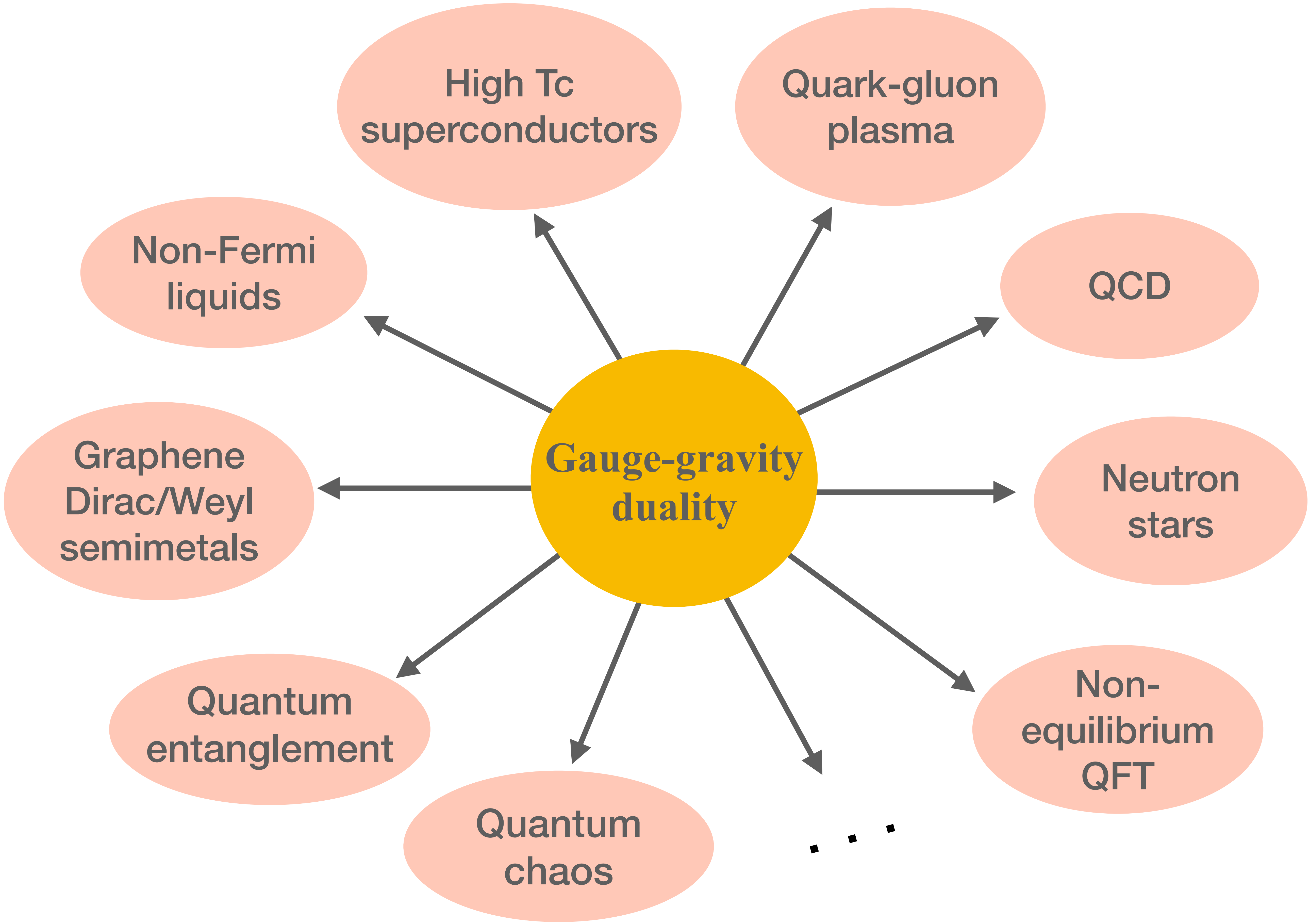
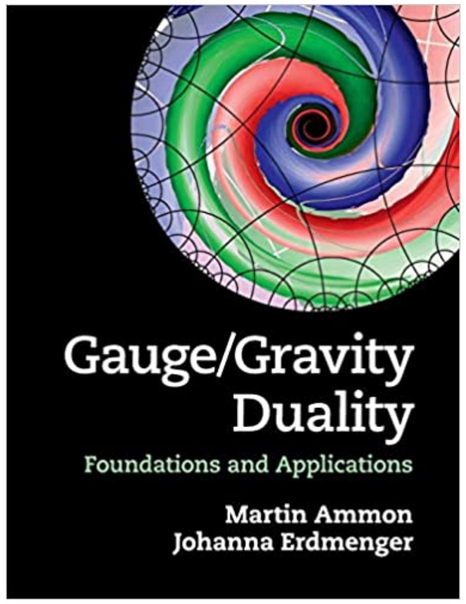
Gauge-gravity duality applied to quantum many body systems

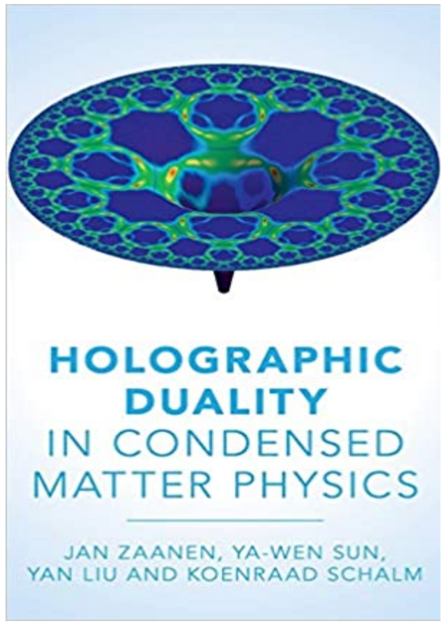
Umut Gürsoy

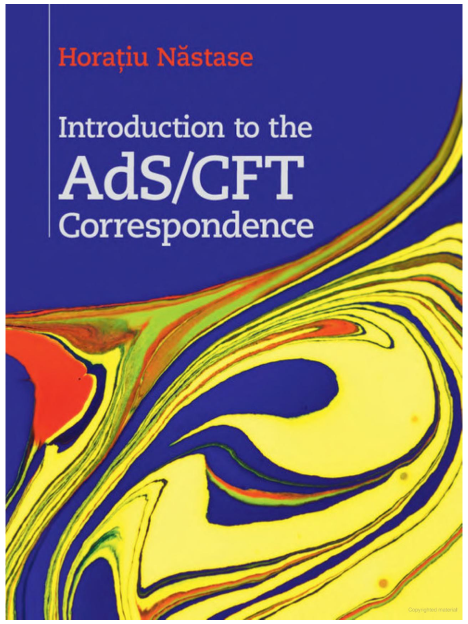
Utrecht University

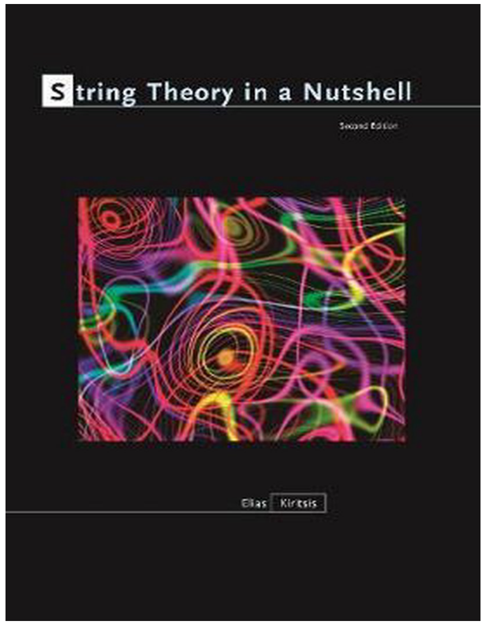
Strings `22 Vienna
21.7.2022


High Tc superconductors
Quark-gluon plasma
Non-Fermi liquids
QCD
Gauge-gravity duality
Graphene Dirac/Weyl semimetals
Neutron stars
Quantum entanglement
Non-equilibrium QFT
Quantum chaos
. . .


Gauge/Gravity Duality
Foundations and Applications
Martin Ammon
Johanna Erdmenger


HOLOGRAPHIC DUALITY IN CONDENSED MATTER PHYSICS
JAN ZAANEN, YA-WEN SUN, YAN LIU AND KOENRAAD SCHALM


Horațiu Năstase
Introduction to the AdS/CFT Correspondence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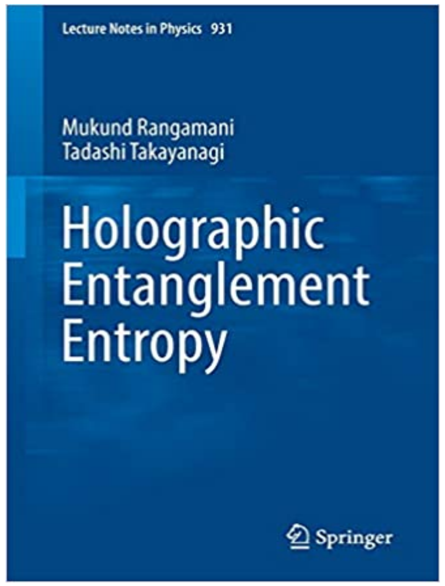
Second Edition
Elias Kiritsis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Mukund Rangamani
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Holographic Entanglement Entropy
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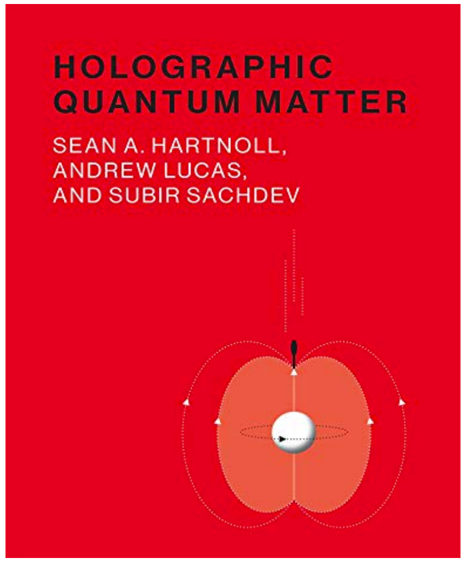
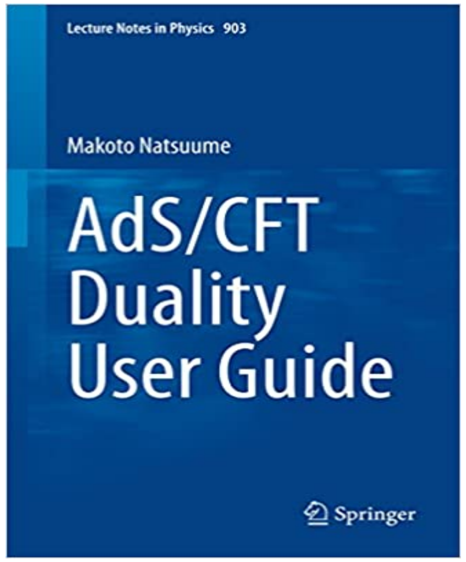

HOLOGRAPHIC QUANTUM MATTER
SEAN A. HARTNOLL,
ANDREW LUCAS,
AND SUBIR SACHDEV


Lecture Notes in Physics 903
Makoto Natsuume
AdS/CFT Duality User Guide
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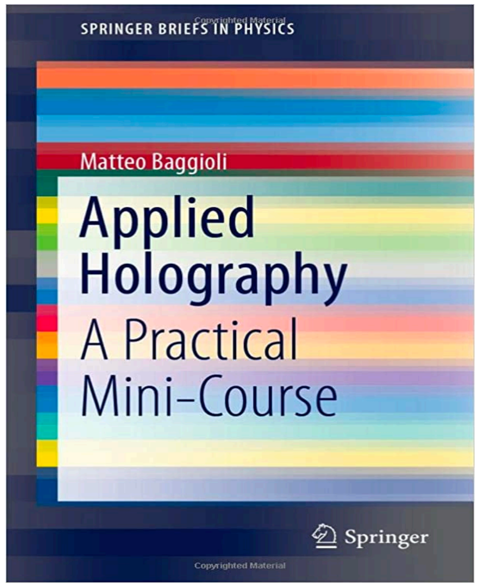
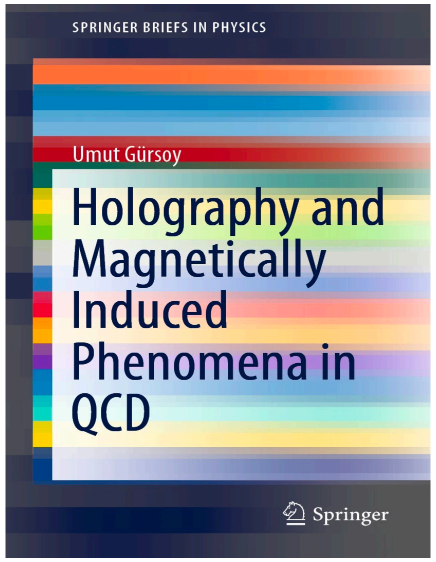

SPRINGER BRIEFS IN PHYSICS
Matteo Baggioli
Applied Holography
A Practical Mini-Course
Springer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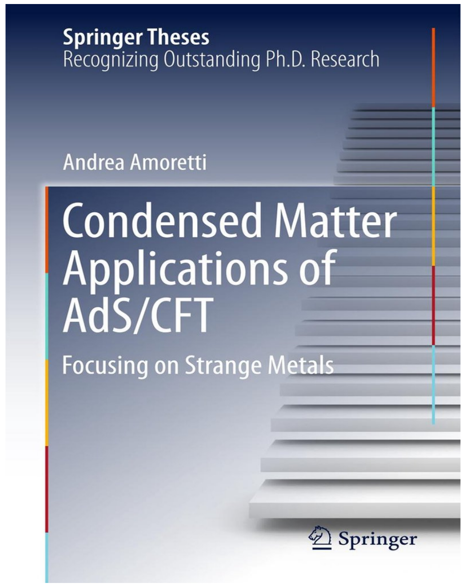
Umut Gürsoy
Holography and Magnetically Induced Phenomena in QCD
Springer


Springer Theses
Recognizing Outstanding Ph.D. Research
Andrea Amoretti
Condensed Matter Applications of AdS/CFT
Focusing on Strange Metals
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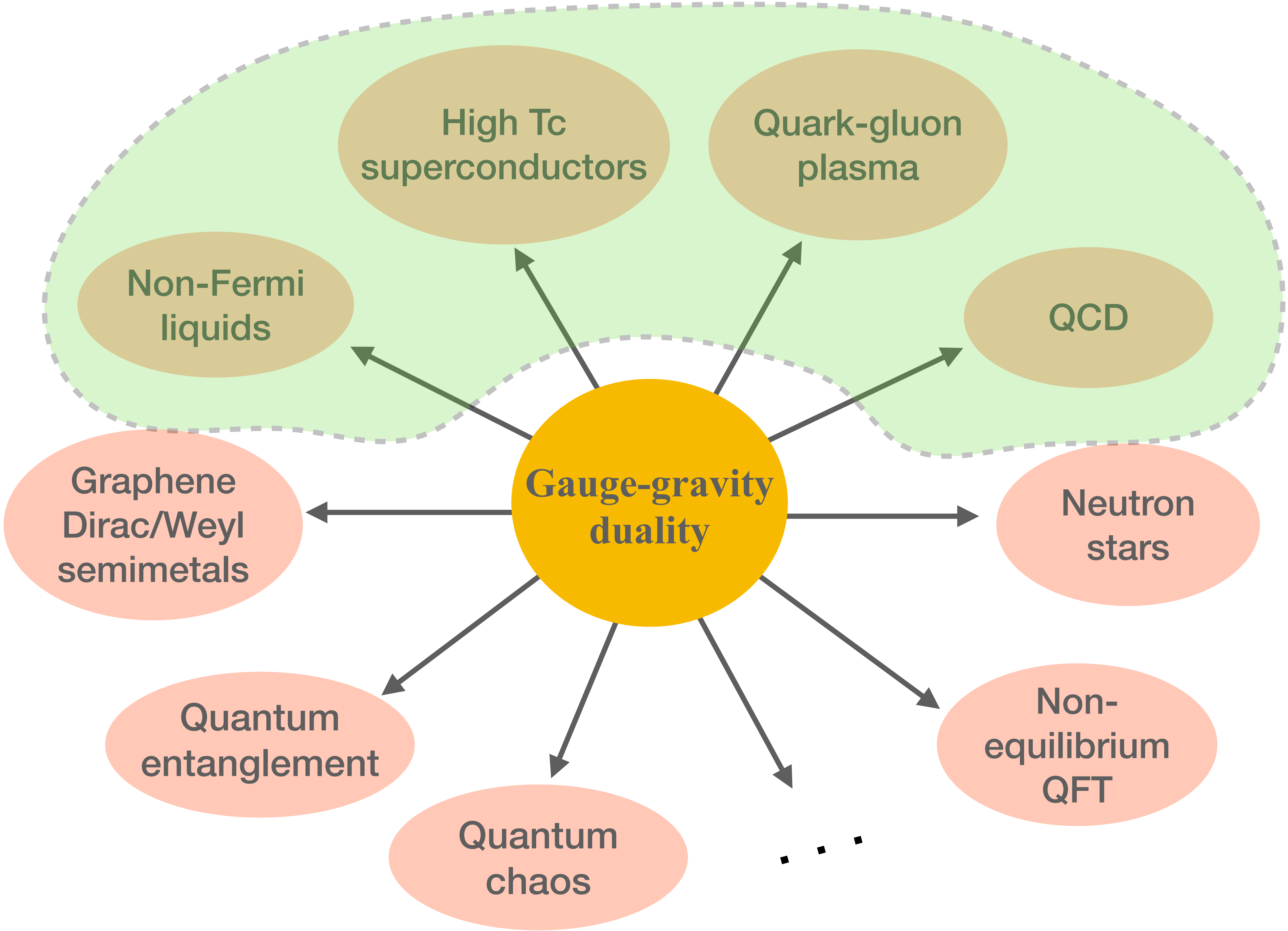
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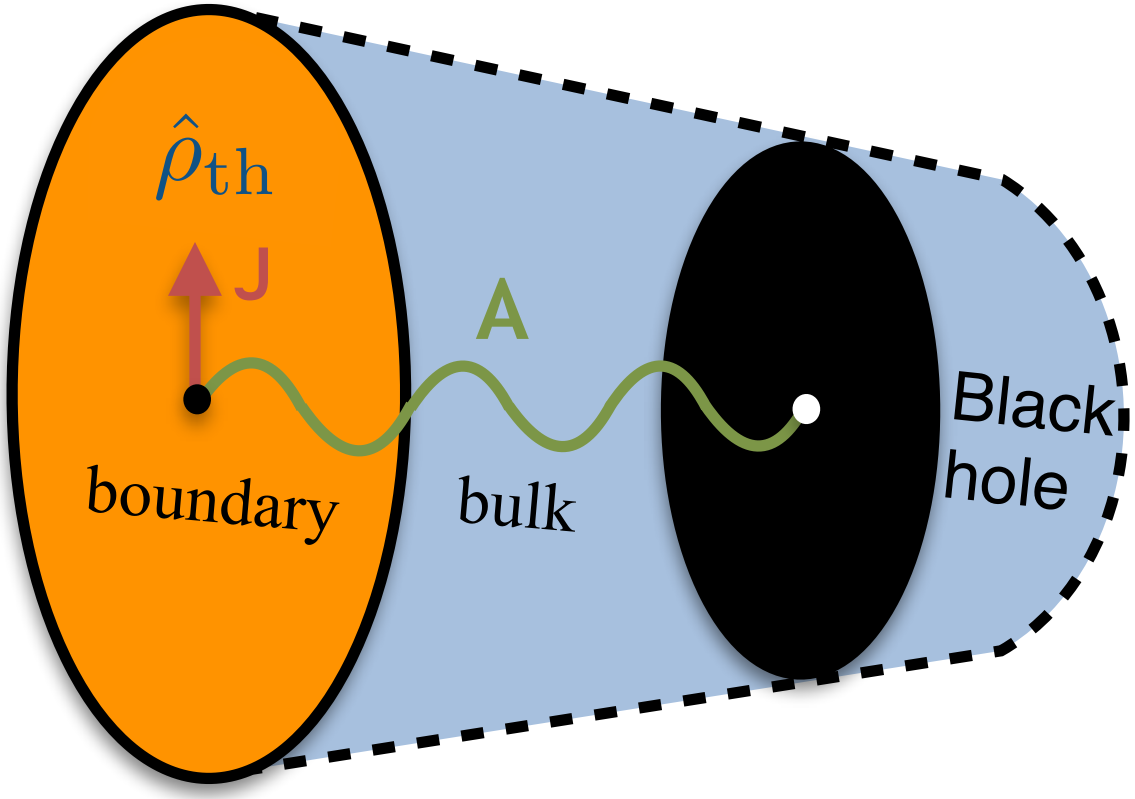
Gauge-gravity duality

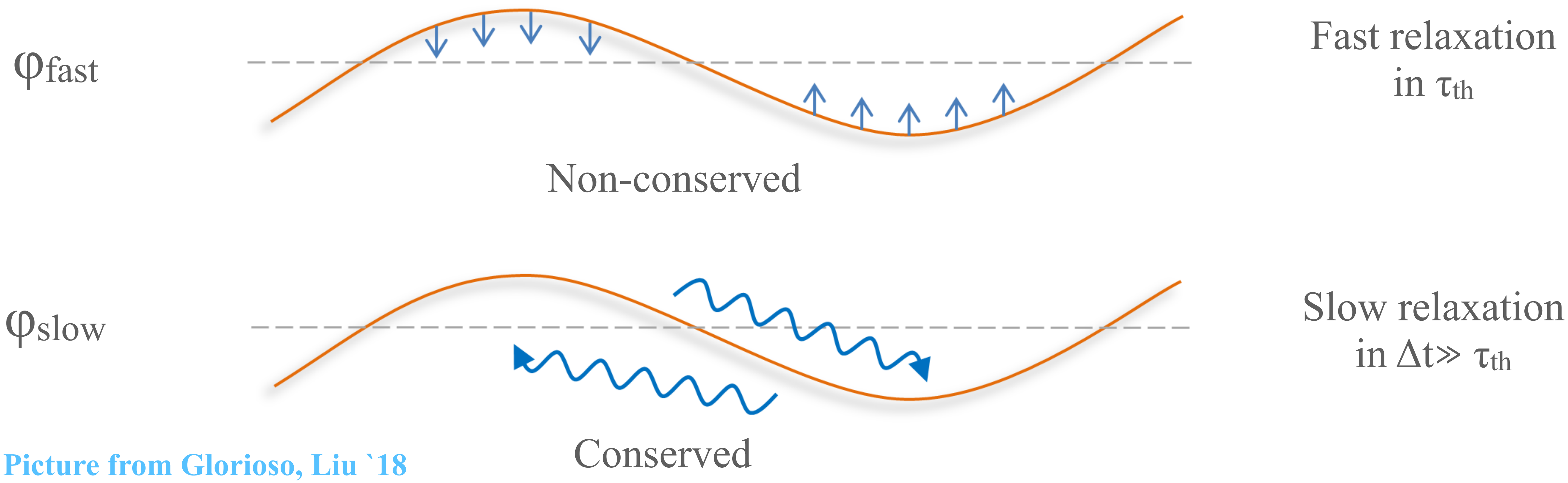


Horizon: thermodynamic, dissipative, short-lived excitations

$$\tau_{\rm th} \sim \frac{\hbar}{k_B T}$$

transport, thermalization, quantum chaos, …
e.g. in high Tc superconductors, quark-gluon plasma, etc

Hydrodynamics



Picture from Glorioso, Liu `18

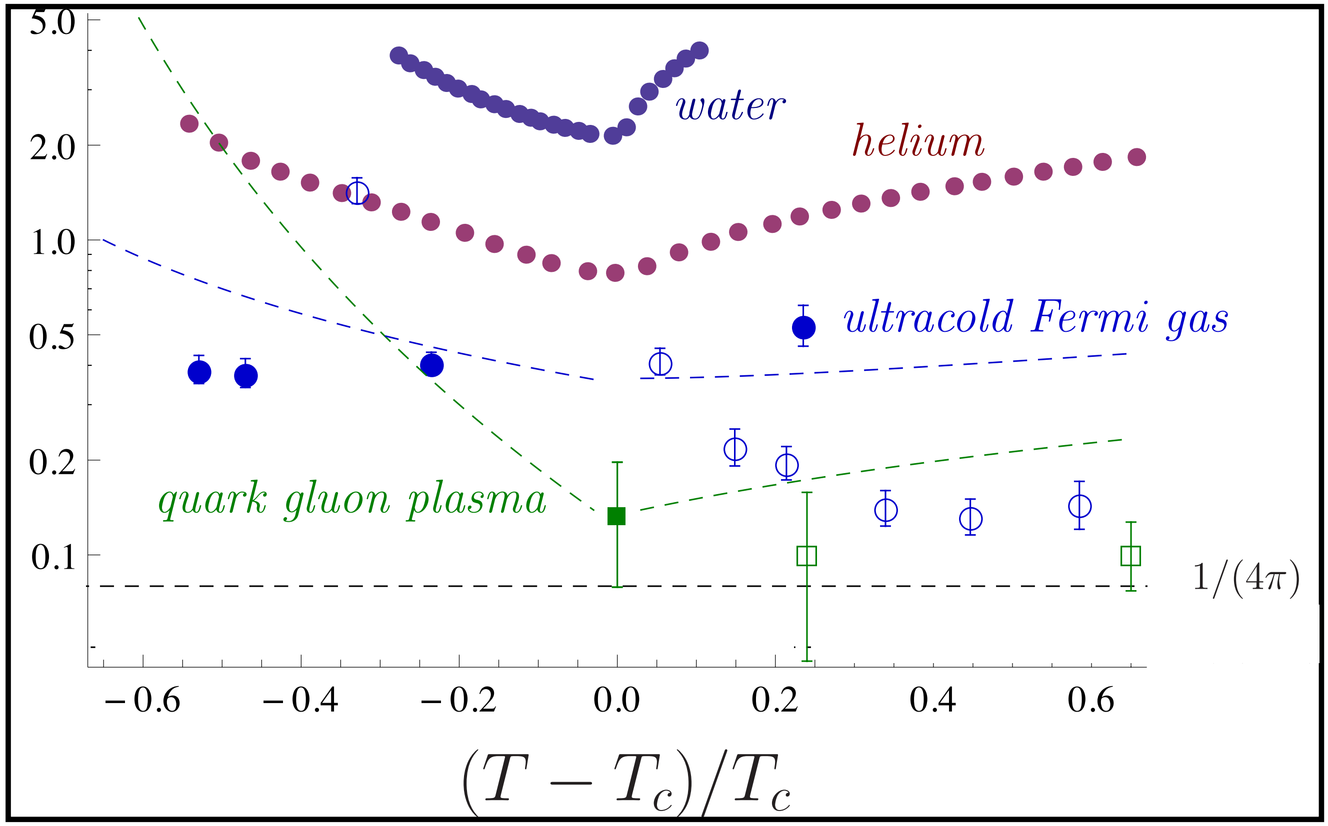
Separation of scales: $\Delta t \gg \tau_{th}$, $\Delta L \gg \lambda_{th} \Longrightarrow$ local theory of conserved charges:

$$\partial_t n_a(x,t) + \nabla \cdot j_a(x,t) = 0, \qquad j_a = \alpha^0_{ab} n_b + \alpha^1_{ab} \nabla n_b + \cdots$$

Transport coefficients

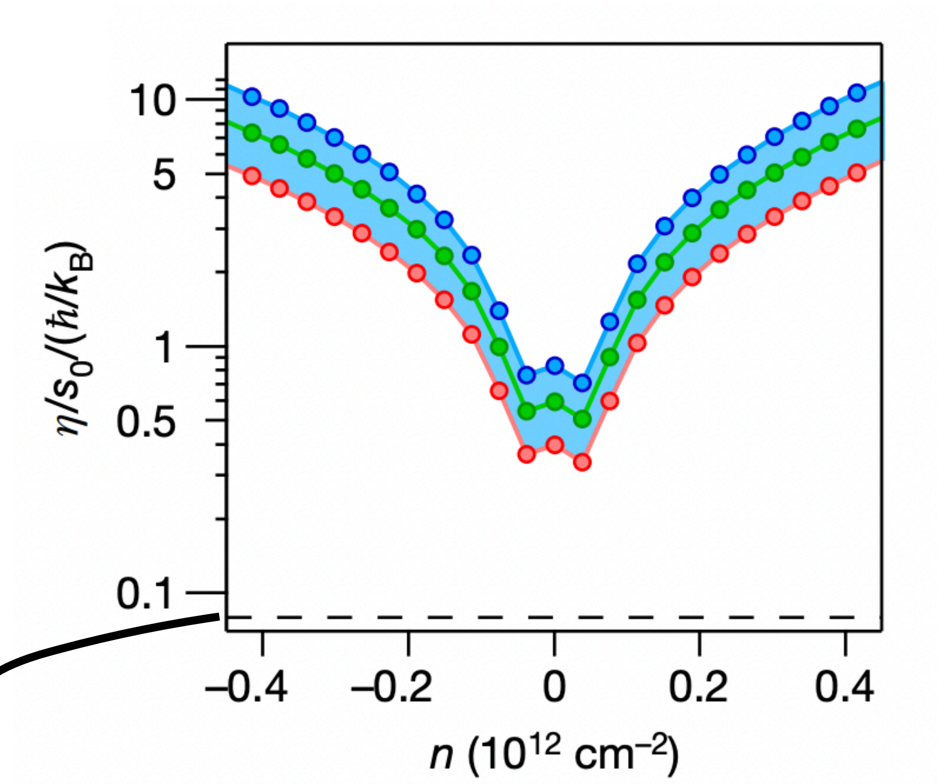
A hallmark of applied holography

Shear viscosity/entropy



Adams et al `12

graphene



Ku et al `19

Universal
holographic answer

Policastro, Son, Starinets `02
Kovtun, Son, Starinets `04

Holographic approach to real systems

I. **Proxies:** e.g. $\mathcal{N} = 4$ sYM for QCD, ABJM for non-Fermi liquids

$\Longrightarrow$ **how good is the proxy?**

Holographic approach to real systems

I. **Proxies:** e.g. $\mathcal{N} = 4$ sYM for QCD, ABJM for non-Fermi liquids

$\Longrightarrow$ **how good is the proxy?**

II. **Bottom-up models:** Einstein's Gravity + matter

e.g. QCD: Gravity + matter in 5D $\Longleftrightarrow$ $T_{\mu\nu}$, tr F^2, $\overline{q}q$, …
Fix by e.g. confinement, chiral symmetry breaking, gapped spectrum,…
However no consistent truncation of QCD to gravity $\Longrightarrow$ full world-sheet

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Fix by e.g. confinement, chiral symmetry breaking, gapped spectrum,…
However no consistent truncation of QCD to gravity $\Longrightarrow$ full world-sheet

$\Longrightarrow$ qualitative understanding, inspire ideas
reproduce by traditional methods ? e.g. hydrodynamics, toy QFT models

