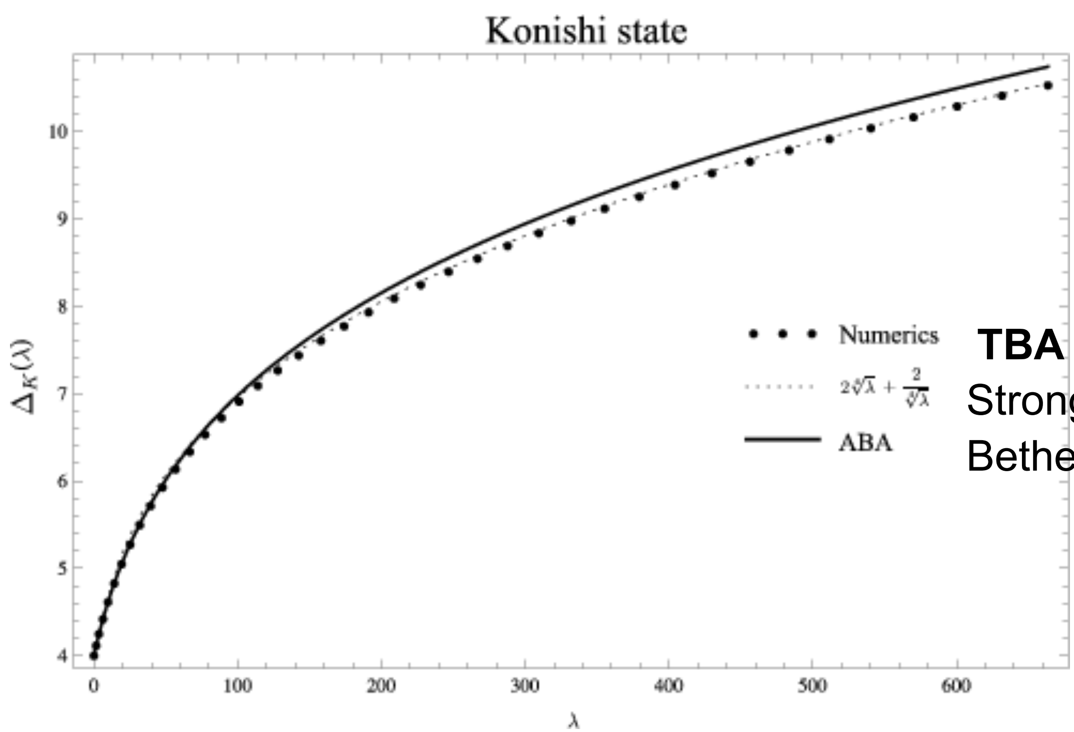
$$P\,|Z\cdots \underset{n}{Y} \cdots\rangle \propto |Z\cdots Z \underset{n+1}{Y} \cdots\rangle - |Z\cdots \underset{n}{Y} Z\cdots\rangle$$

Spin chain

Large N gauge theory / matrix model

Finite L $\mathcal{O} = \mathrm{tr}(\underbrace{XZ\cdots ZX}_{L})$

- $S^{(\lambda)}$ determine all perturbative $1/L$ corrections at finite λ.
- $e^{-L\bullet}$ corrections "can" be computed by **Thermodynamic Bethe ansatz**
  TBA (later)



TBA
Strong coupling
Bethe ansatz

[Gromov, Kazakov, Vieira]

- Works for some operators, but practically hard for most operators…..

