Diagnosing collisions in the interior of a wormhole

Y.Z. arXiv:2011.06016
Felix Haehl and Y.Z. arXiv:2104.02736
Felix Haehl, Alex Streicher, Y.Z. arXiv:2105.12755

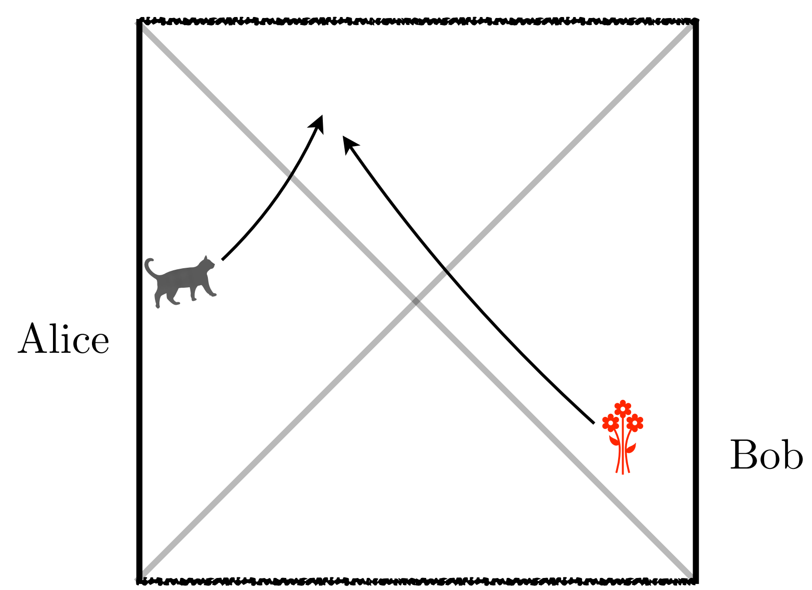
J. Maldacena, hep-th/0106112
J. Maldacena, L. Susskind: arXiv:1306.0533

Motivation

- Two distant black holes can be connected in the interior through a wormhole.

- Such an Einstein-Rosen bridge can be interpreted as an entangled state: ER = EPR

- A mystery: Two signals sent in from the two different boundaries can meet in the interior. On the other hand, there are no boundary Interactions.



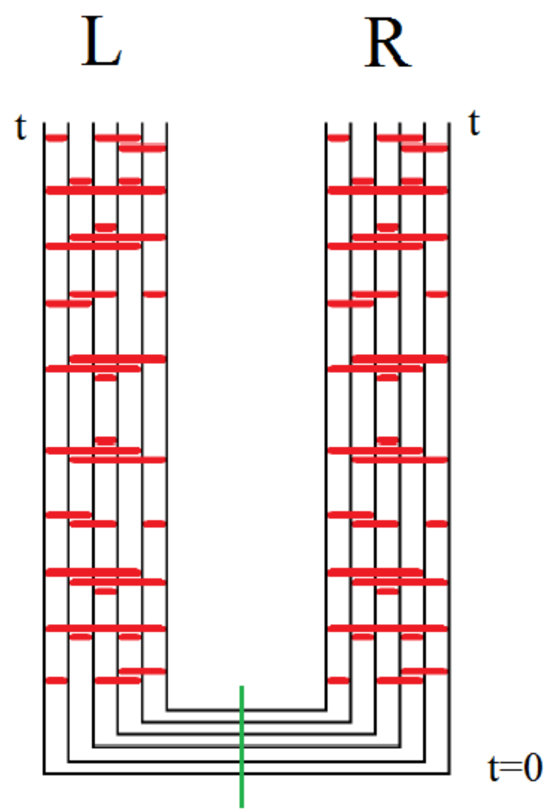
Outline

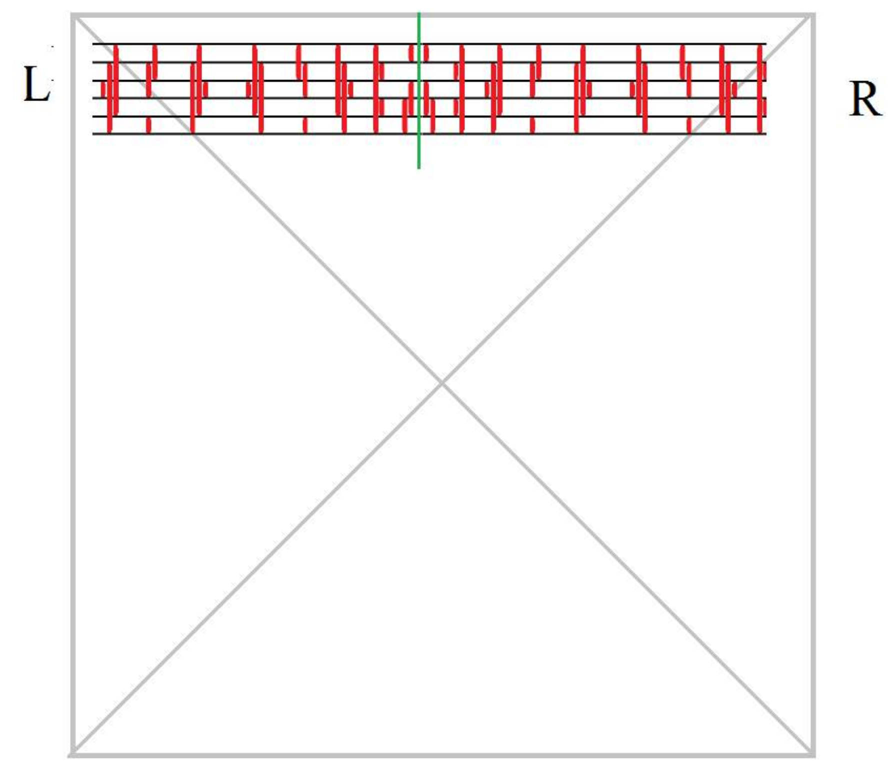
- Thermal field double and quantum circuit in the interior of eternal black hole
- A meeting in the interior of a wormhole and overlap of perturbations in the quantum circuit
- Diagnosing the collision by six-point functions
- The overlap region in the quantum circuit and post-collision region
- Unanswered questions

Bulk tensor network and quantum circuit

B. Swingle arXiv:1209.3304
T. Hartman, J. Maldacena arXiv:1303.1080
L. Susskind arXiv:1411.0690

- The bulk geometry reflects the minimal circuit preparing the state.



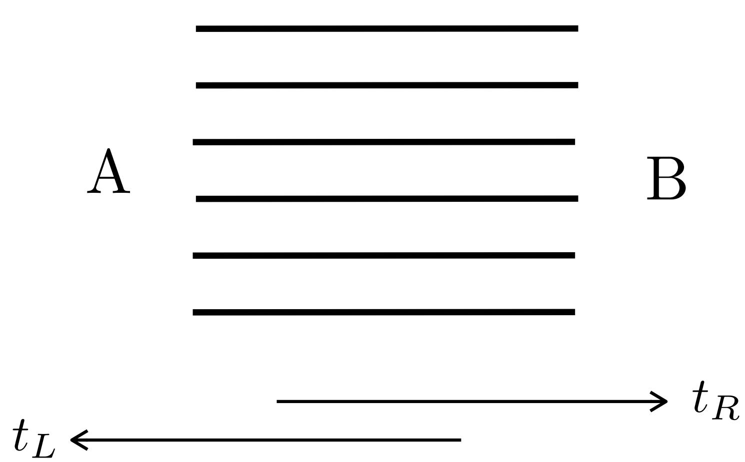


- As we apply unitary time evolution to the circuit, the state gets more complex, the minimal circuit gets longer, and the Einstein-Rosen bridge also gets longer.