Outline

Part I: High Tc superconductors, linear in T resistivity

- Pseudo-Goldstone hydrodynamics
- SYK inspired microscopic models

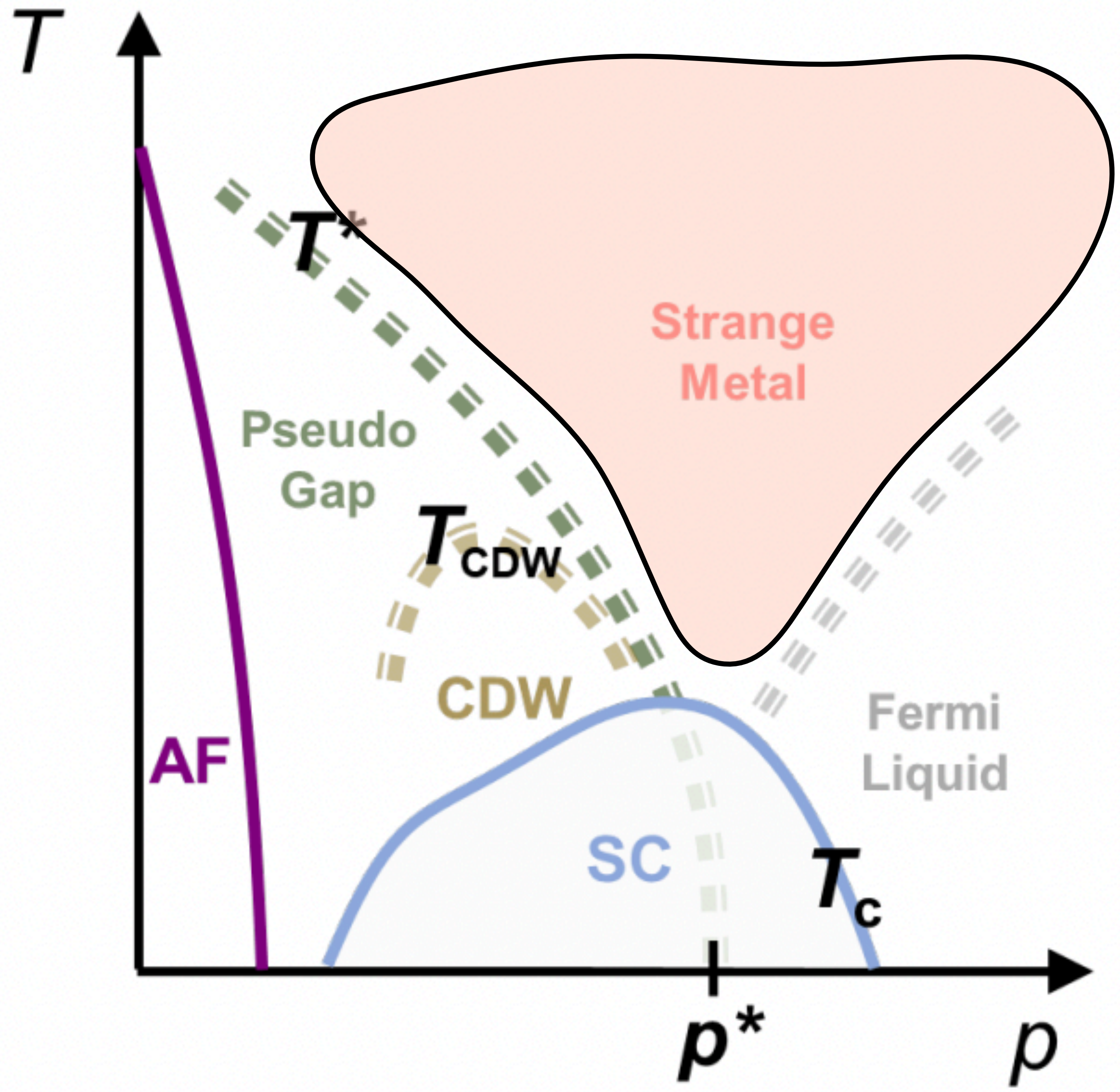
Part II: QCD

- $\mathcal{N} = 4$ sYM vs. QCD
- Spin hydrodynamics
- Magnetic fields and QCD

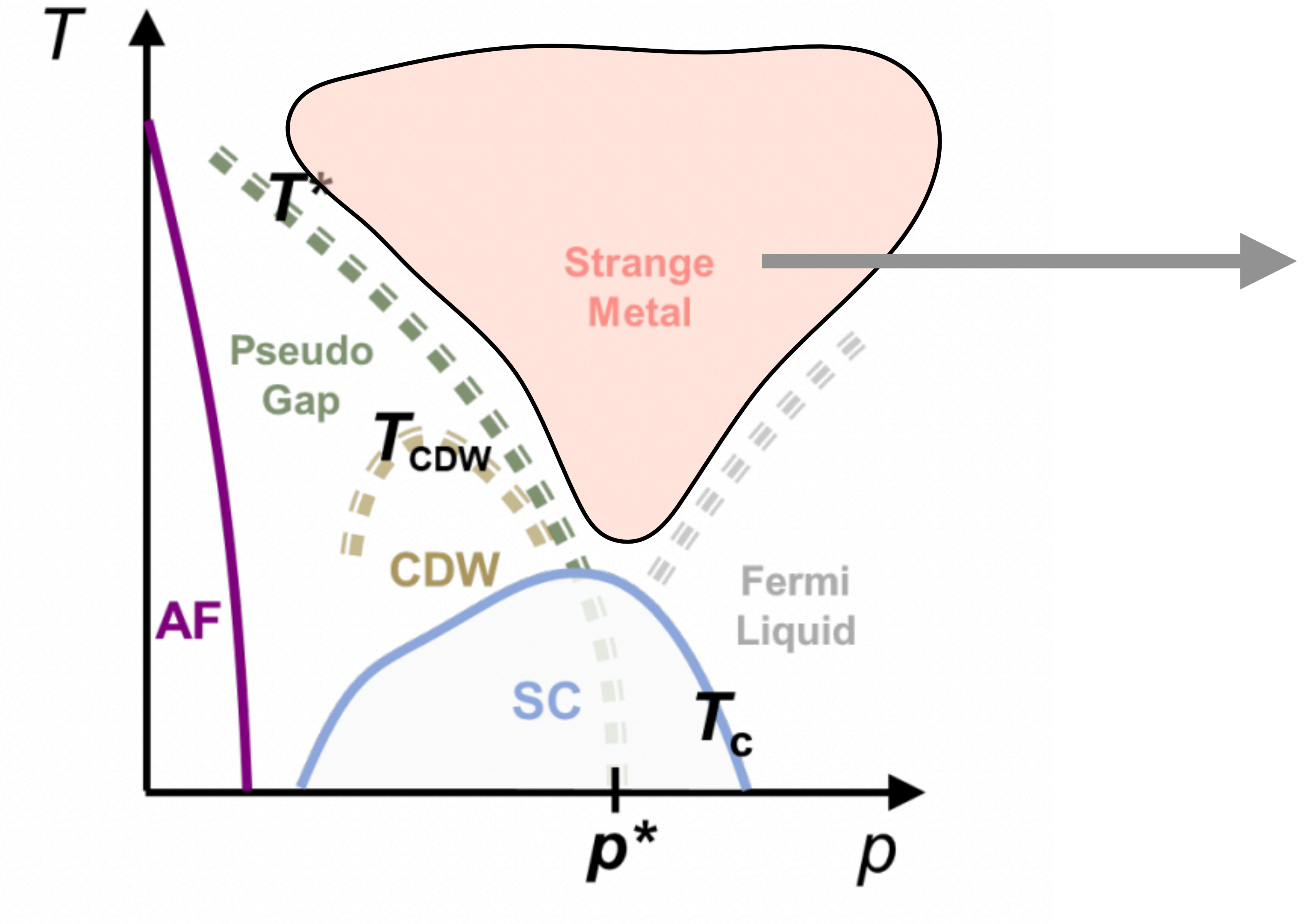
Part I: High Tc superconductors

Pseudo-Goldstone hydrodynamics

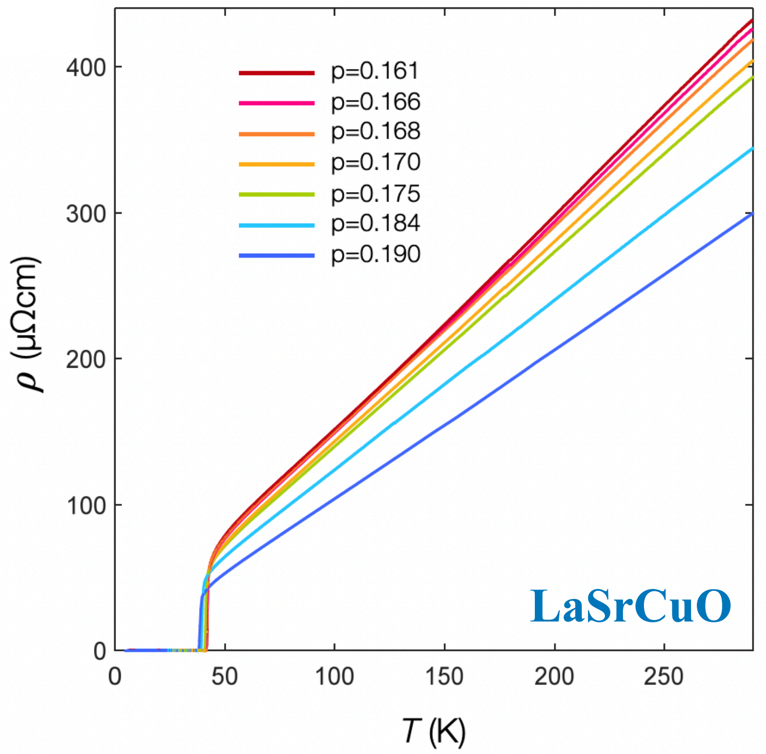
High Tc superconductors



High Tc superconductors

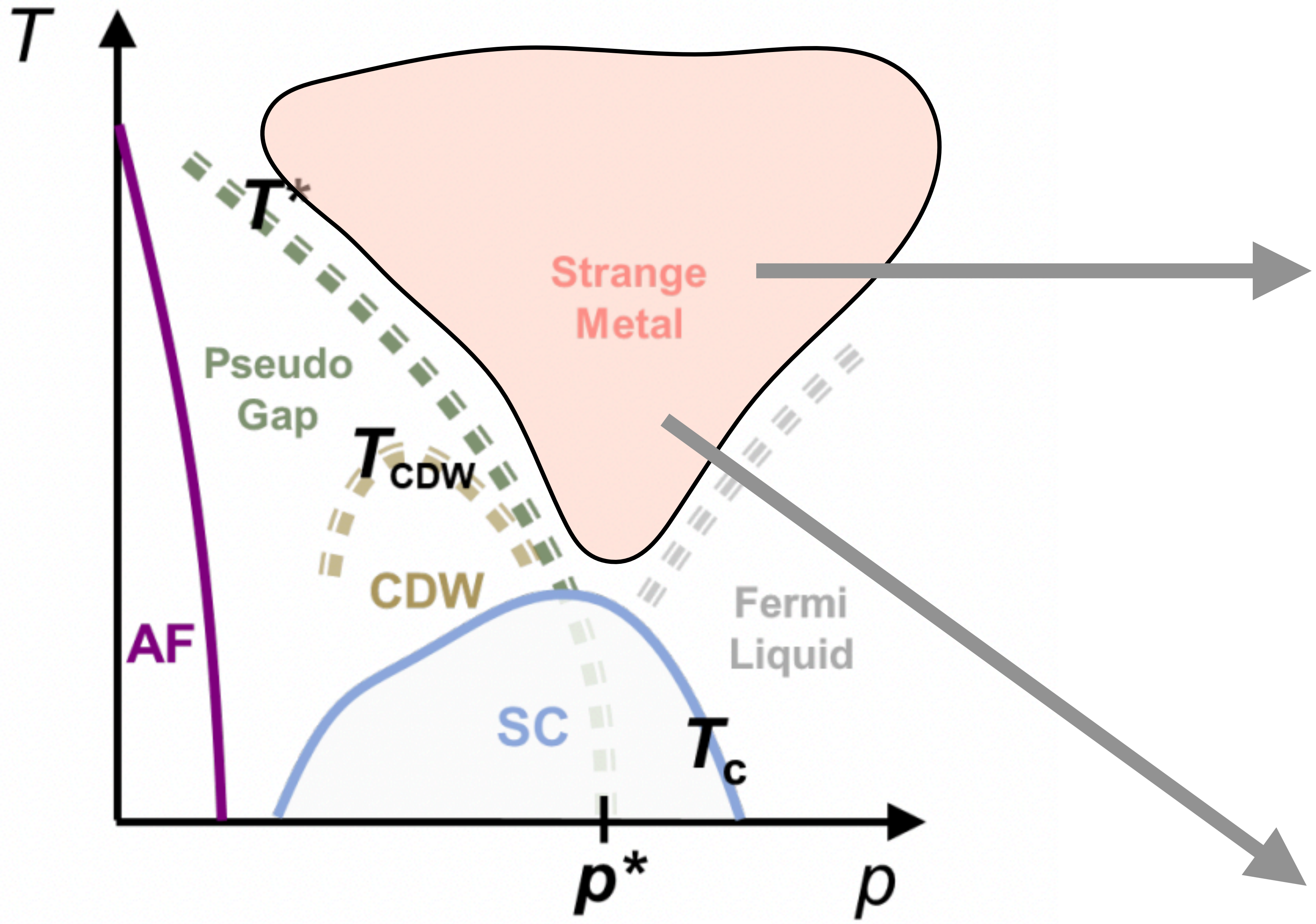


Giraldo-Galdo `18

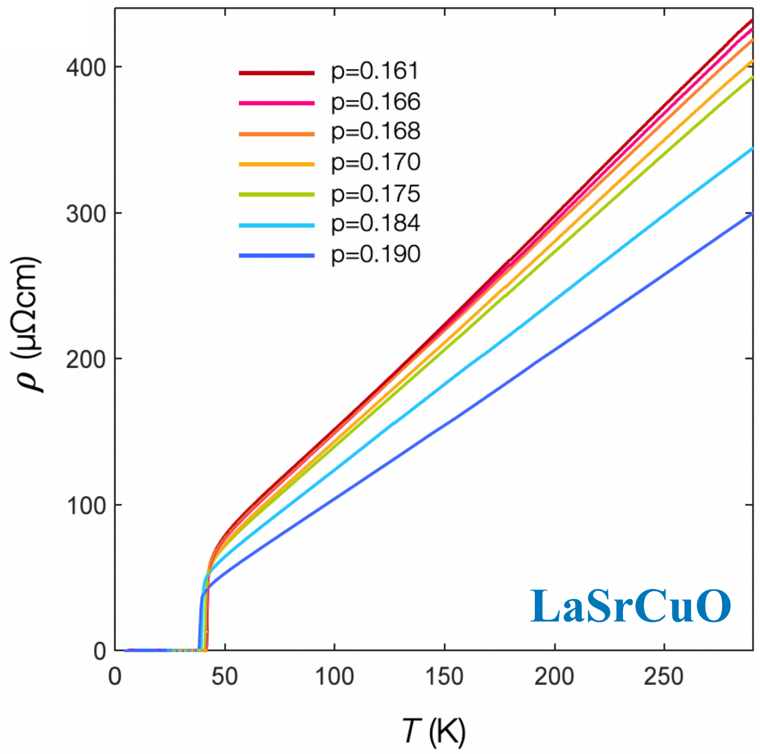


Linear in T resistivity

High Tc superconductors



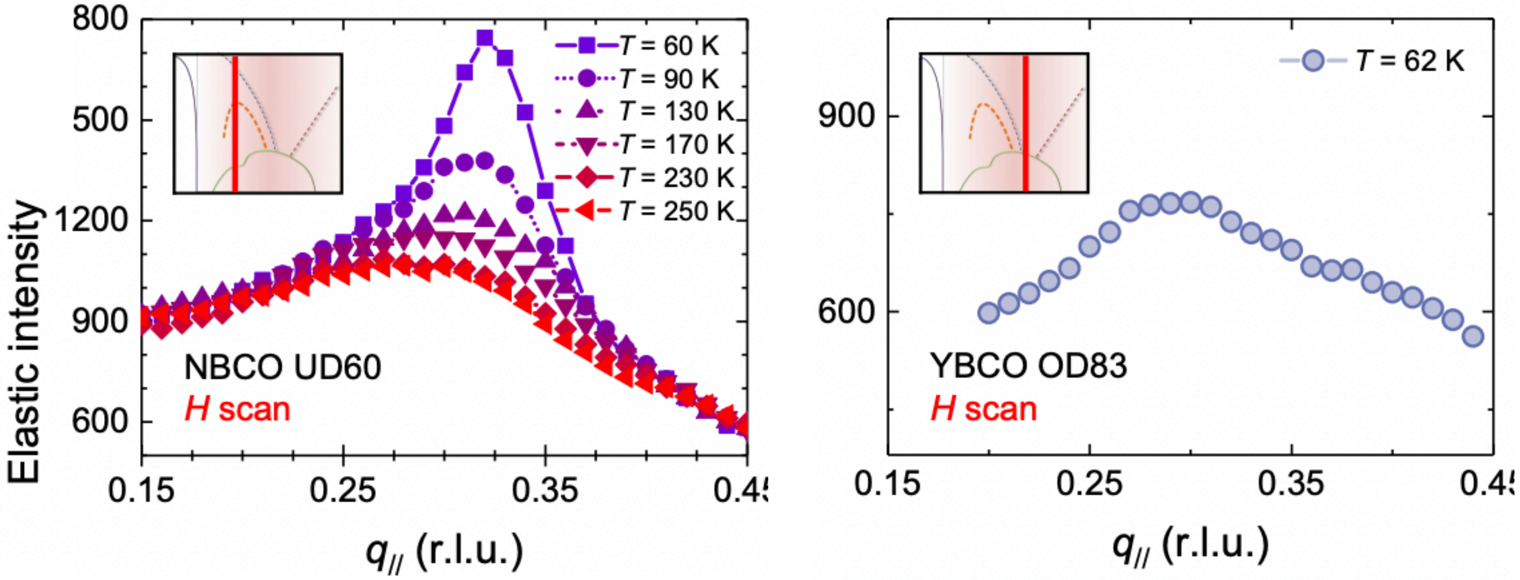
Giraldo-Galdo `18



Linear in T resistivity

Charge density fluctuations

Arpaia, Ghiringhelli `21



Charge density modulations

Spontaneous breaking of translations

$$\omega(k) = \pm c_s k - i D_\phi k^2$$

Charge density modulations

Spontaneous breaking of translations

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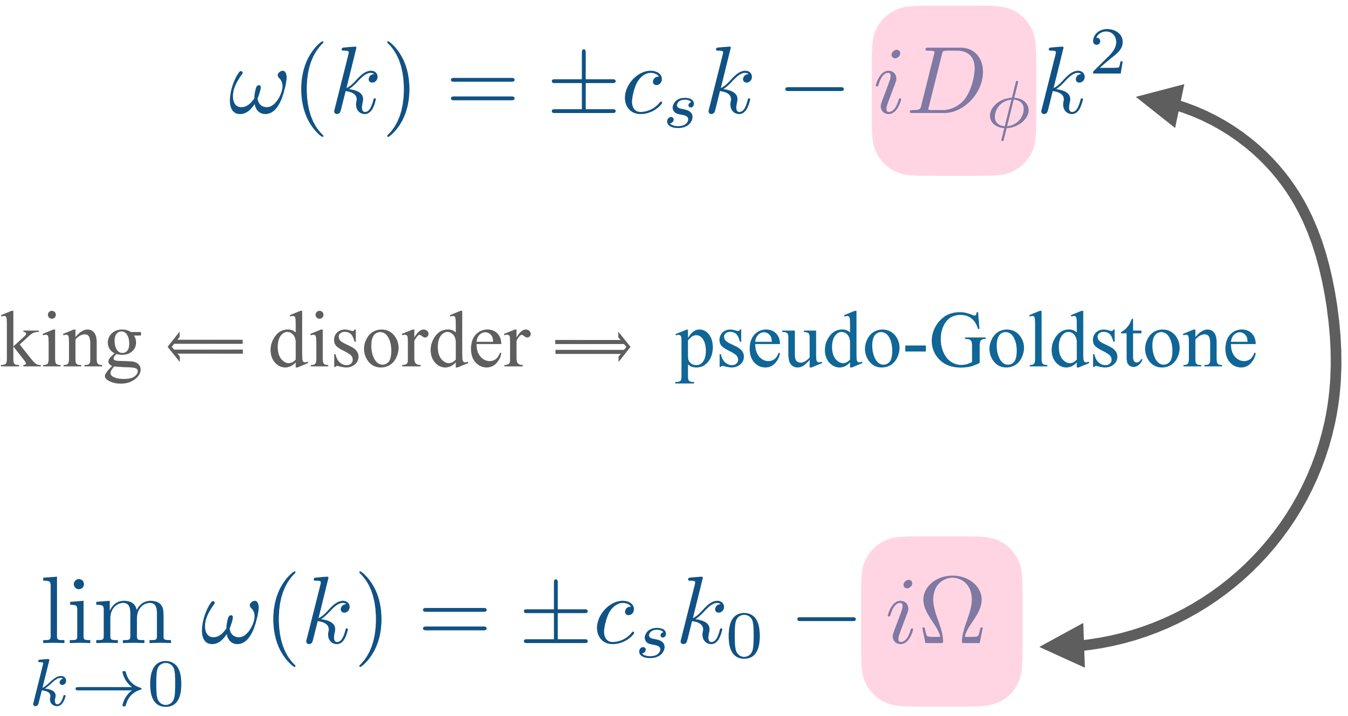
Weak explicit breaking $\Longleftarrow$ disorder $\Longrightarrow$ pseudo-Goldstone

$$\lim_{k\to 0} \omega(k) = \pm c_s k_0 - i\Omega$$

Charge density modulations

Spontaneous breaking of translations



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Weak explicit breaking ⟸ disorder ⟹ pseudo-Goldstone

$$\lim_{k\to 0} \omega(k) = \pm c_s k_0 - i\Omega$$

A curious relation between diffusion and relaxation from holography

Gauge-gravity with charge density waves

Models in 3+1D with explicit and spontaneous breaking of translations

Amadeo, Ammon, Amoretti, Andrade, Arean, Argurio, Baggioli, Domokos, Donos, Erdmenger, Gauntlett, Gouteraux, Greininger, Jarvinen, Jimenez-Alba, Jokela, Krikun, Li, Ling, Lippert, Martin, Meyer, Musso, Nakamura, Niu, Ooguri, Pando-Zayas, Pantelidou, Poovuttikul, Park, Schalm, Vegh, Xian, Wu, Zaenen, Ziogas, Zhang `09 - `22

Holographic Q-lattices **Donos, Gauntlett `13**

$$\mathcal{L} = R - \frac{1}{2}\partial\phi^2 - V(\phi) - \frac{1}{4}Z(\phi)F^2 - \frac{1}{2}\sum_{I=1}^{2} Y(\phi)\partial\psi_I^2$$

Goldstones

Explicit Spontaneous

$$\Phi_I = \phi\, e^{i\psi_I}\,, \qquad \psi_I = k x^I\,, \qquad \phi = k_0\, r + \phi_v r^2 + \mathcal{O}(r^3)$$

Weak explicit breaking $\frac{k_0}{\mu} \ll \frac{\phi_v}{\mu^2}$

$$G_{\phi\phi},\, G_{j\phi},\, G_{jj} \quad \Longrightarrow \quad \boxed{\Omega = k_0^2 D_\phi}$$

Amoretti, Arean, Gouteraux, Musso `18`19; Donos, Martin, Pantelidou, Ziogas `19; Baggioli, Gray, Grieninger `21

Locality in hydrodynamics

Gouteraux, Delacretaz, Ziogas `21

Hydrodynamic equations:

$$\dot{n}_a(k,t) + \underset{\text{local}}{M_{ab}(k)} n_b(k,t) = 0$$

Locality in hydrodynamics

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Hydrodynamic equations:

$$\dot{n}_a(k,t) + \underset{\text{local}}{M_{ab}(k)} n_b(k,t) = 0$$

In presence of external sources: $H_0 \to H_0 - \int \delta\mu_a(x,t) n_a(x,t)$

Kadanoff, Martin `63

$$\dot{n}_a(k,t) + \underbrace{M_{ab}(k)\,(n_b(k,t) - \chi_{bc}}_{\text{local}}(k)\delta\mu_c(k,t)) = 0$$

Susceptibility matrix $\chi_{ab} = -\frac{\delta^2 W}{\delta\mu_a \delta\mu_b}$ local beyond ξ_{th}

Typically $\xi_{th} < \Delta L$ except for (pseudo) Goldstones or near criticality

Pseudo-Goldstone hydrodynamics

Delacretaz, Gouteraux, Hartnoll, Karlsson `21, Gouteraux, Delacretaz, Ziogas `21;
Ammon, Arean, Baggioli, Gray, Grieninger `21

Superfluid as example:

$$e^{-\beta W} = \int \mathcal{D}\phi \; e^{-\beta F[\delta\mu_n, \delta\mu_\phi, \phi]}$$

Weak explicit breaking:

$$F = \int \frac{1}{2}\left(\nabla\phi^2 + k_0^2\phi^2\right) - \delta\mu_\phi\phi + \cdots$$

Susceptibility: $$\chi_{\phi\phi} = \frac{1}{k^2 + k_0^2}$$

Pseudo-Goldstone hydrodynamics

Delacretaz, Gouteraux, Hartnoll, Karlsson `21, Gouteraux, Delacretaz, Ziogas `21;
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Susceptibility: $$\chi_{\phi\phi} = \frac{1}{k^2 + k_0^2}$$

Josephson relation: $$\dot{\phi} = -\Omega\phi + D_\phi \nabla^2 \phi + \cdots$$

$$\implies \qquad M_{\phi\phi}\chi_{\phi\phi} = \frac{\Omega + D_\phi k^2}{k^2 + k_0^2} \dashrightarrow \boxed{\Omega = k_0^2 D_\phi}$$

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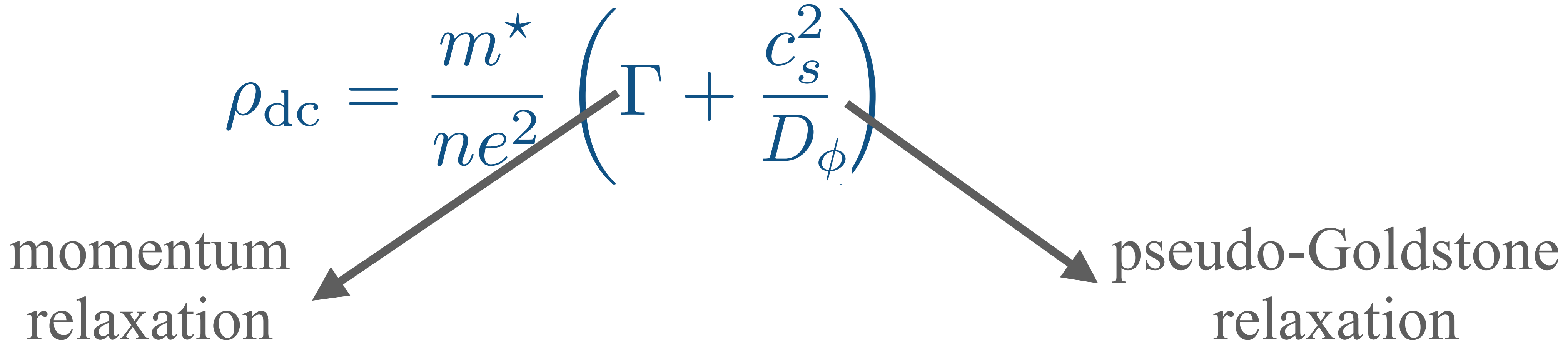
- Can be proved also using Schwinger-Keldish formalism, …

Implications for high Tc superconductors

Spontaneous + weak explicit breaking of translations:



$$\rho_{\mathrm{dc}} = \frac{m^\star}{ne^2}\left(\Gamma + \frac{c_s^2}{D_\phi}\right)$$

momentum
relaxation

pseudo-Goldstone
relaxation

Implications for high Tc superconductors

Spontaneous + weak explicit breaking of translations:

$$\rho_{\rm dc} = \frac{m^\star}{ne^2}\left(\underbrace{\Gamma}_{\text{momentum relaxation}} + \underbrace{\frac{c_s^2}{D_\phi}}_{\text{pseudo-Goldstone relaxation}}\right)$$

Planckian diffusivity bounds: $D_\phi \approx \frac{\hbar}{k_B T} c_s^2$

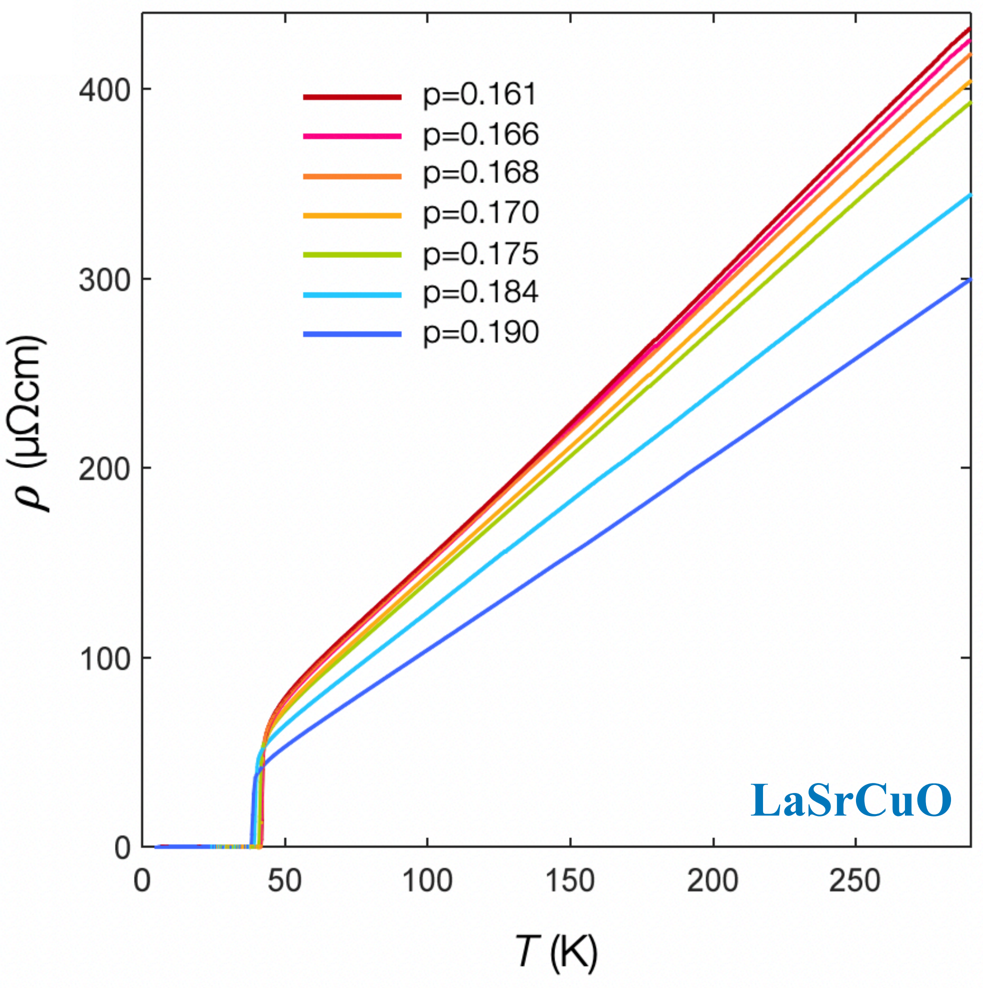
Zaanen `04; Hartnoll `14
Hartman, Hartnoll, Mahajan `17

Gouteraux, Delacretaz, Ziogas `21;

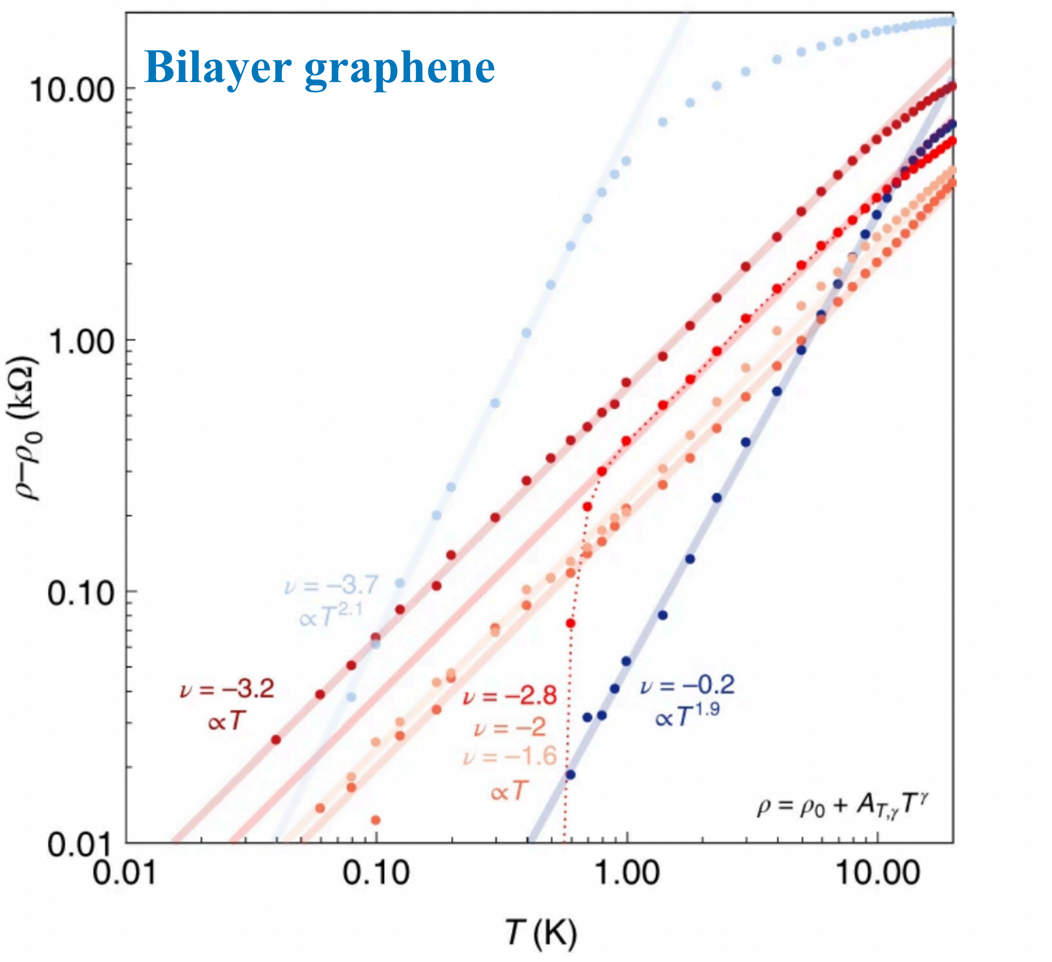
$$\boxed{\rho_{\rm dc} \approx \frac{m^\star}{ne^2}\left(\Gamma + \frac{k_B T}{\hbar}\right)}$$

SYK-Yukawa model

Linear in T resistivity: microscopics



Girallo-Gallo et al `18



Jaoui et al `22

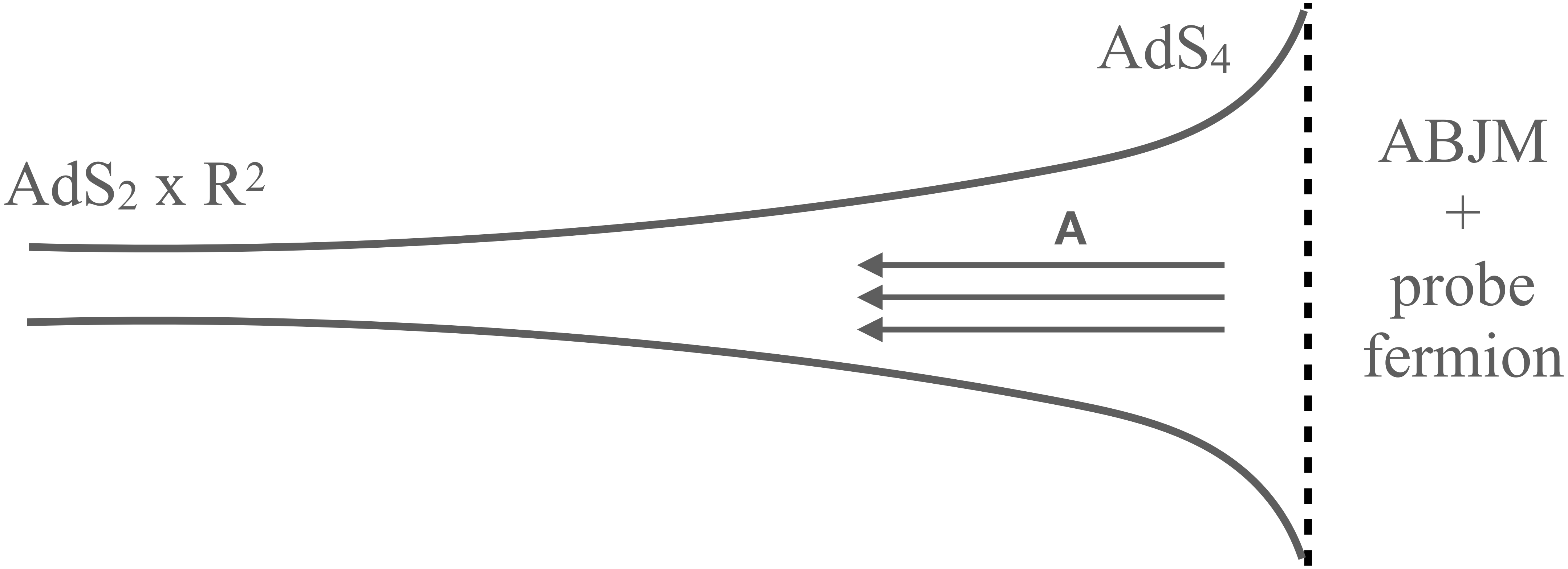
SYK-type model based on quantum criticality

Patel, Guo, Esterlis, Sachdev `22

Generic description: fermi surface + gapless scalar

Chubukov, Kachru, P. A. Lee, S. S. Lee, McGreevy, Metlitski, Raghu, Sachdev, Senthil, Torroba, … `89 - `22

Holographic non-Fermi liquids

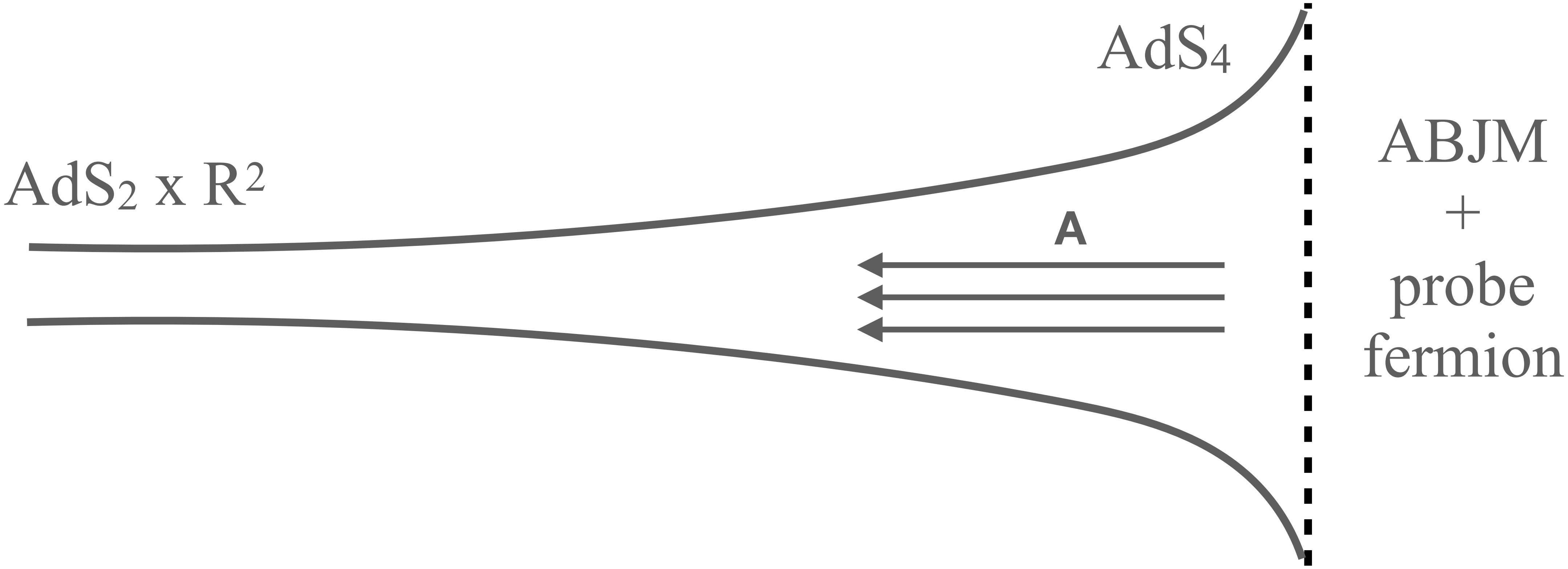


$$G_f = \frac{1}{i\omega + \epsilon(k) + ig|\omega|^{\nu(k)}} \longrightarrow$$

Probe fermion
+ AdS$_2$

Faulkner, Liu, McGreevy, Vegh; Cubrovic, Schalm, Zaanen `09

Holographic non-Fermi liquids



$$G_f = \frac{1}{i\omega + \epsilon(k) + ig|\omega|^{\nu(k)}} \longrightarrow$$ Probe fermion + AdS$_2$

Faulkner, Liu, McGreevy, Vegh; Cubrovic, Schalm, Zaanen '09

AdS$_2$ $\Longrightarrow$ revival of SYK (fermions scattering off SYK)

Linear resistivity: microscopics

SYK-inspired 2D quantum critical "clean" metal at large N

Patel, Guo, Esterlis, Sachdev `21

Fermi surface

$$\mathcal{S}_g = \int d\tau \sum_{\boldsymbol{k}} \sum_{i=1}^{N} \psi^\dagger_{i\boldsymbol{k}}(\tau) \left[\partial_\tau + \varepsilon(\boldsymbol{k})\right] \psi_{i\boldsymbol{k}}(\tau)$$

$$+ \frac{1}{2} \int d\tau \sum_{\boldsymbol{q}} \sum_{i=1}^{N} \phi_{i\boldsymbol{q}}(\tau) \left[-\partial_\tau^2 + K\boldsymbol{q}^2 + m_b^2\right] \phi_{i,-\boldsymbol{q}}(\tau)$$

order parameter

$$+ \frac{g_{ijl}}{N} \int d\tau d^2 r \sum_{i,j,l=1}^{N} \psi_i^\dagger(\boldsymbol{r},\tau)\psi_j(\boldsymbol{r},\tau)\phi_l(\boldsymbol{r},\tau)$$

Randomness in flavor $\Longrightarrow$ systematic expansion in 1/N

$$\overline{g_{ijl}} = 0\,, \quad \overline{g^*_{ijl} g_{abc}} = g^2\, \delta_{ia}\delta_{jb}\delta_{lc}$$

Momentum conservation $\Longrightarrow$ Drude peak without relaxation

$$\mathrm{Re}[\sigma(\omega)] \sim N\delta(\omega)$$

Linear resistivity: microscopics

Patel, Guo, Esterlis, Sachdev `22

Spatially random potential

$$\mathcal{S}_v = \frac{1}{\sqrt{N}} \int d^2 r d\tau \, v_{ij}(\boldsymbol{r}) \psi_i^\dagger(\boldsymbol{r},\tau) \psi_j(\boldsymbol{r},\tau)$$

$$\overline{v_{ij}(\boldsymbol{r})} = 0 \, , \quad \overline{v_{ij}^*(\boldsymbol{r}) v_{lm}(\boldsymbol{r}')} = v^2 \, \delta(\boldsymbol{r} - \boldsymbol{r}') \delta_{il} \delta_{jm}$$

$\Longrightarrow$ constant DC conductivity determined by e$^-$e$^-$ scattering rate $\sim v^2$

Linear resistivity: microscopics

Patel, Guo, Esterlis, Sachdev `22

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$\Longrightarrow$ constant DC conductivity determined by e-e- scattering rate $\sim v^2$

Spatially random interaction

$$\mathcal{S}_{g'} = \frac{1}{N} \int d^2r d\tau \, {g'}_{ijl}(\boldsymbol{r}) \psi_i^\dagger(\boldsymbol{r},\tau) \psi_j(\boldsymbol{r},\tau) \phi_l(\boldsymbol{r},\tau)$$

$$\overline{{g'}_{ijl}(\boldsymbol{r})} = 0\,, \quad \overline{{g'}^*_{ijl}(\boldsymbol{r}) {g'}_{abc}(\boldsymbol{r}')} = {g'}^2 \, \delta(\boldsymbol{r}-\boldsymbol{r}') \delta_{ia} \delta_{jb} \delta_{lc}$$

Linear resistivity: microscopics

Patel, Guo, Esterlis, Sachdev `22

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$\Longrightarrow$ conductivity with non-trivial relaxation

Conductivity $\mathrm{Re}\,\sigma(\omega) \sim \dfrac{1}{v^2 + {g'}^2\omega}$

$\omega \ll T$

Linear resistivity $\rho \propto T$

Discussion

Pseudo-Goldstone hydrodynamics:

Relies on diffusivity bounds D ~ 1/T
Quantum criticality?
Does it apply to realistic materials?