Quantum Spectral Curve

[Gromov, Kazakov, Leurent,Volin]

Reformulation of Bethe equation

$$e^{ip_jL}\prod_{k\neq j}S(p_j,p_k)=1 \iff \left(\frac{u_j+\frac{i}{2}}{u_j-\frac{i}{2}}\right)^L\prod_{k\neq j}\frac{u_j-u_k-i}{u_j-u_k+i}=1$$

- Introduce $Q(u)\equiv\prod_j(u-u_j)$ Q-function
- Bethe eq is equivalent to

$$T(u)Q(u)=(u-i/2)^LQ(u+i)+(u+i/2)^LQ(u-i)$$

$$0=T(u_j)Q(u_j)=(u_j-i/2)^LQ(u_j+i)+(u_j+i/2)^LQ(u_j-i)$$

- It can be further reformulated into

$$u^L\propto Q(u+i/2)\tilde{Q}(u-i/2)-Q(u-i/2)\tilde{Q}(u+i/2)$$

QQ-relation
(Plucker identities of Grassmannian)

with $T(u)\propto Q(u+i)\tilde{Q}(u-i)-Q(u-i)\tilde{Q}(u+i)$

QQ-relation for N=4 SYM

- QQ-relations can be generalized to the full N=4 SYM.

$$\mathbf{Q}_{a|j}(u+i/2)-\mathbf{Q}_{a|j}(u-i/2)=\mathbf{P}_a(u)\mathbf{Q}_j(u) \qquad (a,j=1,\dots,4)$$

e.g.

$$\mathbf{Q}_j(u)=\mathbf{Q}_{a|j}(u\pm i/2)\mathbf{P}^a(u) \qquad \mathbf{P}_{\mathbf{a}}(u)=\mathbf{Q}_{a|j}(u\pm i/2)\mathbf{Q}^j(u)$$

- QQ-relations do not depend on λ. Same relations hold for spin chain at weak coupling.

Spin chain

- To incorporate λ-dependence, we need to use the fact that it is a large N gauge theory / matrix model.

Quantum Spectral Curve for Matrix Model

$$\int dM \ \exp\left(-\frac{N}{2g^2}\mathrm{tr}(M^2)\right) = \int \prod_j dm_j \prod_{j<k} (m_j - m_k)^2 e^{\frac{N}{2g^2}\sum_j m_j^2}$$

- Two-point functions of $\mathrm{tr}(M^L)$ are non-diagonal:

$$\left\langle \mathrm{tr}(M^L)\mathrm{tr}(M^{L'})\right\rangle_{\text{large N, connected}} \neq 0 \qquad \text{for } L \neq L'$$

- Diagonalization [Rodriguez-Gomez, Russo '16]

$$\mathrm{tr}(M^L) = \sum_j m_j^L \quad \mapsto \quad \sum_j T_L(m_j/g)$$

Chebyshev polynomial

- "Q-function/Quantum Spectral Curve"

$$T_L(u/g) = x^L + \frac{1}{x^L} =: Q^{(L)}(u)$$

$$u = g(x + 1/x)$$

Zhukovsky variable

Properties of Q-functions for Matrix Model

$$Q^{(L)}(u) = x^L + \frac{1}{x^L}$$

1. Most naturally defined on x-plane, which is a double cover of u.

$$u = g(x + 1/x) \quad \Longleftrightarrow \quad x(u) = \frac{u + \sqrt{u^2 - 4g^2}}{2g} \left(= \frac{1}{\text{resolvent}} \right)$$

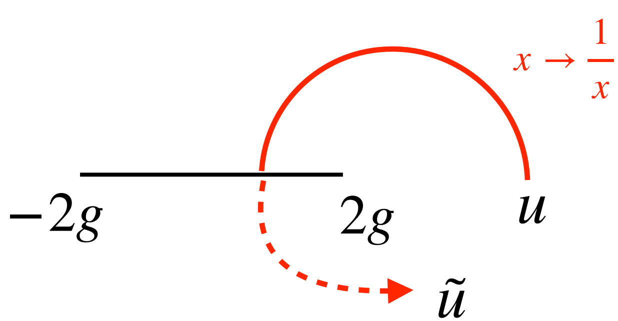
2. Charges (=Length of operators) can be read off from $u \to \infty$.

$$Q^{(L)}(u) \overset{u\to\infty}{\to} u^L$$

degree L branched covering

cf. [Mulase,Penkava], [Razamat],[Gopakumar]

