Wormholes and black hole microstates in AdS/CFT

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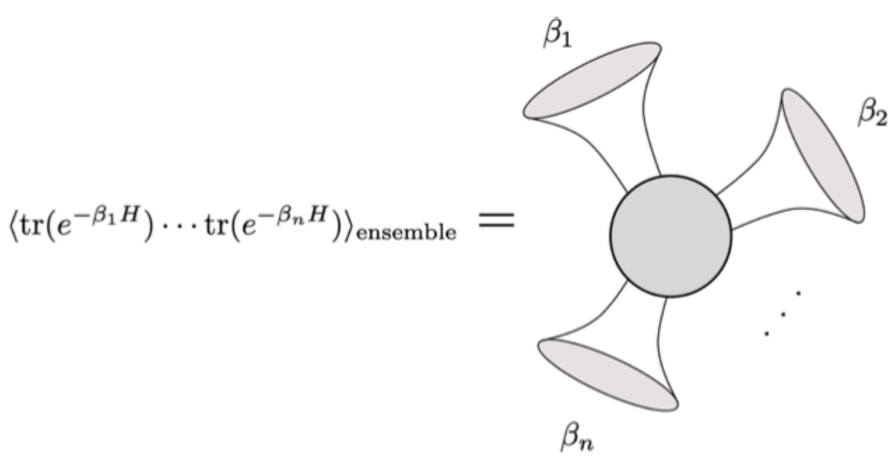
Strings 2021

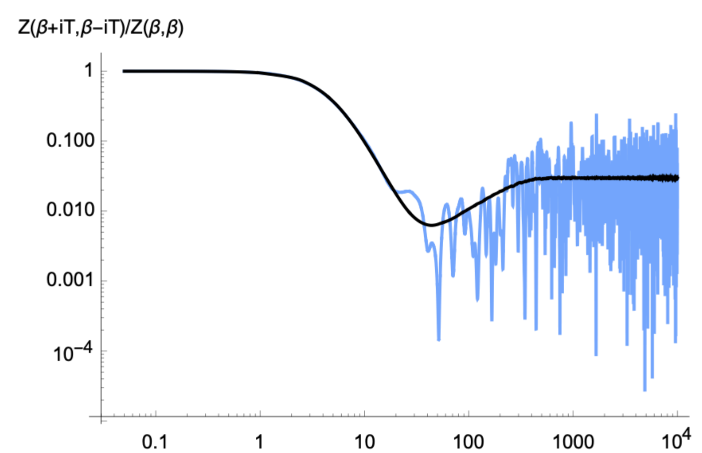
based on joint work with Jordan Cotler:
arXiv:2006.08648
arXiv:2010.02241
arXiv:2104.00601

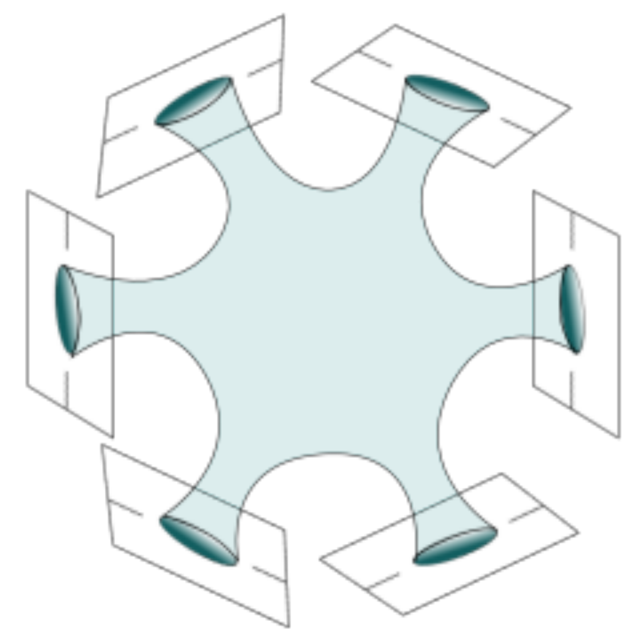
Wormholes are a central player in recent work on soluble models of low-dimensional gravity.

cf Shenker's talk.

- Tractable non-perturbative corrections in G_N.
- Probe fluctuation statistics of dual ensemble, like level statistics of JT black holes.
- Unitarize BH evaporation in models with replica wormholes.
- Blessing and a curse (lead to factorization paradox).
- And more.