SYK-Yukawa model:

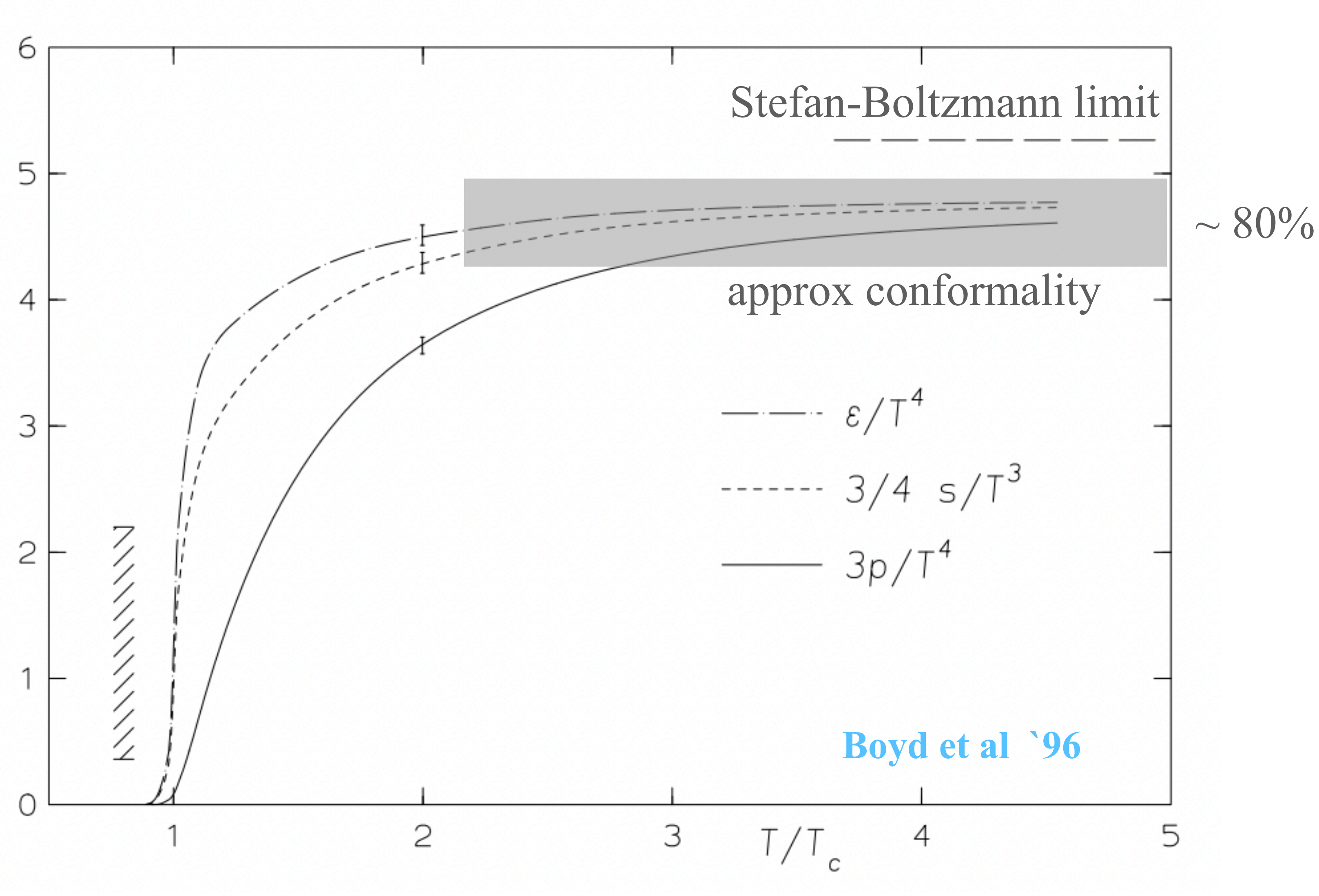
Specific to large N
Charge density waves/fluctuations?
Does it apply to realistic materials?

Common to both: Planckian dissipation, disorder,
long range correlations

Part II: QCD

$\mathcal{N} = 4$ sYM vs. QCD

Approximately conformal EoS



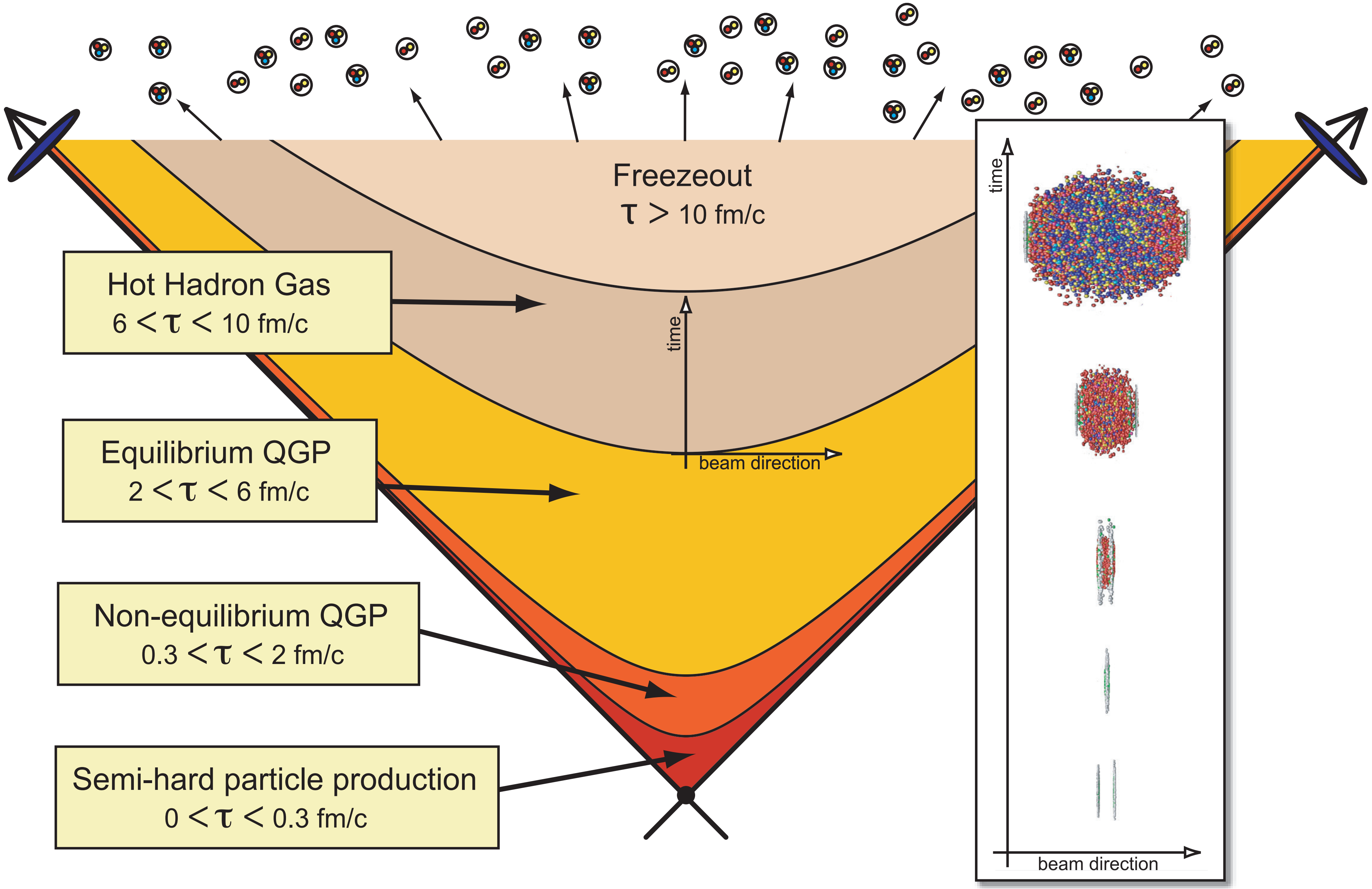
Boyd et al `96

$\mathcal{N} = 4$ super Yang-Mills: s(λ = ∞) / s(λ = 0) = 75%

Gubser, Klebanov, Peet `96; Klebanov `00

⟹ a good proxy for T > 2T$_c$

Quark gluon plasma



Freezeout

$\tau > 10$ fm/c

Hot Hadron Gas

$6 < \tau < 10$ fm/c

Equilibrium QGP

$2 < \tau < 6$ fm/c

Non-equilibrium QGP

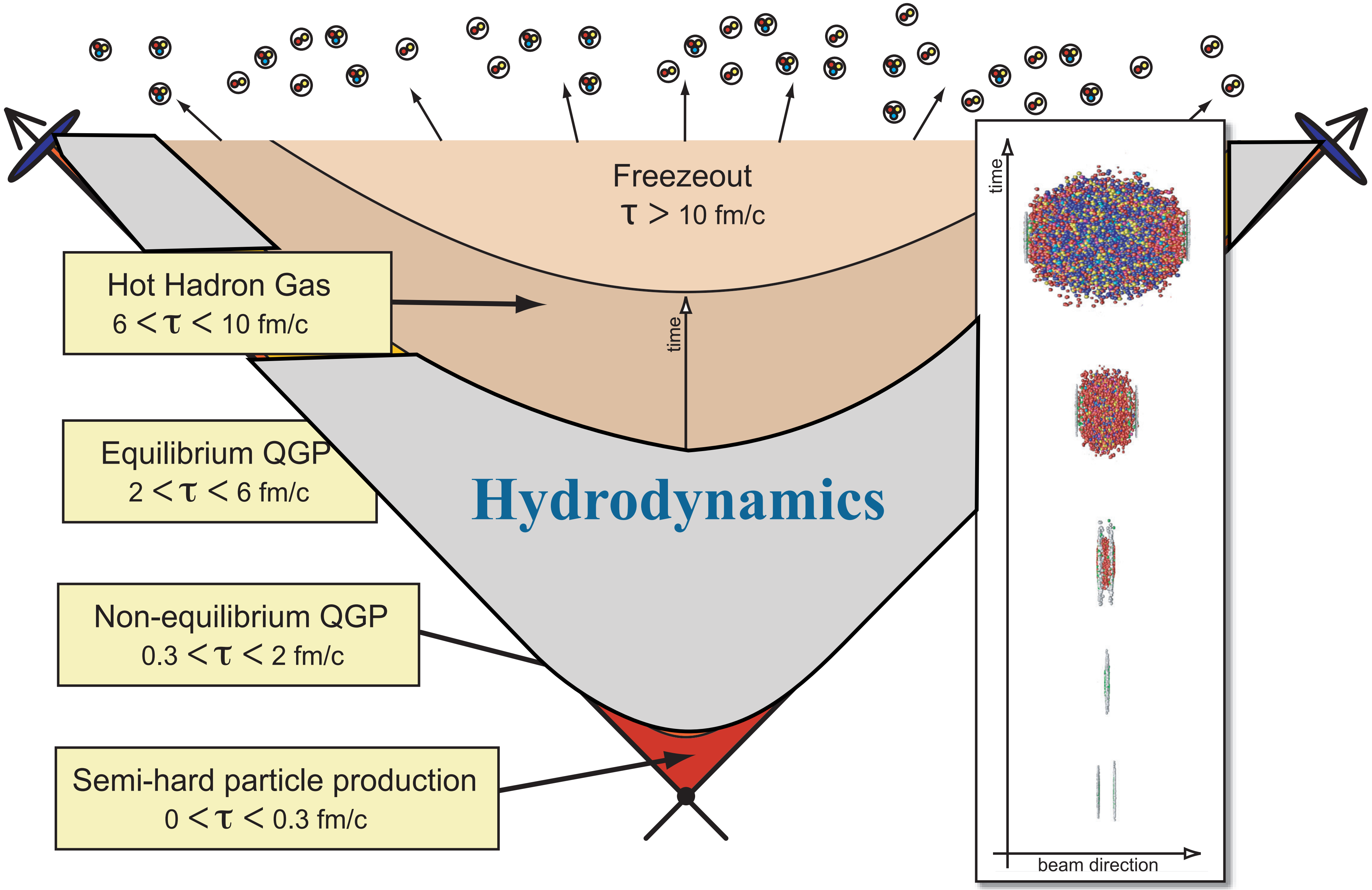
$0.3 < \tau < 2$ fm/c

Semi-hard particle production

$0 < \tau < 0.3$ fm/c

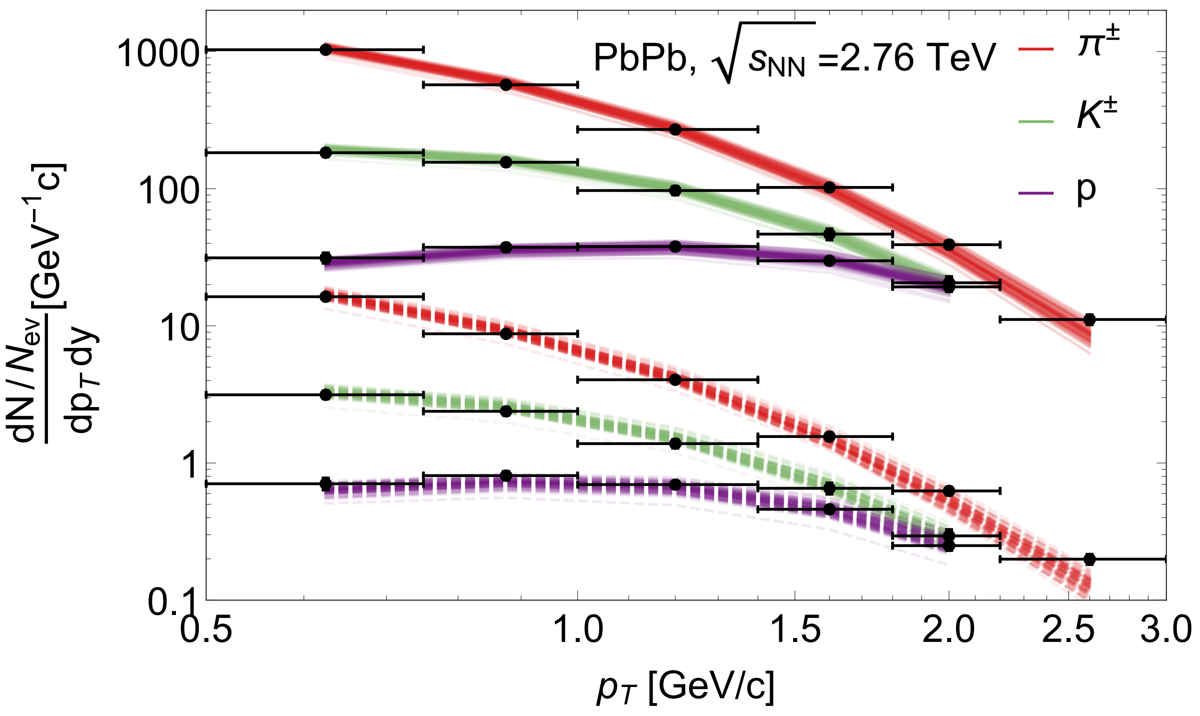
Heavy ion collisions @ RHIC, LHC

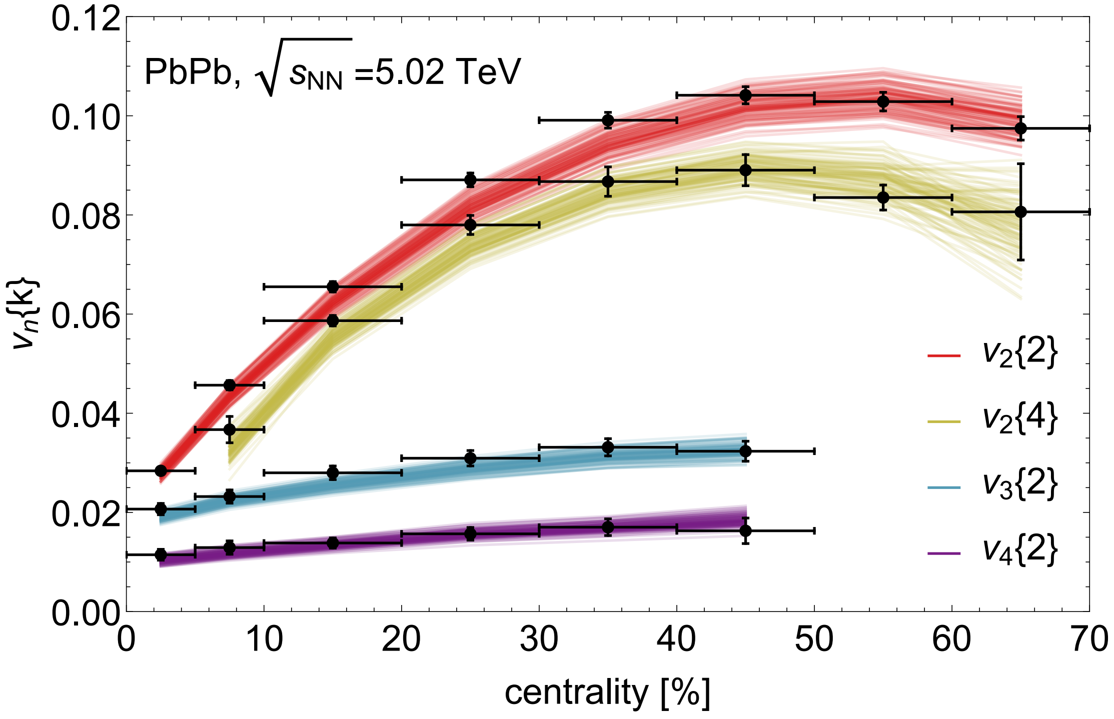
Quark gluon plasma



Heavy ion collisions @ RHIC, LHC

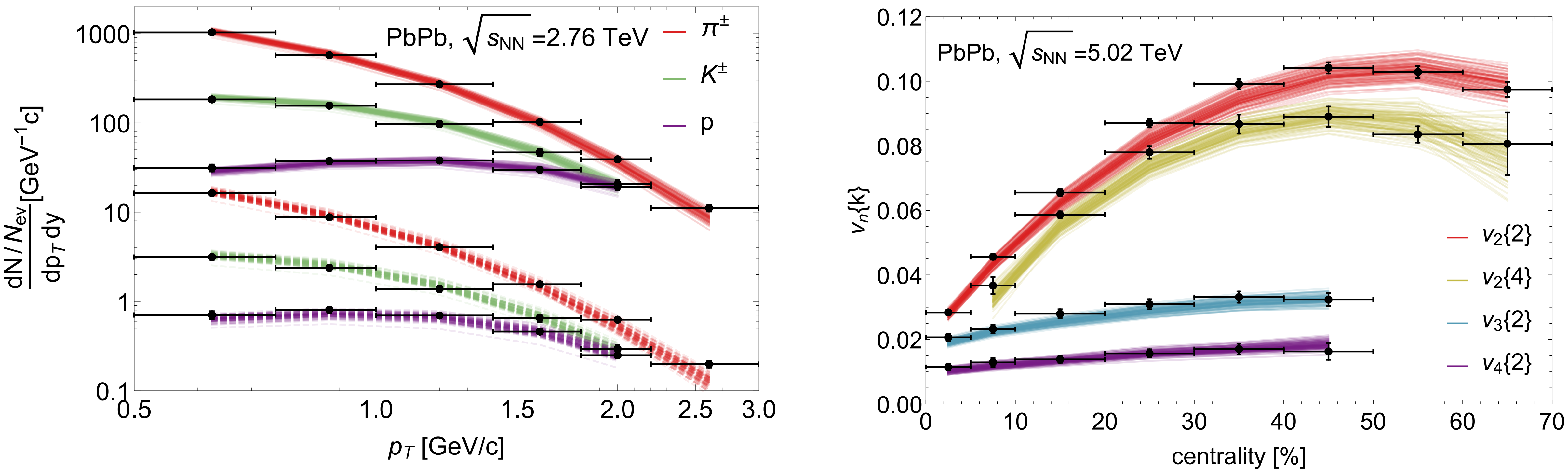
Transport from Bayesian analysis



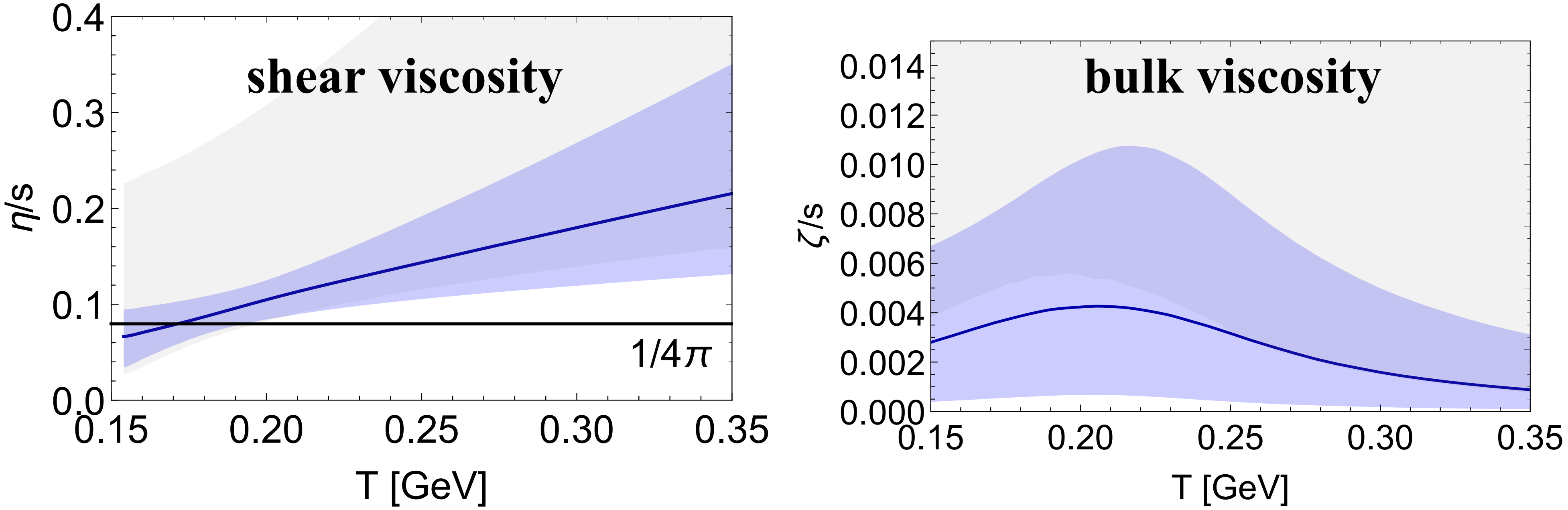


Nijs, van der Schee, Snellings, UG `20

Transport from Bayesian analysis



Nijs, van der Schee, Snellings, UG `20



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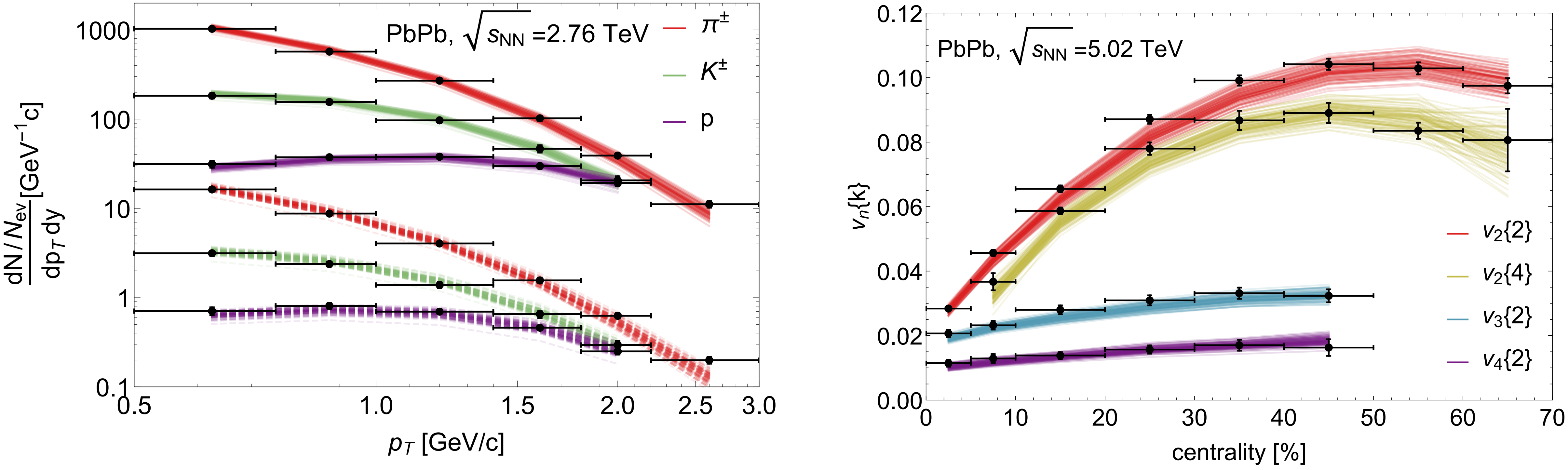

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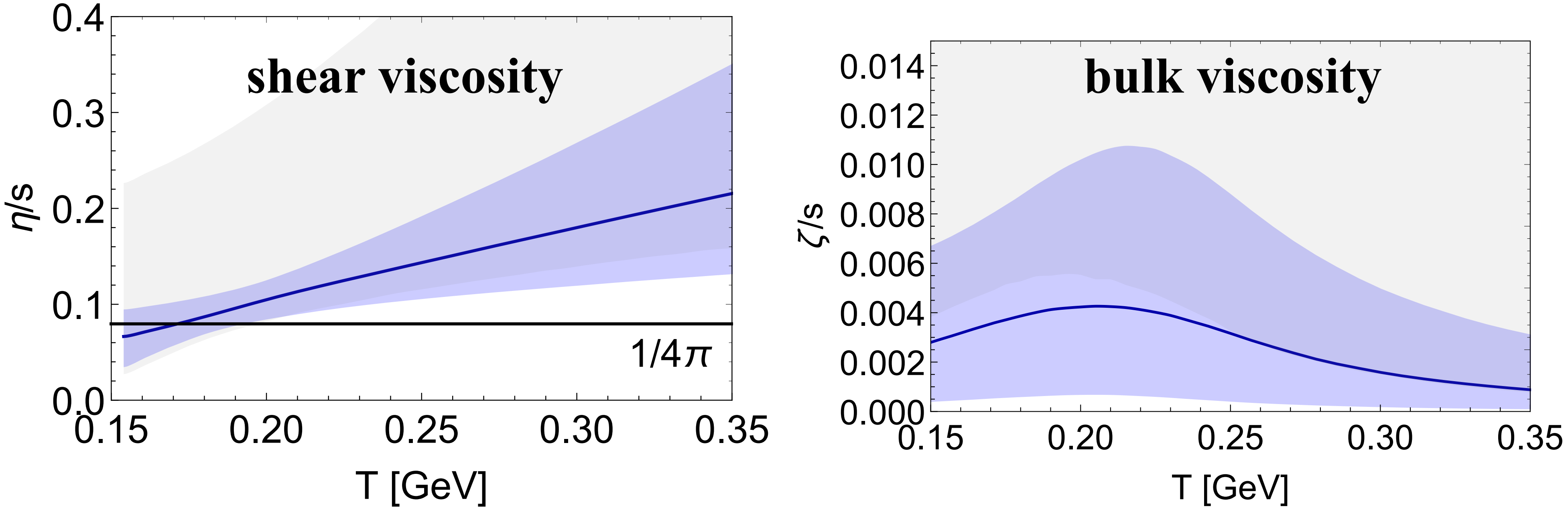
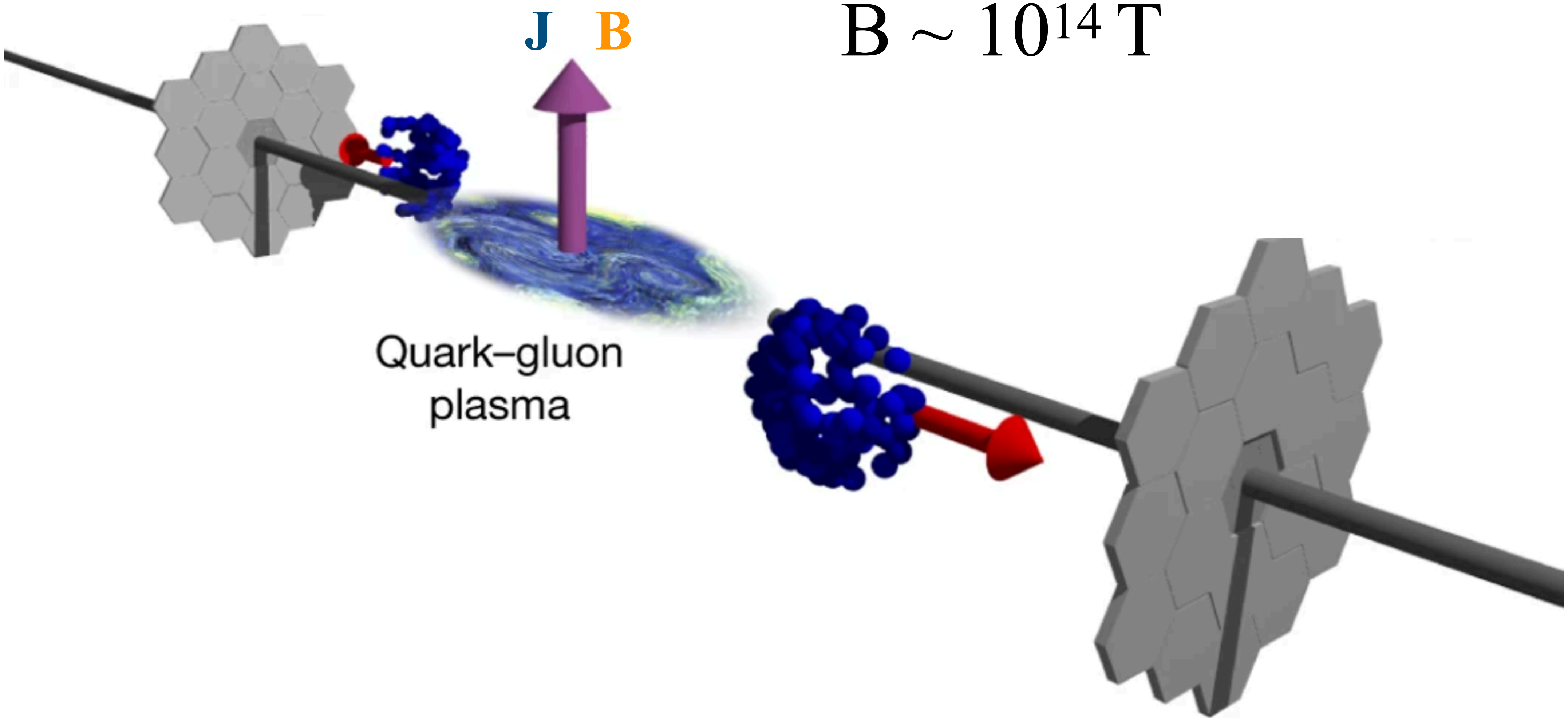


$\mathcal{N} = 4$ sYM at $\lambda = \infty$ is a very good proxy for $T \gtrsim 2T_c$ QCD !