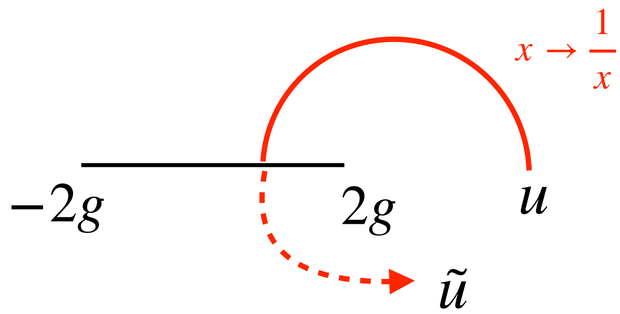
3. Satisfy the gluing condition (monodromy property).



$$Q(u) = Q(\tilde{u})$$

Gluing condition for N=4 SYM

SL(2) sector



$$\mathbf{Q}_1(\tilde{u}) \propto \left(\mathbf{Q}_3(u)\right)^*$$
$$\mathbf{Q}_2(\tilde{u}) \propto \left(\mathbf{Q}_4(u)\right)^*$$

- Together with QQ-relations, they determine Q-functions.

Spin chain

Large N gauge theory /
matrix model

- Δ can be read off from $u \to \infty$:

$$\mathbf{Q}_1(u) \overset{u\to\infty}{\to} u^{(\Delta-S)/2}$$

11-loop anomalous dimension of Konishi

[Marboe, Volin]

$$
\begin{aligned}
\gamma_{11} \quad = \quad & -242508705792 + 107663966208\zeta_3 + 70251466752\zeta_3^2 - 12468142080\zeta_3^3 \\
& +1463132160\zeta_3^4 - 71663616\zeta_3^5 + 180173002752\zeta_5 - 16655486976\zeta_3\zeta_5 \\
& -24628230144\zeta_3^2\zeta_5 - 2895575040\zeta_3^3\zeta_5 + 19278176256\zeta_5^2 - 9619845120\zeta_3\zeta_5^2 \\
& +2504494080\zeta_3^2\zeta_5^2 + \tfrac{882108048384}{175}\zeta_5^3 + 45602231040\zeta_7 + 14993482752\zeta_3\zeta_7 \\
& -12034759680\zeta_3^2\zeta_7 + 1406730240\zeta_3^3\zeta_7 + 30605033088\zeta_5\zeta_7 + 21217637376\zeta_3\zeta_5\zeta_7 \\
& -\tfrac{1309941061632}{275}\zeta_5^2\zeta_7 - 13215327552\zeta_7^2 - 4059901440\zeta_3\zeta_7^2 - 69762034944\zeta_9 \\
& +23284599552\zeta_3\zeta_9 - 3631889664\zeta_3^2\zeta_9 - 11032374528\zeta_5\zeta_9 - 6666706944\zeta_3\zeta_5\zeta_9 \\
& -23148129024\zeta_7\zeta_9 - 10024051968\zeta_9^2 - 54555179184\zeta_{11} + \tfrac{10048541184}{5}\zeta_3\zeta_{11} \\
& -726029568\zeta_3^2\zeta_{11} - 8975463552\zeta_5\zeta_{11} - 22529041920\zeta_7\zeta_{11} - \tfrac{1437993422496}{175}\zeta_{13} \\
& +\tfrac{1504385419392}{35}\zeta_3\zeta_{13} - 30324602880\zeta_5\zeta_{13} - \tfrac{151130039581392}{875}\zeta_{15} - 41375093760\zeta_3\zeta_{15} \\
& -\tfrac{196484147423712}{275}\zeta_{17} + 309361358592\zeta_{19} - 1729880064Z_{11}^{(2)} - \tfrac{1620393984}{5}\zeta_3 Z_{11}^{(2)} \\
& -131383296\zeta_5 Z_{11}^{(2)} + \tfrac{138107420928}{175}Z_{13}^{(2)} + \tfrac{3543865344}{35}\zeta_3 Z_{13}^{(2)} - \tfrac{5716780416}{7}Z_{13}^{(3)} \\
& -\tfrac{674832384}{7}\zeta_3 Z_{13}^{(3)} + \tfrac{48227088384}{175}Z_{15}^{(2)} + \tfrac{3581880576}{25}Z_{15}^{(3)} + 754974720Z_{15}^{(4)} \\
& -\tfrac{854924544}{11}Z_{17}^{(2)} + \tfrac{4963244544}{55}Z_{17}^{(3)} + \tfrac{818159616}{275}Z_{17}^{(4)} + \tfrac{175363688448}{1925}Z_{17}^{(5)}\,.
\end{aligned}
$$