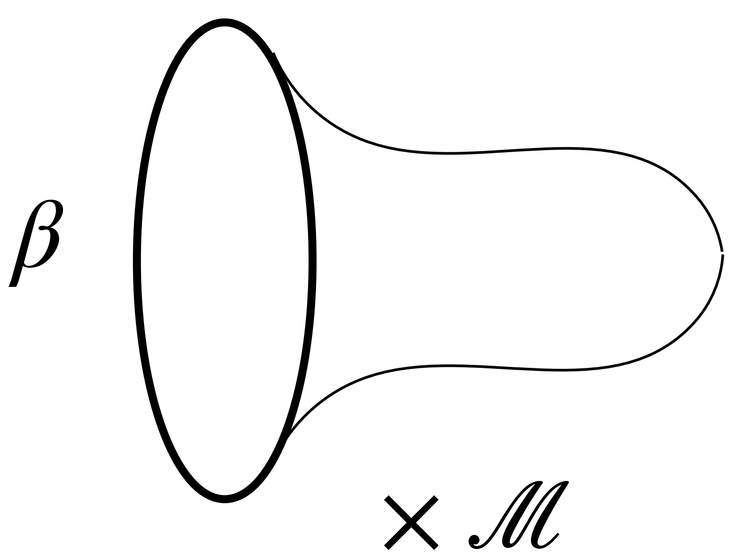


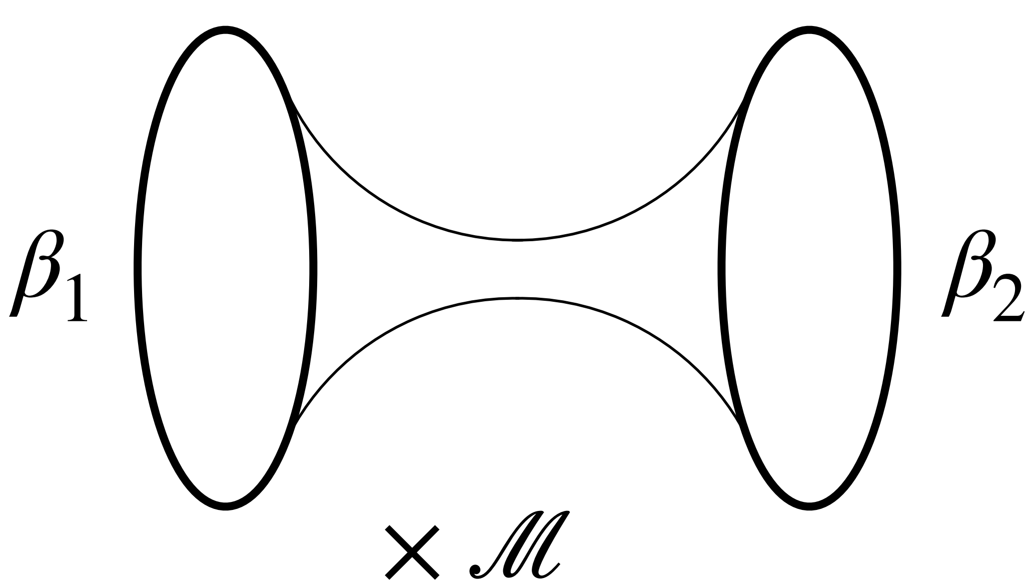
GOAL FOR TODAY:

Develop machinery to study wormholes in $d>2$, embed into proper AdS/CFT.

What do we hope to learn?

In gravity we look for Euclidean wormholes with two asymptotically (Euclidean) AdS regions with $\mathbb{S}^1 \times \mathscr{M}$ boundary (with $\mathscr{M} = \mathbb{T}^{d-1}, \mathbb{S}^{d-1}$).





<u>Punchline:</u>

Euclidean BH amplitude (one boundary) encodes coarse-grained approximation to the black hole density of states.

Euclidean wormhole amplitude (two bdys) encodes coarse-grained approximation to the two-point level statistics of black hole microstates, in particular, <u>long-range</u> repulsion.

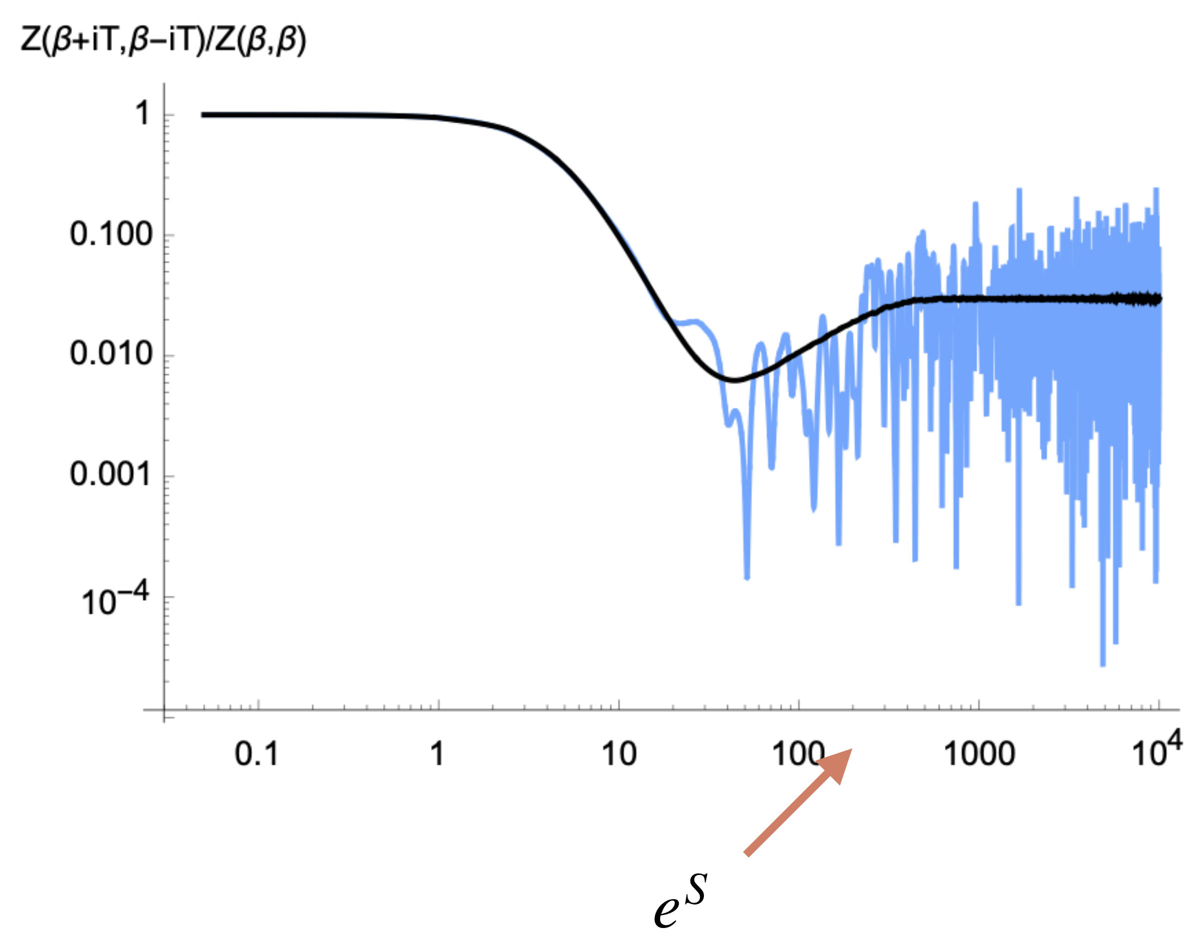
What do we hope to learn?

There is good reason to expect a holographic CFT on $\mathbb{S}^{d-1}$ to have a chaotic spectrum of energy eigenstates. Such a spectrum has short-range and long-range repulsion between energy eigenvalues. Both are well-diagnosed by "spectral form factor."
cf. Shenker's talk. [many papers], [CGHPS⁴T] '16

$$Z(\beta + iT, \beta - iT) = \text{tr}(e^{-(\beta+iT)H})\text{tr}(e^{-(\beta-iT)H})$$

↑ β_1 ↑ β_2

SFF of a single theory has a wildly fluctuating form at late times, but ensemble (or time) average is smooth.



Methods and "philosophy"

The central difficulty:

Wormhole amplitudes are generically non-perturbatively suppressed contributions to path integral with no saddle point approximation.

