Finiteness, Compactness, Desert and the Swampland

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Based on works:

2104.05724 (Y. Hamada, V.)
2106.10839 (H. Tarazi, V.)

Works in progress:

(Y. Hamada, M. Montero, I. Valenzuela, V.)
(C. Long, M. Montero, I. Valenzuela, V.)
(A. Bedroya, Y. Hamada, M. Montero, V.)

Plan for the talk:

1- Finiteness of the string landscape

2- Finiteness in theories with 16 supercharges

3-Compactness of the brane probe and reconstruction of internal geometry (SLP, DC)

4- Finiteness in 6d, N=(1,0) landscape

5- The Desert scenario?

Finiteness in string landscape

Finite number of known Minkowski SUSY vacua (number of CY manifolds of a given dimension believed to be finite [Yau]).

Of course there are infinitely many defects for a given theory. (E.g. N D3 branes in type IIB in 10d).

Examples:

-A theory with 16 supercharges in d dimensions in the string landscape has a bound in rank:

$$rank(G) \leq 26 - d$$

-A theory with 8 supercharges in 6d has a provably finite construction from F-theory (proven that elliptic CY 3-folds are finite)

These examples suggest that for any consistent theory of quantum gravity in any dimension there is an upper bound on the number of massless modes (which may depend on the dimension and the number of supersymmetries). The landscape of consistent QG theories is finite.

Finiteness Conjecture

Theories with N=32

The massless modes are determined by SUSY alone and is unique. So at least the matter content is fixed. Finiteness of the landscape (at least at the level of content of massless field) is trivially satisfied.

Theories with N=16

The chiral versions in 6d and 10d are essentially fixed by anomalies.

For non-chiral versions, using the stronger form of the completeness hypothesis applied to strings and using the strong form of distance conjecture it has been argued [Kim,Tarazi,V]

$$rank(G) \leq 26 - d$$

Completeness

BPS strings

$+8$ $+8$

d	r																			
9	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17		
8	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
7	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

Completeness
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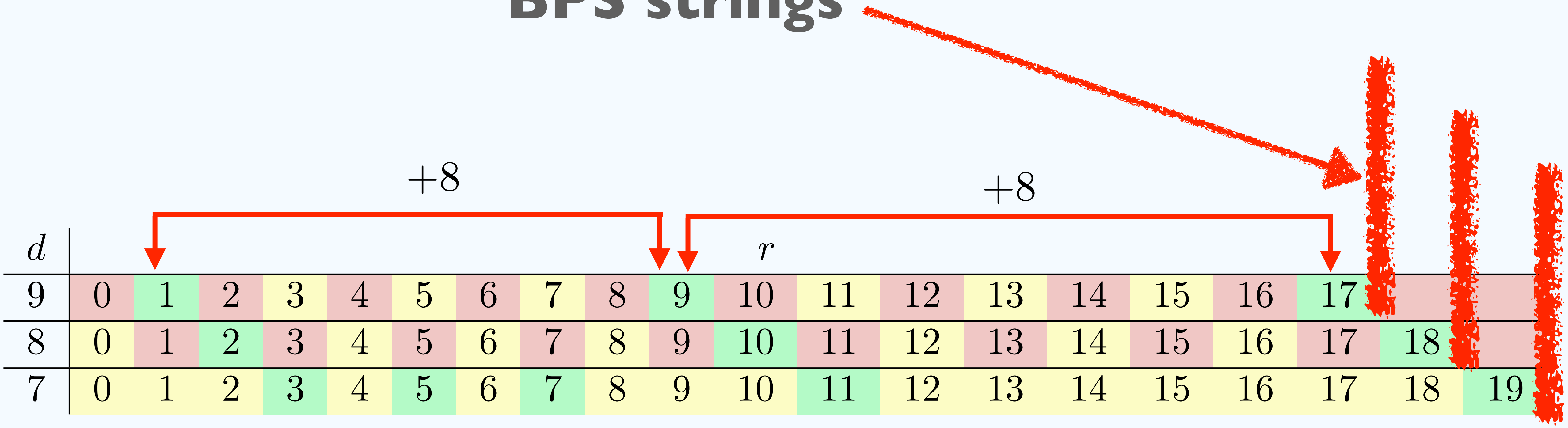
The rank restriction in d=9,8 and the rank parity in d=7 is explained using cobordism conjecture [Montero,V].