$Z_{\bullet}^{(\bullet)}$ = multiple zeta values

Correlation functions from QSC?

- In Gaussian MM, correlation functions are given by integrals of Q-functions:

$$\langle \mathcal{O}_{L_1}\mathcal{O}_{L_2}\rangle_c = \oint d\mu_2\, Q^{(L_1)}(u)Q^{(L_2)}(u) \propto \delta_{L_1,L_2}$$

$$\langle \mathcal{O}_{L_1}\mathcal{O}_{L_2}\mathcal{O}_{L_3}\rangle_c = \oint d\mu_3\, Q^{(L_1)}(u_1)Q^{(L_2)}(u_2)Q^{(L_3)}(u_3)$$

$$\mathcal{O}_L = \mathrm{tr}(M^L) + \cdots \qquad d\mu_2 = \frac{dx}{2\pi i x} \qquad d\mu_3 = \frac{dx_1}{2\pi i}\frac{dx_2}{2\pi i}\frac{dx_3}{2\pi i}\frac{(x_1+x_2+x_3+x_1x_2x_3)(1+x_1x_2+x_2x_3+x_3x_1)}{\prod_j (1-x_j^2)^2}$$

They describe a topological subsector of N=4 SYM cf. [Drukker, Plefka]

- Hope for the same in N=4 SYM….?

$$\langle \mathcal{O}_1\mathcal{O}_2\rangle \sim \oint d\mu_2\, \mathbf{Q}_1(u)\mathbf{Q}_2(u)$$

$$\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\rangle \sim \oint d\mu_3\, \mathbf{Q}_1(u)\mathbf{Q}_2(u)\mathbf{Q}_3(u)$$

?

Correlation functions from QSC?

- Great progress for $d\mu_2$ at weak coupling (= rational spin chain)

Cavaglia, Gromov, Levkovich-Maslyuk, Ryan, Volin, Grabner, Julius…
Derkachov, Olivucci, Manashov, Kozlowski,….

$$\int d\mu_2 \, \mathbf{Q} \, \left((\text{charge}) \cdot \mathbf{Q}'\right) = \int d\mu_2 \, \left((\text{charge}) \cdot \mathbf{Q}\right) \, \mathbf{Q}' \propto \delta_{\mathbf{Q}\mathbf{Q}'}$$

$GL(n)$, non-compact spin chain, general representations….

- Simplest case (SL(2) spin chain of length L)

Derkachov, Korchemsky, Manashov

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \int_{-\infty}^{\infty} \left(\prod_{k=1}^{L-1} \frac{dx_k}{2\pi} \frac{\mathbf{Q}_1(x_k)\mathbf{Q}_2(x_k)}{(\cosh \pi x_k)^L} \right) \prod_{j<k} \sinh\left(\pi(x_j - x_k)\right)(x_j - x_k)$$

Can we derive/generalize using Bethe/gauge correspondence?
Nekrasov, Shatashvili

- Next steps: **1.** Finite λ. **2.** $d\mu_3$.

Difficulties: 1. # of integration variables increases with loops.
2. $d\mu_3 \neq d\mu_2$ (because of double traces)

Results also in fishnet limits and from localization.