Spin hydrodynamics

$J \sim 1000\hbar$

$B \sim 10^{14}\ \mathrm{T}$



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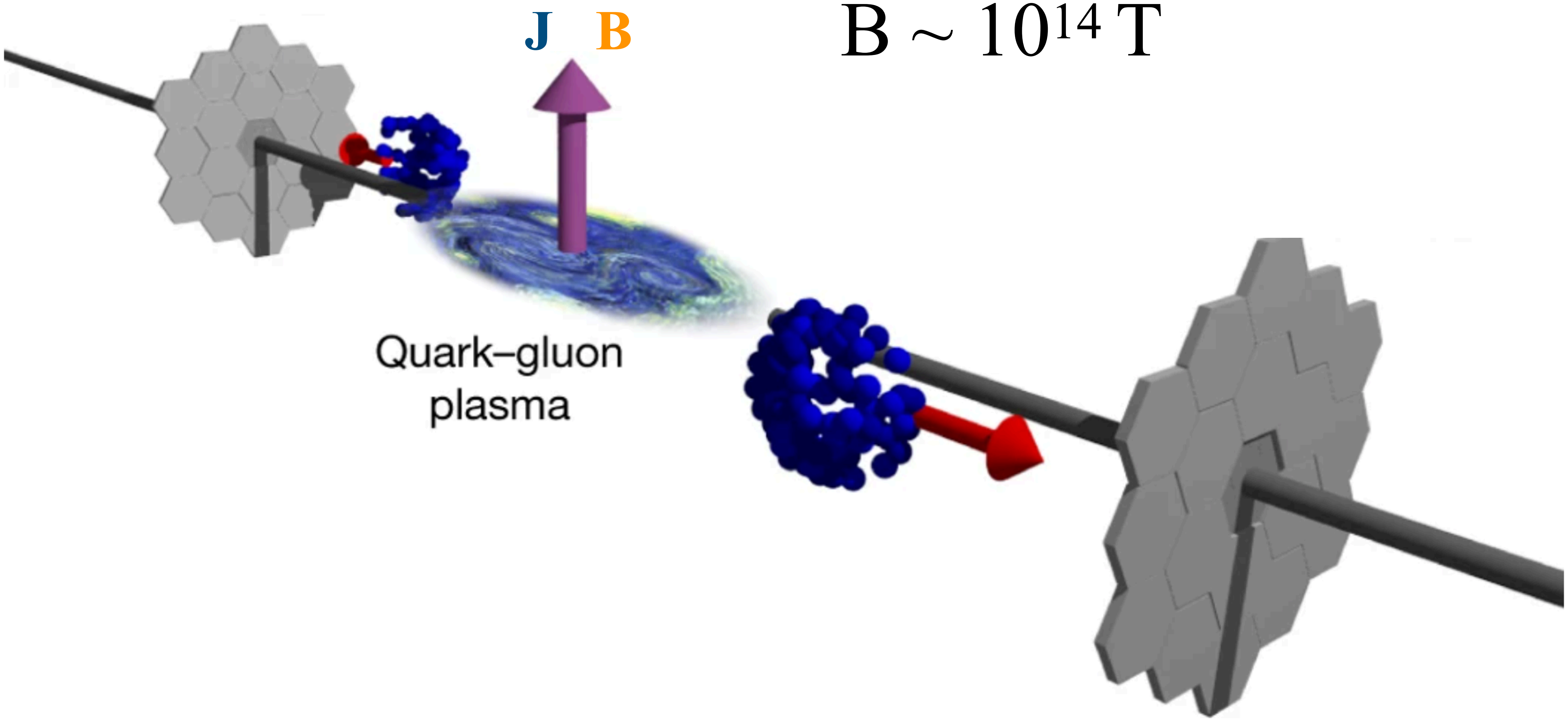
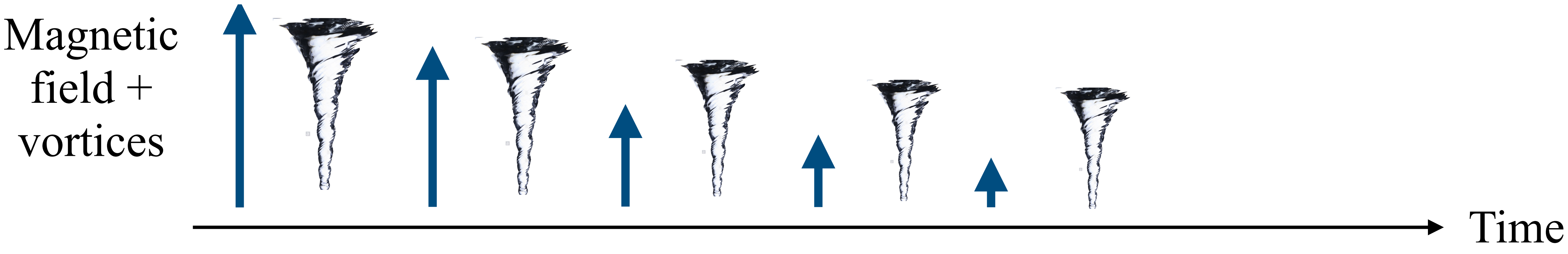
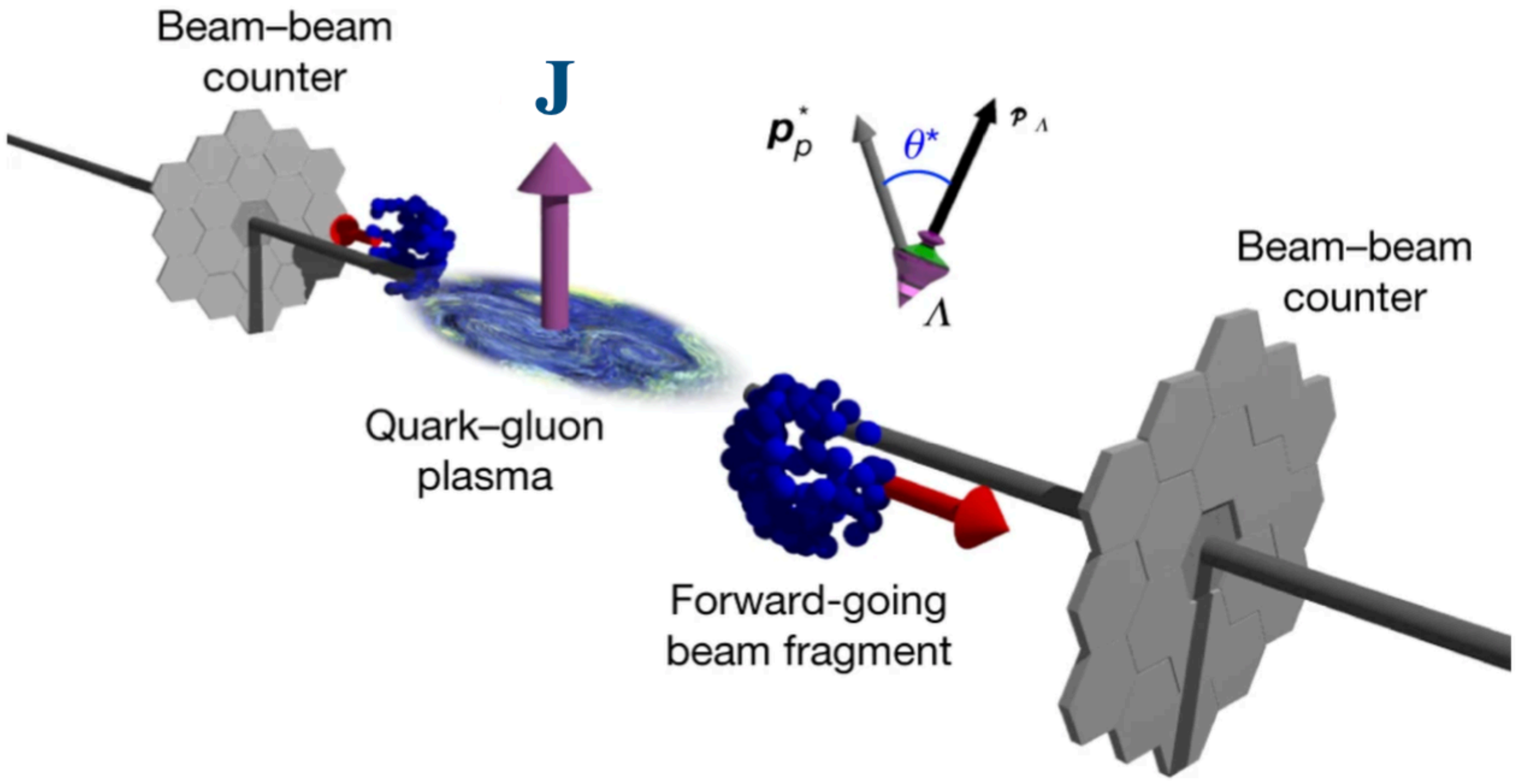
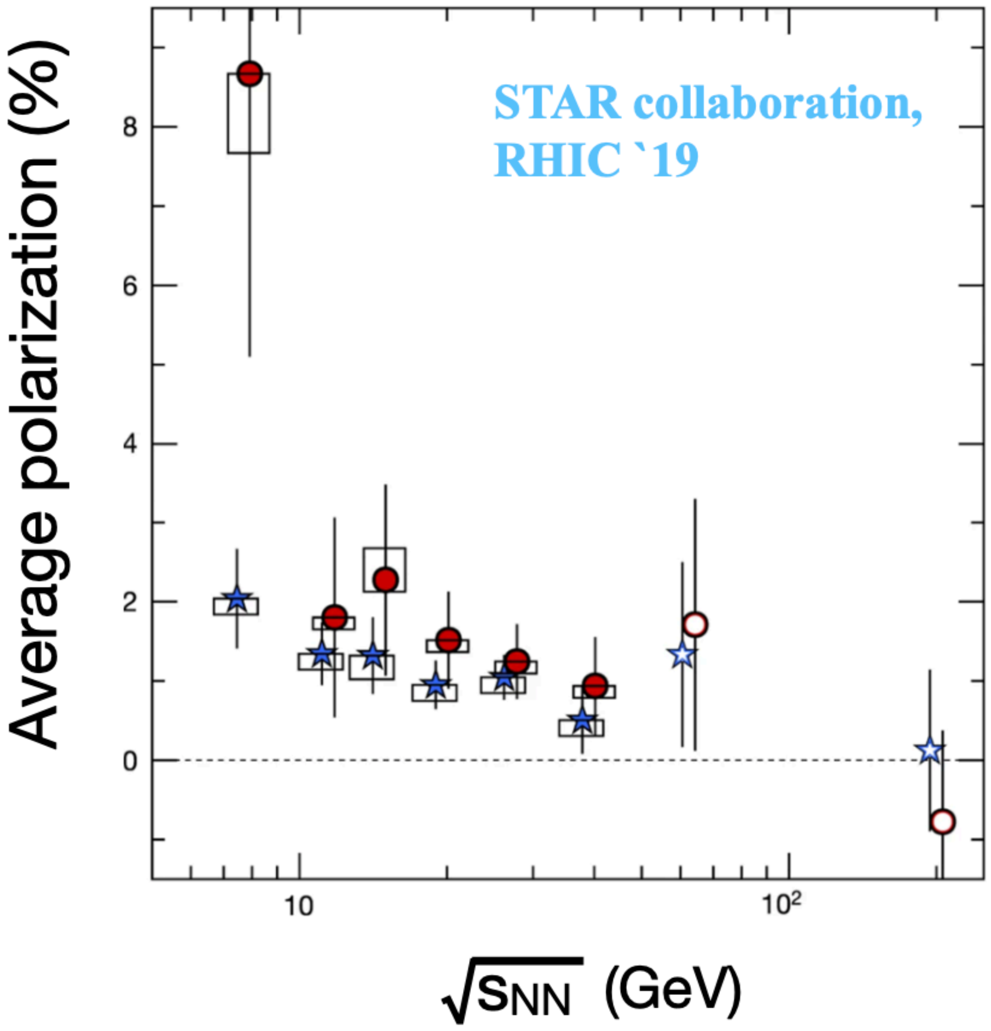




$\Longrightarrow$ relativistic spin hydrodynamics

Global spin polarization in QGP

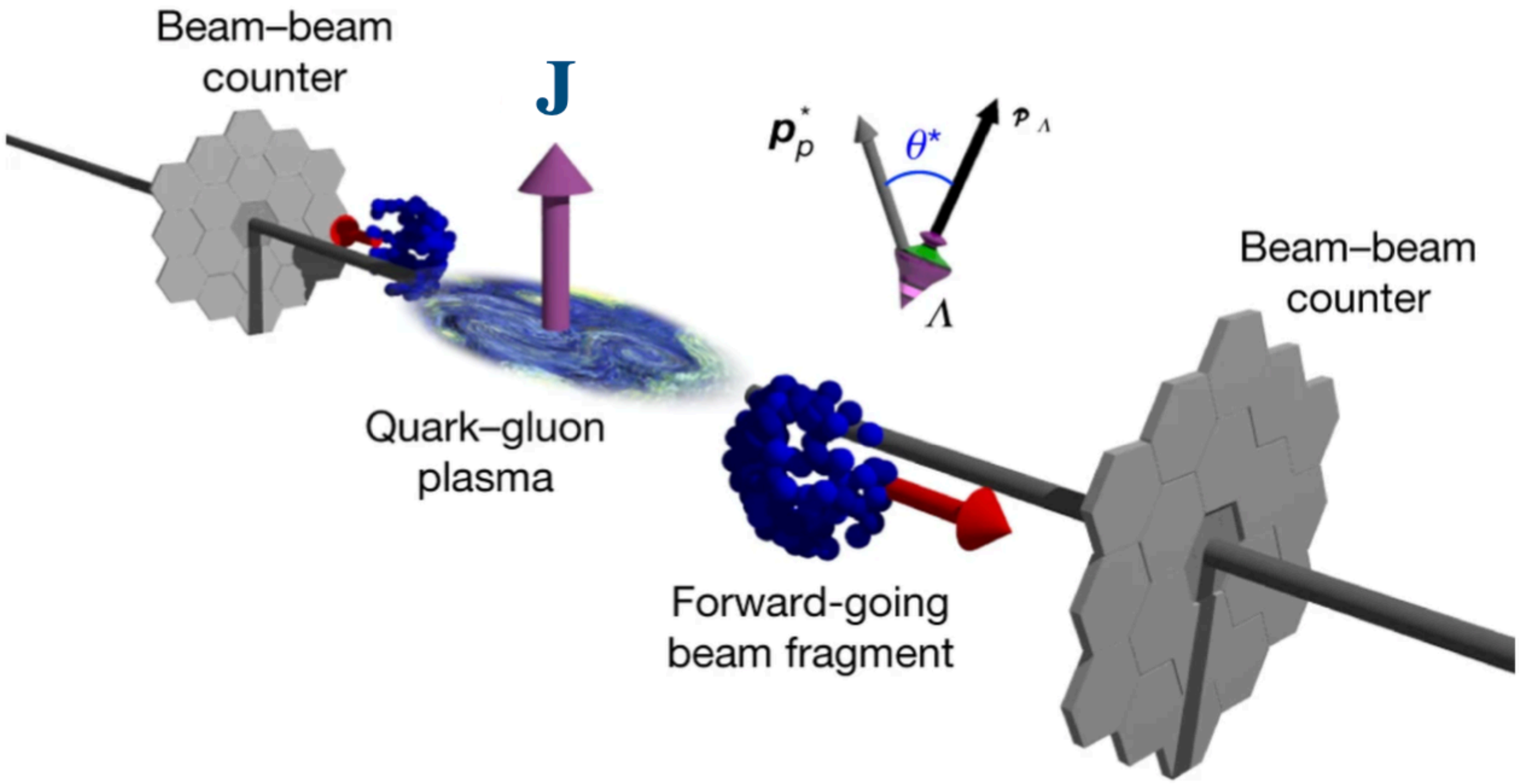


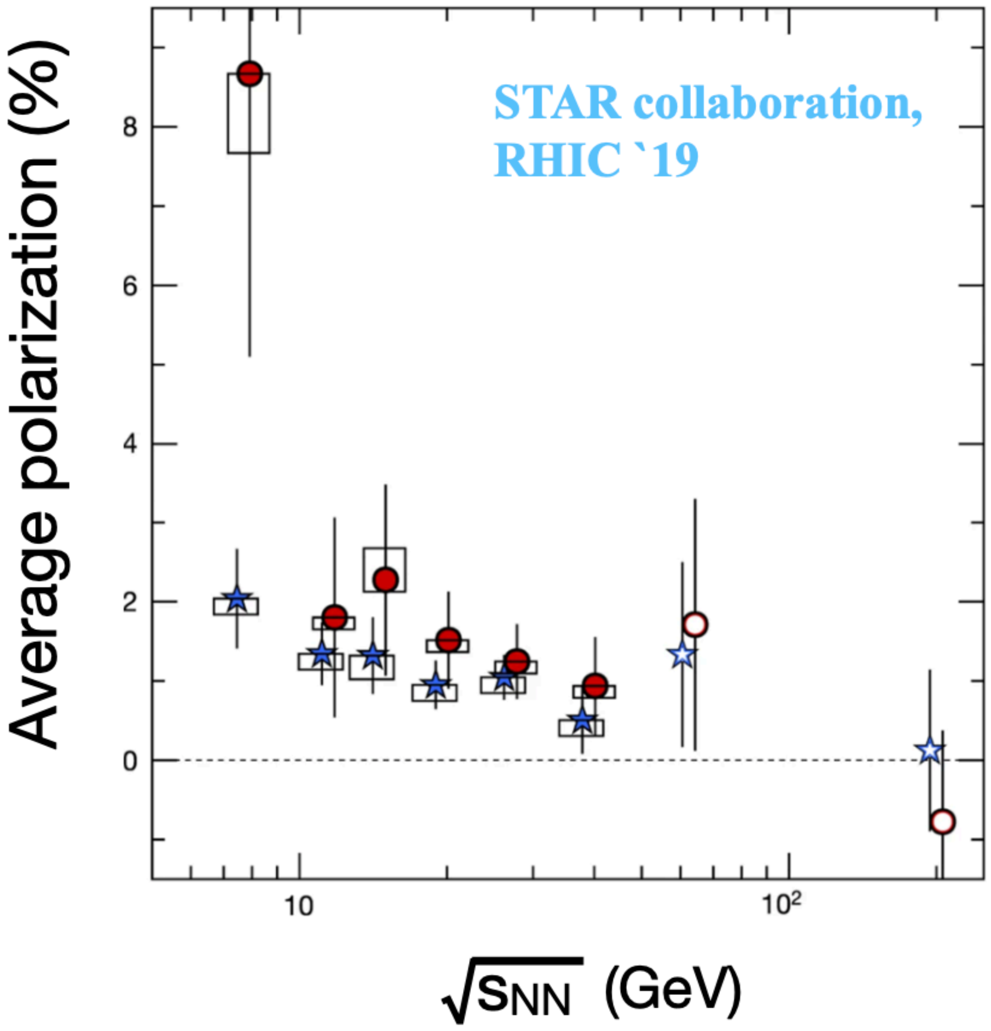


STAR collaboration,
RHIC `19

Global hyperon polarization at RHIC resulting from $\vec{S} \cdot \vec{J}$

Global spin polarization in QGP




STAR collaboration, RHIC `19

Global hyperon polarization at RHIC resulting from $\vec{S} \cdot \vec{J}$

$\Longrightarrow$ holographic/hydrodynamic description?

QFT with spin current

Belinfante-Rosenberg ambiguity:

Total angular momentum

$$J^{\lambda\mu\nu} = \overbrace{x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}}^{\text{orbital}} + \overbrace{S^{\lambda\mu\nu}}^{\text{spin}}$$

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0\,, \qquad \partial_\lambda J^{\lambda\mu\nu} = 0$$

Preserved by

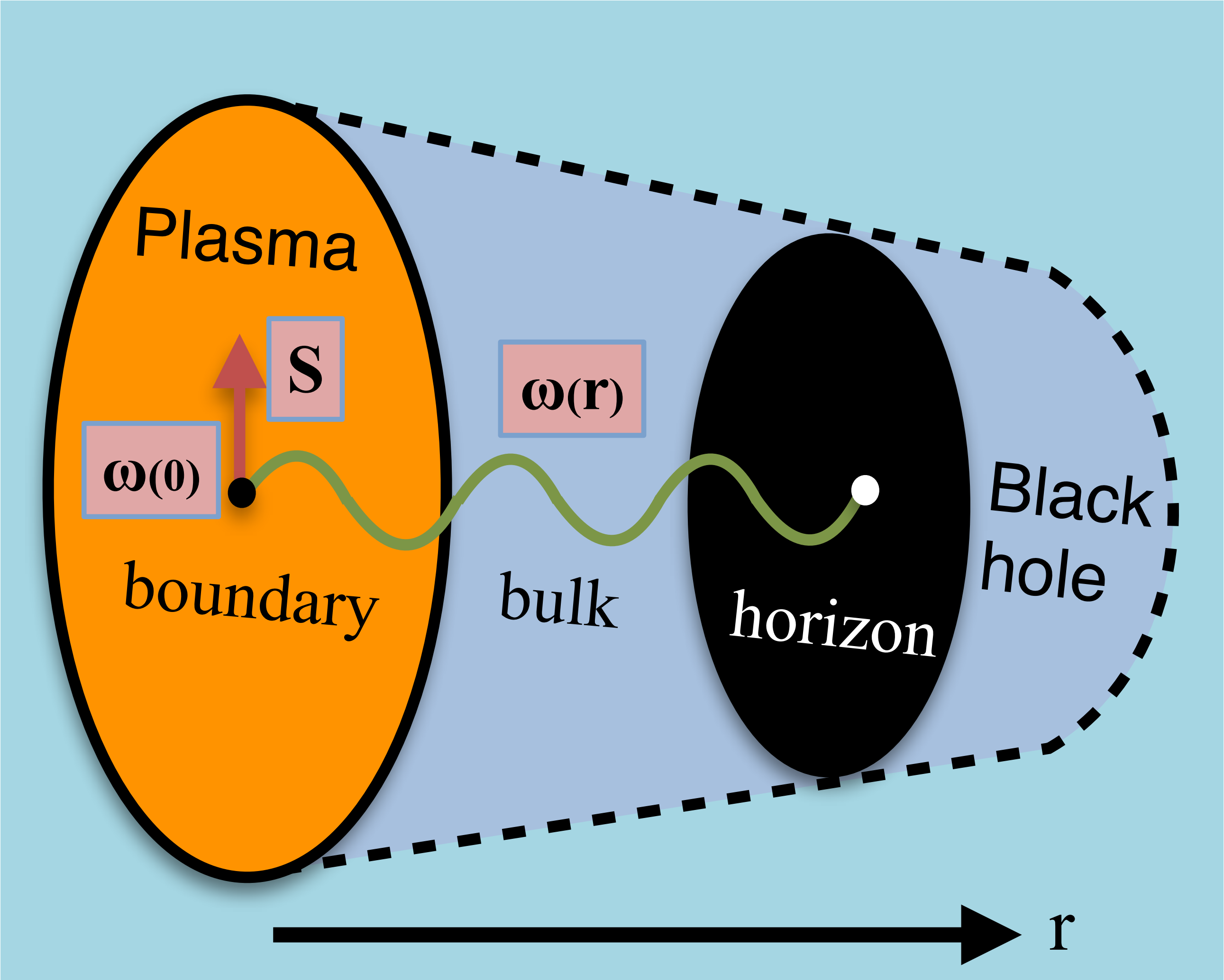
$$T'^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu}\right)\,,$$

$$S'^{\lambda\mu\nu} = S^{\lambda\mu\nu} - \Phi^{\lambda\mu\nu}$$

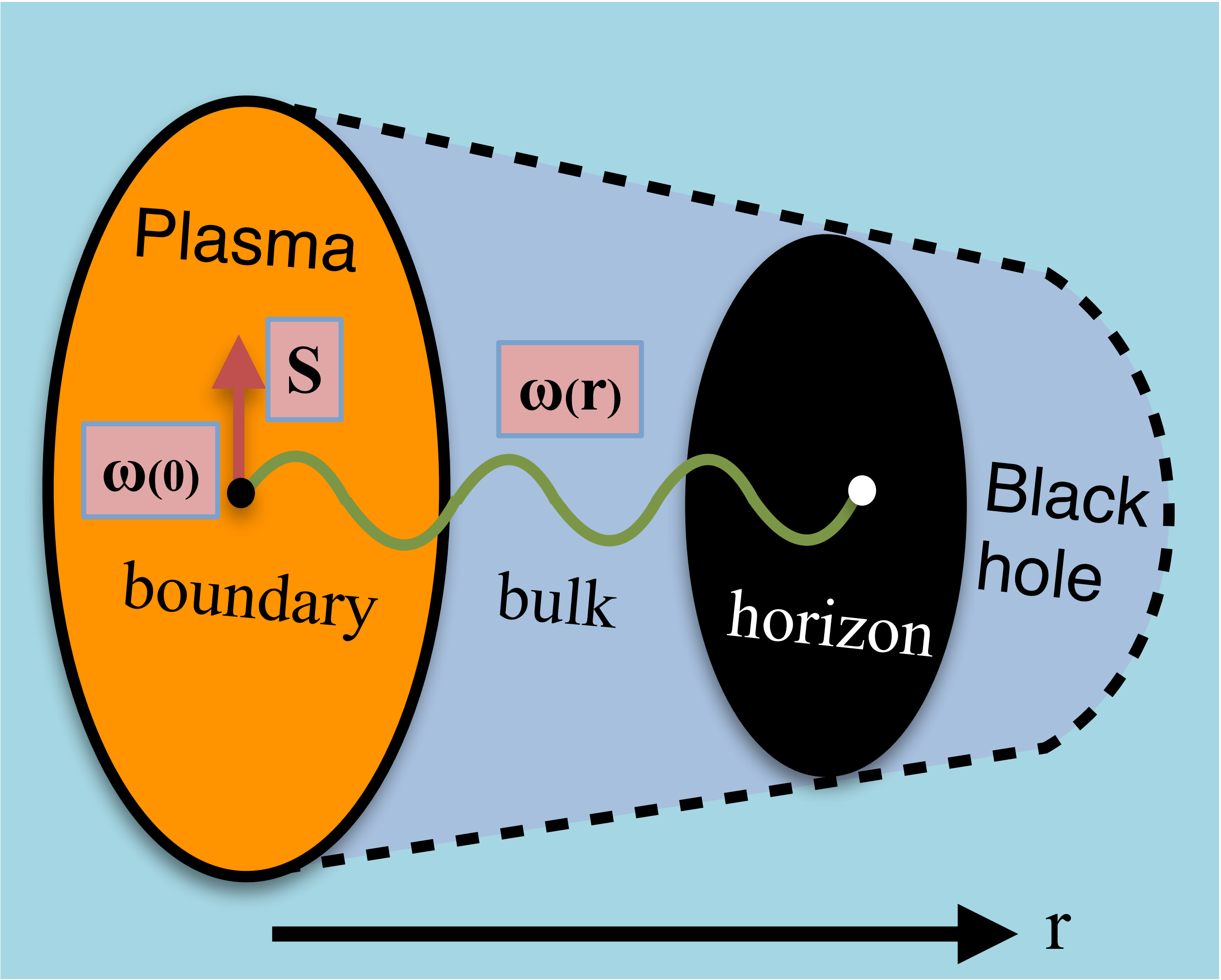
Belinfante-Rosenberg gauge:

$$\Phi^{\lambda\mu\nu} = S^{\lambda\mu\nu}$$

Holography with spin current



Holography with spin current



- 5D Lovelock-Chern-Simons Gallegos, UG `19
- Independent e^a and ω^{ab} in the presence of torsion

⟹ suggests a resolution of BR ambiguity

Quantum field in a nontrivial Lorentz representation

$$e^{iW[e,\omega]} = \int D\Psi e^{iI[e,\omega,\Psi]}$$

Energy-momentum and spin current

$$T^{\mu\nu} = \frac{\delta W}{\delta e^a_\mu} e^\nu_a, \qquad S^\lambda_{ab} = \frac{\delta W}{\delta \omega^{ab}_\lambda}$$

Metric and spin connection are dependent:

$$de^a + \omega^a_b e^b = 0$$ Belinfante-Rosenfeld gauge

Quantum field in a nontrivial Lorentz representation

$$e^{iW[e,\omega]} = \int D\Psi e^{iI[e,\omega,\Psi]}$$

Energy-momentum and spin current

$$T^{\mu\nu} = \frac{\delta W}{\delta e^a_\mu} e^\nu_a, \qquad S^\lambda_{ab} = \frac{\delta W}{\delta \omega^{ab}_\lambda}$$

Metric and spin connection are independent in presence of torsion:

$$de^a + \omega^a_b e^b = T^a$$

Torsion fixes Belinfante-Rosenberg ambiguity $\Phi^{\lambda\mu\nu} \Leftrightarrow T^a_{\mu\nu}$

$\Longrightarrow$ Keep T^a as external source, $T^a \to 0$ at the end.

Hydrodynamics with torsion

Gallegos, Yarom, UG `21`22; Huang, Hongo, Kaminski, Stephanov, Yee `22

$$\omega_\mu^{ab} = \mathring{\omega}_\mu^{ab} + K_\mu^{ab}, \qquad \mathring{\omega} \sim \partial e$$

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Diffeomorphism and local Lorentz invariance

$$\mathring{\nabla}_\mu T^{\mu\nu} = \frac{1}{2} R^{\rho\sigma\nu\lambda} S_{\rho\lambda\sigma} - T_{\rho\sigma} K^{\nu ab} e^\rho{}_a e^\sigma{}_b \qquad \text{4 equations}$$

$$\mathring{\nabla}_\lambda S^\lambda{}_{\mu\nu} = 2T_{[\mu\nu]} - 2S^\lambda{}_{\rho[\mu} e_{\nu]}{}^a e_\rho{}^b K_{\lambda ab}\,, \qquad \text{6 equations}$$

Hydrodynamics with torsion

Gallegos, Yarom, UG `21`22; Huang, Hongo, Kaminski, Stephanov, Yee `22

$$\omega_\mu^{ab} = \mathring{\omega}_\mu^{ab} + K_\mu^{ab}, \qquad \mathring{\omega} \sim \partial e$$

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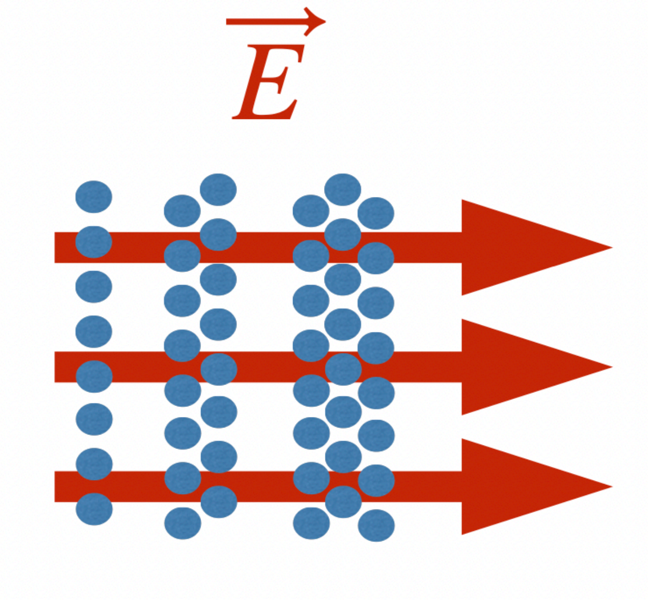
10 dynamical variables:

$$T \qquad u^\mu \qquad \mu^{ab} = \omega_\mu^{ab} u^\mu$$

Spin "chemical"
potential

Hydrostatic equilibrium

Thermal equilibrium in presence of time-independent external forces

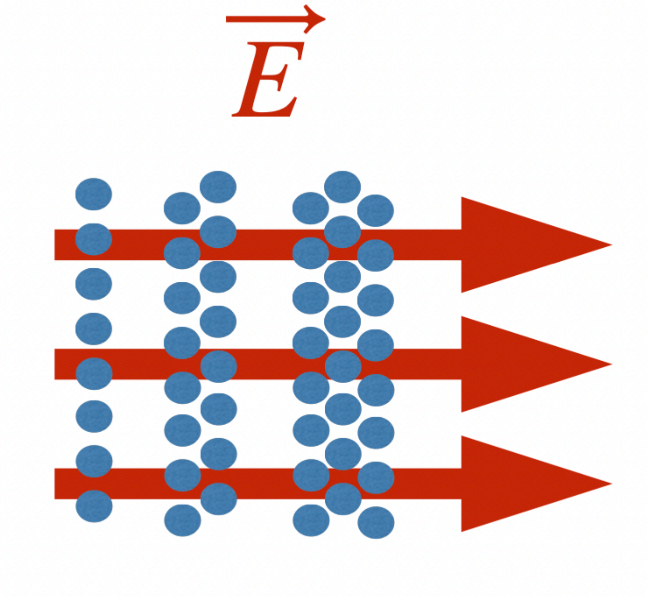


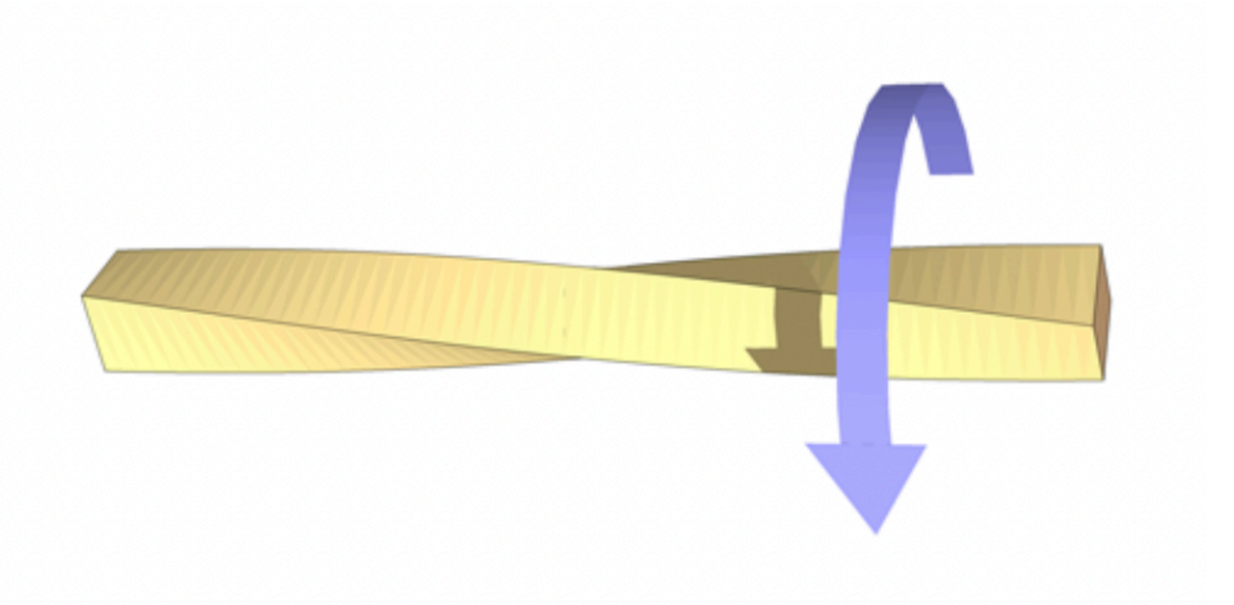
$$
\begin{aligned}
a^\mu &= u^\alpha \partial_\alpha u^\mu \\
&= -\frac{\partial^\mu T}{T}
\end{aligned}
$$

$$
\vec{E} = -T \vec{\nabla} \frac{\mu}{T}
$$

Hydrostatic equilibrium

Thermal equilibrium in presence of time-independent external forces





$$a^\mu = u^\alpha \partial_\alpha u^\mu$$
$$= -\frac{\partial^\mu T}{T}$$

$$\vec{E} = -T\vec{\nabla}\frac{\mu}{T}$$

$$\underbrace{u^\mu K_\mu^{ab}}_{\text{torsion}} = \mu^{ab} - 2u^{[a}\underbrace{a^{b]}}_{\text{acceleration}} + \underbrace{\Omega^{ab}}_{\text{vorticity}}$$

Beyond hydrostatics

Spin is “slave” to background flow:

$$\underbrace{\mu^{ab}}_{\text{spin potentials}} = -2u^{[a}\underbrace{a^{b]}}_{\text{acceleration}} + \underbrace{\Omega^{ab}}_{\text{vorticity}}$$