Completeness

BPS strings



The rank restriction in d=9,8 and the rank parity in d=7 is explained using cobordism conjecture [Montero,V]. But not all combinations of gauge groups with correct rank appear.

Can we explain that?

The Swampland bounds up to now had focused mainly on unitarity of string probes. Basic new ingredient:
Consider magnetic version of strings.
For example in 8d, this is a 3-brane. Gauge group instantons lead to 3-brane charges. This preserves half of the SUSY leading to N=2 in 4d. Small instantons of gauge group lead to rank 1 Coulomb branch. We now argue why this Coulomb branch should be compact based on Black Hole entropy!

In fact we argue:

In a d dimensional quantum gravity the moduli of any p-brane probe with $p < d - 2$, is compact. More precisely, the spectrum of the Laplacian on it is discrete.

The basic idea is to compactify the theory on T^p and wrap the p-brane on it leading to a black hole in d-p dimensions. The eigenstates of Laplacian translate to states of black hole. The number of states in a given mass range is

$$\int dM\, e^{S(M)}$$

And eigenstates of Laplacian in a given range do correspond to a subset of these and so it cannot be infinite. So the moduli of the brane is compact.

Side remark:

If we could apply this argument to spacetime filling branes having compact moduli, this in effect would mean that the bulk scalar fields should be ``morally compact''. Indeed this is the spirit of the Distance Conjecture [Ooguri,V]. This fits with the idea of ``emergence'' of infinite distance in field space [Grimm,Palti,Valenzuela;Heidenreich,Reece,Rudelius] from finite ones in the presence of an infinite tower of light modes. One can compute the one-loop correction to the field metric from the tower (+mild assumptions) to show mass of the tower decreases exponentially with distance in field space as demanded by DC.

Going back to d=8, N=16 theories and moduli of 3-brane probe, we conclude that the Coulomb branch moduli space of gauge instantons is a compact one-dimensional space. Considering the coupling constant represented by an elliptic curve of the corresponding moduli, leads to a hyperKahler geometry, due to N=2 SUSY. We end up with a compact 4d space: $T^4 \quad or \quad K3$. It has to be $K3$ if we have any non-trivial gauge group. (Disjoint union of $K3$'s is ruled out by a strong form of the cobordism conjecture).

This is consistent with F-theory realization of these theories. In this context the 3-brane is nothing but the D3 brane probing the internal F-theory compactification on elliptic K3!

We have thus reconstructed the internal string geometry from the perspective of IR effective theory!

Moreover by studying the structure of Coulomb branch for gauge instantons one recovers the dictionary between K3 singularities and enhanced gauge symmetries.

For maximal rank theories with 16 supercharges for d>6 this argument reconstructs the observed gauge symmetries in the string landscape, providing another strong evidence for SLP.

For the lower rank theories this study provides additional restrictions on gauge groups that appear, but does not completely fix the allowed set to the known examples.

Theories with N=8

The first dimension this occurs is d=6, with (1,0) supersymmetry. In these cases anomaly considerations greatly reduce the available possibilities for matter. But still allow for unbounded number of massless modes. It can be argued that at least all such cases belong to the Swampland by a careful study of unitarity properties of the BPS strings in such theories. So again it seems we have a landscape with an upper bound on number of massless modes.

An Explanation of finiteness: Desert Scenario?

One idea for trying to explain the finiteness conjecture is based on Species problem for BH. If the number N of light modes is arbitrarily large then the entropy of a fixed (say Planck sized) black hole cannot account for all these states.

An Explanation of finiteness: Desert Scenario?

One idea for trying to explain the finiteness conjecture is based on Species problem for BH. If the number N of light modes is arbitrarily large then the entropy of a fixed, say Planck sized black hole cannot account for all these states. However a proposal to avoid the species problem and get arbitrarily large N is based on noting that any effective theory has a cutoff that may depend on N.