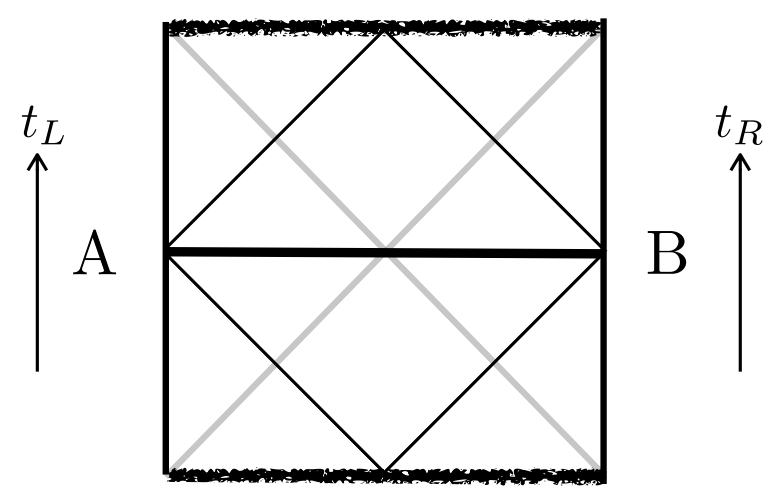
- Identify circuit time with boundary time: $d\tau = \frac{2\pi}{\beta} dt$

Thermofield Double

$$|\mathrm{TFD}\rangle = \sum_k e^{-\frac{\beta}{2}E_k} \, |E_k\rangle_L \otimes |E_k\rangle_R$$



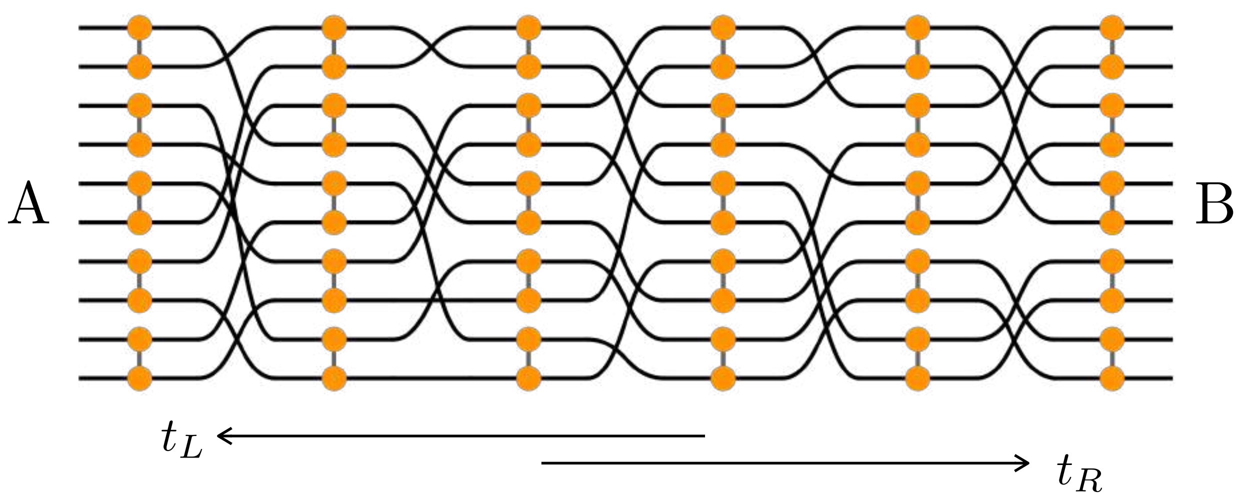
(a) Quantum circuit



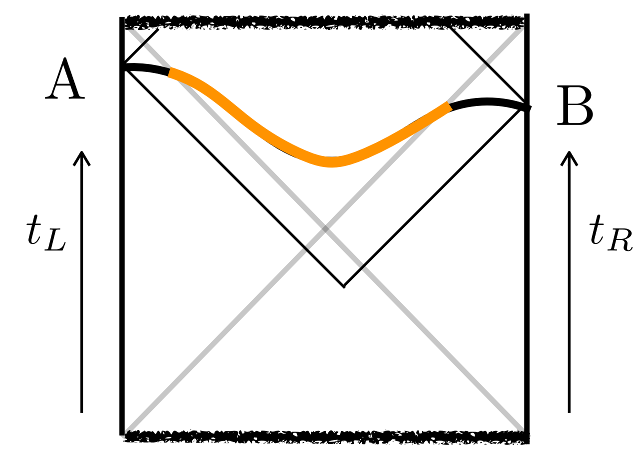
(b) Wormhole geometry

We represent thermofield double by S Bell pairs. The corresponding wormhole geometry has minimal length.

Time-evolved thermofield Double



(a) Quantum circuit



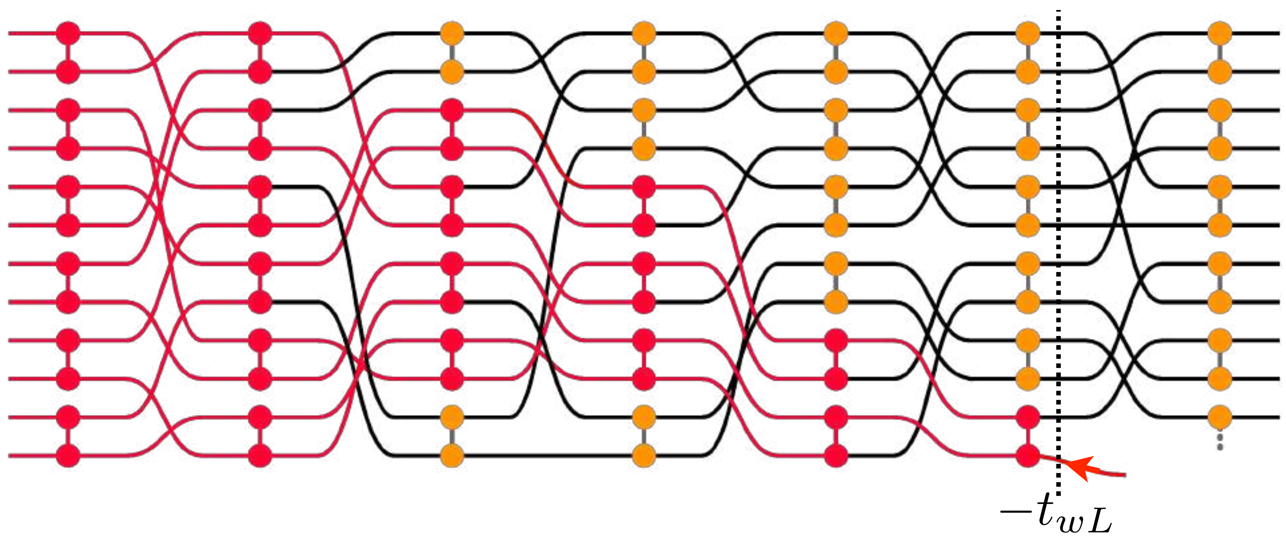
(b) Wormhole geometry

- The orange gates can be undone from either side. We call them healthy gates.