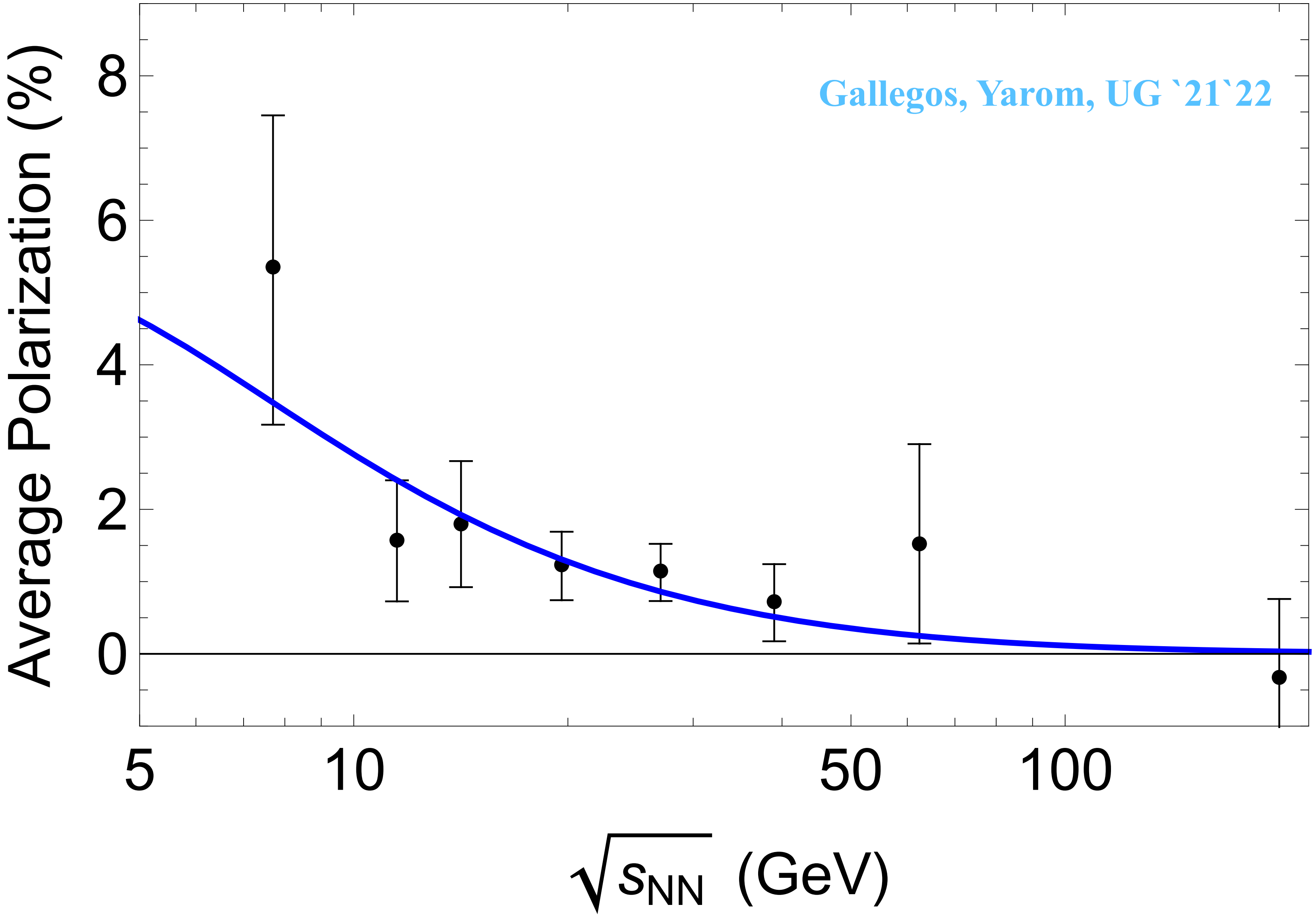
up to $O(\nabla^2)$

Beyond hydrostatics

Spin is "slave" to background flow:

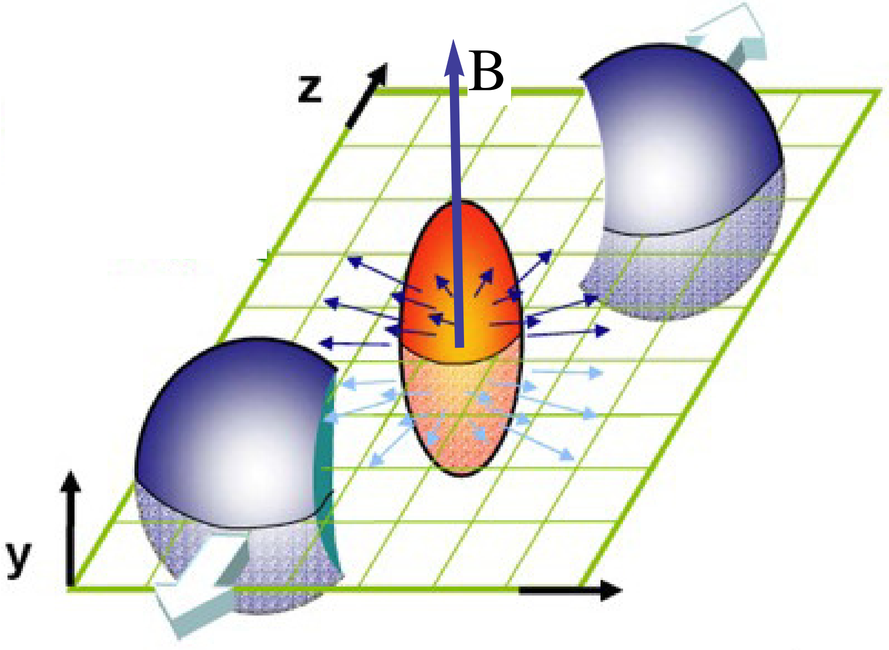
$$\underbrace{\mu^{ab}}_{\text{spin potentials}} = -2\underbrace{u^{[a}a^{b]}}_{\text{acceleration}} + \underbrace{\Omega^{ab}}_{\text{vorticity}}$$

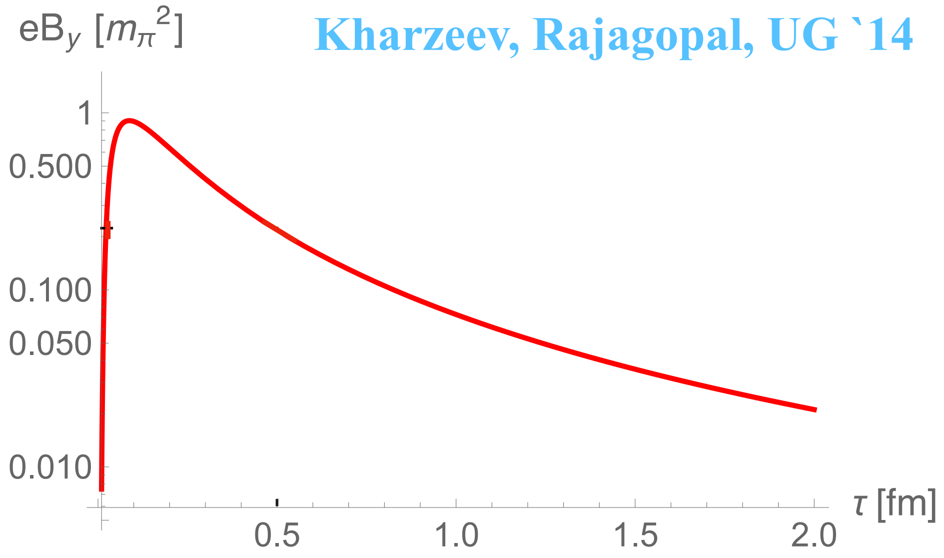
Bjorken flow: conformal, boost and parity invariant



QCD and magnetic fields

QCD and magnetic fields



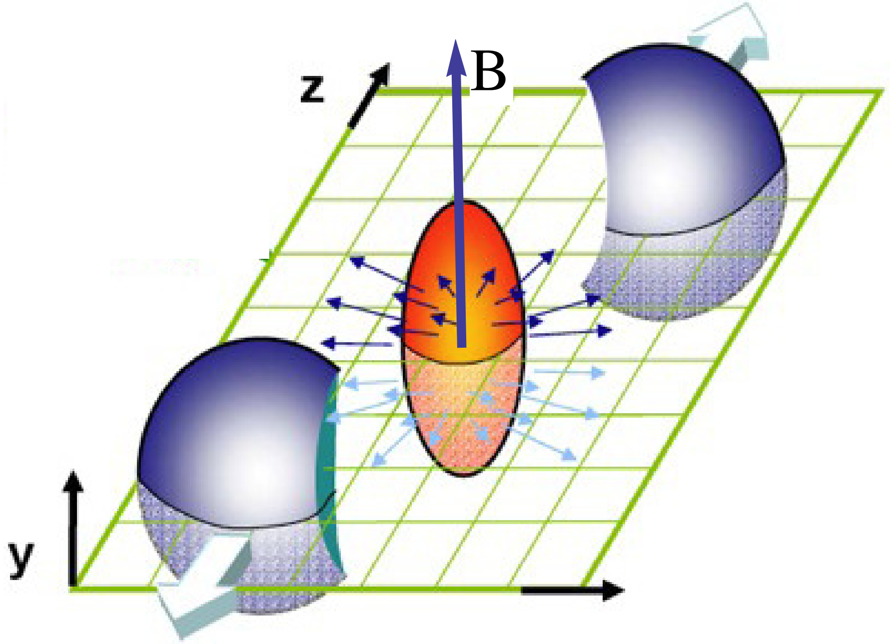


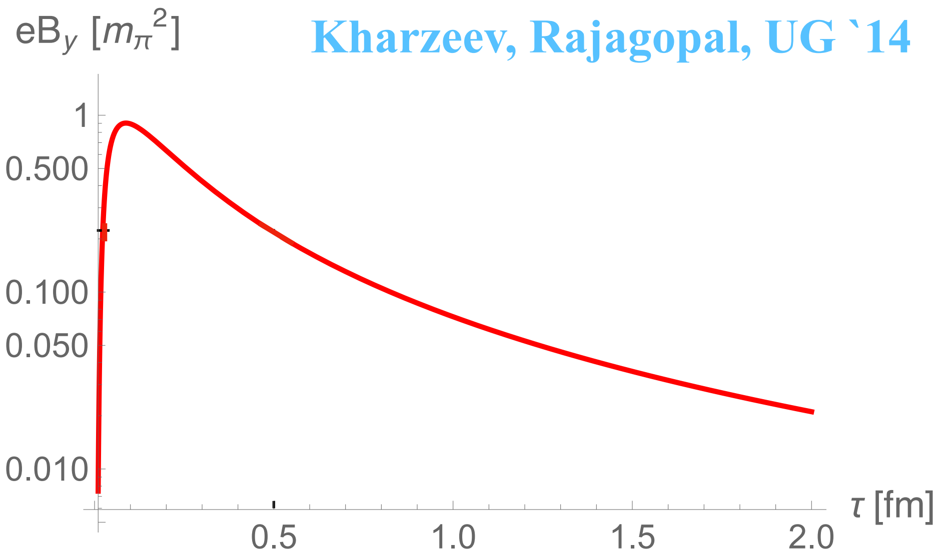
Kharzeev, Rajagopal, UG `14

Charge, spin and chiral dynamics; matter antimatter asymmetry

Chiral magnetic effect Kharzeev, Warringa, McLerran `08; Vilenkin `80

QCD and magnetic fields





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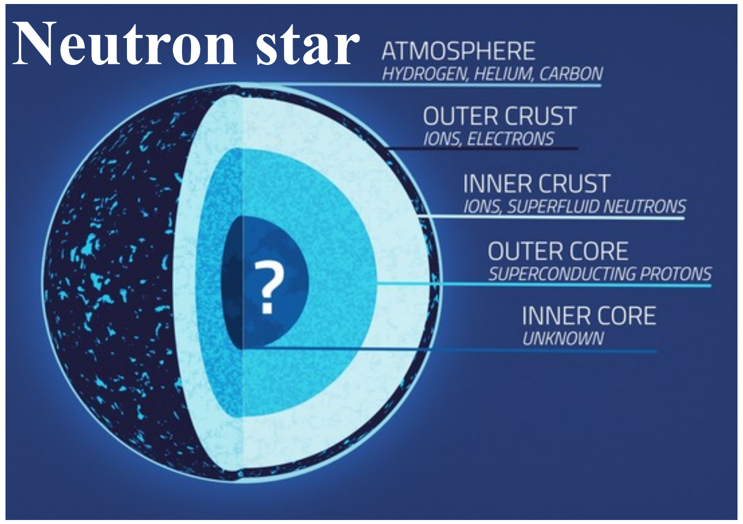
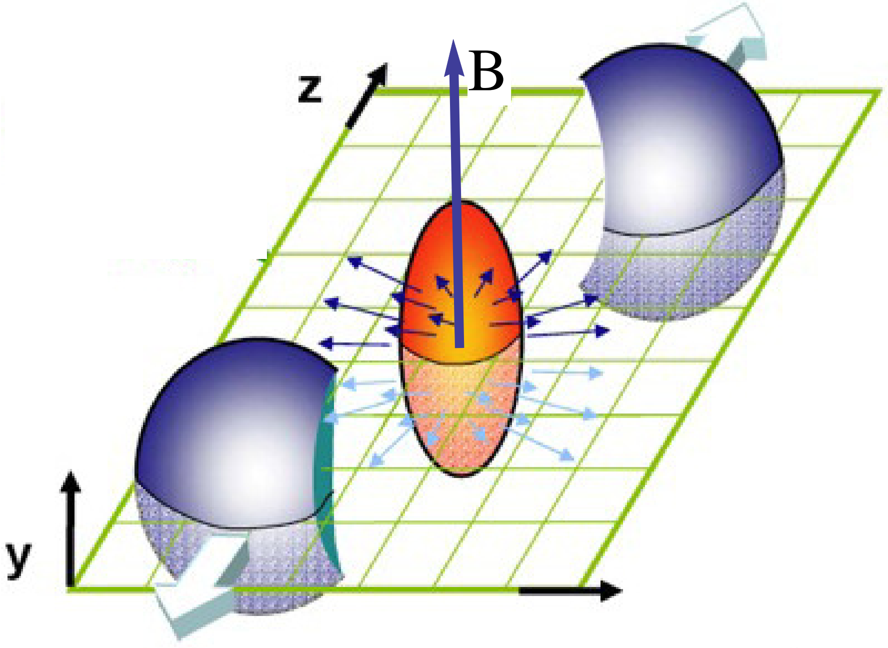
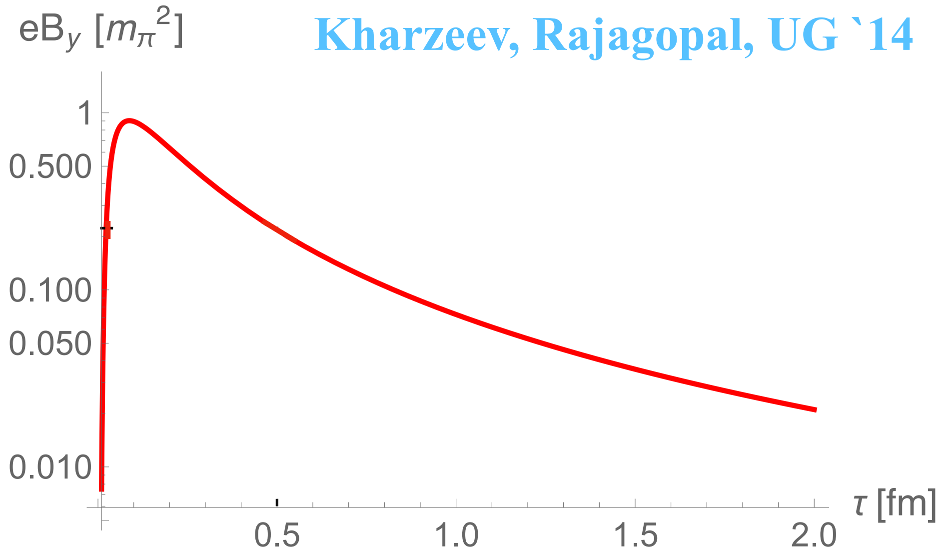


Holographic modeling of neutron stars
Jarvinen `22

QCD and magnetic fields





Kharzeev, Rajagopal, UG `14

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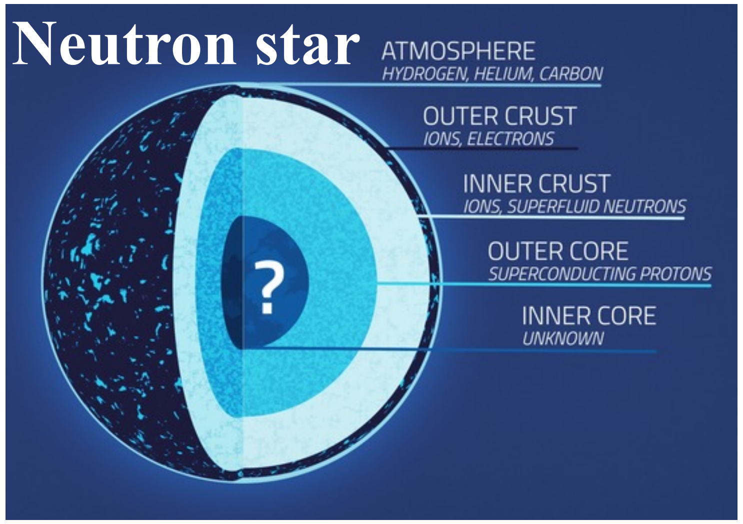
This talk



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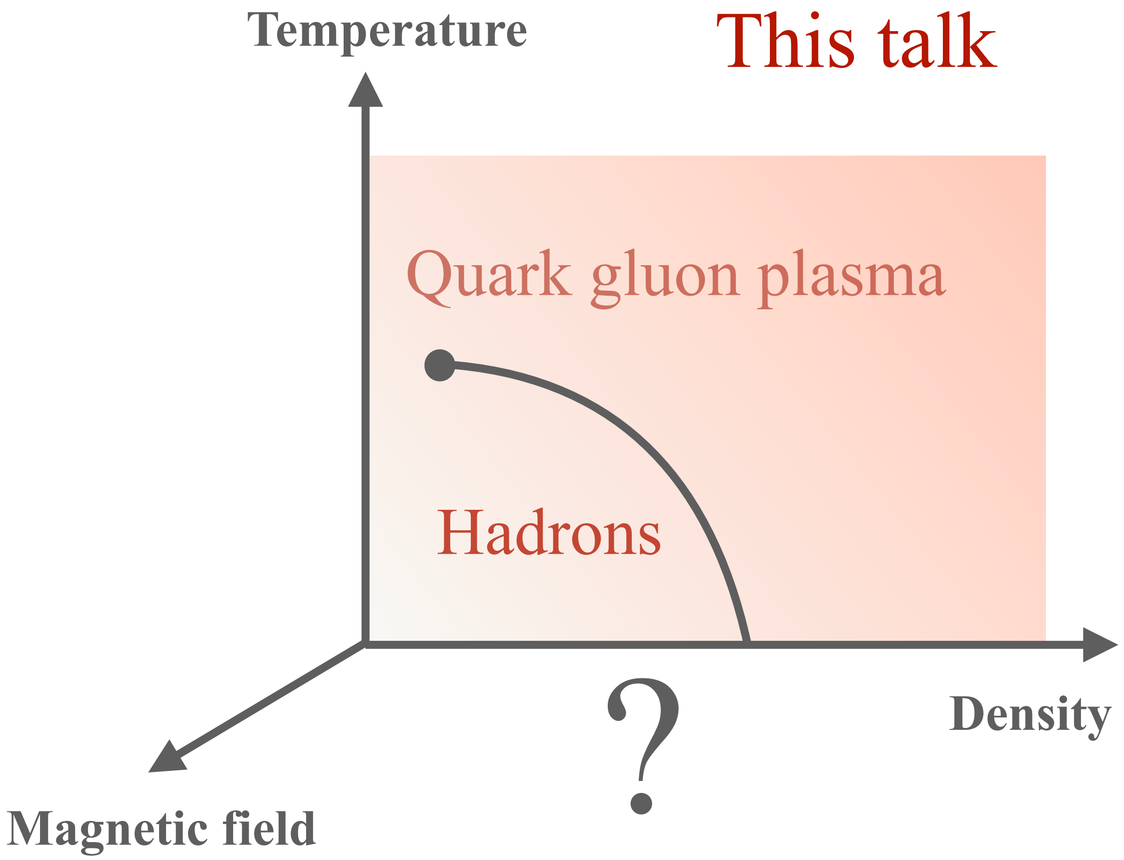

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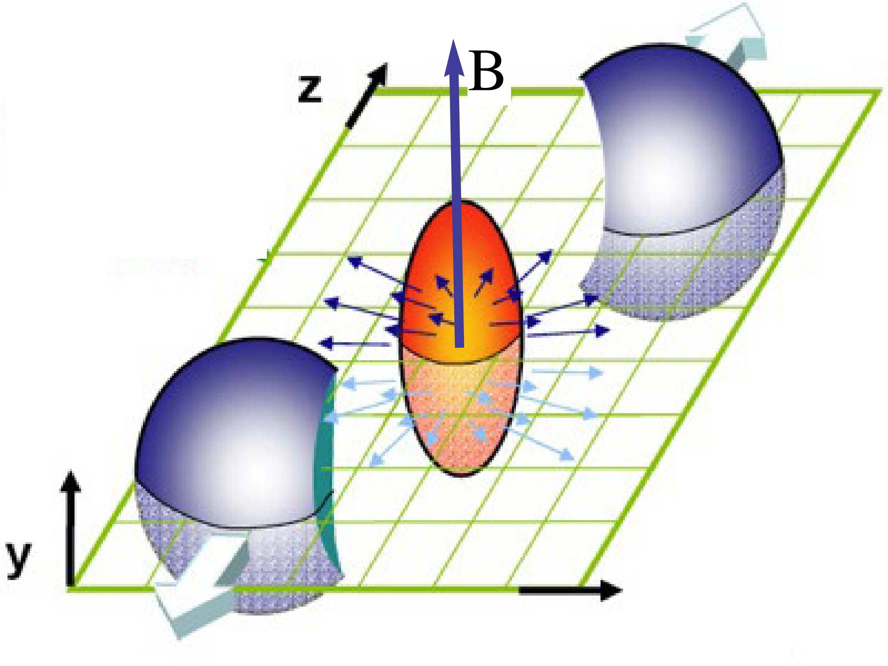

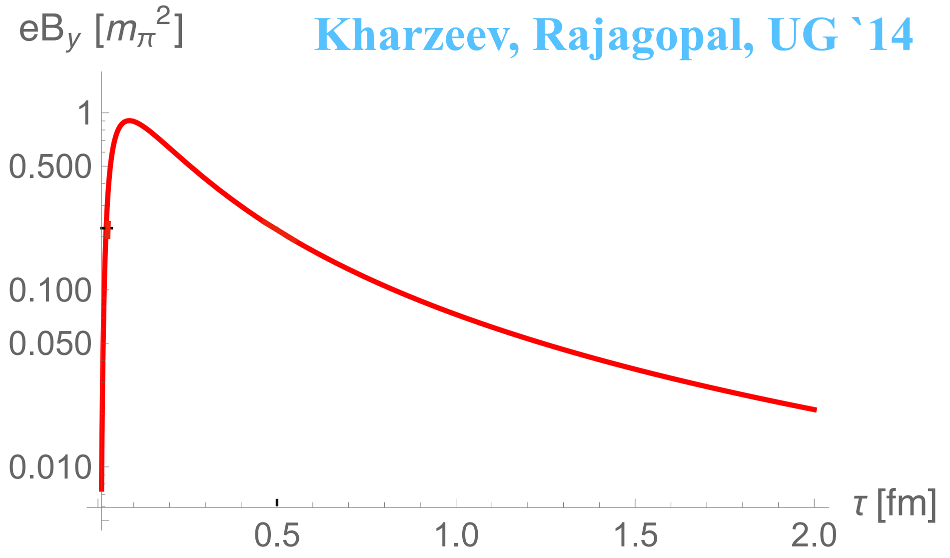

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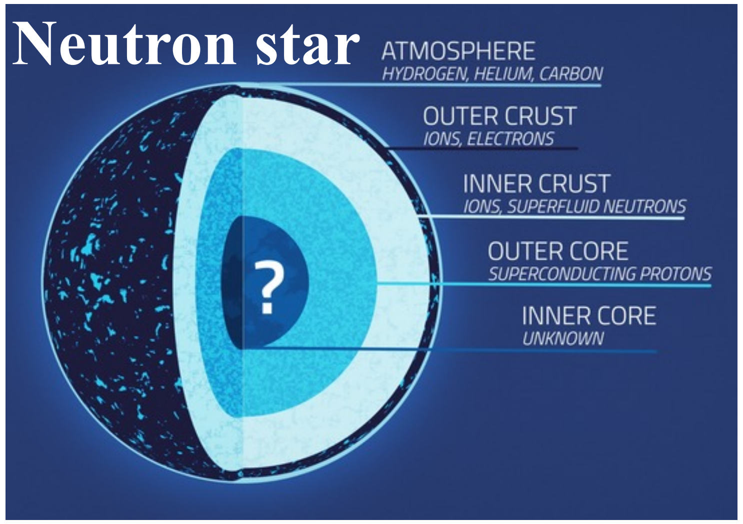

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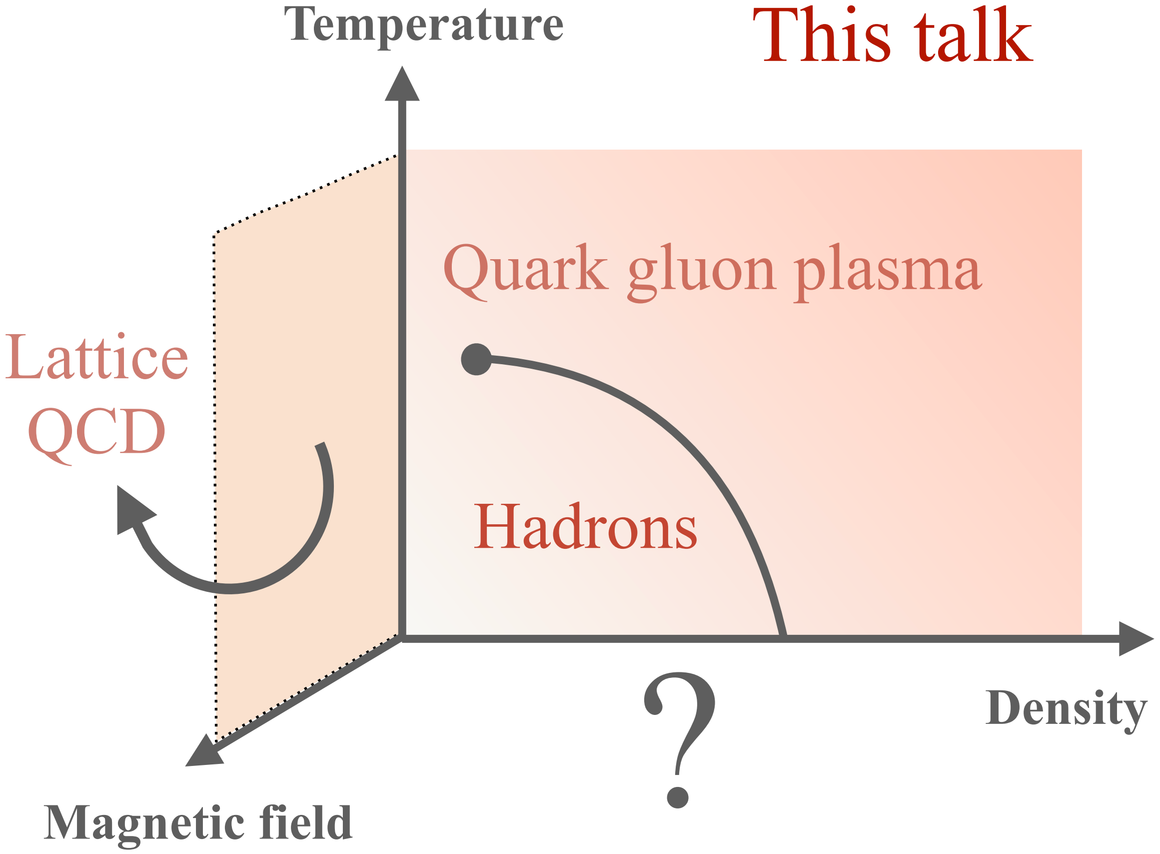


Temperature

Lattice QCD

Quark gluon plasma

Hadrons

Density

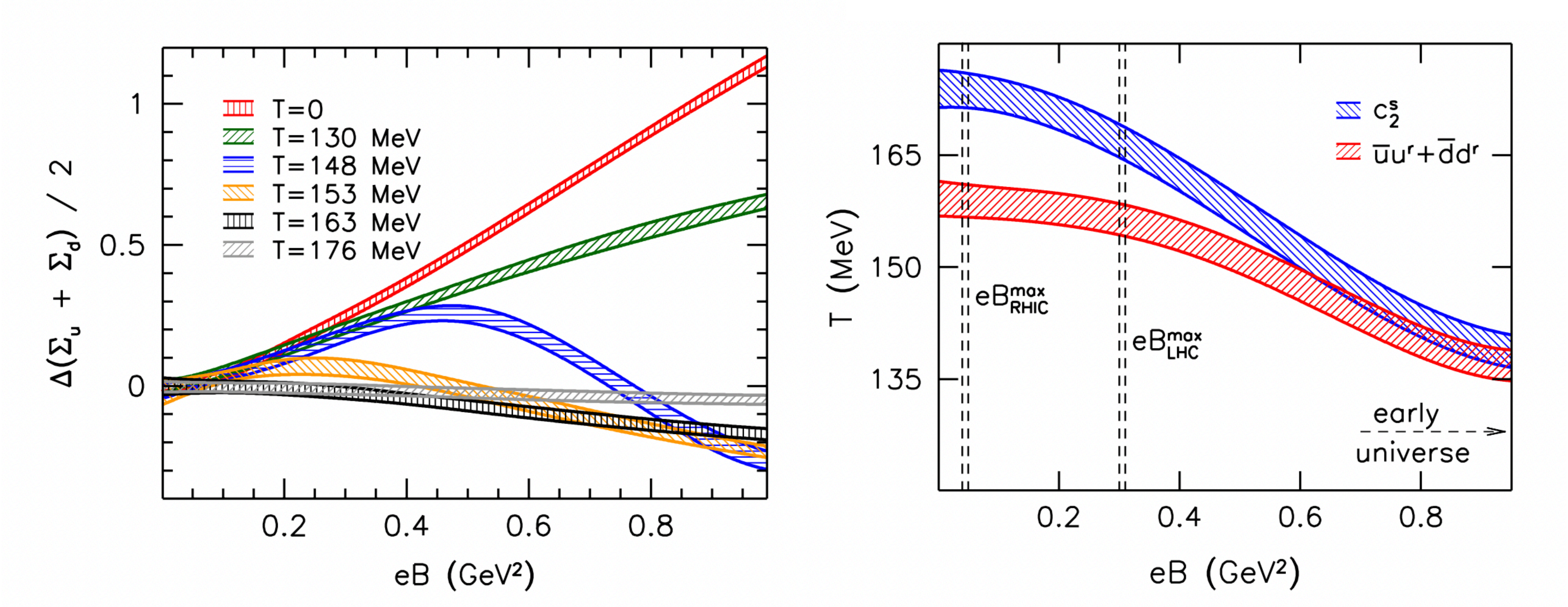
?