Cavaglia, Gromov, Levkovich-Maslyuk

Giombi, Jiang, SK

D-branes, determinants, g-function

Jiang, SK, Vescovi, Yang, Wu, Wang

de Leeuw, Kristjansen, Zarembo, Wilhelm, Buhr-Mortensen, Ispen, Linardopoulos, Muller

Bajnok, Gombor

Determinant correlators

Jiang, SK, Vescovi '19

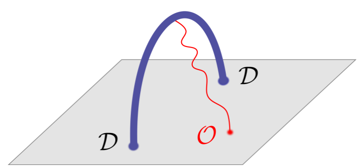
- A new class of solvable correlation functions.

$$\langle \det(Z)(x_1)\ \det(\bar{Z})(x_2)\ \mathcal{O}(x_3)\rangle$$

non-BPS single-trace

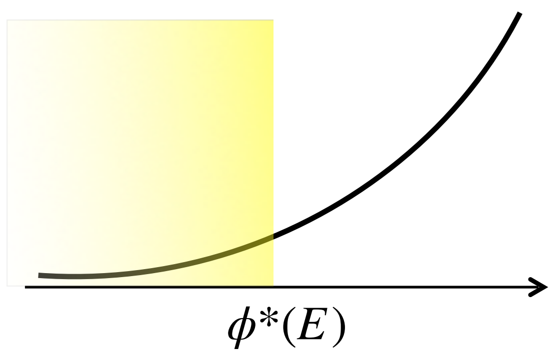
Analog of "**baryon-baryon-meson** vertex"

D-brane in AdS (**giant graviton**)

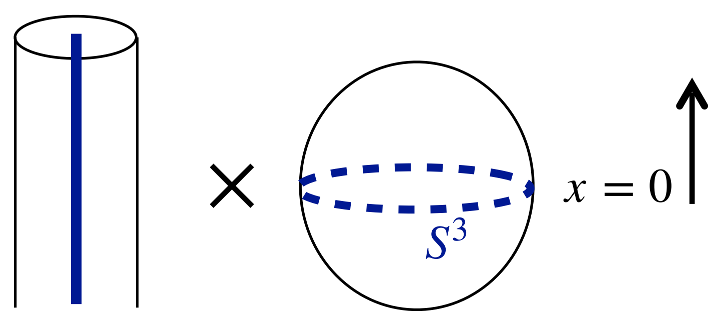


Matrix product state at weak coupling
Worldsheet g-function at finite coupling

- Analogy with FZZT brane in minimal string : $\det(E-H)$



$\det(E-H)$ **: FZZT brane**



$\det(x-Z)$ **: giant graviton**

Weak coupling: Matrix Product State

- Represent det's by fermion integrals: $\det(Z(x_k)) = \int d\bar{\chi}_k d\chi_k \exp(\bar{\chi}_k Z(x_k)\chi_k)$

cf. talk by Shenker

- Integrate out N=4 SYM fields (Z)

→effective action for "mesons" $\rho_{kj} \sim (\bar{\chi}_k \chi_j)$: $\int d\rho \exp(-NS_{\text{eff}}[\rho])$

2 × 2 matrix

cf. [Budzik, Gaiotto **today**]

Saddles in chiral algebra sector = D-branes in BCOV

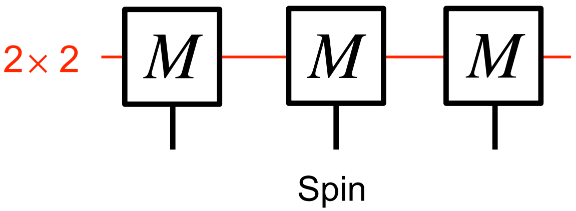
- Single-trace operator: $\mathcal{O} = \text{tr}_{N\times N}(YZYZZ\cdots)$

$$\langle \det Z \; \det \bar{Z} \; \mathcal{O} \rangle \quad \mapsto \quad \text{tr}_{2\times 2}(M_Y M_Z M_Y M_Z \cdots)$$

$$= \langle \text{MPS} | \mathcal{O} \rangle$$

$$M_Y \sim \text{diag}\left(\frac{1}{|x-x_1|^2}, \frac{-1}{x-x_2|^2}\right) \cdot \rho^{-1}_{\text{saddle}}$$

$$\langle \text{MPS} | = \sum_{\bullet_s = Y,Z} \langle \bullet_1 \bullet_2 \cdots | \text{tr}\left(M_{\bullet_1} M_{\bullet_2} \cdots\right)$$



cf. [Chen, de Mello Koch, Kim, Van Zyl]