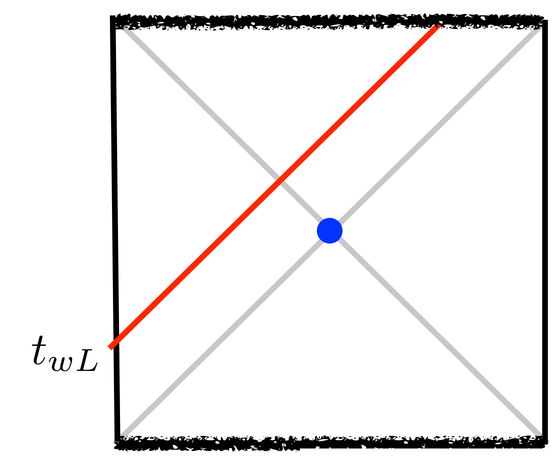
- The number of healthy gates per unit time is constant.

P. Hayden, J. Preskill arXiv:0708.4025v2
L. Susskind, Y.Z. arXiv:1408.2823
A. Brown, L. Susskind, Y.Z. arXiv:1608.02612

Perturbed thermofield double



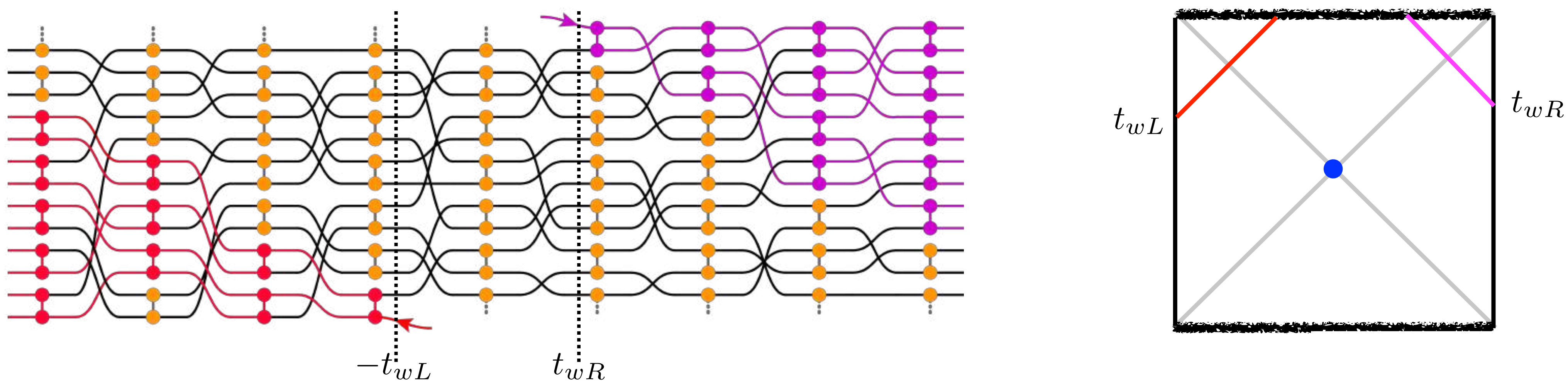
(a)



(b)

- Epidemic model: The red arrow represents the extra qubit due to Alice's perturbation. Any qubits that interact directly or indirectly with the perturbation will get perturbed relative to the original circuit describing the thermofield double state.

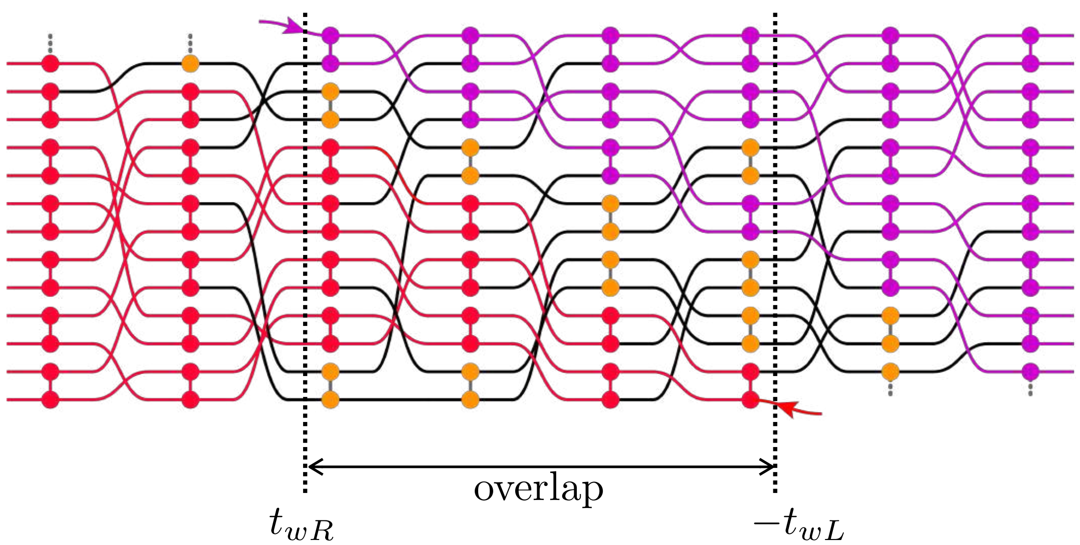
Quantum circuit with perturbations from both sides



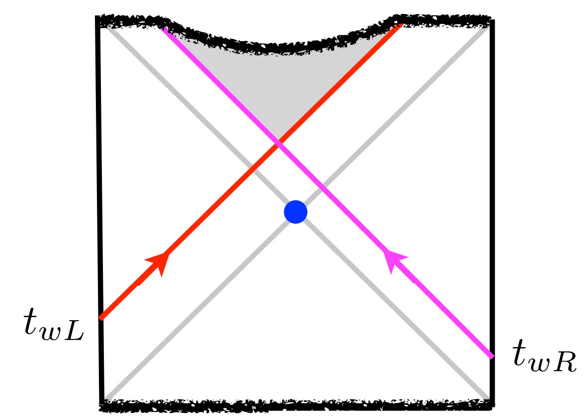
(a) (b)

- When $t_{wL} + t_{wR} > 0$, the two perturbations do not have overlap in the quantum circuit. Correspondingly, the signals sent into the bulk hit the singularity before they have a chance to meet in the wormhole.