In "vanilla" case with $\mathbb{S}^1_\beta \times \mathbb{S}^{d-1}$ or $\mathbb{S}^1_\beta \times \mathbb{T}^{d-1}$ boundary, Witten-Yau theorem implies no Euclidean wormhole solutions to Einstein gravity with negative cc.
(Long history of non-vanilla wormholes; recently, [Marolf, Santos] '21)

Basic idea:

Take bulk effective field theory as seriously as we can and imitate the rules of QFT.

Borrow methods (mainly constrained instantons) from QFT.

Collect "data." Remain agnostic with regard to factorization, α parameters…

The Plan:

1. Introduction
2. Wormholes in pure 3d gravity
3. Wormholes in $\text{AdS}_{>2}$
4. Wormholes in $\text{AdS}_5 \times \mathbb{S}^5$

Pure AdS_3 gravity

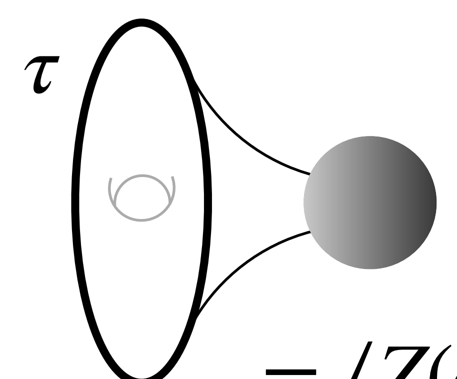
Old expectation:

"Torus Z" of pure 3d gravity with negative cc has negative dos near spectral edge.
So pure 3d gravity is inconsistent.
[Maloney, Witten] '07, [Keller, Maloney] '14

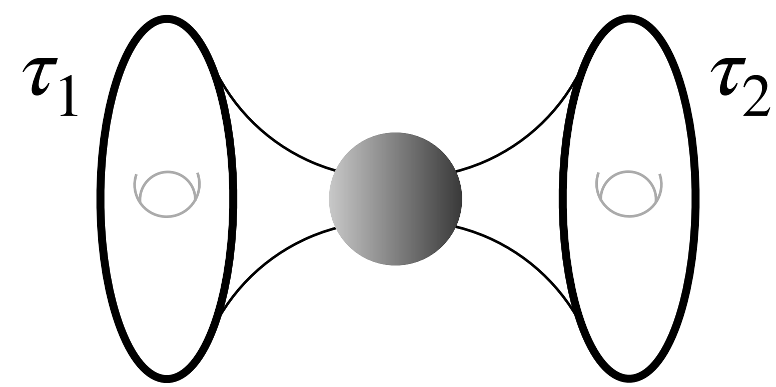
(see also [Benjamin, Ooguri, Shao, Wang] '19, [Alday, Bae], [Benjamin, Collier, Maloney] '20)

A conjecture:

AdS_3 gravity *is* consistent,
dual to an ensemble.*



$= \langle Z(\tau) \rangle$



$= \langle Z(\tau_1) Z(\tau_2) \rangle$

This conjecture appeared in print last summer:
[Belin, de Boer], [Cotler, KJ], [Maxfield, Turiaci]

*Of CFTs? Focus on gravity.

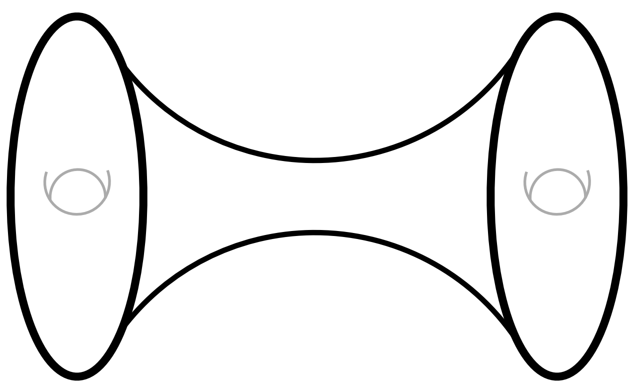
See also Narain ensembles:
[Maloney, Witten],
[Afkhami-Jeddi, Cohn, Hartman, Tadjini]

Two boundaries in AdS$_3$

Goal: compute two-boundary amplitudes $\sim_? \langle ZZ \rangle_{\text{conn}}$

Simplest examples: $\Sigma_g \times I$

$g = 0,1$

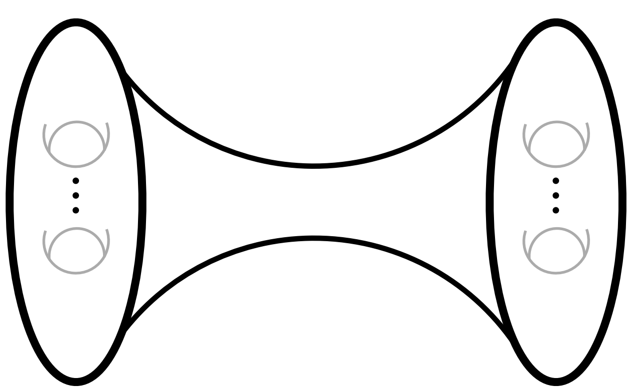


No saddle point

g=1: [Cotler, KJ] '20
g=0: [Cotler, Iliesiu, KJ, Rayhaun] WIP

Focus on g=1

$g > 1$



$\mathbb{H}^3/\Gamma$ Saddle point + 1-loop

[Maldacena, Maoz] '04
[Giombi, Maloney, Yin] '07

The computation

Our approach: [Cotler, KJ] 2006.08648

- Work in first-order formulation, with a careful choice of continuation from Lorentzian signature.
- $\mathbb{T}^2 \times I = \text{annulus} \times \mathbb{S}^1_y$
- "Constrain first": works to 1-loop

$$A_\mu = \left(\frac{1}{2}\varepsilon^{abc}\omega_{bc\mu} + e^a_\mu\right) J_a$$

$$S = \frac{ik}{4\pi}\int d^3x\, \varepsilon^{ij}\mathrm{tr}\left(A_i\partial_y A_j - A_y F_{ij}\right) - (A \to \bar{A}) + S_{\mathrm{bdy}}$$

- Residual path integral over $A_i = \tilde{G}^{-1}\partial_i\tilde{G}$.
  (with bdy dofs ~ AdS_3 version of Schwarzian modes [Cotler, KJ] '18 and pseudomoduli)

$$\widetilde{Z}_{\mathbb{T}^2\times I}(\tau_1,\tau_2) = V_\varnothing \int_{\frac{c-1}{24}}^{\infty} dh d\bar{h}\, \chi_{h,c}(\tau_1)\bar{\chi}_{\bar{h},c}(\bar{\tau}_1)\chi_{h,c}(\tau_2)\bar{\chi}_{\bar{h},c}(\bar{\tau}_2)$$

The result

$$Z_{\mathbb{T}^2\times I}(\tau_1,\tau_2) = \frac{1}{2\pi^2}\frac{1}{\sqrt{\mathrm{Im}(\tau_1)}\,|\eta(\tau_1)|^2}\frac{1}{\sqrt{\mathrm{Im}(\tau_2)}\,|\eta(\tau_2)|^2}\sum_{\gamma\in PSL(2;\mathbb{Z})}\frac{\mathrm{Im}(\tau_1)\mathrm{Im}(\gamma\tau_2)}{|\tau_1+\gamma\tau_2|^2}$$

Disconnected contribution is $O(e^c)$; this is $O(1)$. So normalized variance in Z is $O(e^{-c})$.