Weak coupling: Result $\langle \mathrm{MPS} | p_1, \cdots \rangle$

- Studied in stat-mech in the context of quench dynamics:

  Tuchiya, Brockmann, De Nardis, Wouters, Caux, Pozsgay, Vernier, Calabrese, Piroli,…

  de Leeuw, Kristjansen, Zarembo, Foda, Wilhelm, Buhr-Mortensen, Ispen, Linardopoulos, Muller

1. $\langle \mathrm{MPS} | p_1 . \cdots \rangle \neq 0$ **iff** $\{p_1, \cdots\} = \{p_1, -p_1, p_2, -p_2, \cdots\}$

   $\Rightarrow \langle \mathrm{MPS} | Q_{2s+1} = 0 \quad \rightarrow \quad \langle \mathrm{MPS} |$: integrable bdy state

   Ghoshal, Zamolodchikov

   Pozsgay, Piroli, Vernier

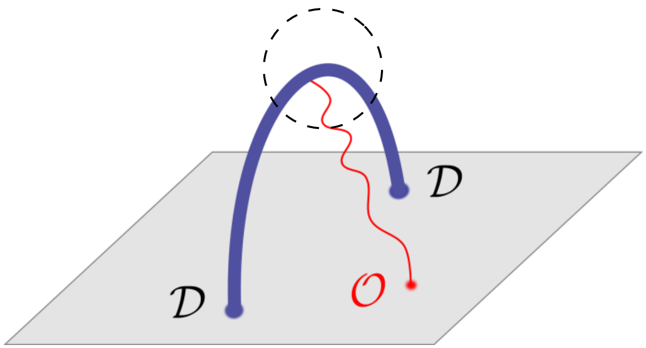
2. $\langle \mathrm{MPS} | p_1, \cdots \rangle \sim \prod_j F(p_j) \times \sqrt{\dfrac{\det G_+}{\det G_-}}$

$$G^{\pm}_{ab} \sim \frac{\partial \log(\text{Bethe eq for } p_a)}{\partial p_b} \pm \frac{\partial \log(\text{Bethe eq for } p_a)}{\partial(-p_b)}$$

- N=4 SYM also gives generalizations of known formulae.

  $SL(2)$ spin chain, supergroup spin chain….

Finite coupling: Worldsheet g-function



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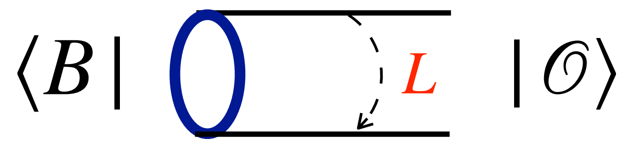


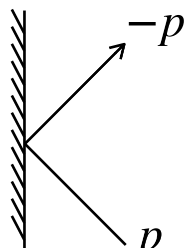
(excited state) g-function

Affleck, Ludwig

Step 1: Determine reflection amplitudes using (integrable) bootstrap

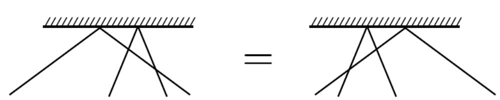
Ghoshal, Zamolodchikov

$R(p) :=$



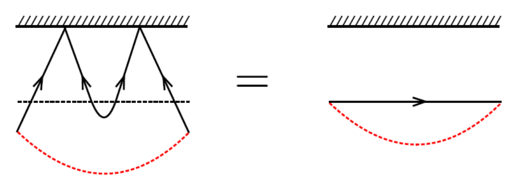
$$\mathrm{SU}(2|2)_{\mathrm{diag}} \subset \mathrm{SU}(2|2)^2$$