A meeting in the interior of the wormhole



(a)



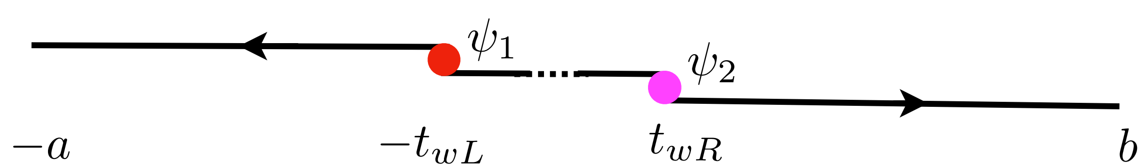
(b)

- When $t_{wL} + t_{wR} < 0$, the two perturbations have overlap in the quantum circuit. Correspondingly, the signals collide inside the wormhole. The larger the overlap is, the stronger the collision is.

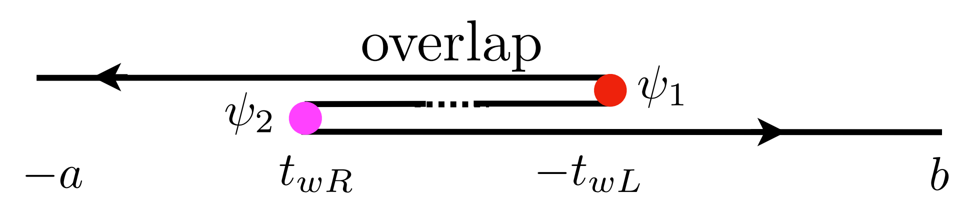
Diagnosing the collision in the wormhole interior

- We want to quantify the overlap in the quantum circuit.

  Imagine experimentalists Alice and Bob try to build the quantum state in their laboratory. They start from some shared EPR pairs and implement the following circuits.



$(t_{wL} + t_{wR} > 0)$



$(t_{wL} + t_{wR} < 0)$

- In the second case, when the perturbations have overlap, before inserting the perturbations both Alice and Bob have to do backward time evolution. This makes the procedure complex. At the end there is a timefold in the circuit. As the fold grows, the collision gets stronger. In order to diagnose it, we think of the entire circuit as a single (complicated) operator preparing a state, and we quantify the “size” of this operator.

L. Susskind arXiv:1802.01198
A. Brown, H. Gharibyan, A. Streicher, L. Susskind, L. Thorlacius, Y.Z. arXiv:1804.04156
D. Roberts, D. Stanford, A. Streicher arXiv: 1802.02633
X-L. Qi, A. Streicher arXiv: 1810.11958

Size of an operator

- Operator growth in SYK model, infinite temperature

$$n_\infty[\mathcal{O}] \equiv \frac{1}{4}\sum_j \mathrm{tr}\big(\{\mathcal{O},\psi_j\}^\dagger\{\mathcal{O},\psi_j\}\big)$$

- At finite temperature

$$\frac{n_\beta[\mathcal{O}]}{n_{max}} \equiv \frac{n_\infty[\mathcal{O}\rho^{\frac{1}{2}}] - n_\infty[\rho^{\frac{1}{2}}]}{n_{max} - n_\infty[\rho^{\frac{1}{2}}]} \qquad n_{max} = \frac{N}{2}$$

- The thermal size of a single fermion $n_\beta[\psi_1(t)]$ can be related to an out-of-time-ordered four point function, which grows exponentially until it saturates to n_{max} at a time of order t_*

$$\mathcal{F}_4(t) \equiv 1 - \frac{n_\beta[\psi_1(t)]}{n_{max}} = -\frac{\sum_j \mathrm{tr}\Big(\psi_1(t)\psi_j\psi_1(t)\rho^{\frac{1}{2}}\psi_j\rho^{\frac{1}{2}}\Big)}{\sum_j \mathrm{tr}(\psi_1(t)\psi_1(t)\rho)\,\mathrm{tr}\Big(\rho^{\frac{1}{2}}\psi_j\rho^{\frac{1}{2}}\psi_j\Big)}$$

- $\mathcal{F}_4$ is the probability of a gate being healthy with one perturbation present

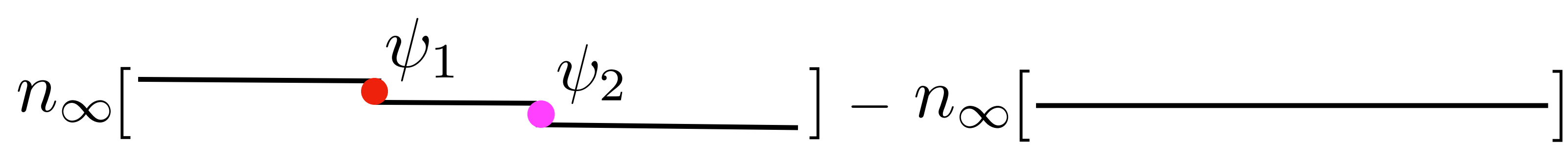
- In our case, the operator that builds the circuit described above is composed of two perturbations:

$$e^{iHa}\psi_1(-t_{wL})\rho^{\frac{1}{2}}\psi_2(t_{wR})e^{iHb}$$

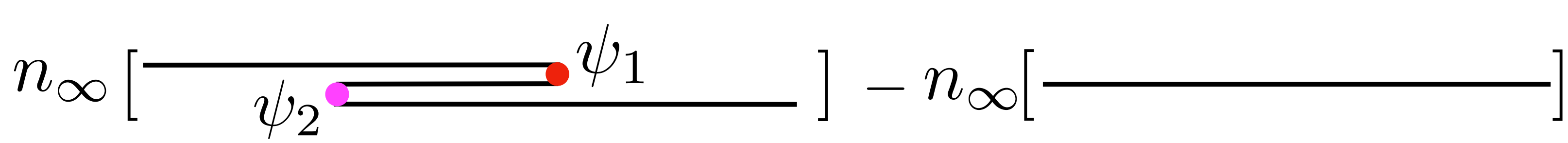
- In order to detect the presence of the fold in the circuit, we define the renormalized size of the operator with two perturbations as follows:

$$\frac{n_{\rm ren}}{n_{\rm max}} \equiv \frac{n_\infty[e^{iHa}\psi_1(-t_{wL})\rho^{\frac{1}{2}}\psi_2(t_{wR})e^{iHb}] - n_\infty[e^{iHa}\rho^{\frac{1}{2}}e^{iHb}]}{n_{\rm max} - n_\infty[e^{iHa}\rho^{\frac{1}{2}}e^{iHb}]}$$

$t_{wL} + t_{wR} > 0 :$



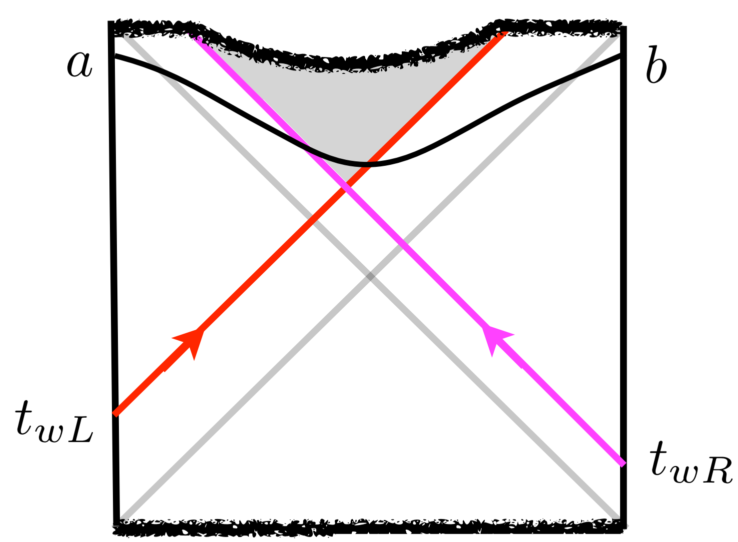
$t_{wL} + t_{wR} < 0 :$



- Without the fold the renormalized size does not grow. In the case where $t_{wL} + t_{wR} < 0$, the fold makes a non-trivial contribution to the renormalized size. This contribution grows as the fold grows.