Magnetic field

Chiral condensate

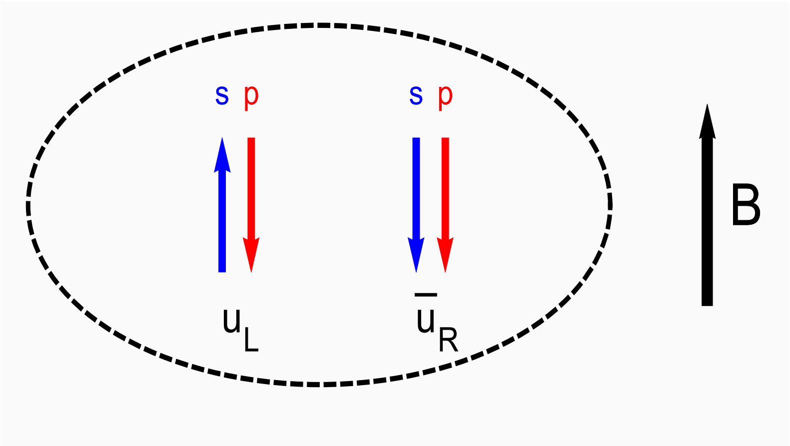
Lattice QCD

Bali et al `11 `12



Chiral condensate

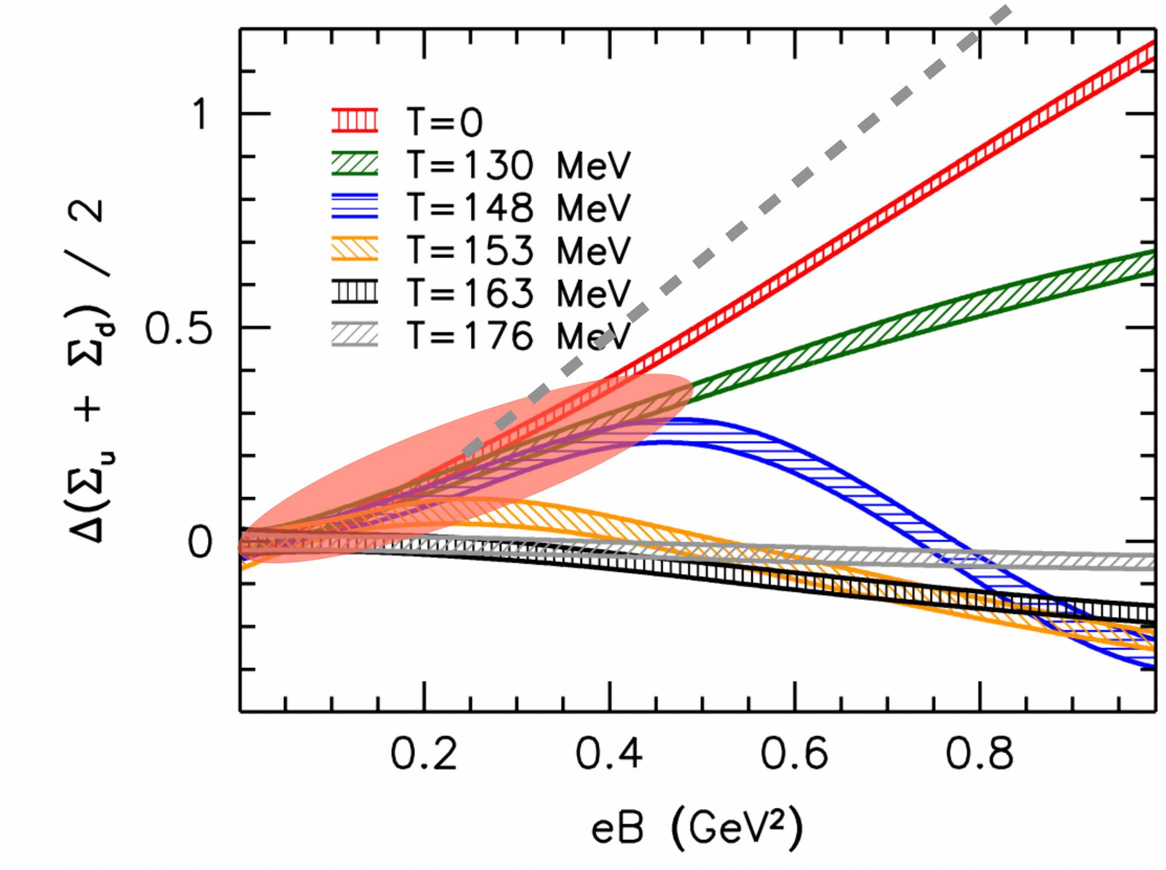
magnetic catalysis

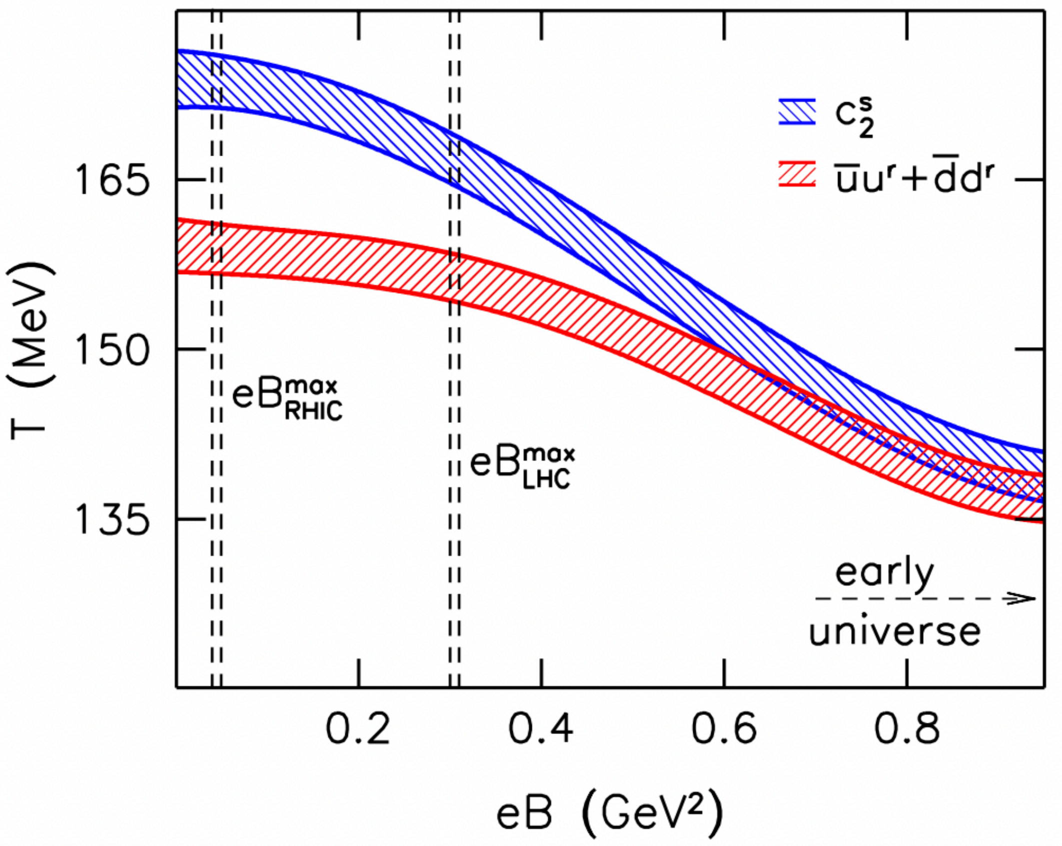


Gusynin, Miransky, Shovkovy `94

Lattice QCD

Bali et al `11 `12





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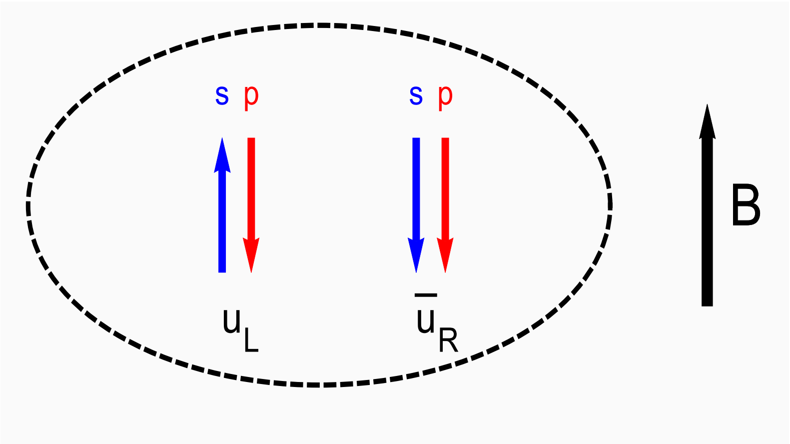

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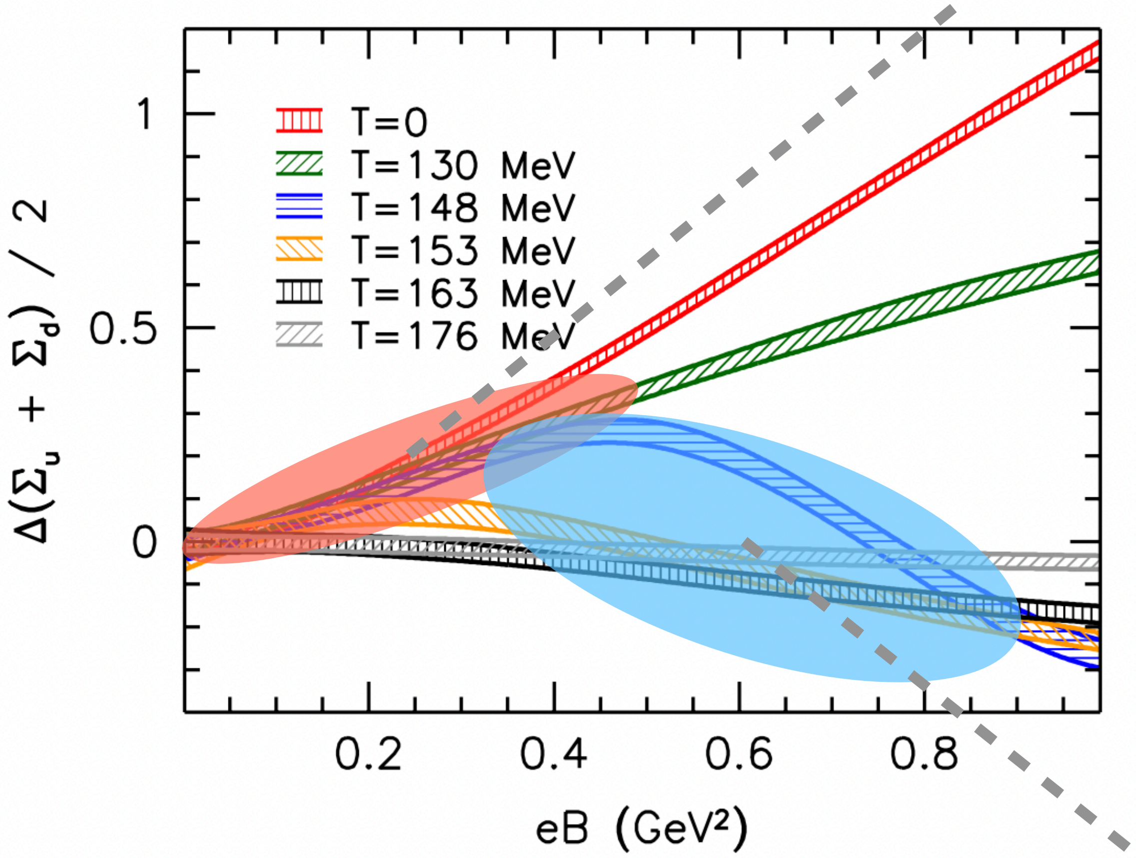
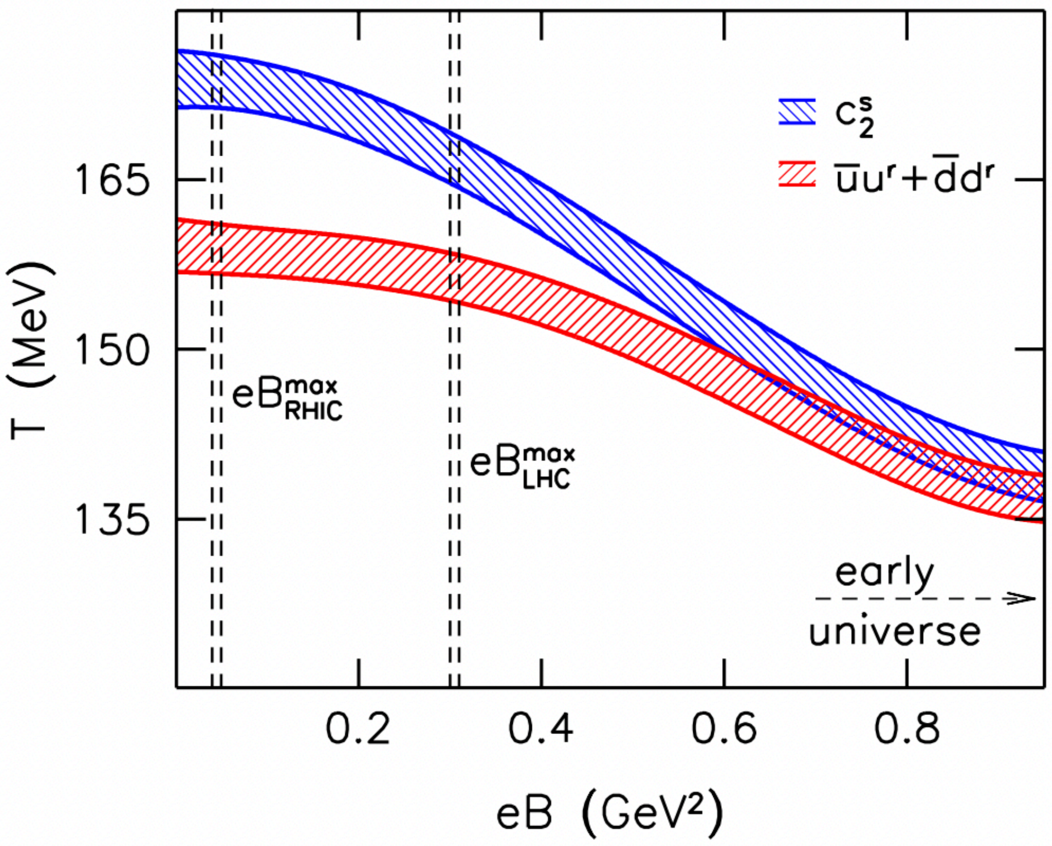
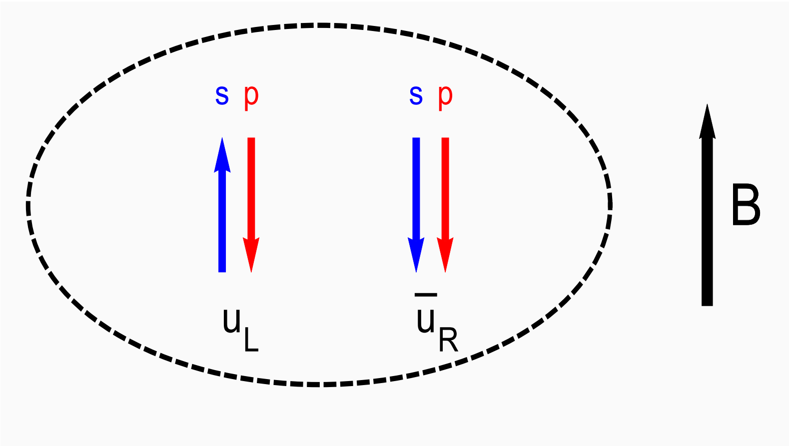




inverse magnetic catalysis

Chiral condensate

magnetic catalysis



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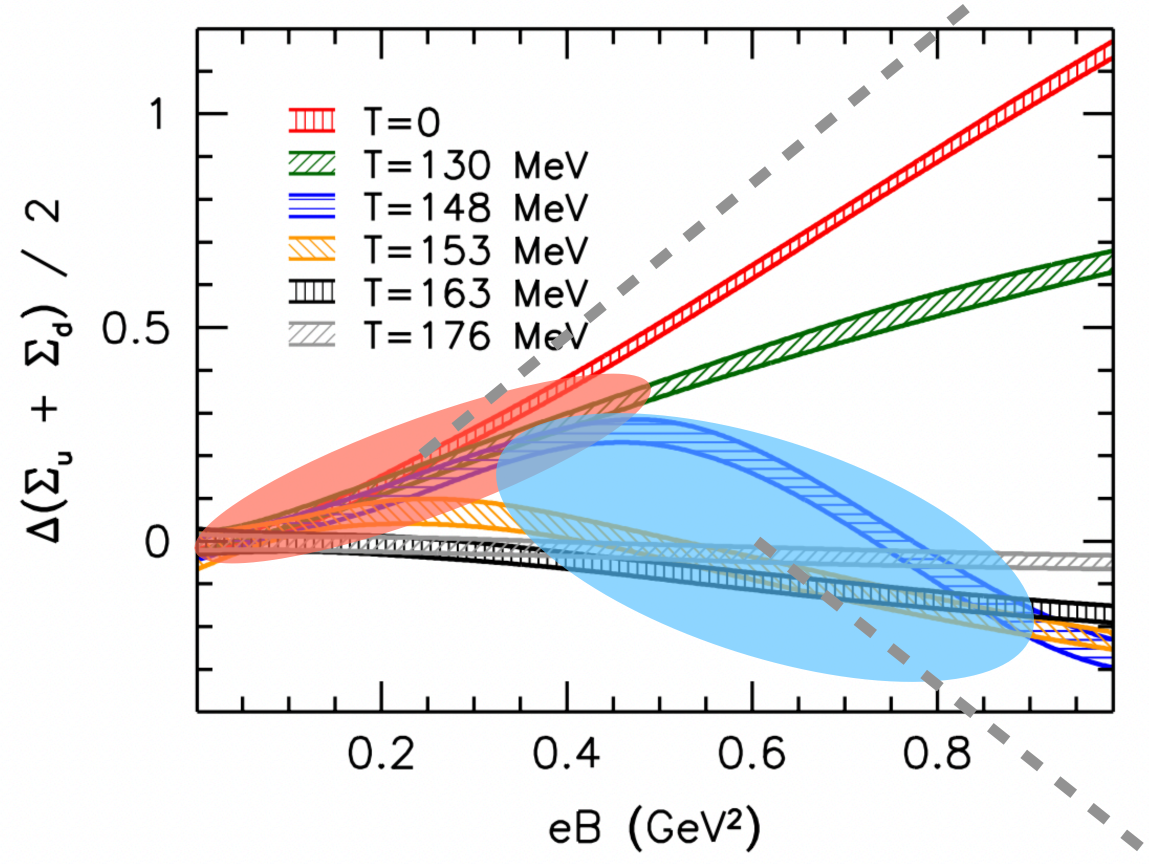
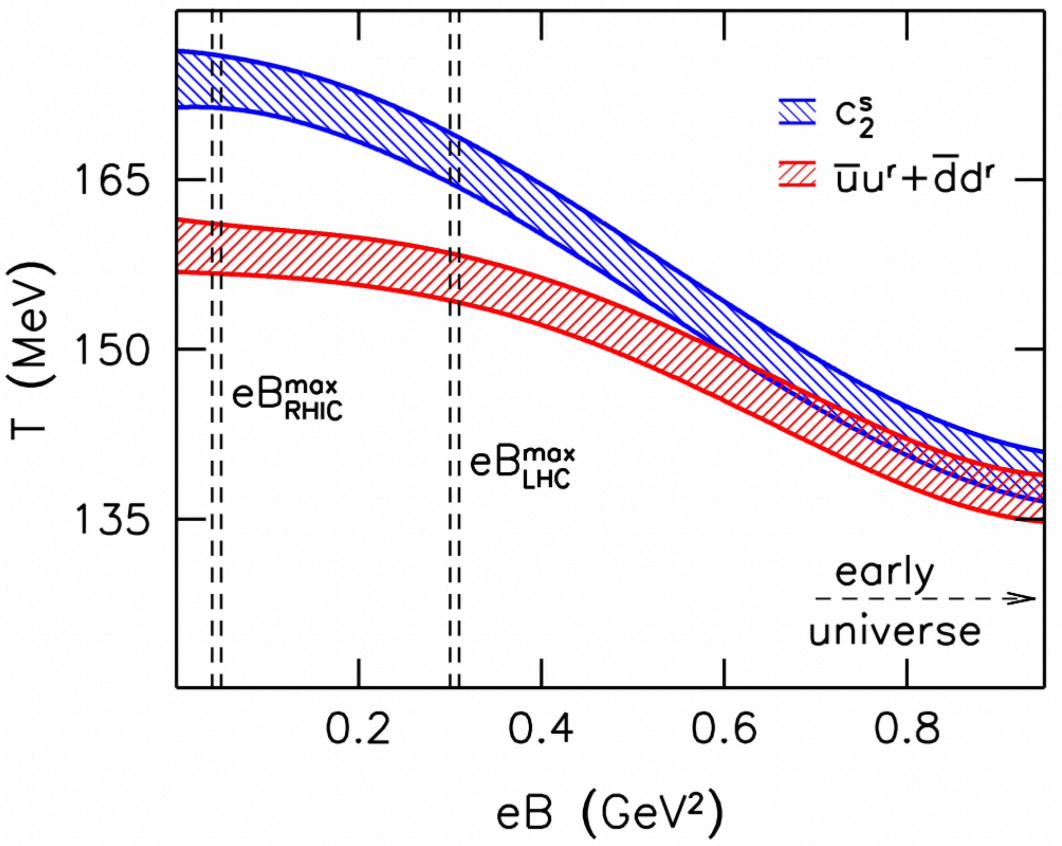
Lattice QCD

Bali et al ’11 ’12





Witten-Sakai-Sugimoto model ⟹ inverse magnetic catalysis

Preis, Rebhan, Schmitt `10

Inverse magnetic catalysis and holography

Preis, Rebhan, Schmitt `10; Mamo `15; Noronha et al `15; Evans et al `16; Iatrakis, Jarvinen, Nijs, UG `16

Improved holographic QCD in Veneziano limit:

$$x = N_f/N = \mathcal{O}(1)$$

Kiritsis, UG `07; Kiritsis, Nitti, UG `07;
Jarvinen, Kiritsis ’11; Alho et al `12;
Iatrakis, Jarvinen, Nijs, UG `16`18;

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$$x = N_f/N = \mathcal{O}(1)$$

$$S_g = M^3N^2\int d^5x\sqrt{-g}\left(R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi)\right)$$

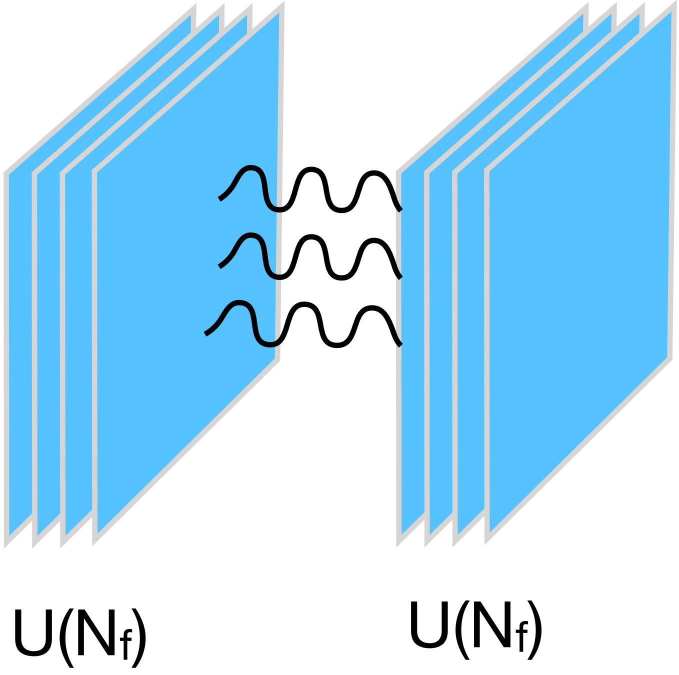
$$\phi \leftrightarrow \mathrm{tr}\, F^2$$

dilaton

+ fundamental flavor in Veneziano limit

tachyon $T \leftrightarrow \bar{q}q$

vector $U(1)_B \leftrightarrow B$



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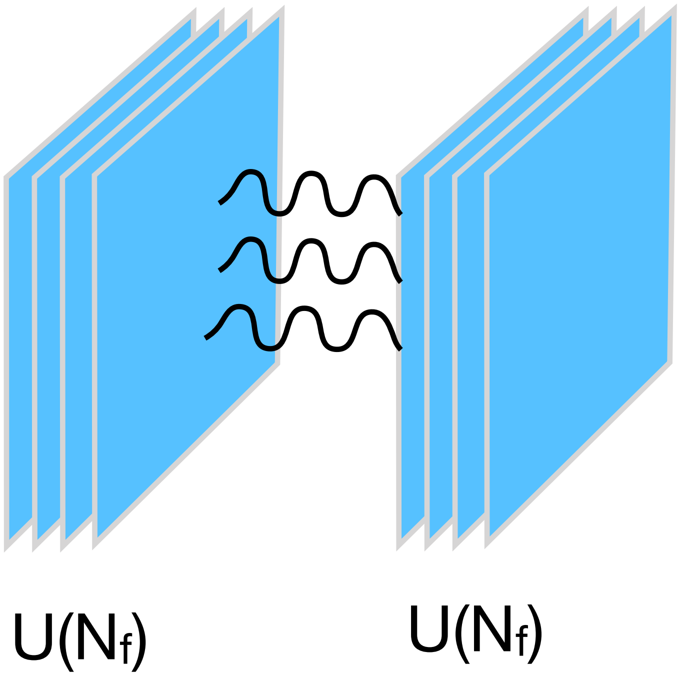

Confinement, χSB, gapped hadron spectrum

Inverse magnetic catalysis in holographic QCD

Iatrakis, Jarvinen, Nijs, UG `16; Jarvinen, Nijs, UG `18; Jarvinen, Nijs, Pedraza, UG `20;

$$T'' + C_1[B;g,\phi]T'^3 + C_2[B;g,\phi]T' + C_3[B;g,\phi] = 0$$

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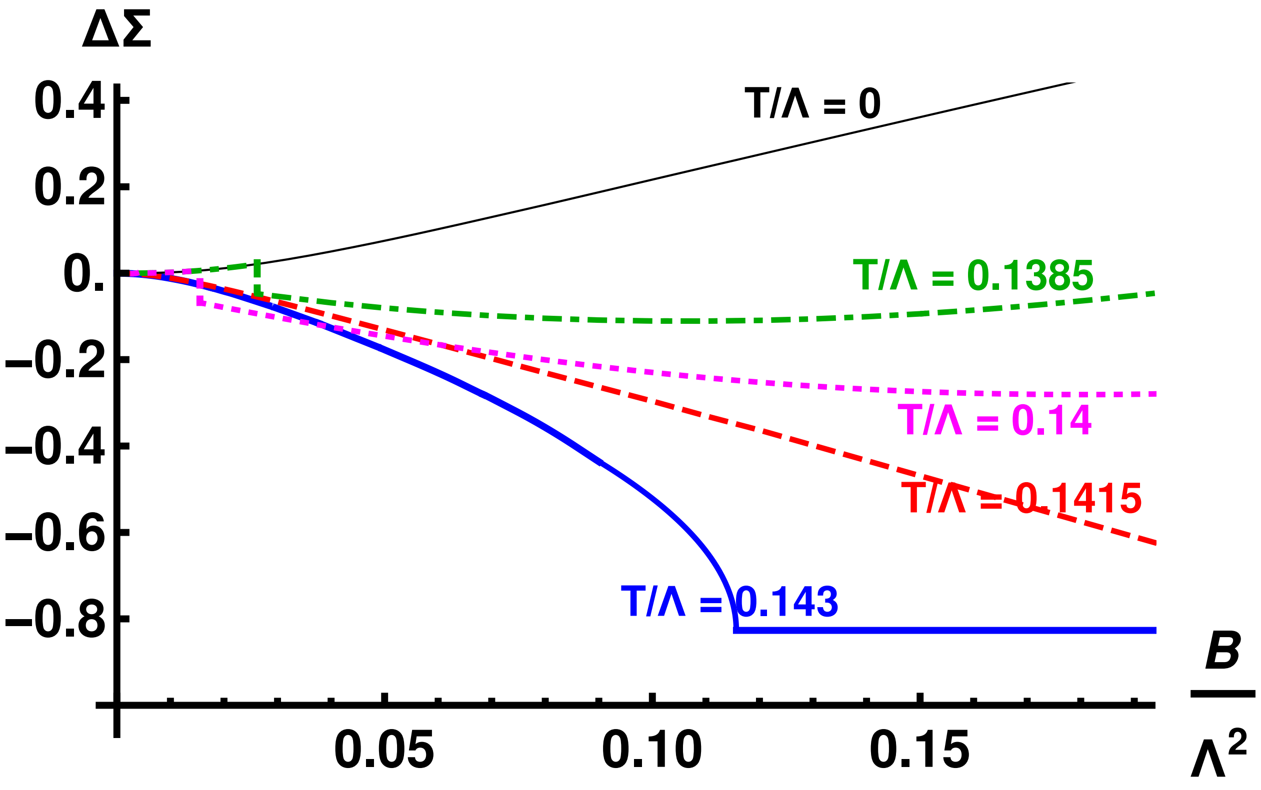

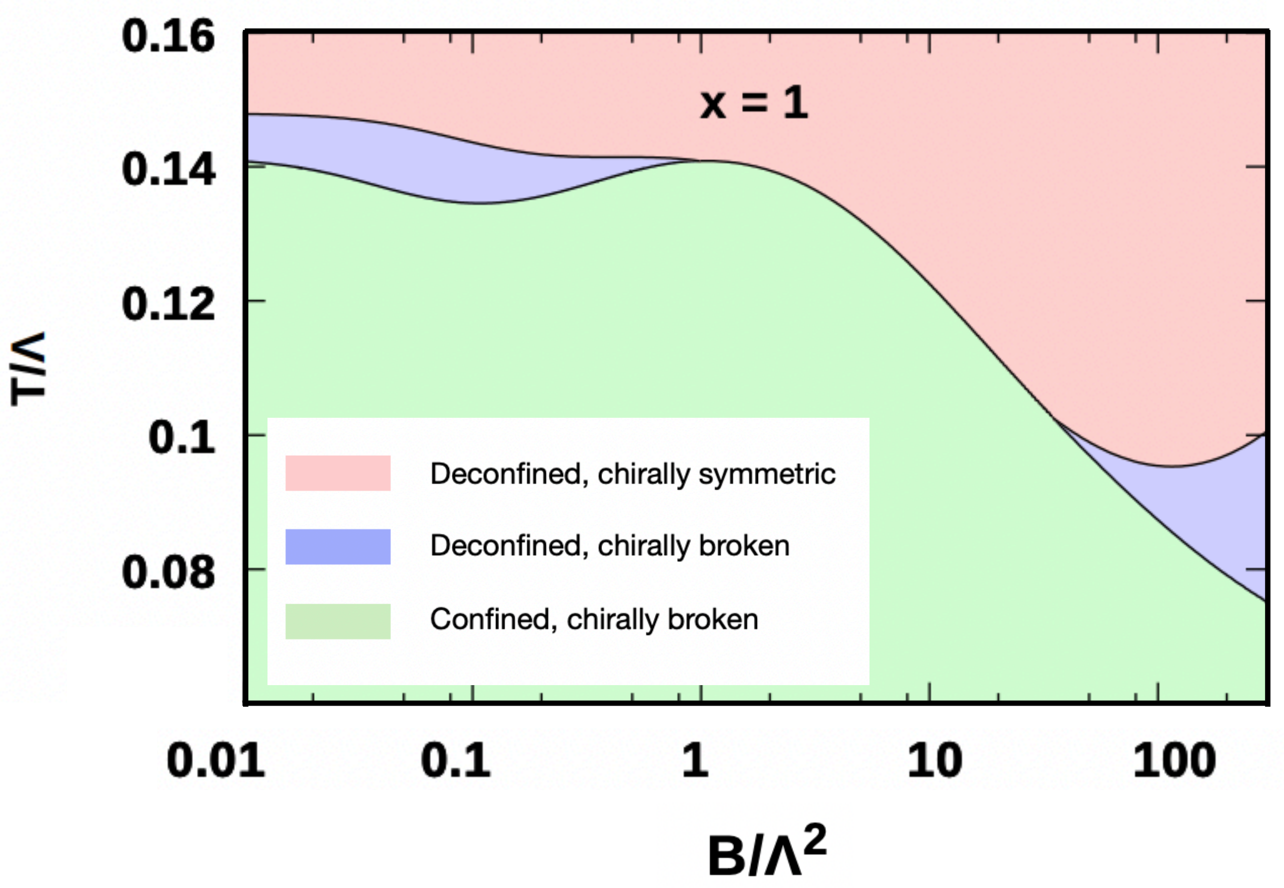

$$T'' + C_1[B; g(B), \phi(B)]T'^3 + C_2[B; g(B), \phi(B)]T' + C_3[B; g(B), \phi(B)] = 0$$

Two distinct
dependence on B:

$$\underbrace{\text{Explicit dependence}}_{\text{catalysis}} \qquad \underbrace{\text{Implicit dependence}}_{\text{inverse catalysis}}$$

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Two distinct dependence on B:

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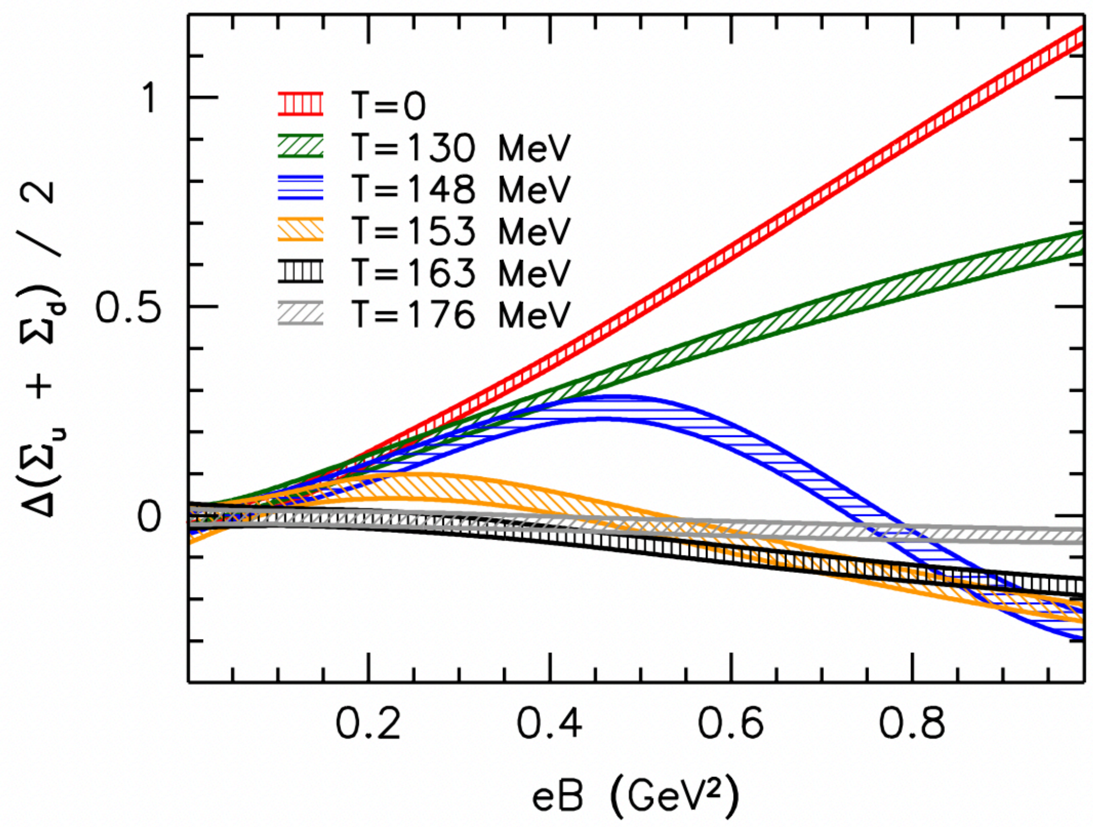

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$$\langle \bar{q}q \rangle = \int \mathcal{D}A e^{-S[A]} \times \mathrm{tr}(D(A,B)+m)^{-1} \times \mathrm{det}(D(A,B)+m)$$

Discussion

N=4 super Yang-Mills vs QCD

Can we understand rapid thermalization?

$\Longrightarrow$ Planckian time scales

Incorporate B and chiral imbalance

Chesler, Yaffe `08
Cassalderrey-Solana, Heller, Mateos, van der Schee `13, …

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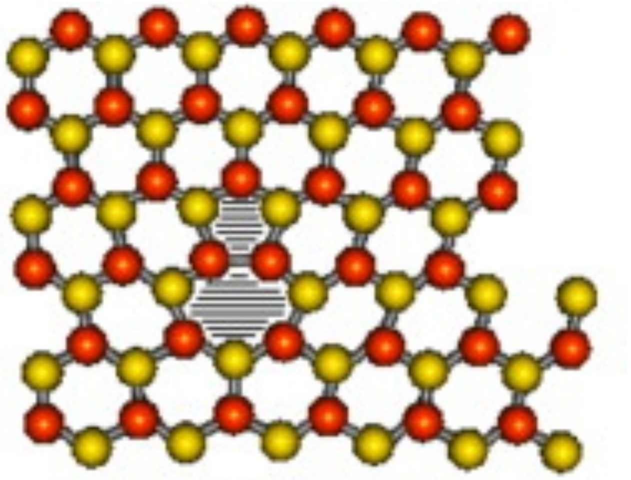
Spin hydrodynamics

Torsion-hydrodynamics
New means of spin transport?

dislocations in graphene



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Holography with torsion:
$\Longrightarrow$ spin transport coefficients

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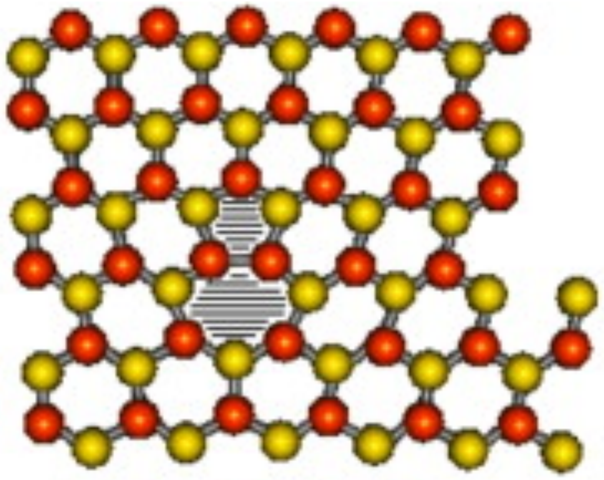

Uncovered: fractons, neutron stars, holographic thermalization, chiral anomalous transport, Graphene, Dirac/Weyl semimetals, …

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Applied gauge-gravity duality:
great source of inspiration for quantum many-body physics

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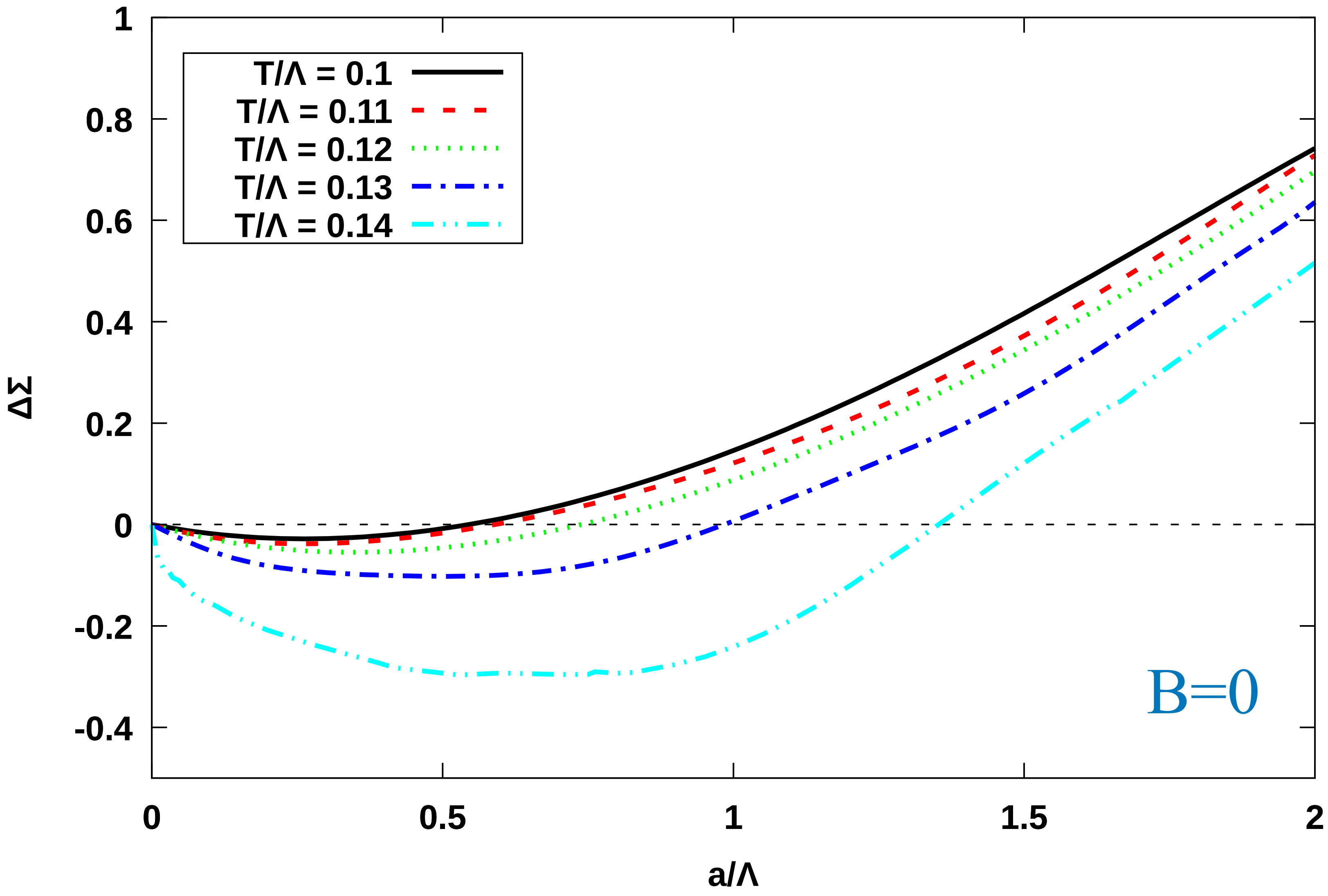
Thank you

Magnetic fields & QCD: how about anisotropy?