Symmetry



Boundary Yang-Baxter

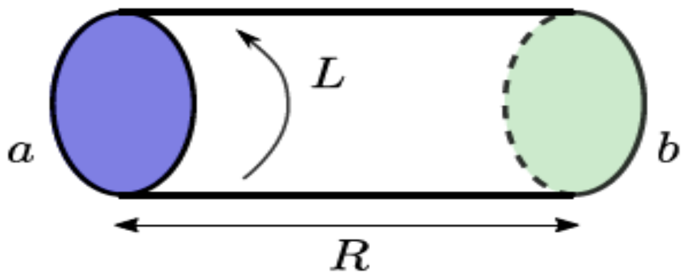
cf. Hofman, Maldacena, …



Boundary unitarity

Finite coupling: Worldsheet g-function

Step 2: Consider a cylinder partition function.



closed string $\underset{R\gg 1}{=} \sum_{\psi_n} |\langle B|\psi_n\rangle|^2 e^{-RE_n}$

- Open string: Thermal partition function at $\beta = L$:

$$Z_{\text{open}}(L) \overset{R\gg 1}{\to} \int D\rho_{\text{open}}\, e^{-L\,R\, s_{\text{eff}}[\rho_{\text{open}}]}$$

determined by reflection amplitude

Density of excitations

different saddles = $\sum_{\psi_n}$	Saddle pt eq = TBA eq	1-loop det = $|\langle B|\psi_n\rangle|^2$

Finite coupling: Result and Fredholm det

$$\langle B|\psi_n\rangle = \underline{(\text{prefactor})}\frac{\sqrt{\text{Det}\, 1+\hat{G}}}{\text{Det}\, 1+\hat{G}_-}$$

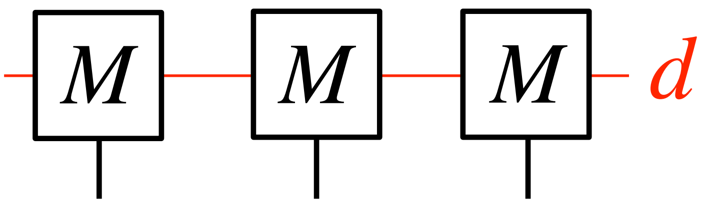
depends on reflection amplitudes

$$\hat{G}\cdot\vec{f}(u) = \int_{-\infty}^{\infty}\frac{dv}{2\pi i}\frac{\partial_v \log S^{(\lambda)}(u,v)}{1+1/Y(v)}\vec{f}(v)$$

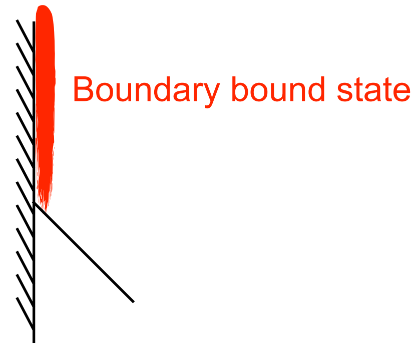
cf. talk by Johnson

- Reproduces and generalizes the weak-coupling answer.
- A variety of quantities can (in principle) be studied by the same methods:
  - 1-point function in the presence of domain wall SK, Wang, Bajnok, Gombor, de Leeuw, Kristjansen, Zarembo, Foda, Wilhelm, Buhr-Mortensen, Ispen, Linardopoulos, Muller
  - D-instanton Caetano, SK, Wang to appear
  - $\langle \mathcal{O} W\rangle$ W : Wilson loop Jiang, SK, Vescovi, to appear
  - 1-pt function in the Coulomb branch Cordova, Coronado, SK, Zarembo, Ivanovsky, Mishnyakov, Terziev, Zaigreev, in progress
  - Other theories. Jiang, SK, Wu, Yang
  - And more….