Invariant under independent modular transformations.*

To study spectral statistics, decompose into
superselection sectors = primaries at fixed spin.
(No repulsion between states related by symmetry.)

Provisional interpretation: $\langle Z(\tau_1)Z(\tau_2)\rangle_{\mathrm{conn}} = Z_{\mathbb{T}^2\times I}(\tau_1,\tau_2) +$ (other connected)

*A modular bootstrap, using stringent inputs from 3d gravity, yields the same result up to normalization. [Cotler, KJ] 2007.15653

Spectral statistics

After stripping off descendants, transform to fixed spin:

$$\left\langle \mathrm{tr}\left(e^{-\beta_1 H_{s_1}}\right)\mathrm{tr}\left(e^{-\beta_2 H_{s_2}}\right)\right\rangle_{\mathrm{conn}} = e^{-\beta_1 E_{s_1}-\beta_2 E_{s_2}}\left(\frac{1}{2\pi}\frac{\sqrt{\beta_1\beta_2}}{\beta_1+\beta_2}\delta_{s_1 s_2}+O\left(\frac{1}{\beta}\right)\right)+\dots$$

$E_s = 2\pi\left(|s| - \frac{1}{12}\right)$ = threshold energy of BTZ with spin s.

1. Leading low T result is precisely what one would find from an RMT ansatz at genus 0. This is as close as we can get to a "smoking gun" for an ensemble dual, which evidently generalizes random matrix theory.
2. Corrections come from PSL(2;Z) sum, higher topologies.
3. Lorentzian SFF exhibits a ramp; eigenvalue repulsion!

$$\left\langle \mathrm{tr}\left(e^{-(\beta+iT)H_{s_1}}\right)\mathrm{tr}\left(e^{-(\beta-iT)H_{s_2})}\right)\right\rangle_{\mathrm{conn}} \simeq \frac{T}{4\pi\beta}e^{-2\beta E_{s_1}}\delta_{s_1 s_2}$$

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Constrained instantons

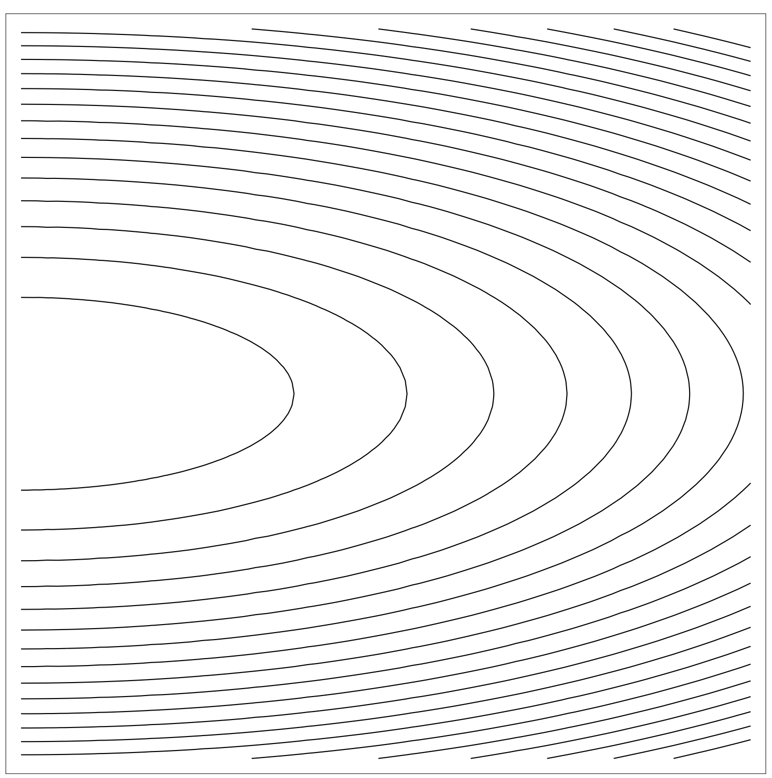
Problem: "Vanilla" wormhole amplitudes have no saddle point approx.

There is some precedent in QFT to calculate "instanton" amplitudes which have no saddle-point approximation.

"Method of constrained instantons."[Affleck] '81, [Affleck, Dine, Seiberg] '83

e.g. "instantons" in 4d YM in a Higgs phase (like $N_f = N - 1$ SQCD on Higgs branch).

Φ



Basic idea:

Foliate field space by slices of constant $\mathscr{C}[\Phi]$.

For intelligent $\mathscr{C}[\Phi]$, integral along slice admits a saddle-point approximation.

This saddle = "constrained instanton."

Constrained instantons

In practice, one inserts $1 = \int \frac{d\zeta d\lambda}{2\pi} e^{i\lambda(\mathscr{C}[\Phi]-\zeta)}$ into the path integral.

Strategy:

Find a line of constrained instantons $(\lambda_\zeta, \Phi_\zeta)$.

Integrate over fluctuations of (λ, Φ) around a fixed constrained instanton.

Integrate over the family of constrained instantons.