Just as the size of a single-perturbation operator is related to an out-of-time-ordered four point function, the renormalized size of a two-perturbation operator can be written as a six-point correlation function: $\frac{n_{ren}}{n_{max}} = 1 - \mathcal{F}_6$

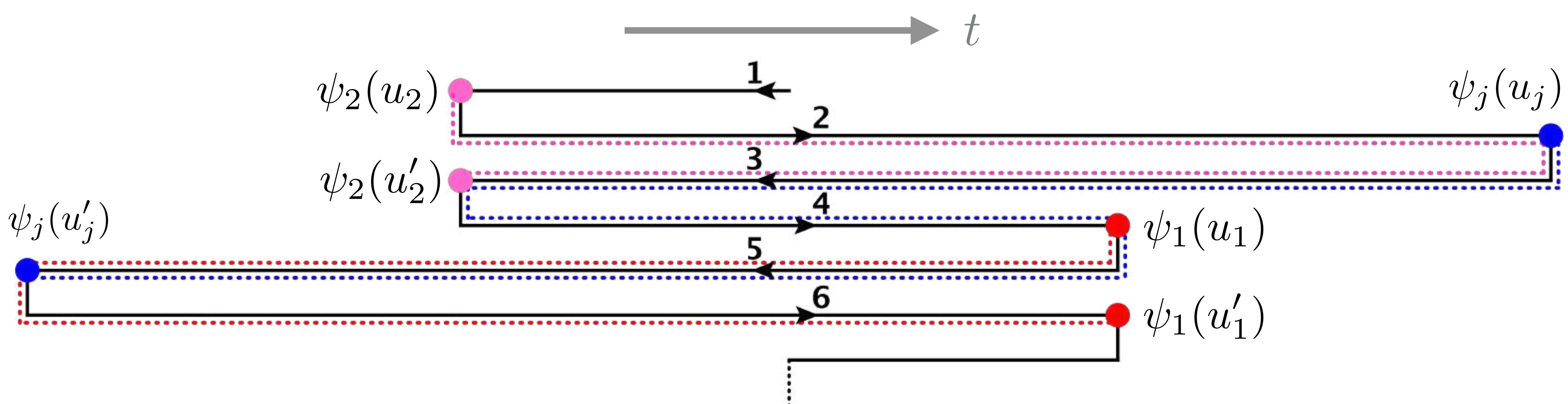
$$\mathcal{F}_6 = \frac{\sum_{j=1}^{N} \Big(\langle \mathrm{TFD}| \, \psi_1^L(t_{wL}) \psi_2^R(t_{wR}) \Big) \; \psi_j^L(a) \psi_j^R(b) \; \Big(\psi_1^L(t_{wL}) \psi_2^R(t_{wR}) \, |\mathrm{TFD}\rangle \Big)}{\sum_{j=1}^{N} \langle \mathrm{TFD}| \, \psi_j^L(a) \psi_j^R(b) \, |\mathrm{TFD}\rangle}$$



J. Maldacena, D. Stanford, Z. Yang arXiv: 1606.01857

Calculation of six-point function in Schwarzian theory

- A contour representation of the six-point function



- We will compute the correlator by performing a path integral over time reparametrizations $t(u)$ weighted by the Schwarzian action

$$\mathcal{F}_6 = \mathcal{N} \int [\mathcal{D}t]\, e^{iS[t(u)]}\, G_{\Delta_1}(u_1, u_1')\, G_{\Delta_2}(u_2, u_2')\, G_{\Delta_j}(u_j, u_j')$$

Result

- With large a, b, the correlator $\mathcal{F}_6$ is only a function of $t_{wL} + t_{wR}$

- Integral representation of the correlator with large a, b:

$$\mathcal{F}_6 = \frac{1}{\Gamma(2\Delta_1)\Gamma(2\Delta_2)} \int_0^{+\infty} dp_1 \int_0^{+\infty} dp_2 \; p_1^{2\Delta_1 - 1} \; e^{-p_1} \; p_2^{2\Delta_2 - 1} \; e^{-p_2} \; \left(\frac{1}{1 + \frac{p_1 p_2}{16C^2 \delta_1 \delta_2} e^{-\frac{2\pi}{\beta}(t_{wL} + t_{wR})}} \right)^{2\Delta_j}$$

- Saddle point approximation:

$$\mathcal{F}_6 \approx \left(1 + \frac{\Delta_1 \Delta_2}{4C^2 \delta_1 \delta_2} e^{-\frac{2\pi}{\beta}(t_{wL} + t_{wR})} \right)^{-2\Delta_j}$$

- Geodesic approximation:

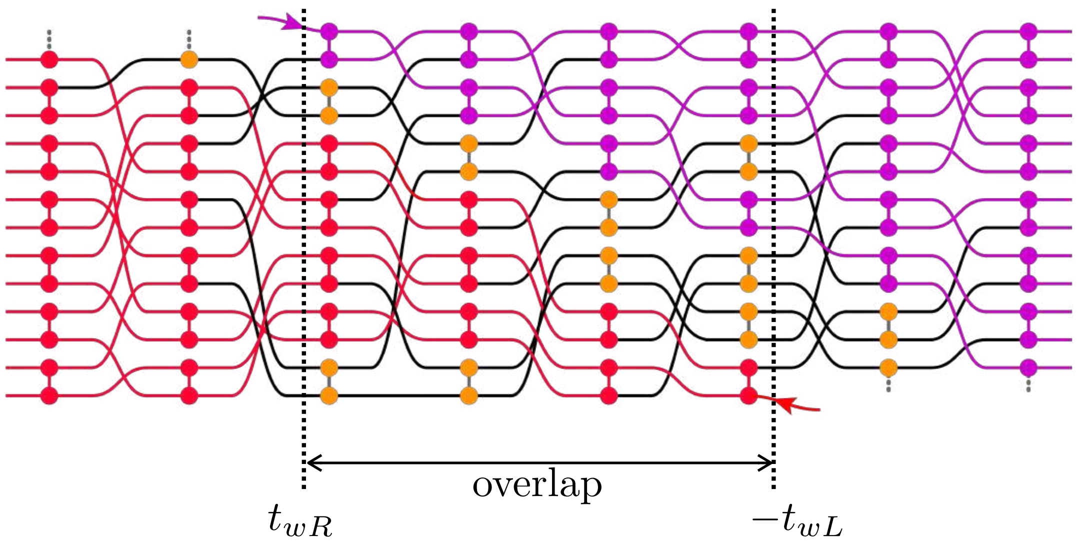
$$\mathcal{F}_6 \approx \frac{e^{-\Delta\, d_{\text{pert.}}}}{e^{-\Delta\, d_{\text{unpert.}}}} = \left(1 + \frac{1}{4} \frac{\delta S_1 \delta S_2}{(S - S_0)^2} \, e^{-\frac{2\pi}{\beta}(t_{wL} + t_{wR})} \right)^{-2\Delta_j}$$