Add $\Psi = a\,|\vec{x}\,|$ to improved holographic QCD



Giataganas, Pedraza, UG `18

Jarvinen, Nijs, Pedraza, UG `19

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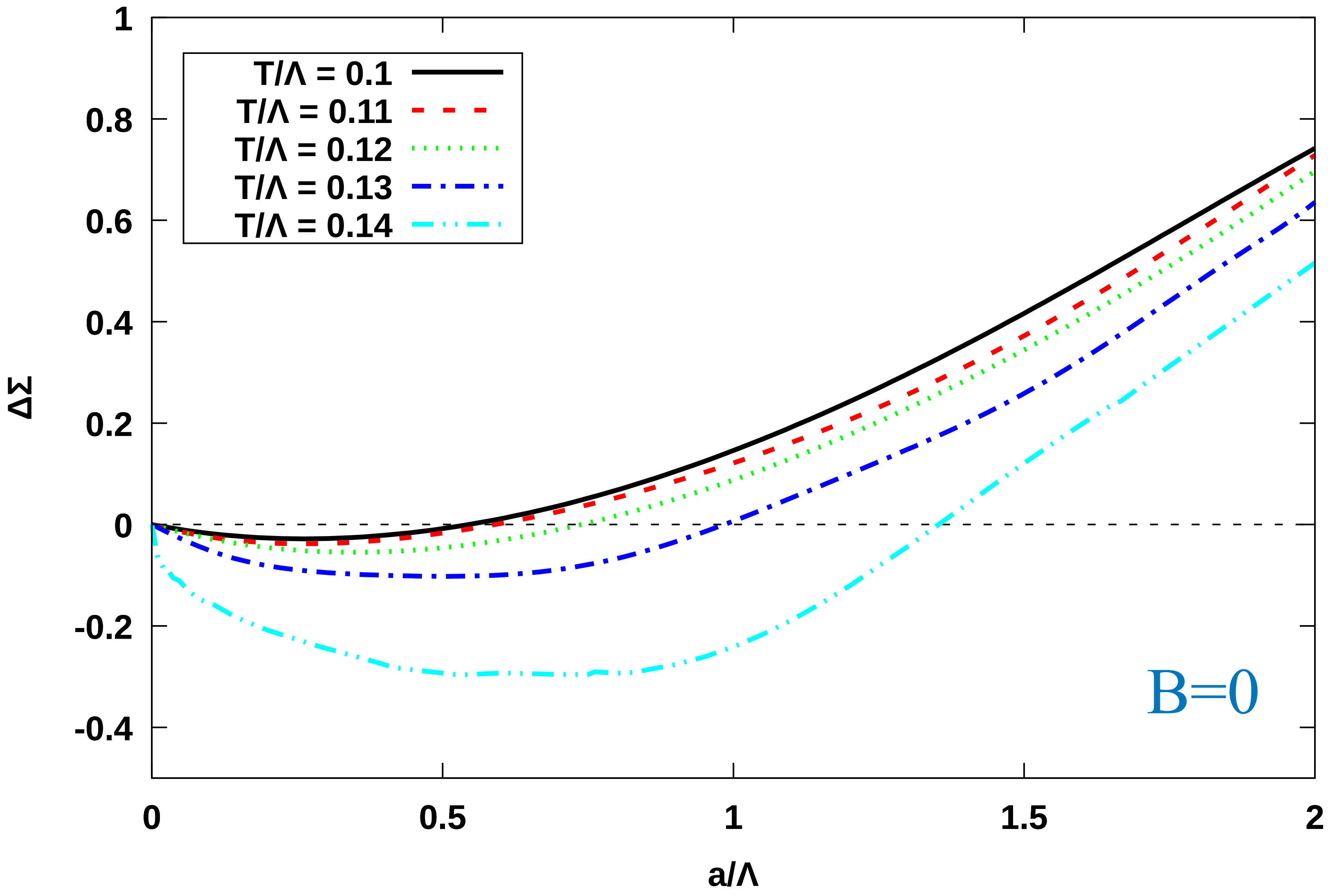

Giataganas, Pedraza, UG `18

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IMC might be due to anisotropy induced by B rather than electromagnetism?

$\Longrightarrow$ a question for lattice QCD!

Pseudo-Goldstone hydrodynamics

Delacretaz, Gouteraux, Hartnoll, Karlsson `21, Gouteraux, Delacretaz, Ziogas `21;
Ammon, Arean, Baggioli, Gray, Grieninger `21

Superfluid as example:

$$e^{-\beta W} = \int \mathcal{D}\phi \; e^{-\beta F[\delta\mu_n, \delta\mu_\phi, \phi]}$$

Weak explicit breaking:

$$F = \int \frac{1}{2}\left(\nabla\phi^2 + k_0^2\phi^2\right) - \delta\mu_\phi\phi - \frac{1}{2}\chi_{nn}\delta\mu_n^2 + \cdots$$

Susceptibility matrix: $\chi = \begin{pmatrix} \chi_{nn} & 0 \\ 0 & \frac{1}{k^2+k_0^2} \end{pmatrix}$

Hydrodynamics should be local for $k_0^{-1} \gg \lambda_{th}$

Pseudo-Goldstone hydrodynamics

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$$e^{-\beta W} = \int \mathcal{D}\phi \; e^{-\beta F[\delta\mu_n, \delta\mu_\phi, \phi]}$$

$$\dot{n} = D_n \nabla^2 n - \nabla^2 \phi - \Gamma n + f_s k_0^2 \phi + \cdots$$

$$\dot{\phi} = -\Omega\phi - \frac{n}{\chi_{nn}} + D_\phi \nabla^2 \phi + \cdots$$

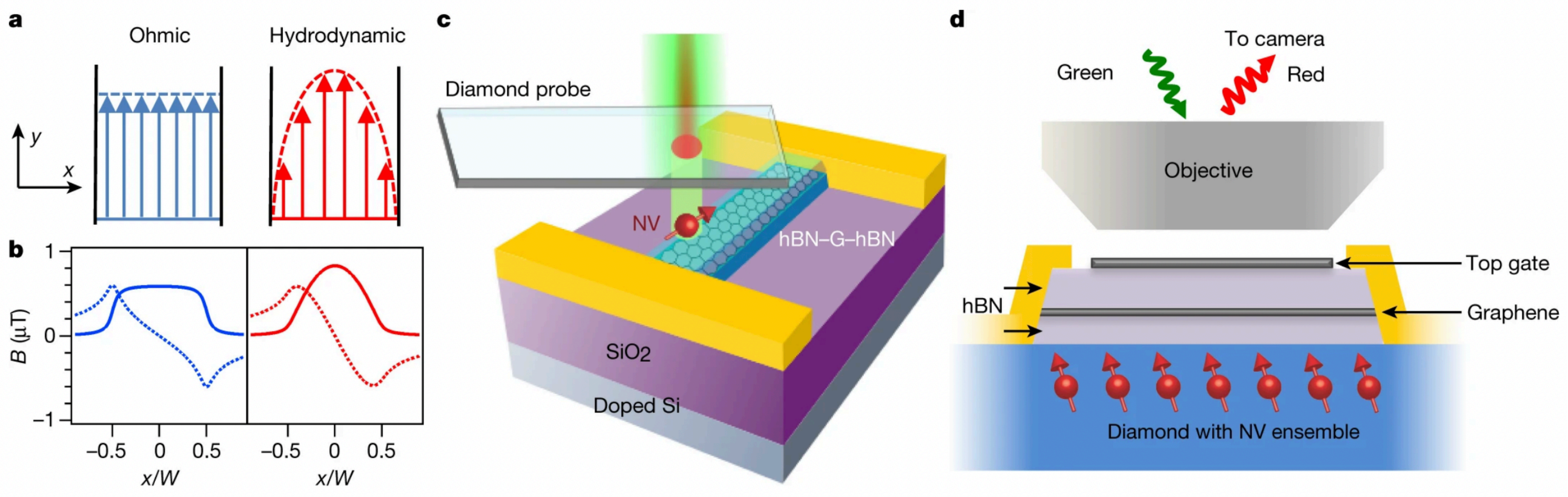
$$\Longrightarrow \qquad M \cdot \chi = \begin{pmatrix} \Gamma + D_n k^2 & -1 \\ 1 & \frac{\Omega + D_\phi k^2}{k^2 + k_0^2} \end{pmatrix}$$

Locality:

$$\boxed{\Omega = k_0^2 D_\phi}$$

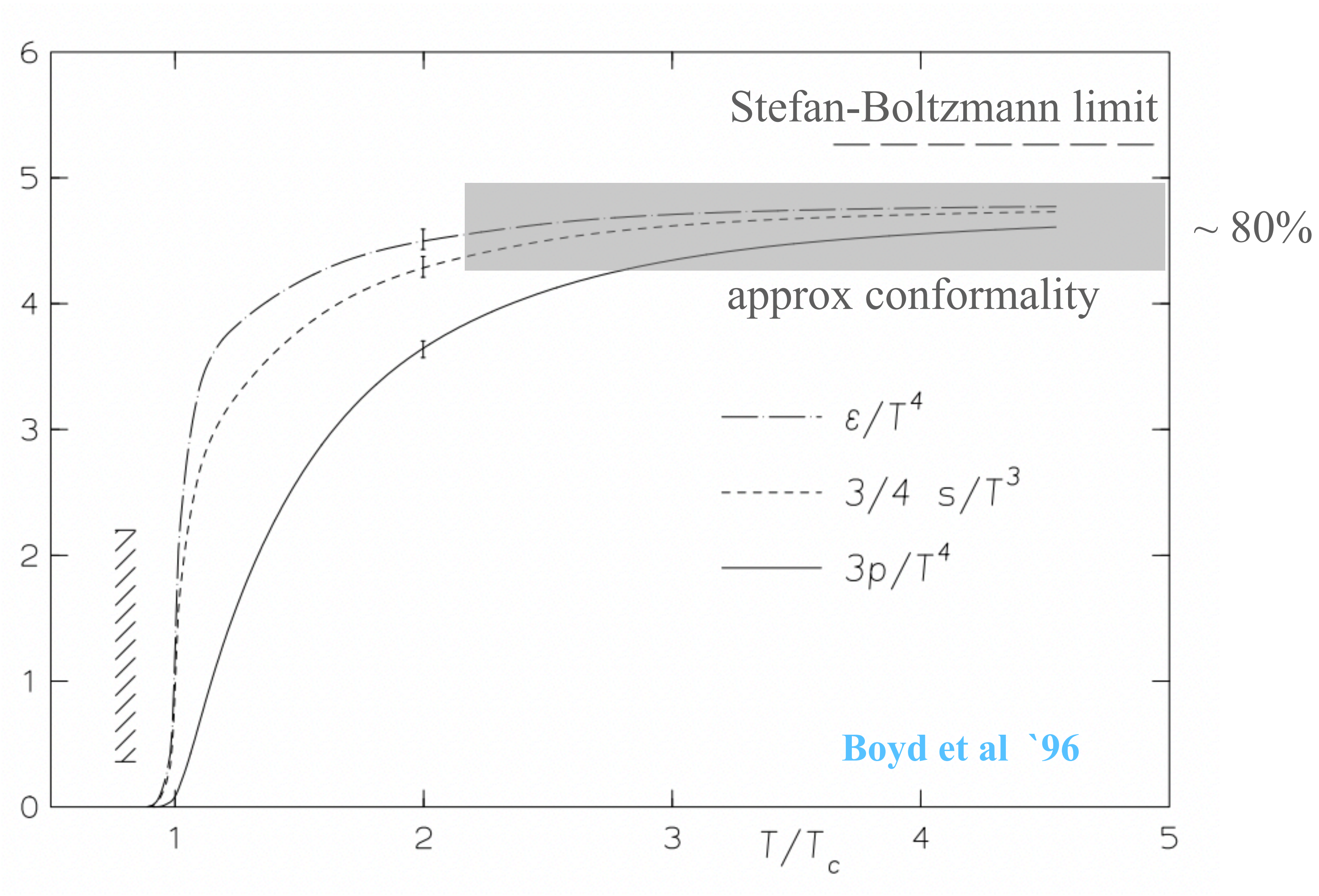
- Proved also using Schwinger-Keldish formalism
- Applies to QCD in the chiral-limit, antiferromagnets, nematic phases, …

Grossi, Soloviev, Teaney, Yan `21



Direct magnetic field imaging of graphene in a high mobility channel

Approximately conformal EoS



$\mathcal{N} = 4$ super Yang-Mills: s(λ = ∞) / s(λ = 0) = 75%

Gubser, Klebanov, Peet `96; Klebanov `00

⟹ a good proxy for T > 2T$_c$

Constructing hydrodynamics action

Most general scalar from T, u, e, K and derivatives ➔ W

$$T^{\mu\nu} = \frac{\delta W}{\delta e^a_\mu} e^\nu_a, \qquad S^\lambda_{ab} = \frac{\delta W}{\delta \omega^{ab}_\lambda}$$

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$$\mu^{ab} = 2u^{[a}m^{b]} + M^{ab} \equiv 2u^{[a}m^{b]} + \epsilon^{abcd}u_c\tilde{M}_d$$

$$W = P(T, m^2, \tilde{M}^2, m\cdot\tilde{M}) + \mathcal{O}(\partial u, \partial T, \partial m, \partial\tilde{M})$$

ideal fluid pressure

gradient corrections

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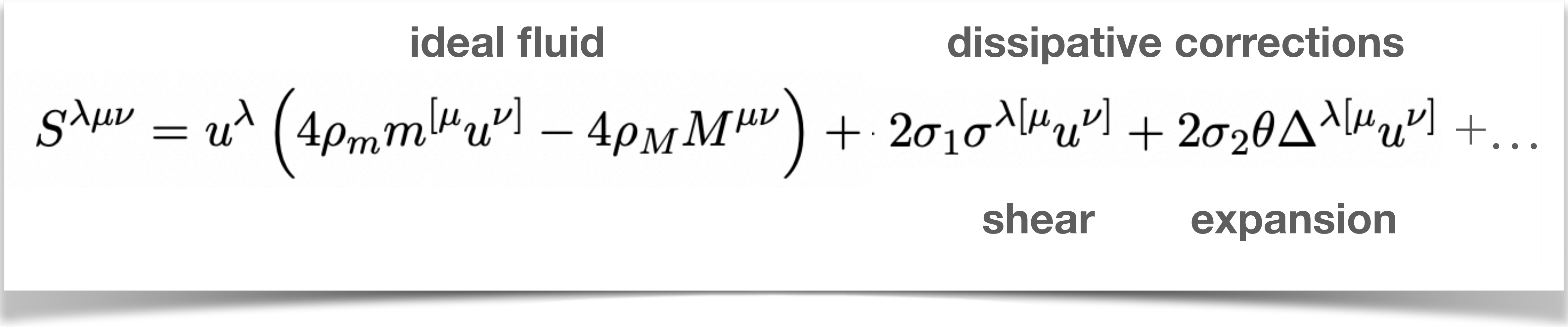

ideal fluid dissipative corrections

$$S^{\lambda\mu\nu} = u^\lambda\left(4\rho_m m^{[\mu}u^{\nu]} - 4\rho_M M^{\mu\nu}\right) + 2\sigma_1\sigma^{\lambda[\mu}u^{\nu]} + 2\sigma_2\theta\Delta^{\lambda[\mu}u^{\nu]} + \dots$$

shear expansion

Ideal spin fluid

Pressure of ideal spin fluid: $P(T, m^2, \tilde{M}^2, m \cdot \tilde{M})$

Constitutive relations:

$$T_i^{\alpha\beta} = \epsilon u^\alpha u^\beta + P\Delta^{\alpha\beta} - 2\left(\frac{\partial P}{\partial m^2} \underbrace{+\,4}_{\text{susceptibilities}} \frac{\partial P}{\partial M^2}\right) u^\alpha \underbrace{M^{\beta\gamma} m_\gamma}_{m \times \tilde{M}\ \text{“spin Poynting”}}$$

$$S_i^\lambda{}_{\alpha\beta} = u^\lambda \underbrace{\rho_{\alpha\beta}}_{\text{spin density}} \,,$$

$$\left(\; M^{ab} \equiv \epsilon^{abcd} u_c \tilde{M}_d \,, \qquad \tilde{m}^{ab} \equiv \epsilon^{abcd} u_c m_d \; \right)$$

$$\epsilon = -P + \frac{\partial P}{\partial T} T + \frac{1}{2} \rho_{ab} \mu^{ab}$$

$$\rho_{\alpha\beta} = 8 \frac{\partial P}{\partial M^2} M_{\alpha\beta} + \ \ldots$$

Gibbs-Duhem relations for ideal spin fluid

Beyond hydrostatics

Spin is "slave" to background flow:

$$m^\mu = a^\mu \qquad M^{ab} = \Omega^{ab}$$

up to O(∇^2)

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Parity invariant, conformal, spin fluid:

ideal fluid **dissipative corrections**

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shear **expansion**

$$\theta = \partial \cdot u$$

Earlier studies

Becattini et al '08; Becattini, Piccinini '08;
Karabali, Nair '14
Florkowski et al '18 '19; Hattori,
X.-G. Huang et al '19;

Application to HIC

Bjorken flow: conformal, boost and parity invariant

$$\delta m^x(\tau) \propto \tau^{-\frac{8}{3}} e^{-\frac{9q^2\eta_0\tau_0}{16T_0\epsilon_0}\left(\frac{\tau}{\tau_0}\right)^{\frac{4}{3}}}, \qquad \delta M^{x\eta}(\tau) \propto \tau^{-\frac{5}{3}} e^{-\frac{9q^2\eta_0\tau_0}{16T_0\epsilon_0}\left(\frac{\tau}{\tau_0}\right)^{\frac{4}{3}}}$$

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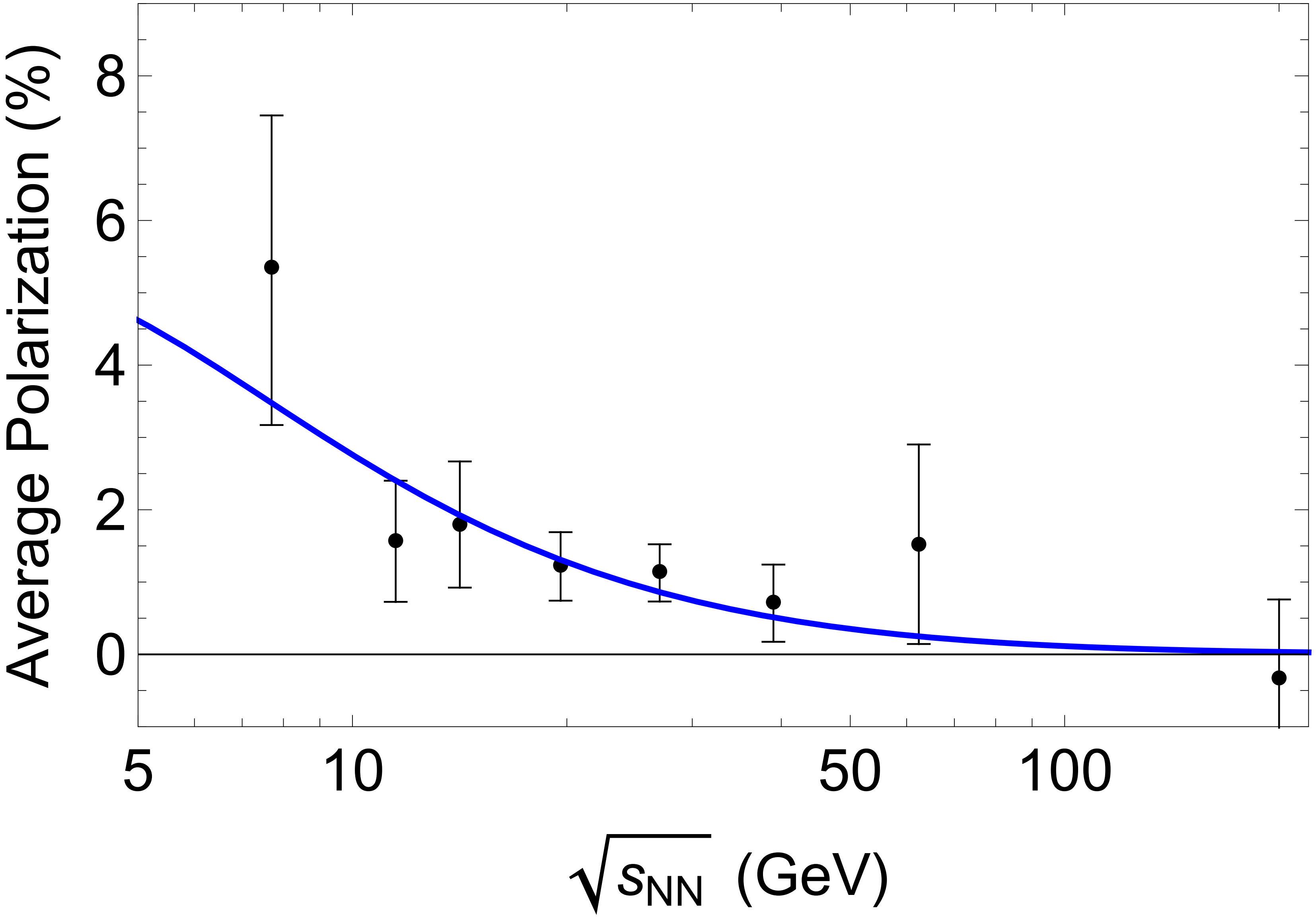

Gallegos, Yarom, UG `21`22

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Belinfante-Rosenberg

$$- 2\chi_1^{(2)} M^{\lambda[\mu}u^{\nu]} + 2\chi_2^{(2)} u^{\lambda}M^{\mu\nu} - 2\chi_3^{(2)} u^{\lambda}u^{[\mu}m^{\nu]} + 4\chi_4^{(2)}\Delta^{\lambda[\mu}m^{\nu]}$$

$$+ 2\sigma_1\sigma^{\lambda[\mu}u^{\nu]} + 2\sigma_2\theta\Delta^{\lambda[\mu}u^{\nu]} ,$$

dissipative corrections

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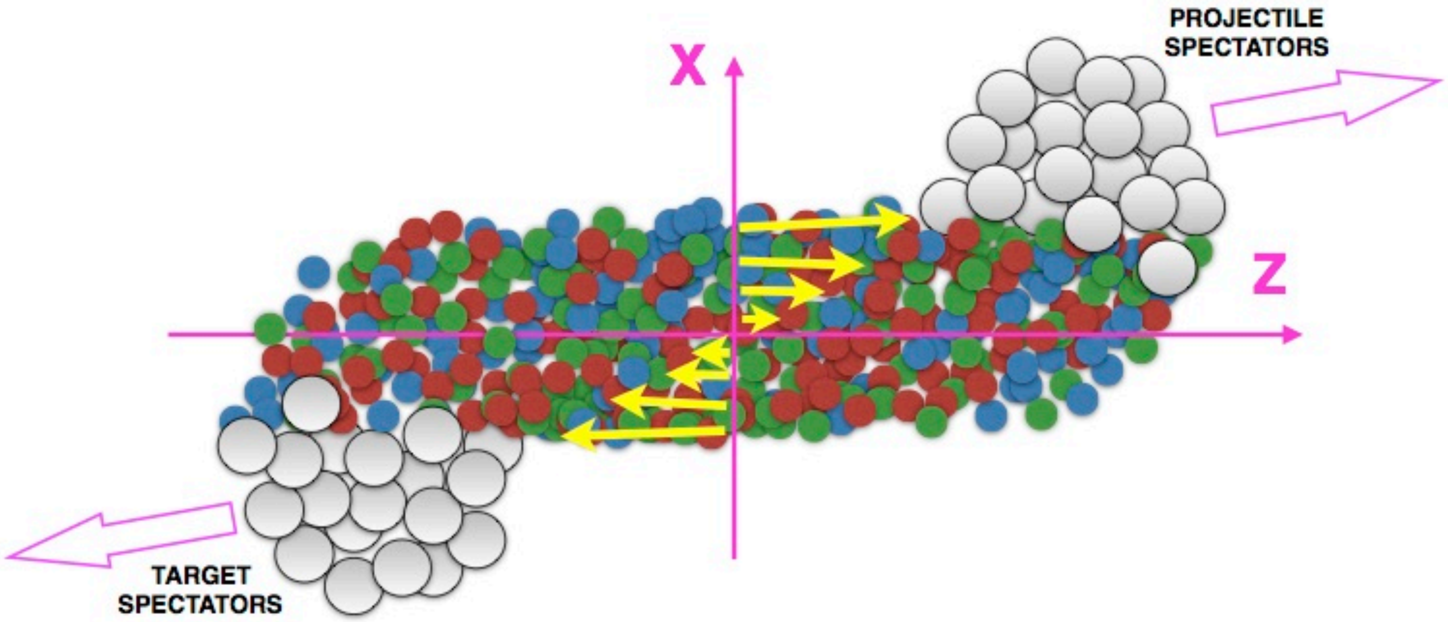

Polarization of hyperon:

$$\Pi_{\mu}(p) = -\frac{1}{4}\epsilon_{\mu\rho\sigma\beta}\frac{p^{\beta}}{m}\frac{\int d\Sigma_{\lambda}p^{\lambda}B(x,p)\mu^{\rho\sigma}}{2\int d\Sigma_{\lambda}p^{\lambda}n_{F}}$$

spin potential

Boltzmann type distribution

freezout surface

Bjorken flow: conformal, boost invariant

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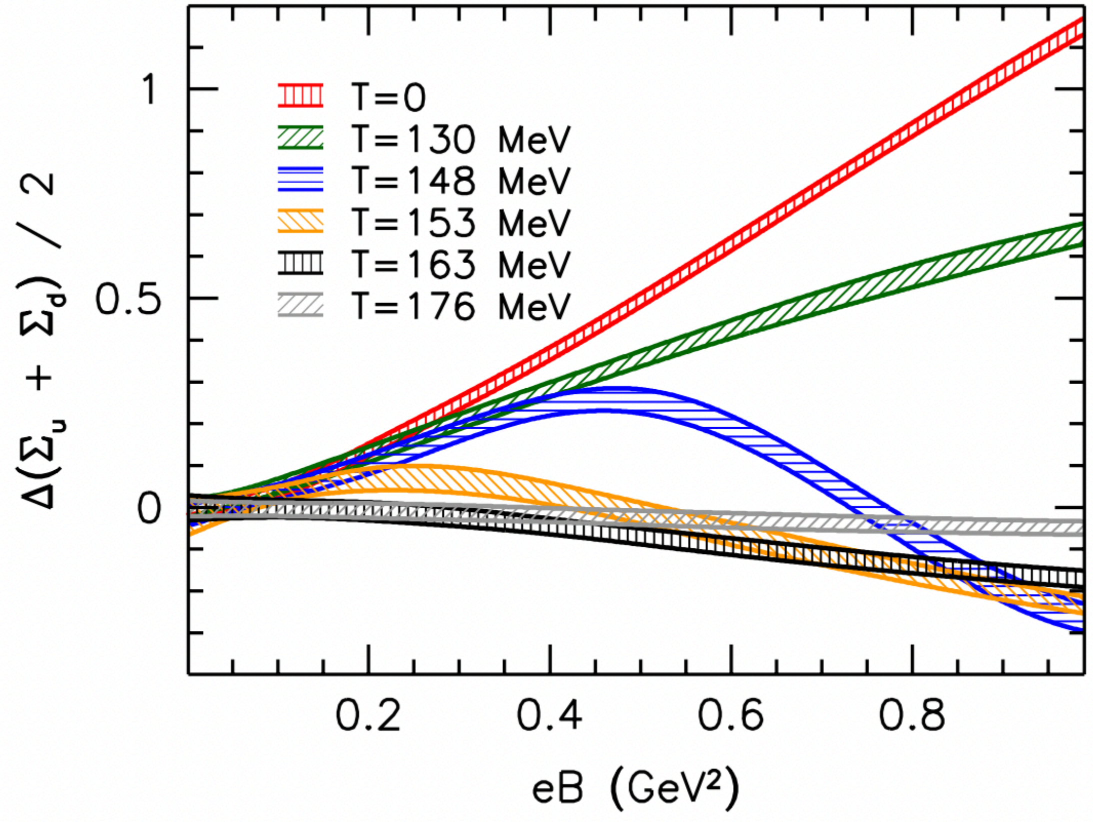
Controlled by x

Two distinct dependence on B:

Explicit dependence on B in tachyon equation $\Longrightarrow$ catalysis
Implicit dependence through background fields $\Longrightarrow$ inverse catalysis



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$$\langle \bar{q} q \rangle = \int \mathcal{D}A e^{-S[A]} \mathrm{det}(D(A,B)+m) \mathrm{tr}(D(A,B)+m)^{-1}$$

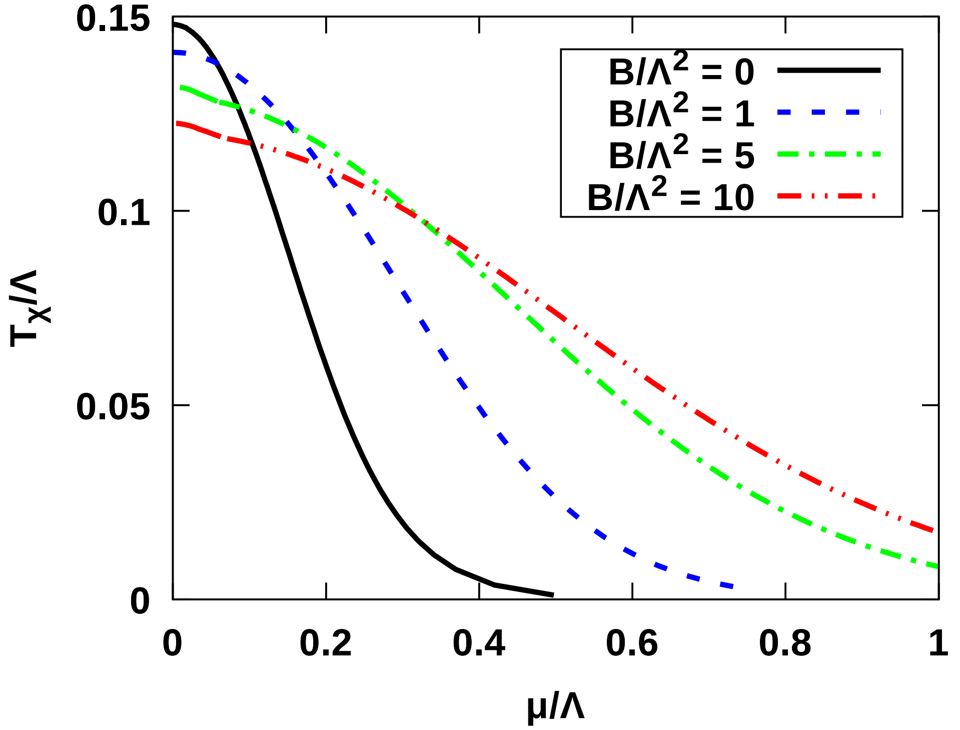
Orders Polyakov loop near T_c
$\Longrightarrow$ punishes A with
small Dirac eigenvalues

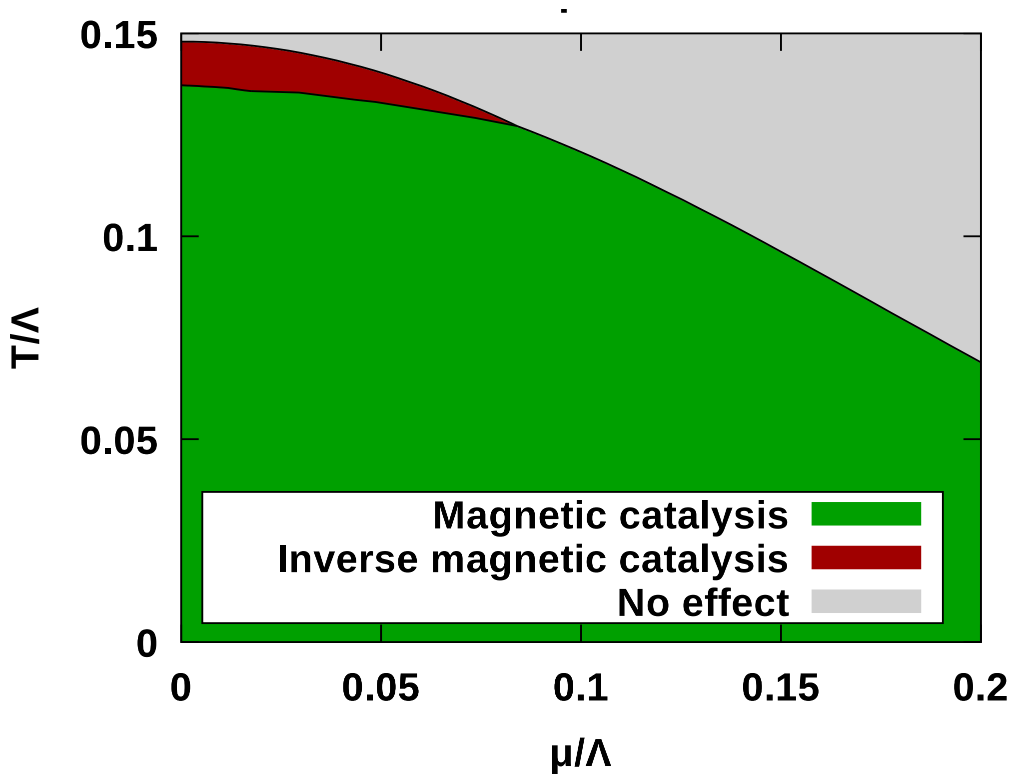
$\sim \rho(0) \propto e|B|$

Banks, Casher `80

D'Elia, Negro `11; Bruckmann, Endrodi, Kovacs `13

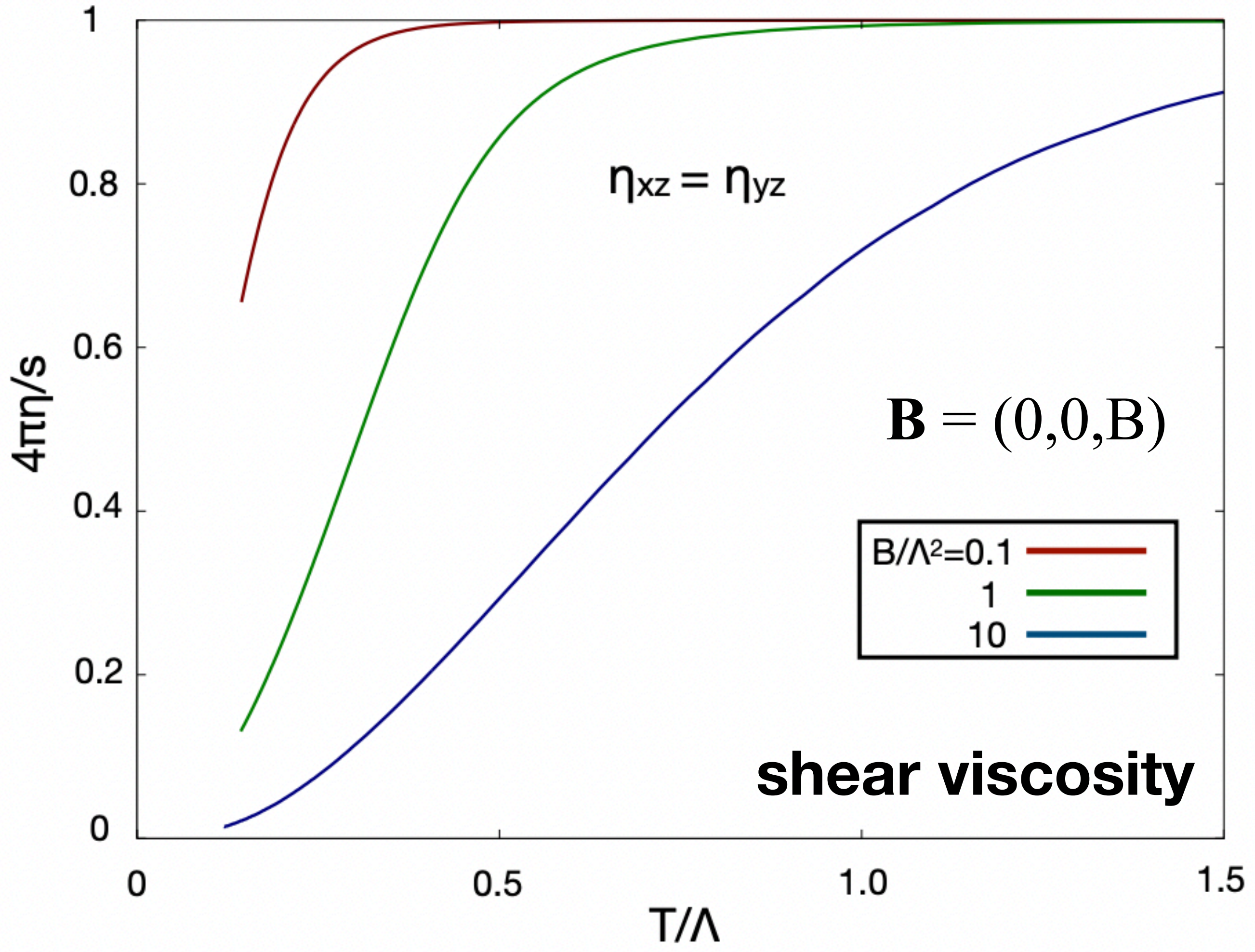
Chiral condensate at finite μ





- B facilitates the chiral transition for $\mu < 0.1 \Rightarrow$ inverse catalysis for small μ
- Magnetic catalysis instead at $\mu > 0.1$
- A small region of inverse magnetic catalysis in the phase diagram