In a theory of quantum gravity we expect a cutoff scale Λ for any effective theory to be always less than Planck scale

$\Lambda < 1$ (in Planck units)

Thus radius of all BH's described by effective theory: $R > \dfrac{1}{\Lambda}$

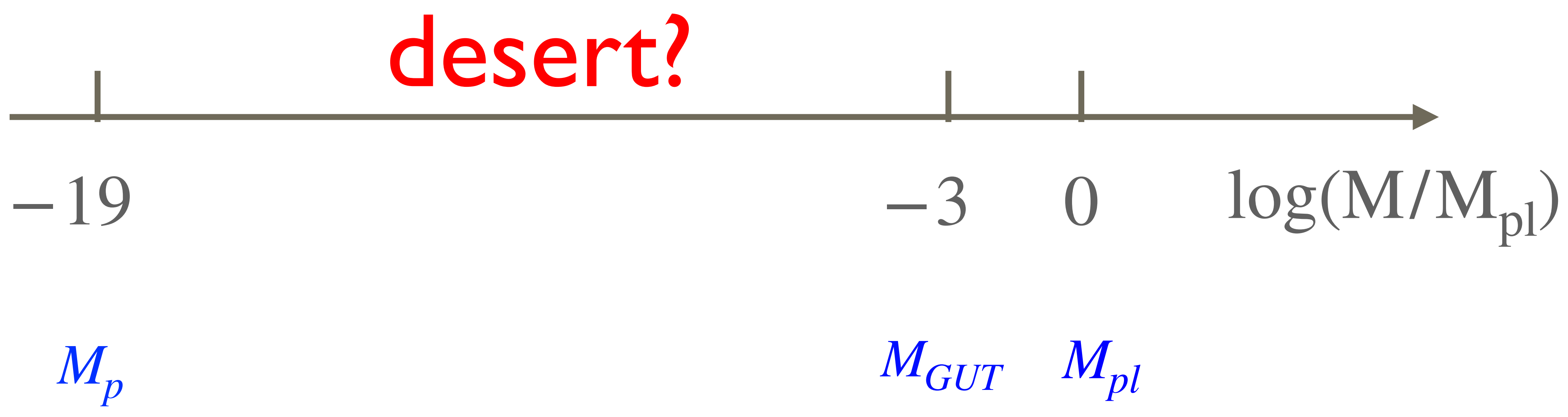
Thus as long as $S \sim R^{d-2} \sim \Lambda^{-d+2} > N$