Size of the two-perturbation operator

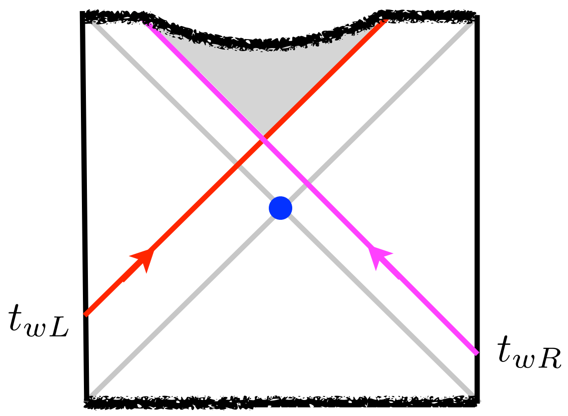
$$\frac{n_{\rm ren}}{n_{\rm max}} = 1 - \mathcal{F}_6 \approx \begin{cases} 0 & -(t_{wL}+t_{wR}) < 0 \\ \frac{\Delta}{2}\frac{\delta S_1 \delta S_2}{(S-S_0)^2}\, e^{-\frac{2\pi}{\beta}(t_{wL}+t_{wR})} & 0 \le -(t_{wL}+t_{wR}) < 2t_* \\ 1 - e^{-2\Delta\frac{2\pi}{\beta}[-(t_{wL}+t_{wR})-2t_*]} & 2t_* \le -(t_{wL}+t_{wR}) \end{cases}$$

- The size only grows for negative $t_{wL}+t_{wR}$. This is expected as there is no collision for positive $t_{wL}+t_{wR}$.
- As we inject the perturbation earlier and $t_{wL}+t_{wR}$ becomes more negative, the size starts to grow exponentially in $-\frac{2\pi}{\beta}(t_{wL}+t_{wR})$. This is consistent with the fact that the collision energy grows exponentially in $-\frac{2\pi}{\beta}(t_{wL}+t_{wR})$.
- When $-(t_{wL}+t_{wR}) > 2t_*$, the size saturates at its maximal value. On the gravity side, a large black hole has formed and the collision happens exponentially close to the singularity.

Properties of a quantum circuit with overlapping perturbations



(a)



(b)

- The healthy gates in the overlap region in the quantum circuit can be undone from both sides. They are meaningless for one boundary system alone.

Y.Z. arXiv:1711.03125

- These healthy gates are stored in the post-collision region in the shared interior.

Estimating the number of healthy gates in the overlap region

- $$\mathcal{F}_6'(t_1,t_2) \equiv 1 - \frac{n_\infty[\psi_1(-t_1)\rho^{\frac{1}{2}}\psi_2(t_2)] - n_\infty[\rho^{\frac{1}{2}}]}{n_{max} - n_\infty[\rho^{\frac{1}{2}}]}$$

  $\mathcal{F}_6'$ depends on both times t_1 and t_2

  $\mathcal{F}_6'$ is the probability of a gate being healthy with two perturbations present

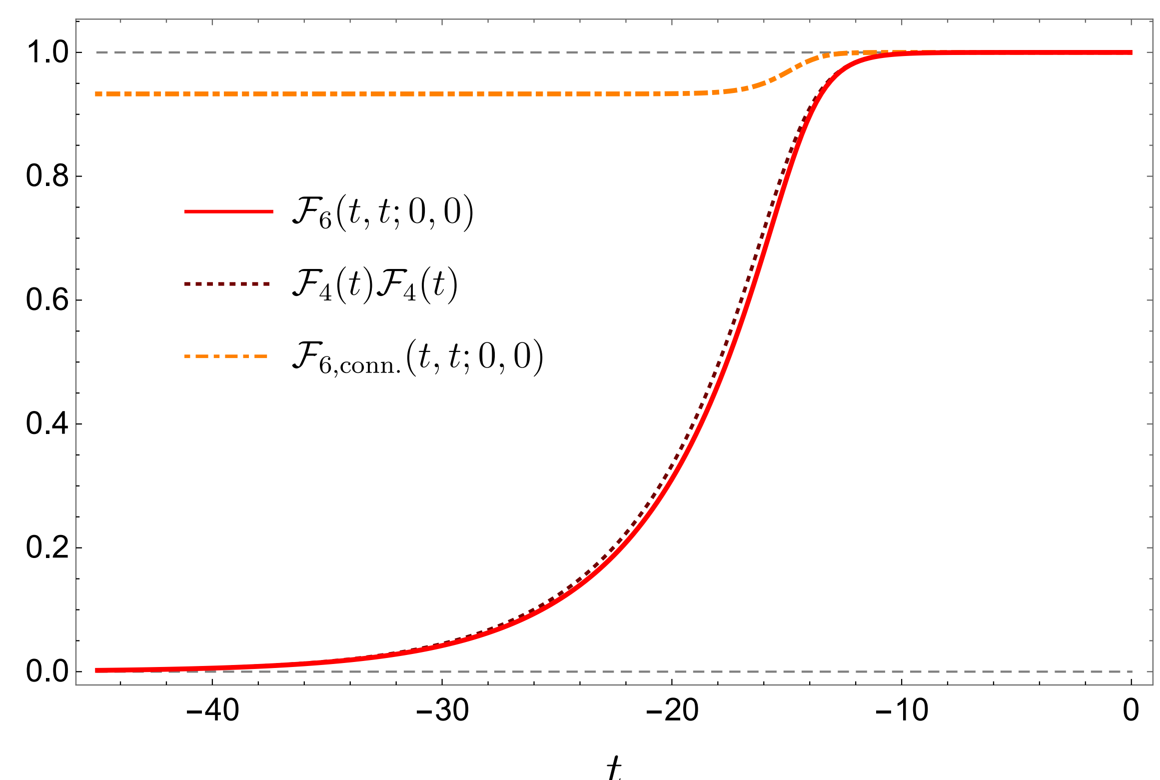
- Recall that $\mathcal{F}_4$ is the probability of a gate being healthy with one perturbation.