Intriguing observation



Weak coupling
(spin chain)



Finite coupling
(String)

Bond dimension d = # of boundary bound states

- (A part of) world sheet d.o.f can be seen already at weak coupling!

*determinant $d=2$, D-instanton $d=1$, Wilson loop $d=\infty$, domain wall $d\geq 1,\ldots$

Lessons and Future….

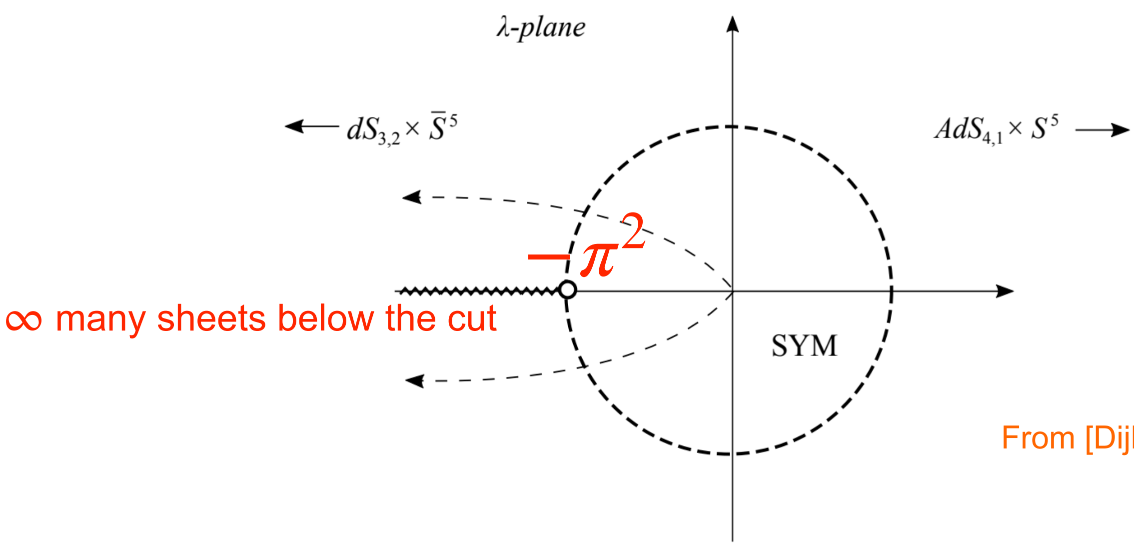
General lessons

- Lesson 1

Large N Feynman diagrams	=	String world sheet in AdS

Can we see the same at finite temperature? How do we see a horizon from large N diagrams?

- Lesson 2 : λ expansion has a finite radius of convergence.



From [Dijkgraaf, Heidenreich, Jefferson, Vafa '18]

Double-scaling near the singularity at $-\pi^2$? Connection to de Sitter? [Polyakov]

Open questions

- Can we see a horizon from large N Feynman diagrams?

  [Festuccia, Liu ’05, ’06] (obstacle: [Linde ’80])

  cf. [Jafferis, Schneider ’21]

- Analyticity in λ: double scaling, connection to de Sitter?

  cf. [Basso, Dixon, Kosower, Krajenbrink, Zhong ’21] [Polyakov ’07], [Dijkgraaf, Heidenreich, Jefferson, Vafa ’18]

- Rewrite g-function in terms of Q-function / Quantum Spectral Curve (straightforward)

- Quantum Spectral Curve from 4d Chern-Simons? (How do we derive gluing conditions?)

  [Costello, Gaiotto, Yagi ’21]

- Single-trace three-point functions from Quantum Spectral Curve?

  (= Plucker coordinates of Grassmannian)

- Quantum Spectral Curve in chiral algebra sector?

- N=2 SCFT? Veneziano limit? [Pomoni, Rabe, Zoubos ’21]

- How to efficiently compute non-planar corrections? Analog of topological recursion? (Perhaps start from topological subsectors) [Giombi, SK ’19]