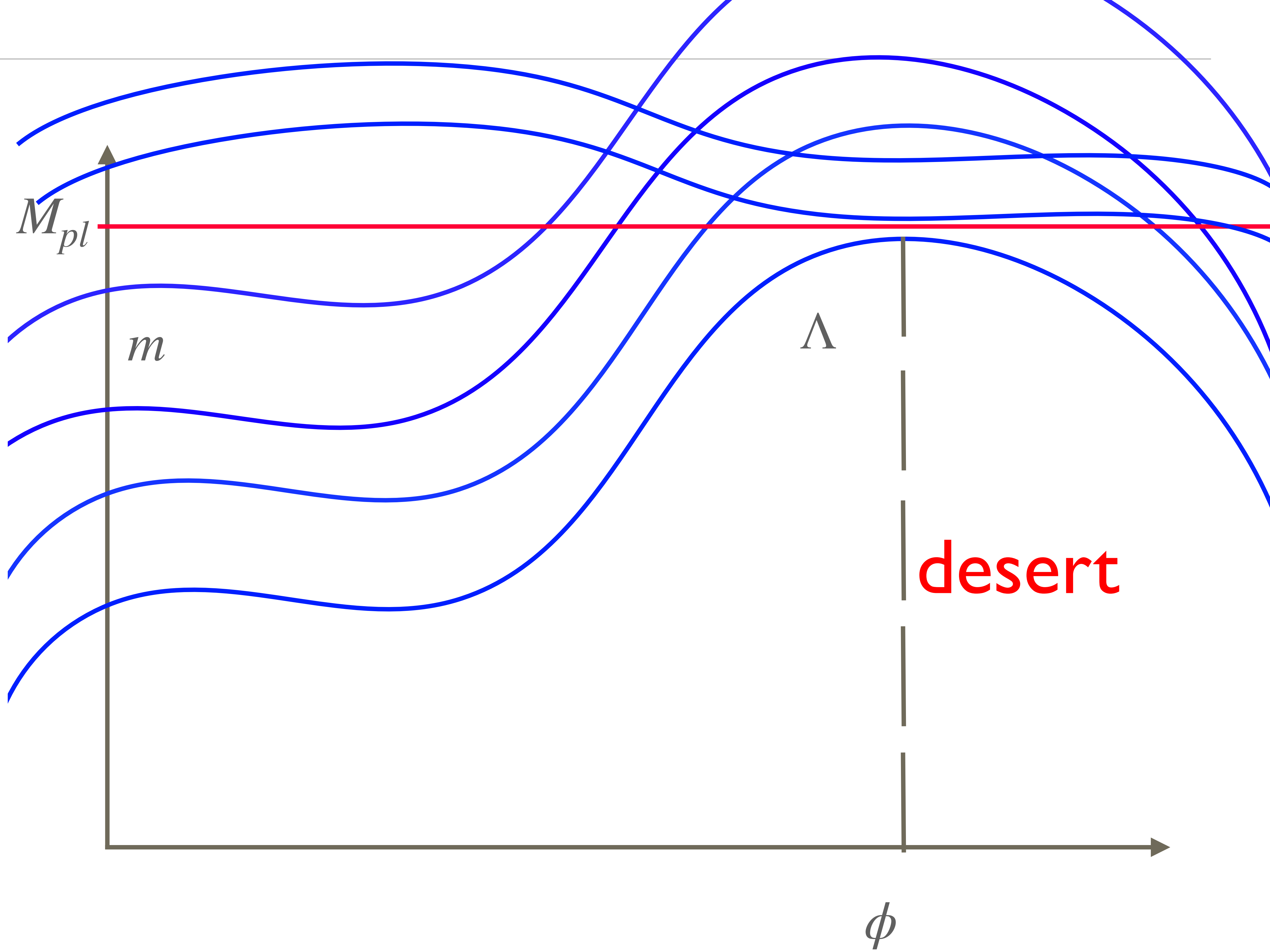
$\Lambda < N^{\frac{-1}{d-2}}$ avoids it.

(called the species scale) [Dvali,Gomez,Lust]

So at first sight it appears that we do not have an argument for boundedness of N and finiteness of the landscape. However, if we can argue Λ should be close to Planck scale then we may have an argument. More precisely we know that Λ changes as we change massless moduli. So the question is whether there are points on moduli where Λ is close enough to Planck for all QG theories. In other words do we always have a desert?

Just as is speculated for our universe?