  Epidemic model: $\mathcal{F}_6'(t_1,t_2) \approx \mathcal{F}_4(t_1)\mathcal{F}_4(t_2)$

- Evaluation of the six point function $\mathcal{F}_6'$ in Schwarzian:

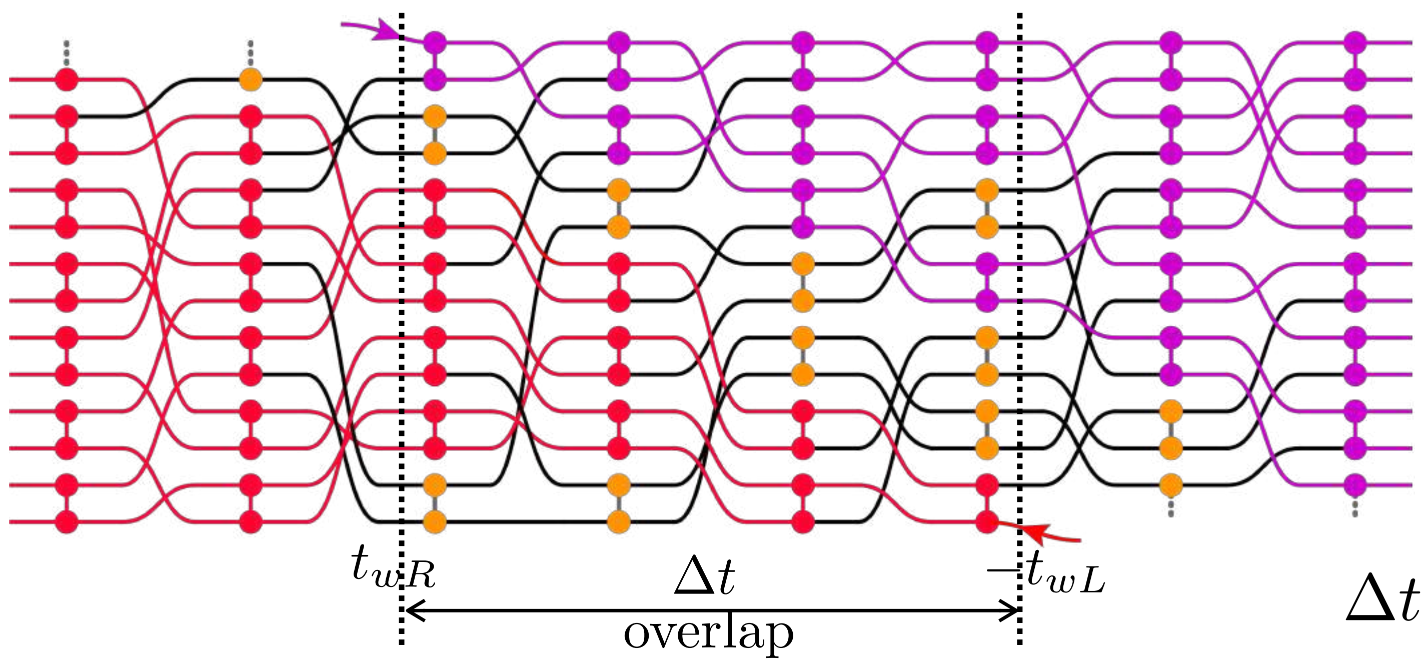
$$\begin{aligned}\mathcal{F}_6'(t_1,t_2) &= \mathcal{F}_4(t_1)\,\mathcal{F}_4(t_2)\,\mathcal{F}_{6,\mathrm{conn.}}(t_1,t_2)\\ &\approx \mathcal{F}_4(t_1)\mathcal{F}_4(t_2)\end{aligned}$$

T. Anous, F. Haehl arXiv:2005.06440



$$\mathcal{F}_6'(t_1,t_2) \approx \left[\frac{1}{1+G\Delta_1\,z_1}\right]^{2\Delta_j}\left[\frac{1}{1+G\Delta_2\,z_2}\right]^{2\Delta_j}\left[\frac{1}{1+\frac{G^2\Delta_1\Delta_2\,z_1\,z_2}{(1+G\,\Delta_1\,z_1)(1+G\,\Delta_2\,z_2)}}\right]^{2\Delta_j}$$

where $z_1 = \frac{e^{-t_1}}{8\,\sin\delta_1}\,, \qquad z_2 = \frac{e^{-t_2}}{8\,\sin\delta_2}$



$$\Delta t \equiv -t_{wL} - t_{wR}$$

$$\frac{N_{\text{healthy}}}{S} = \int_{t_{wR}}^{-t_{wL}} dt\;\mathcal{F}_6'(t-t_{wR}, -t_{wL}-t) = \frac{N_{\text{healthy}}}{S}(\Delta t)$$

Computing the spacetime volume in the post-collision region

- Post-collision region: A larger black hole forms

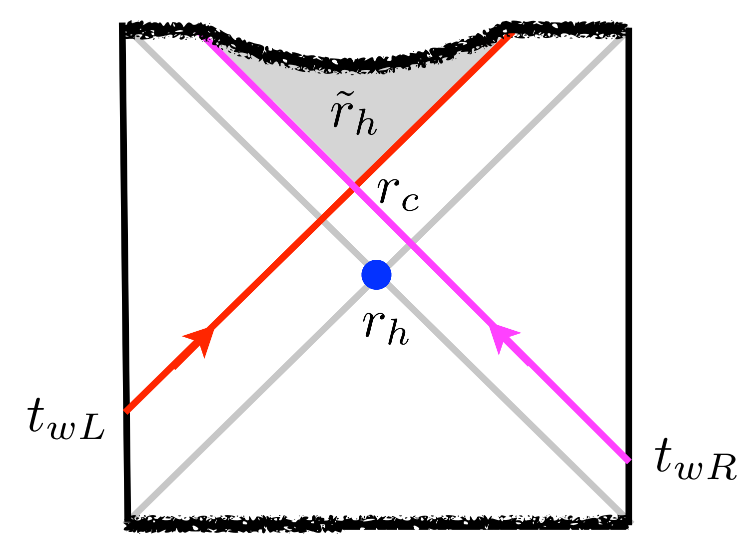
$$\frac{\tilde{r}_h^2}{r_h^2} = 1 + \frac{2\delta S_1}{S} + \frac{2\delta S_2}{S} + \frac{4\delta S_1 \delta S_2}{S^2} \cosh^2\left(\frac{\pi}{\beta}\Delta t\right)$$