Results in an integral representation for the path integral:

$$Z = \int d\zeta \, e^{-S[\Phi_\zeta]} \widetilde{Z}_1(\zeta)(1 + \text{(two loops)})$$

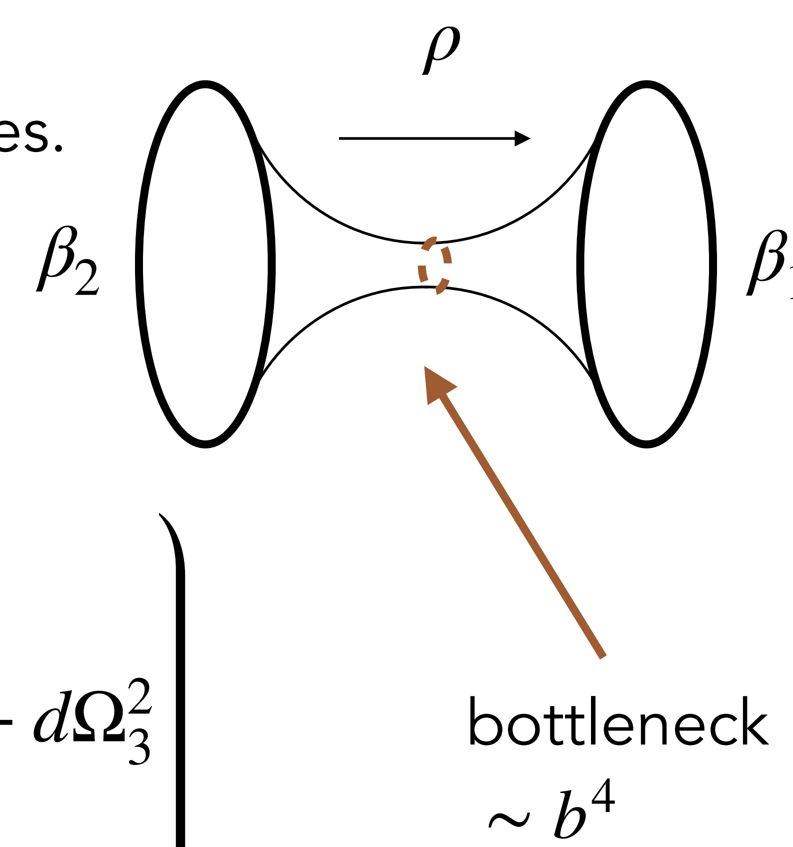
loop expansion around fixed constrained instanton

Wormholes as constrained instantons

To adapt these techniques to Einstein gravity, we need a useful constraint.

On-shell, length between two boundaries at finite cutoff is infinite.
What if we fix it to be finite? (Also useful in dilaton gravity [Stanford] '20)

The ensuing constrained saddles are the desired wormholes.



e.g. a 5d wormhole with $\mathbb{S}^1 \times \mathbb{S}^3$ boundaries

$\ell \sim \ln b/\varepsilon$

$$ds^2 = d\rho^2 + \frac{b^2\cosh(2\rho) - 1}{2}\left(\left(\frac{\beta_1 e^{2\rho} + \beta_2 e^{-2\rho}}{2\left(\cosh(2\rho) - \frac{1}{b^2}\right)}\right)^2 d\tau^2 + d\Omega_3^2\right)$$

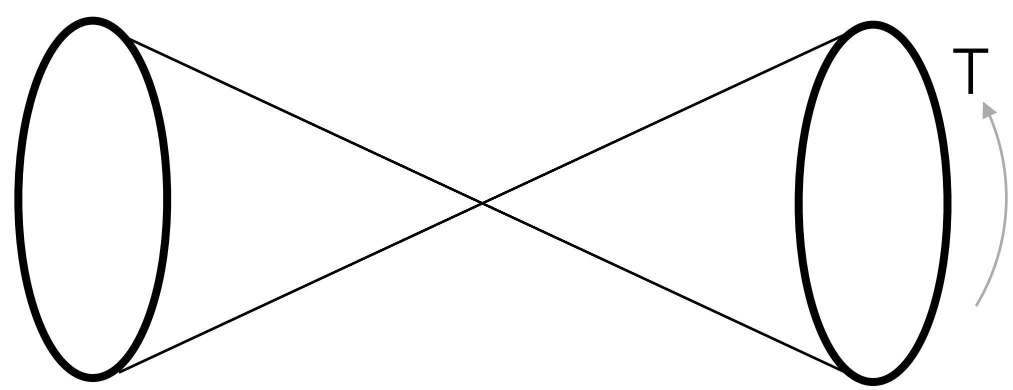
$b \geq 1$

Other examples…

[Cotler, KJ] 2010.02241

Some features

1. All wormholes we find become:
   - Euclidean black hole when $\beta_1 = -\beta_2 = \beta$ (for the value of b so no deficit at $\rho = 0$).
   - "Double cone" of [Saad, Shenker, Stanford] '18 when $\beta_1 = -\beta_2 = iT$.



So these wormholes are analytic continuations of black holes.

2. Boundary energies are $E_1 = E_2 = E \propto \dfrac{b^d}{G}$. Renormalized action is $S_{\rm ren} = (\beta_1 + \beta_2)E$.

3. Minimum b corresponds to E_0 = energy of lightest BH.

4. Study of perturbations shows the wormhole is labeled by a single parameter b (pseudomodulus) + twist zero modes.

The wormhole amplitude

This leads to an integral representation for Z_{wormhole}:

$$Z_{\text{wormhole}}(\beta_1, \beta_2) = \int_{E_0}^{\infty} dE' \, e^{-(\beta_1+\beta_2)E'} Z_1(\beta_1, \beta_2; E')\big(1 + GZ_2(\beta_1, \beta_2; E') + O(G^2)\big)$$

Classical gravity

Loop expansion around a fixed wormhole

"Boltzmann factor" implies the amplitude is dominated by "small wormholes" with $E \to E_0$.

Full amplitude is UV sensitive.

But we would like to extract reliable physics at $E = O(G^{-1})$…

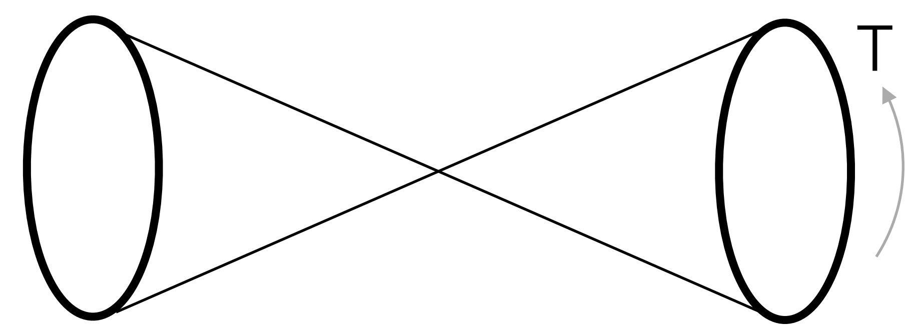
Fixing the energy, method 1

The wormhole amplitude is a "canonical ensemble" quantity, with β_1, β_2.