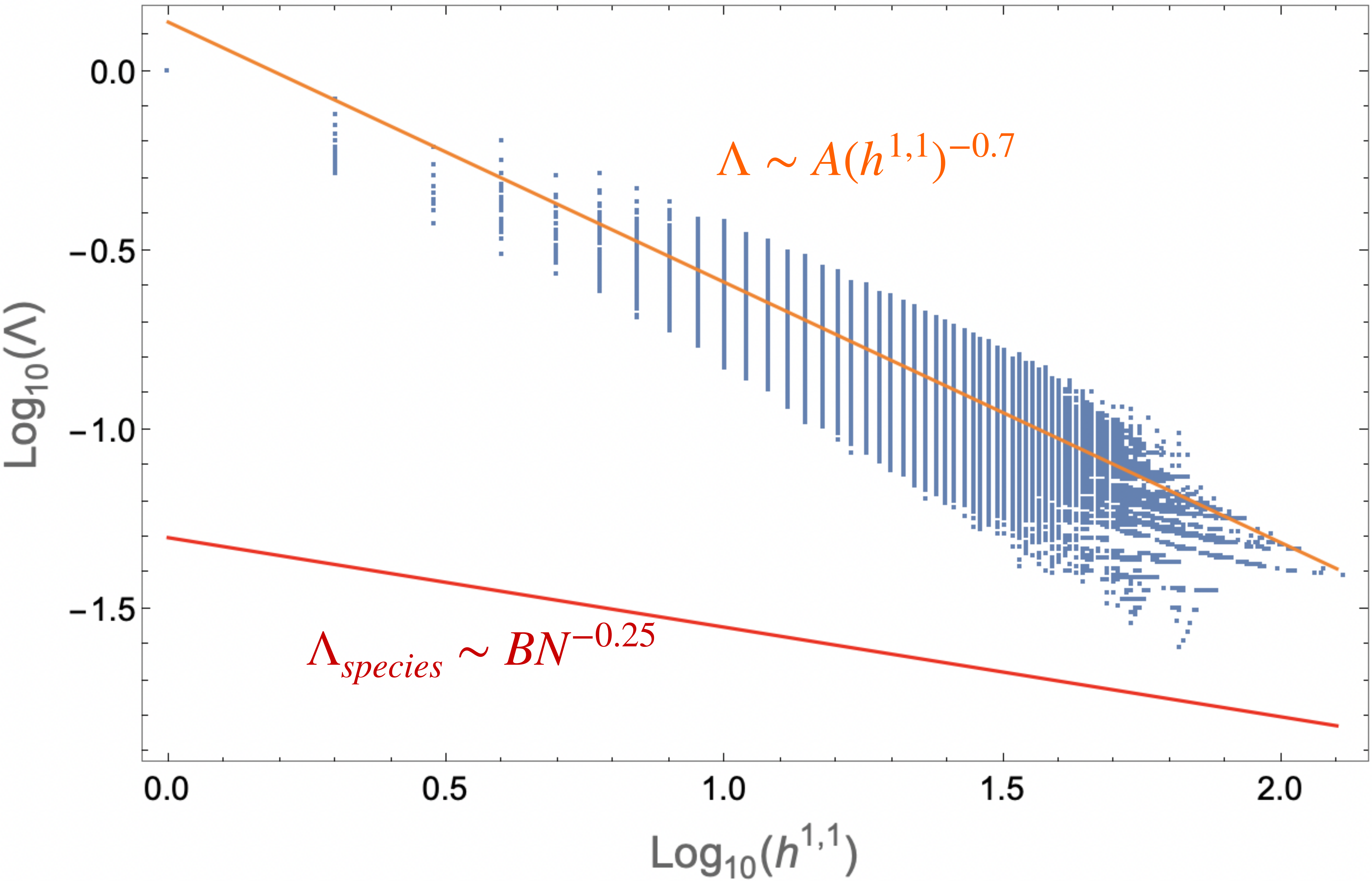
M_{pl}
m
Λ
desert
ϕ

This seems to be the case for all theories realized in the string landscape with at least 16 supercharges (for example, heterotic string at self-dual moduli and string coupling $g_s \sim 1$). In these cases one can plausibly argue that Λ at some point in moduli is close to the Planck scale at least for BPS states. How about lower SUSY, like N=8?

In 6d (1,0) theories mass scale associated to BPS strings gives an estimate for the cutoff Λ as a function of moduli.

F-theory on elliptic CY 3-fold.

We examined 34,871 toric bases to estimate the gap, by going to a point in moduli with Λ as close as possible to the Planck mass.



$\Lambda \sim A(h^{1,1})^{-0.7}$
$\Lambda_{species} \sim BN^{-0.25}$
0.0
-0.5
-1.0
-1.5
$Log_{10}(\Lambda)$
0.0
0.5
1.0
1.5
2.0
$Log_{10}(h^{1,1})$

Conclusion

We have seen various examples which reinforce the picture that the landscape of consistent quantum gravities are finite. A refined version of `desert scenario' may imply bound on the number of massless modes as expected from finiteness.

Compactness of the brane probe leads to reconstruction of internal geometry in some cases. Provides further evidence for SLP as well as further support for the emergence proposal of the DC.