$$\Delta t \equiv -t_{wL} - t_{wR}$$

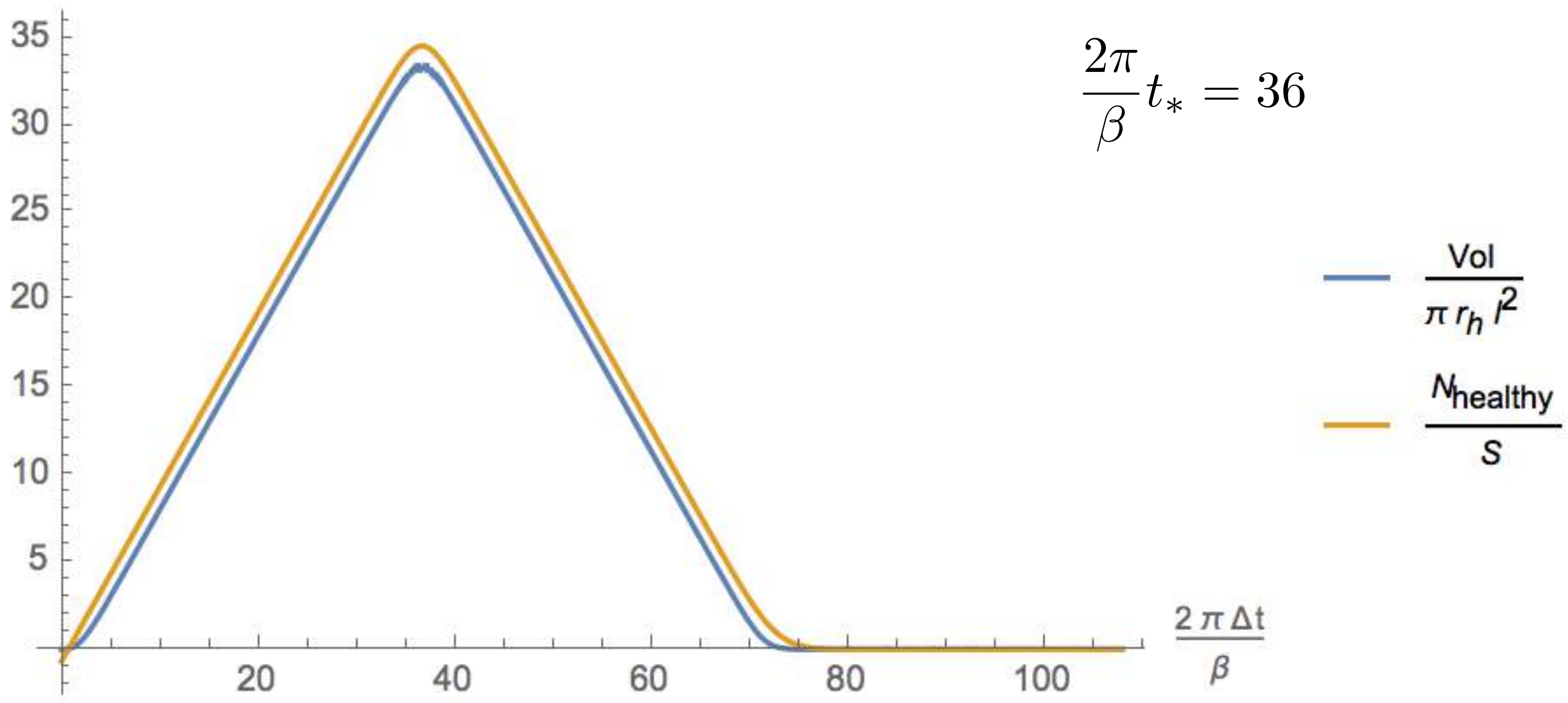
T. Dray, G. 't Hooft 1985
S. Shenker, D. Stanford arXiv:1312.3296

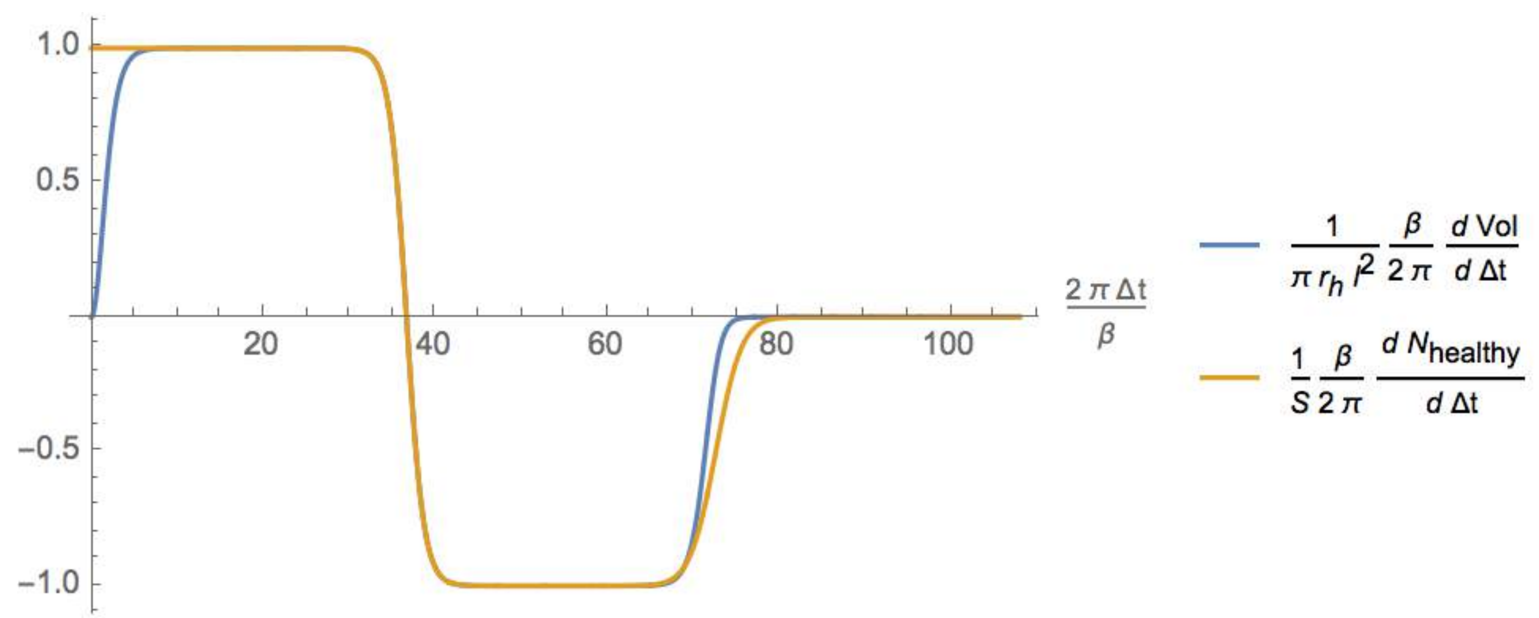
- Spacetime volume of post-collision region in BTZ black hole

$$V = 2\pi \tilde{r}_h l^2 \left(\frac{1}{2}\log\frac{\tilde{r}_h + r_c}{\tilde{r}_h - r_c} - \frac{r_c}{\tilde{r}_h}\right)$$
$$= V(\Delta t)$$



Comparison of the total number of healthy gates in the overlap region $N_{\text{healthy}}(\Delta t)$ and the spacetime volume of post collision region $V(\Delta t)$





Unanswered questions

- The role of singularity: the existence of spacelike singularity prevents the meeting of signals that are sent in too late. What about collision near the inner horizon for charged black holes?

Y. Lensky, X-L. Qi, arXiv:2012.15798

- Our story is only sensitive to the portion of trajectory of infalling object when it is close to the horizon. From the picture of neutral black holes, one tends to say that at the singularity the perturbation has smallest size and becomes simple, but this picture doesn't work for charged black holes.

- How broadly does this “meeting” occur in general quantum systems? We expect the discussion holds when the system has a bulk dual. For more general quantum systems, the meaning of “meeting” is not clear, but we expect it should be as general as the notion of ER = EPR.

- If we want to use the growth of size to diagnose the interior property, how to incorporate the microstate information in the definition of size?

Felix Haehl, Y.Z. arXiv:2102.05697
A. Kar, L. Lamprou, M. Rozali, J. Sully arXiv:2106.02046

Thank you.