What if we pass to "microcanonical ensemble" with $E_1, E_2 = O(1/G)$?

We can isolate a wormhole of fixed energy by fixing the boundary energies within a Gaussian bin of width Δ. (This was used by S[3] '18 in their study of SYK/JT.)

Leads to double cone of S[3] '18 (with moduli stabilized).



$$E' = \frac{E_1 + E_2}{2}, \;\; \beta_1 = \frac{E_2 - E_1}{2\Delta^2} + iT = -\beta_2$$

$$\left\langle \text{tr}\left(e^{-\frac{(H-E_1)^2}{2\Delta^2}} e^{-iHT}\right) \text{tr}\left(e^{-\frac{(H-E_2)^2}{2\Delta^2}} e^{iHT}\right) \right\rangle_{\text{conn}} \approx \begin{cases} T\Delta e^{-\frac{(E_1-E_2)^2}{4\Delta^2}} f_{1-\text{loop}}, & \text{gravity}, \\ \frac{T\Delta}{2\sqrt{\pi}} e^{-\frac{(E_1-E_2)^2}{4\Delta^2}}, & \text{RMT}. \end{cases}$$

[Cotler, KJ] 2104.00601

Evidence for a ramp in this observable, contingent on quantum corrections.

Fixing the energy, method 2

Another option:

Boundary energy is a gauge-invariant bulk observable.

We can simply insert a constraint $\delta\left(E_{\text{avg}} - E\right)$ into the bulk path integral.

At the classical level, this selects the wormhole with $E' = E$, and modifies the spectrum of fluctuations at loop level.

For $E = O(1/G)$ this gives us a way, within bulk EFT, to select a particular macroscopic Euclidean wormhole, but the boundary interpretation is unclear.

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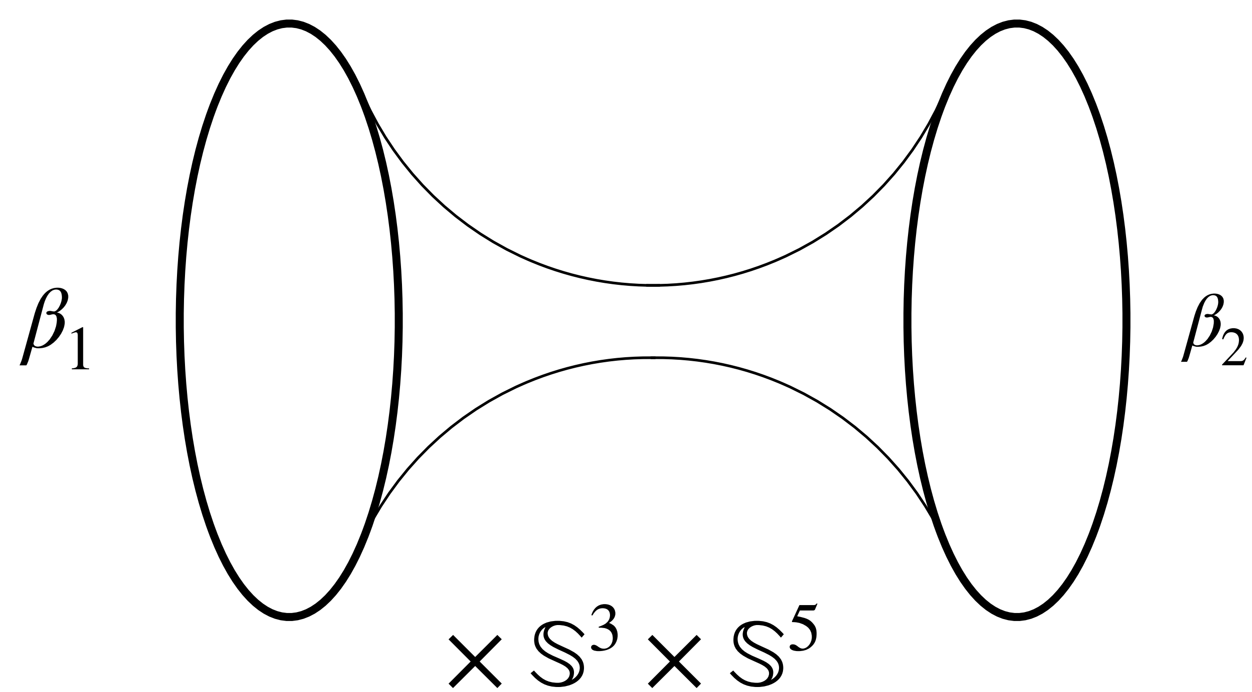
Uplifting to $\mathrm{AdS}_5 \times \mathbb{S}^5$

We easily uplift AdS_5 wormholes to 10d:* [Cotler, KJ] 2104.00601

$$ds_E^2 = e^{-\frac{10}{3}\varphi} ds_5^2 + e^{2\varphi} d\Omega_5^2 = ds^2_{\text{wormhole}} + d\Omega_5^2,$$

(along with constant axiodilaton, RR flux background)

Fix 5d length.



*These methods uplift AdS wormholes to ones in Freund-Rubin compactifications.

Perturbative stability

Important:

There is a sense of perturbative stability for a constrained instanton. Fluctuations in the non-constraint direction must increase the total action.

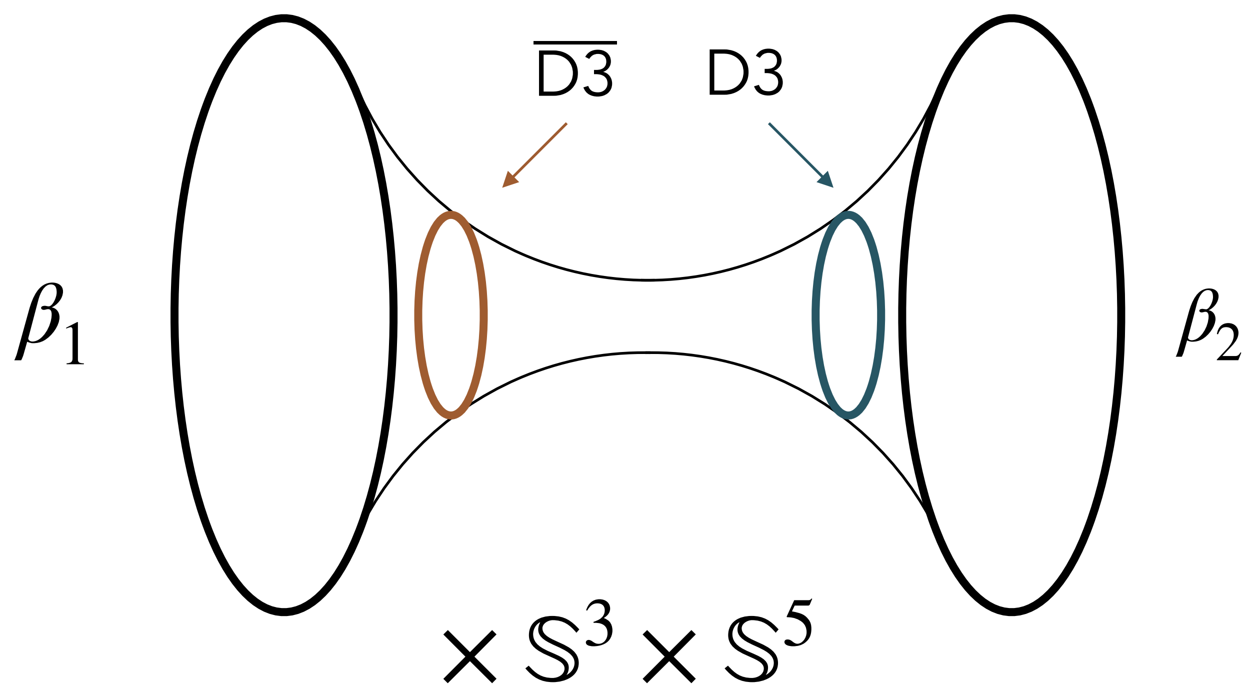
$$\frac{\delta^2 S_{\rm tot}}{\delta\phi^2} \geq 0$$

We have performed a partial stability analysis, focusing on the most dangerous instability channels, modes with small angular momentum on the $\mathbb{S}^3$ and $\mathbb{S}^5$.

This analysis is both conceptually and technically involved, on account of the conformal mode, which mixes with other fluctuations.

Result: No instabilities in the channels studied.

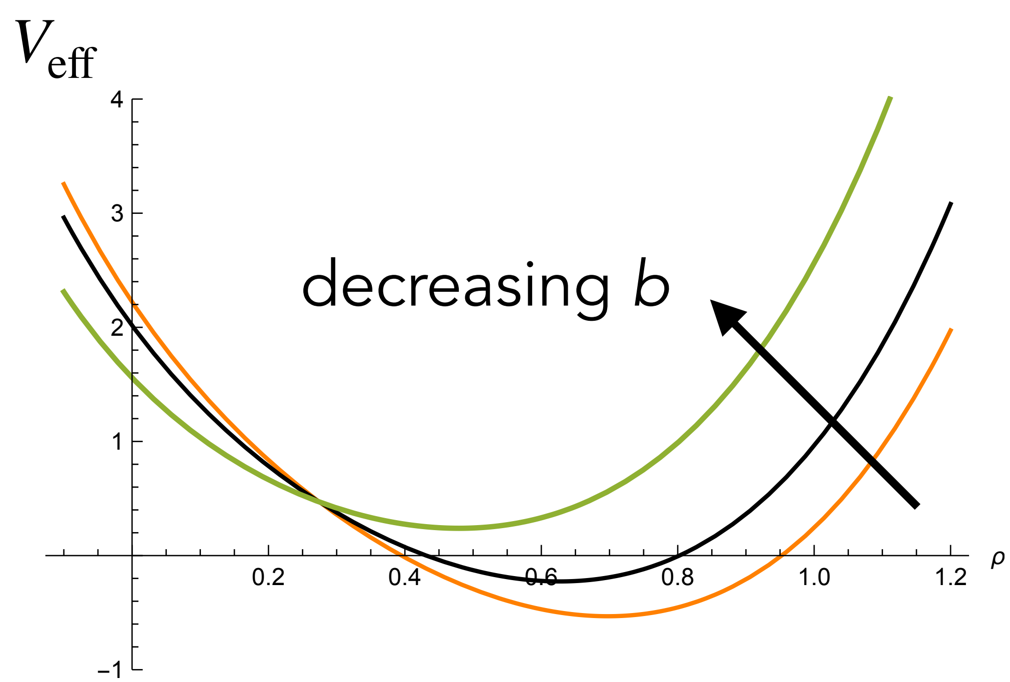
Brane nucleation

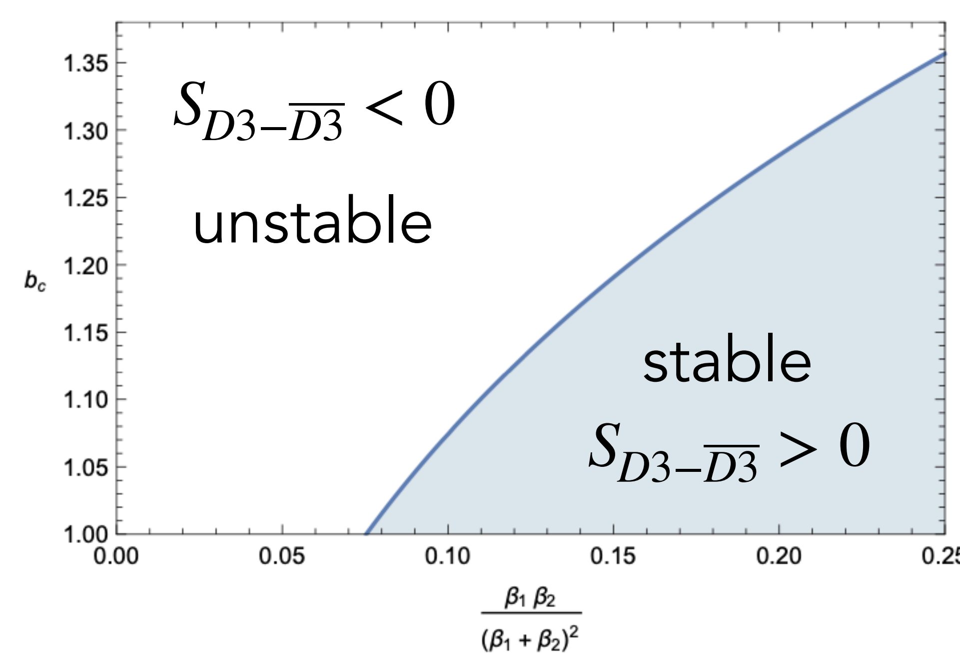


The RR flux background introduces the possibility of D3 nucleation.

We find a stable solution for probe D3-$\overline{D3}$ brane pair in the wormhole, wrapping $\mathbb{S}^1 \times \mathbb{S}^3$'s

RR repulsion = gravitational attraction.





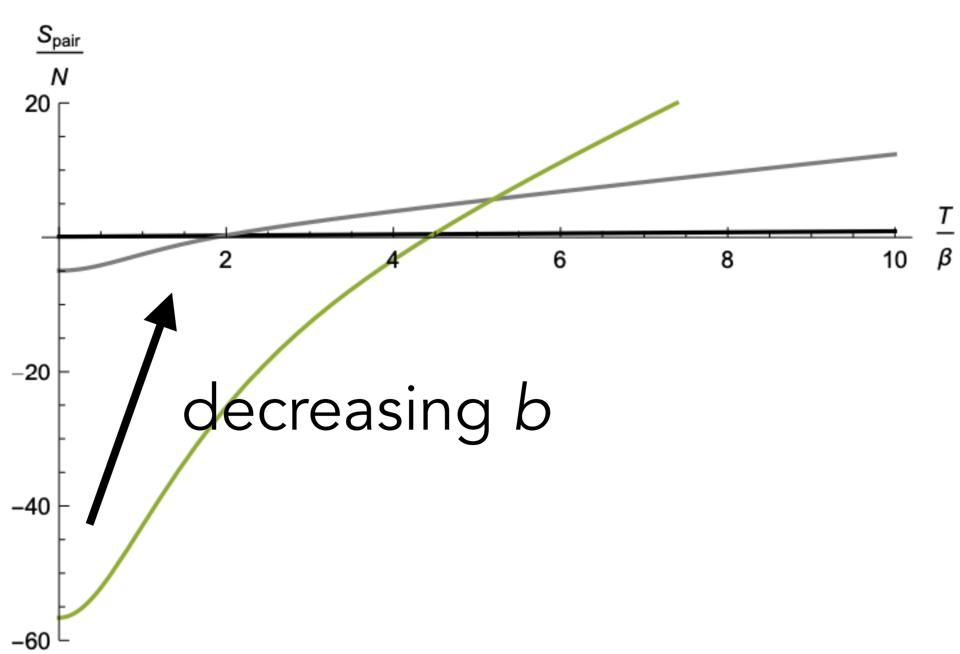
Lifting brane nucleation instabilities

So there is a non-perturbative instability to $D3 - \overline{D3}$ pair creation for large wormholes.

Wormholes with macroscopic, but sufficiently small bottleneck are <u>stable</u>.

<u>Another mechanism</u>:

Consider the continuation required for SFF. $\beta_1 = \beta + iT\,,\ \beta_2 = \beta - iT$



The instability is lifted at $T = O(b)$!

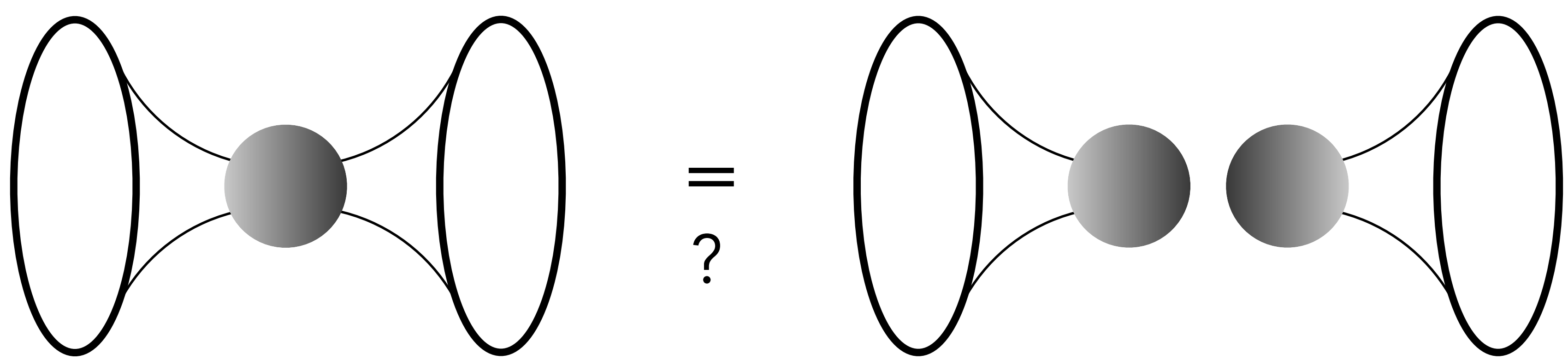
Conclude the double cone with $T/\beta \to \infty$ is stable against brane nucleation.

[Mahajan, Marolf, Santos] '21 also found that the double cone is stable.

Whence factorization?

I have postponed the discussion of factorization until now.

One reason is to stress that wormholes can be a feature, and not only a bug.



Is there a factorization paradox or not in $AdS_5 \times \mathbb{S}^5$?

Do we not sum over these wormholes because of a "stringy exclusion principle?"

Recent progress in 0+0 [Saad, Shenker, Stanford, Yao] '21.
Can those methods be adapted to holographic gauge theories? Not yet clear.

Our techniques also allow us to construct wormholes in asy flat 10d SUGRA. WIP

Summary

1. Computed $Z_{\mathbb{T}^2\times I}$ amplitude in pure 3d gravity.

   Repulsion in spectrum of BTZ microstates, described by RMT.

$$Z_{\mathbb{T}^2\times I}(\tau_1,\tau_2) = \frac{1}{2\pi^2}\frac{1}{\sqrt{\mathrm{Im}(\tau_1)}\,|\eta(\tau_1)|^2}\frac{1}{\sqrt{\mathrm{Im}(\tau_2)}\,|\eta(\tau_2)|^2}\sum_{\gamma\in PSL(2;\mathbb{Z})}\frac{\mathrm{Im}(\tau_1)\mathrm{Im}(\gamma\tau_2)}{|\tau_1+\gamma\tau_2|^2}$$

2. Developed machinery to study Euclidean wormholes in $\mathrm{AdS}_{>2}$.

   Ensuing wormholes are analytic continuation of Euclidean BHs,

   appear to describe a ramp in (a cousin of) SFF.

3. Can uplift to $\mathrm{AdS}_5\times\mathbb{S}^5$, perform partial stability analysis.

4. Lots more to do!

Thank you!