B and anisotropy dependence of transport



shear viscosity

Jarvinen, Nijs, Pedraza, UG `20

Observed earlier

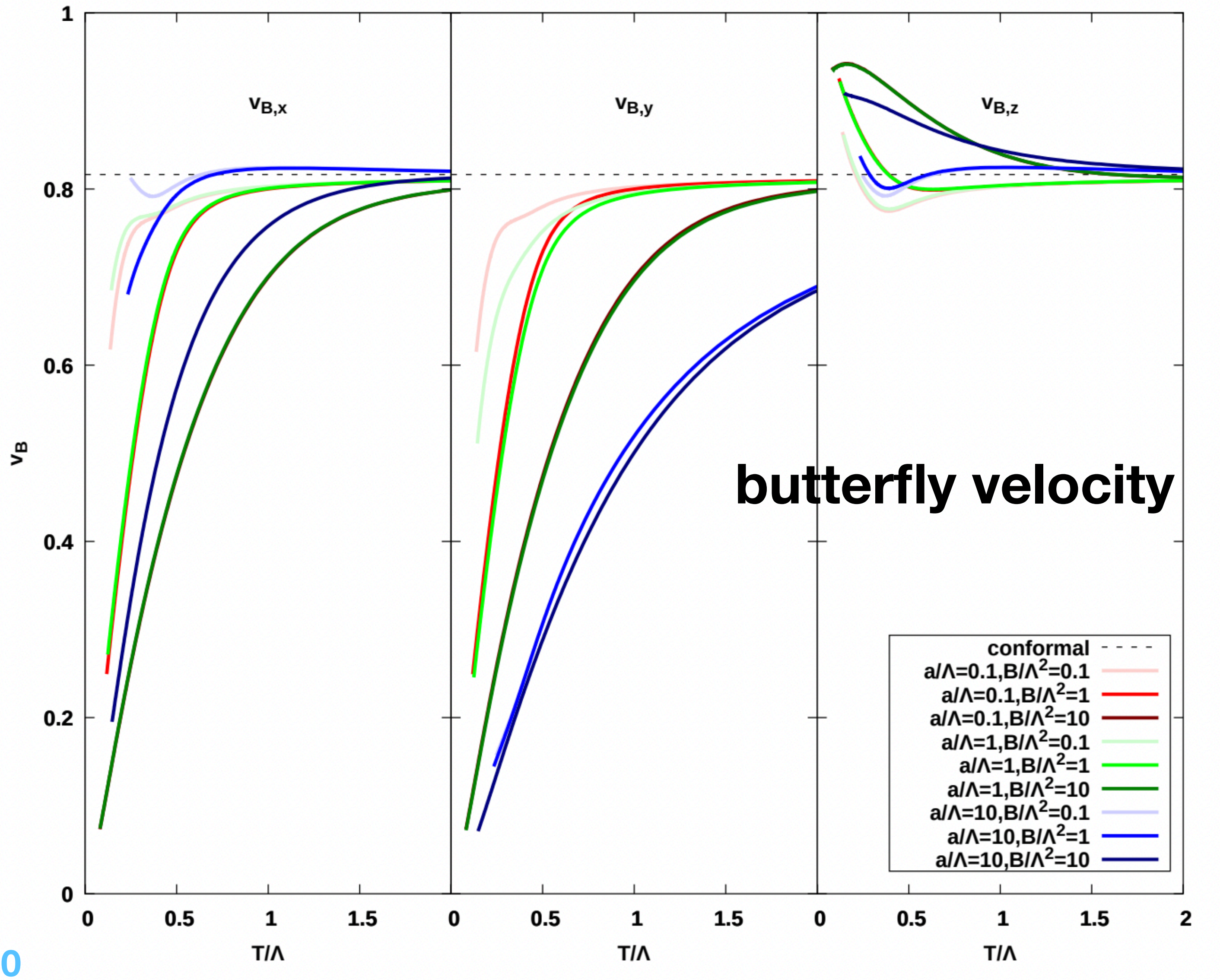
Erdmenger, Kerner, Zeller ’10; Mateos, Trancanelli `11; Cremonini `11 Rebhan, Steineder `11; Giataganas `12; Mamo `12; Jain, Samanta, Trivedi `14; Critelli, Finelli, Noronha, Rougemont `15`16

OTOC Douglas, Maldacena, Shenker ’15

$$\langle [W(t,\vec{x}),V(0,0)]^2 \rangle \sim \frac{1}{N^2} \exp\left[\lambda_L \left(t - \frac{|\vec{x}|}{v_B}\right)\right]$$

$\mathbf{v_B}$ bounds rate of information transfer

Roberts, Swingle `16



butterfly velocity

Giataganas, Pedraza, UG ’18
Jarvinen, Nijs, Pedraza, UG ‘20

Holography for T=0 QCD

$\mathcal{N} = 4$ sYM is not a good proxy for T=0 QCD:

Confinement
Asymptotic freedom
Gapped spectrum
Chiral symmetry breaking

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Top-down: decoupling limit of D-brane setups in IIA/B

Witten '98; Klebanov, Strassler '00; Polchinski, Strassler '00; Maldacena, Nunez '00; Girardello et al '99; Sakai, Sugimoto '04, …

KK modes ~ 1/R contaminate QFT and decouple only beyond the SG approx

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Polchinski, Strassler ’02; Erlich et al ‘05; Brodsky, de Teramond ’05; da Rold, Pomarol ’05; Reece, Csaki ’06, Kiritsis, Nitti, UG ’07; Gubser, Nellore ’08, …

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➔ This talk: Robust (universal) aspects of duality in bottom-up approach

Dynamical bottom-up models

Dual of QCD: noncritical string theory in 5D on dilaton-gravity background (Possibly with Ramond sector e.g. 0B) Polyakov ’96; Gubser, Klebanov Polyakov ‘98

$\Longrightarrow$ **see Aharony and Dubovsky @ strings 2019**

- Integrating out massive string modes $\Longrightarrow$ effective 5D gravity + matter
- IR sum-rules in QCD: OPA semi-closed on relevant/marginal operators

Shifman, Vainshtein, Zakharov ’79

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$$S_g = M^3 N^2 \int d^5x \sqrt{-g}\left(R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi)\right)$$

$T_{\mu\nu}$ $\quad$ $\mathrm{Tr}F^2$ $\quad$ $\beta(\lambda)$

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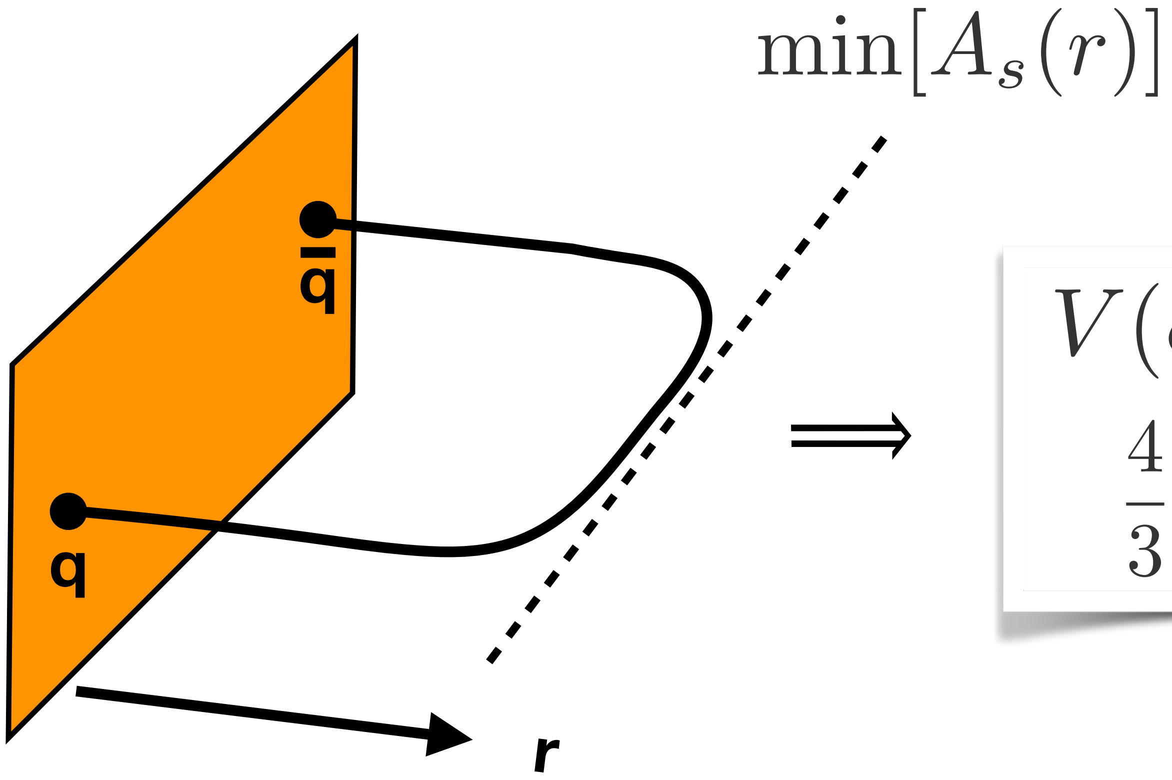
$$T_{\mu\nu} \qquad \mathrm{Tr}F^2 \qquad \beta(\lambda)$$

Determine $V_g(\phi)$ by confinement, asymptotic freedom Kiritsis, UG '07; Gubser, Nellore '08

Universal results: confinement⟹gapped spectrum

Linear confinement: $ds^2 = e^{2A_s(r)}(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$



$$\Longrightarrow$$

$$V(\phi) \to e^{Q\phi}$$

$$\frac{4}{3} \leq Q < \frac{8}{3}$$

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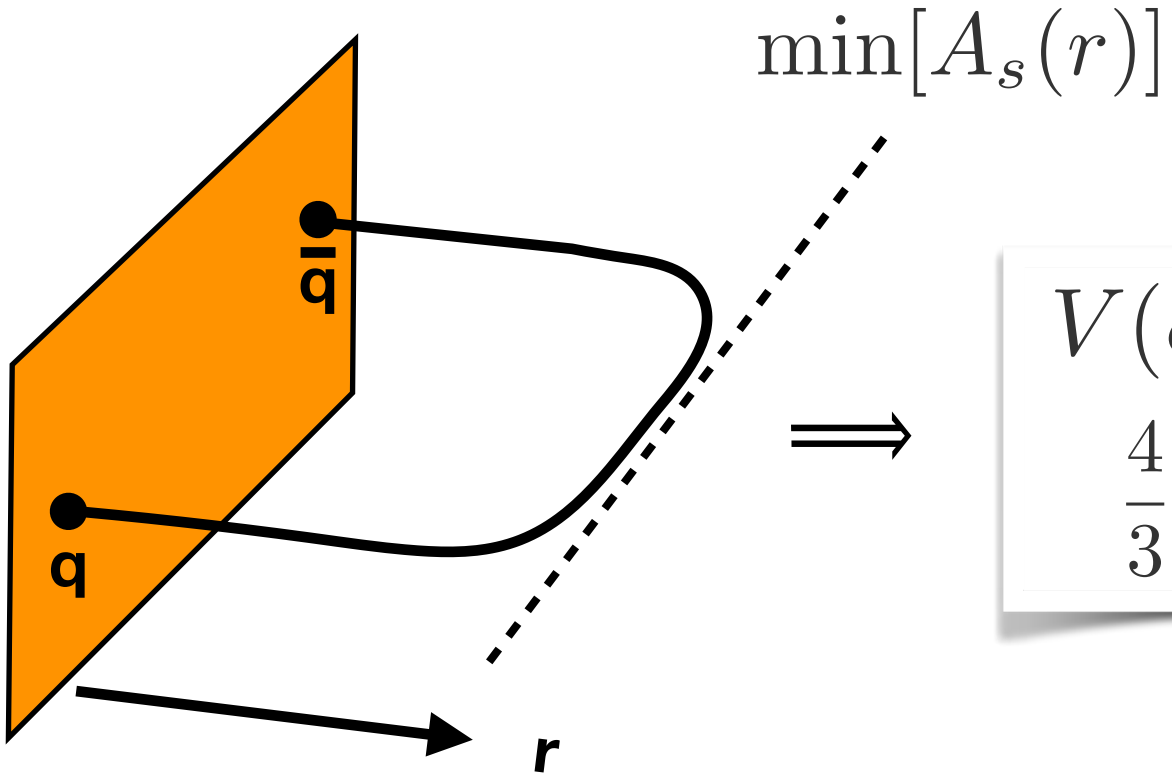
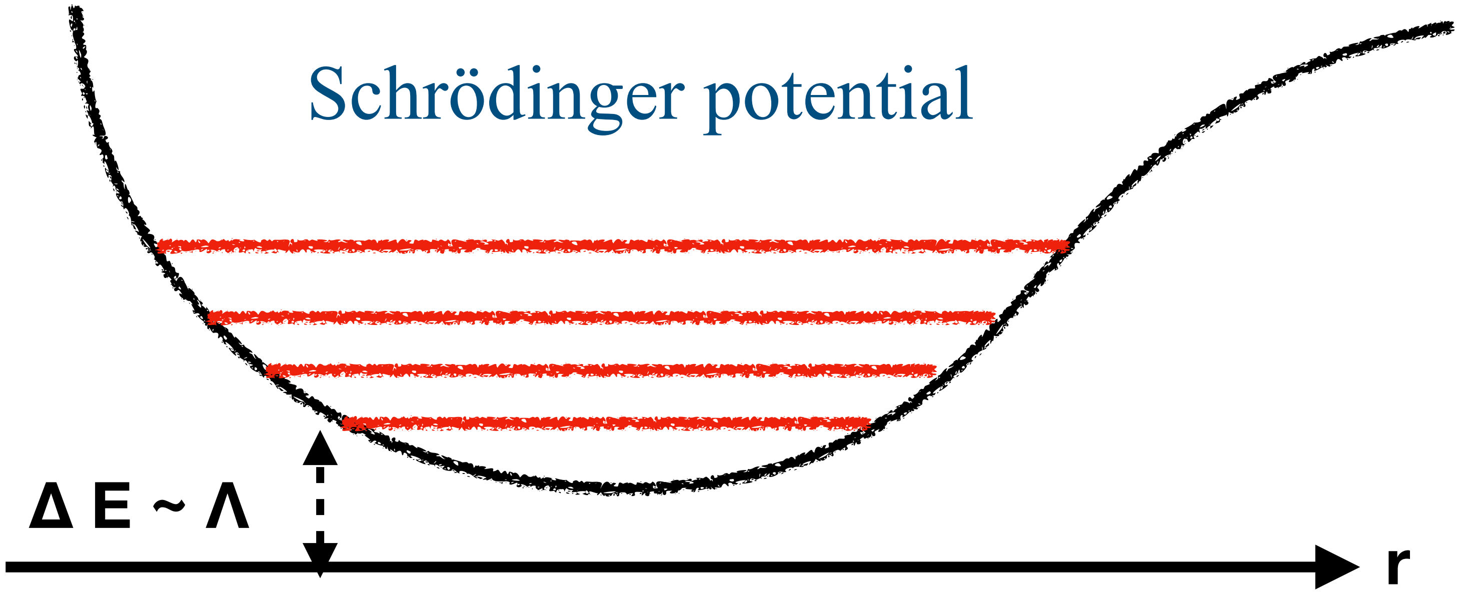


$\Longrightarrow$

$$V(\phi) \to e^{Q\phi}$$
$$\frac{4}{3} \leq Q < \frac{8}{3}$$

$\Longrightarrow$ Gapped spectrum:



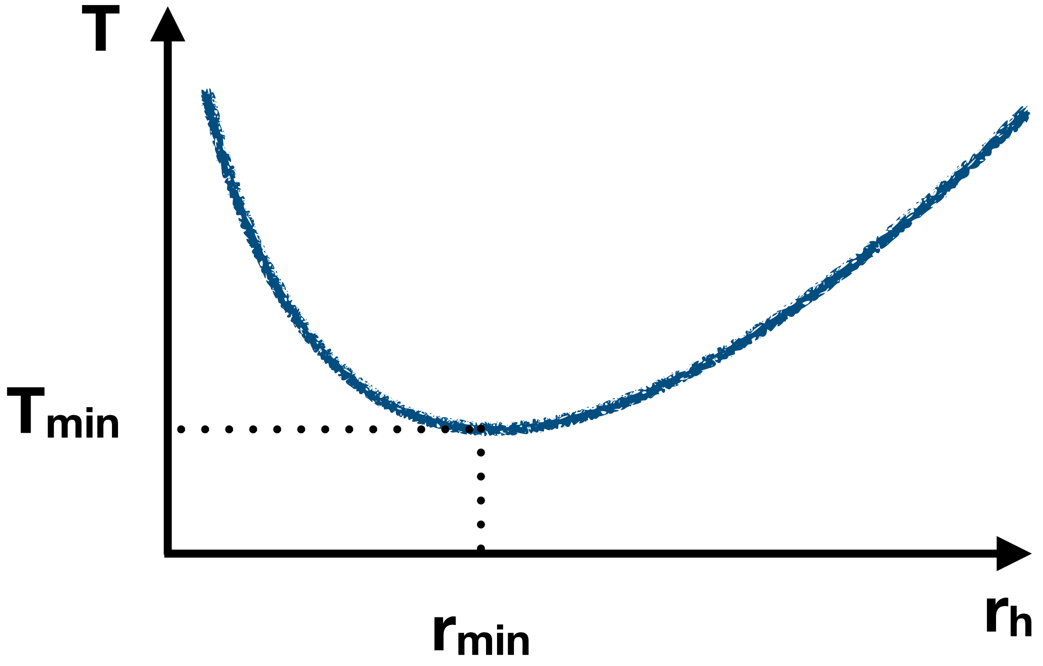
Kiritsis, Nitti, UG ’07

Universal results:1st order deconfinement

Confined $\Longleftrightarrow$ thermal gas $$ds^2 = e^{2A_0(r)}(dr^2 + \delta_{\mu\nu}dx^\mu dx^\nu), \quad \tau \sim \tau + \frac{1}{T}$$

Plasma $\Longleftrightarrow$ black brane $$ds^2 = e^{2A(r)}(\frac{dr^2}{f(r)} + f(r)d\tau^2 + \delta_{ij}dx^i dx^j)$$

- **$\exists$ min(A_s) $\Longrightarrow$ $\exists$ T_{min}**



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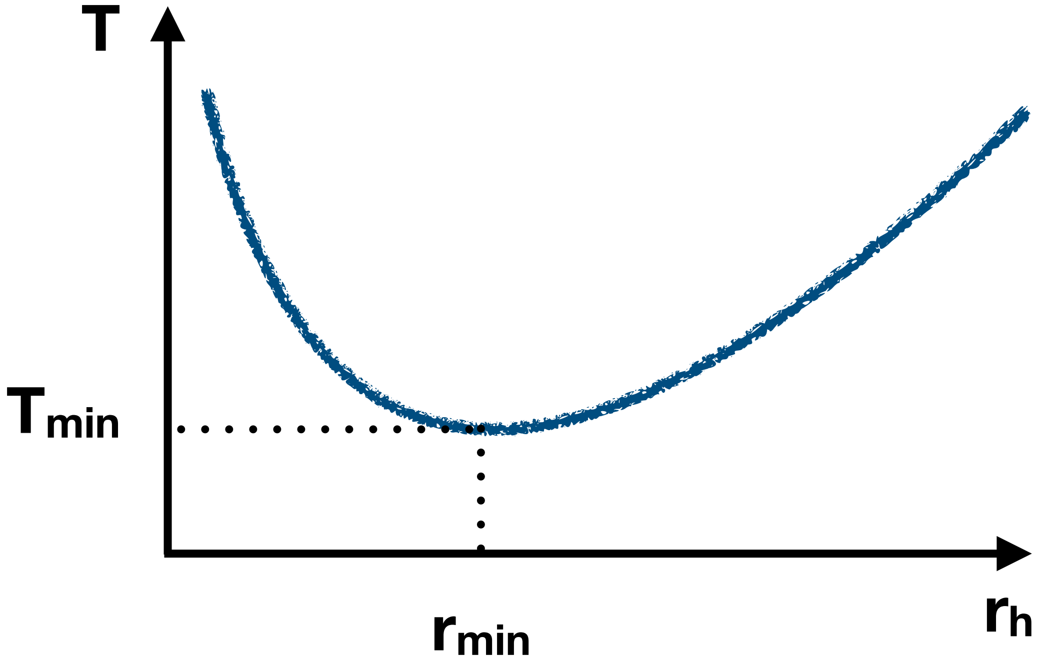
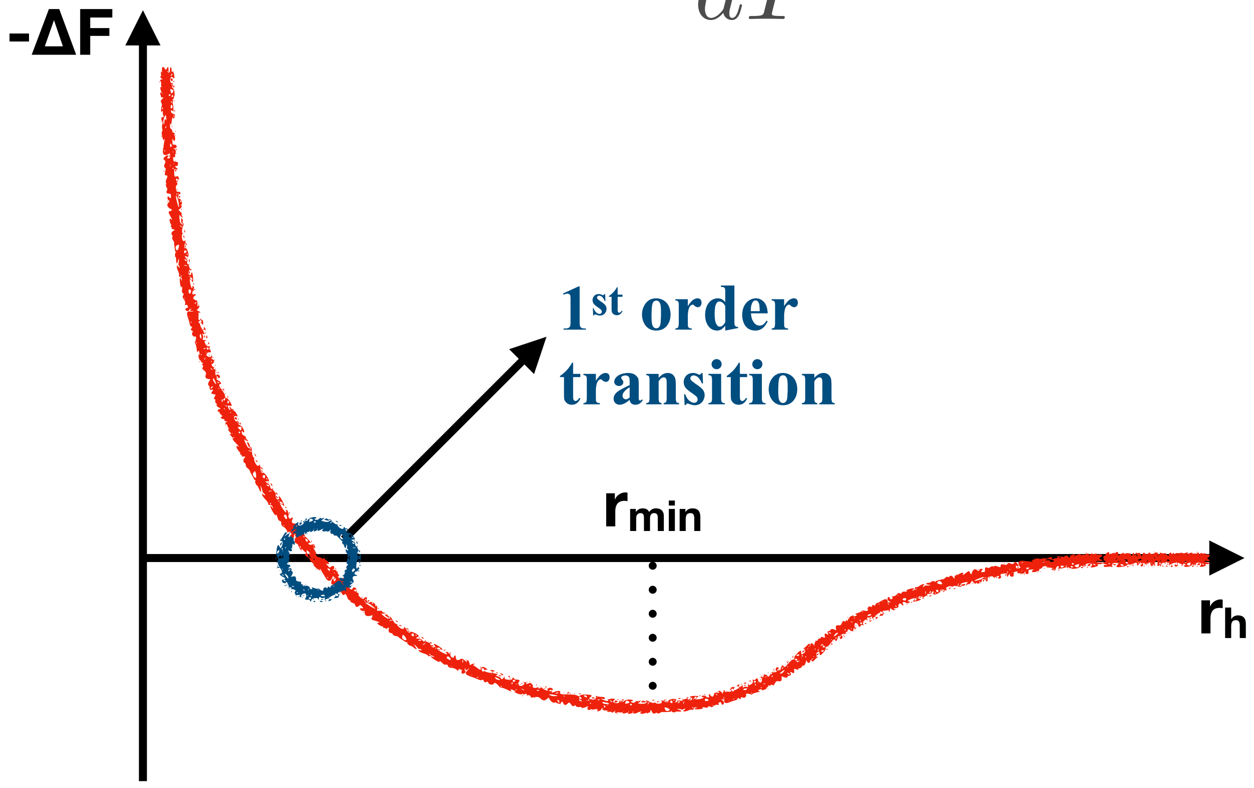
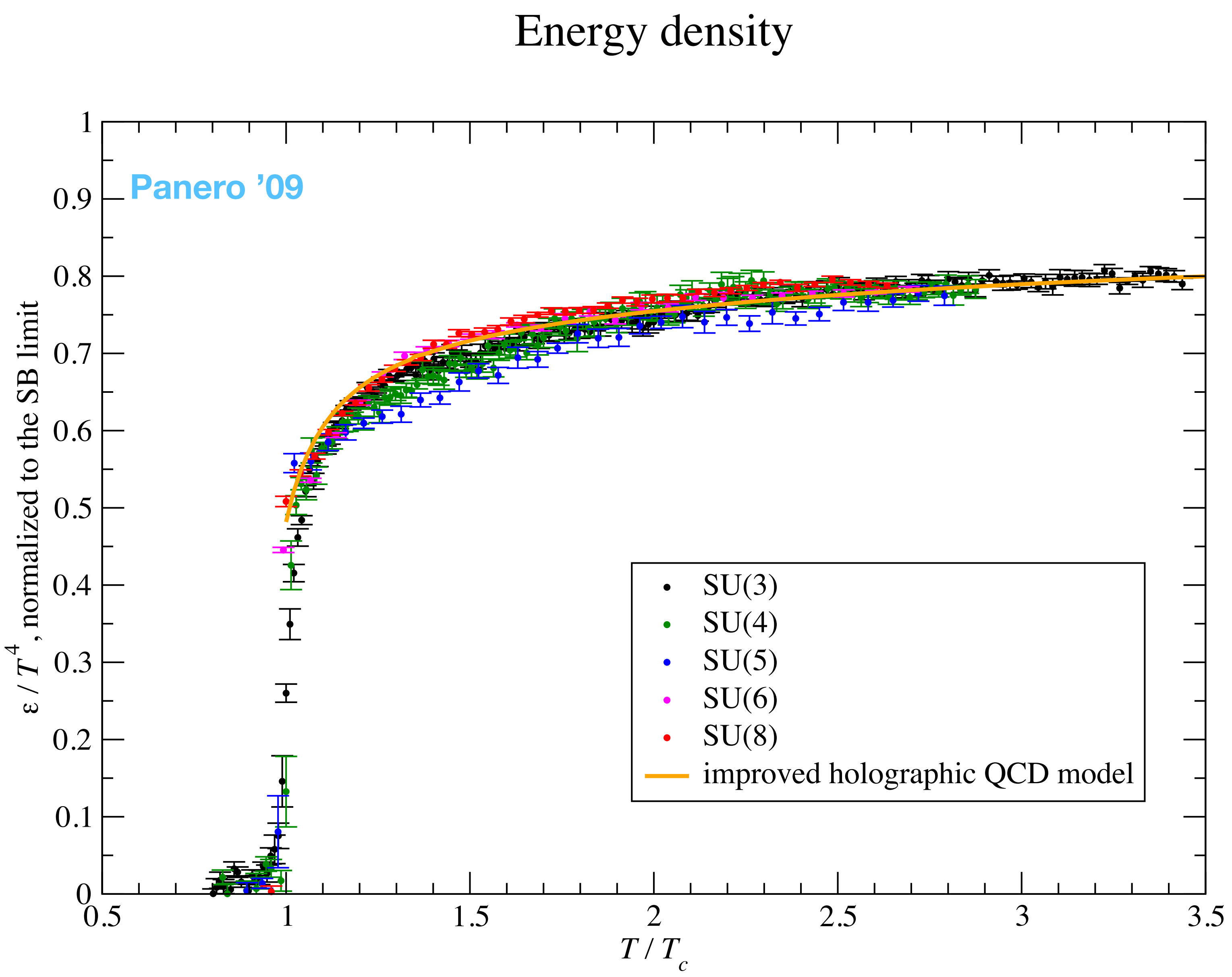
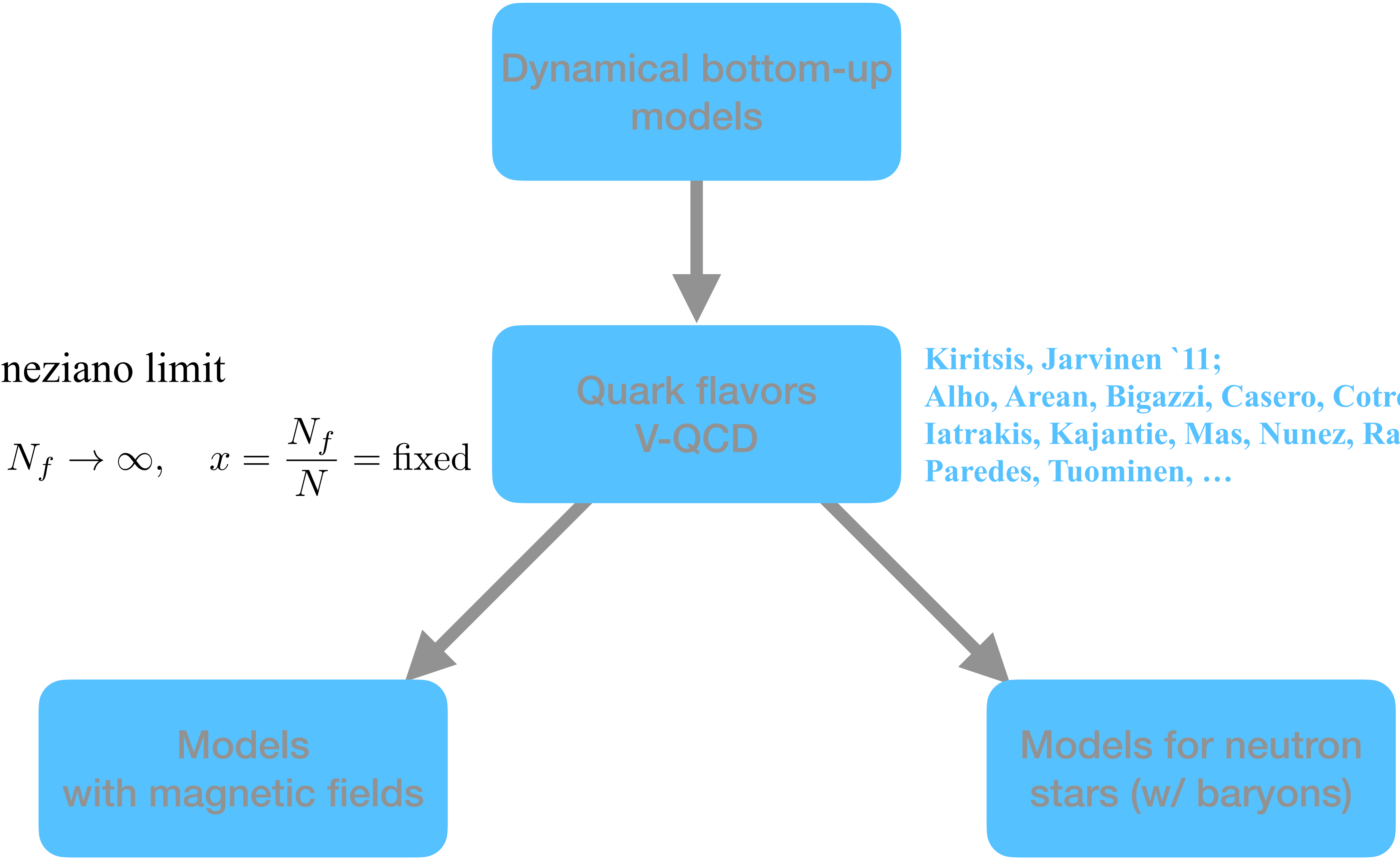
$$S = -\frac{dF}{dT} > 0$$





Non-universal results





Dynamical bottom-up models

Quark flavors V-QCD

Veneziano limit

$$N \to \infty, \quad N_f \to \infty, \quad x = \frac{N_f}{N} = \text{fixed}$$

Kiritsis, Jarvinen `11;
Alho, Arean, Bigazzi, Casero, Cotrone,
Iatrakis, Kajantie, Mas, Nunez, Ramallo, Rosen,
Paredes, Tuominen, …

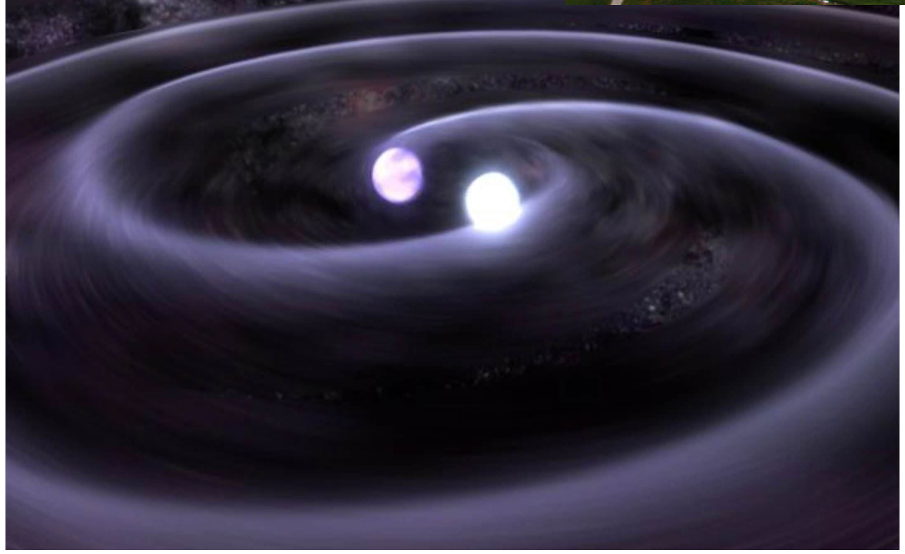
Models with magnetic fields

Models for neutron stars (w/ baryons)

Critelli, Demircik, Erdmenger, Finazzo, Jarvinen, d'Hoker,
Iatrakis, Kaminski, Kraus, Nijs, Noronha, Noronha-Hostler,
Preis, Rebhan, Rougemont, Schmitt, UG, …

Demircik, Ecker, Jarvinen, Jokela,
Hoyos, Ishii, Nijs, Remes, van der Schee,
Tarrio, Subils, Vuorinen, …

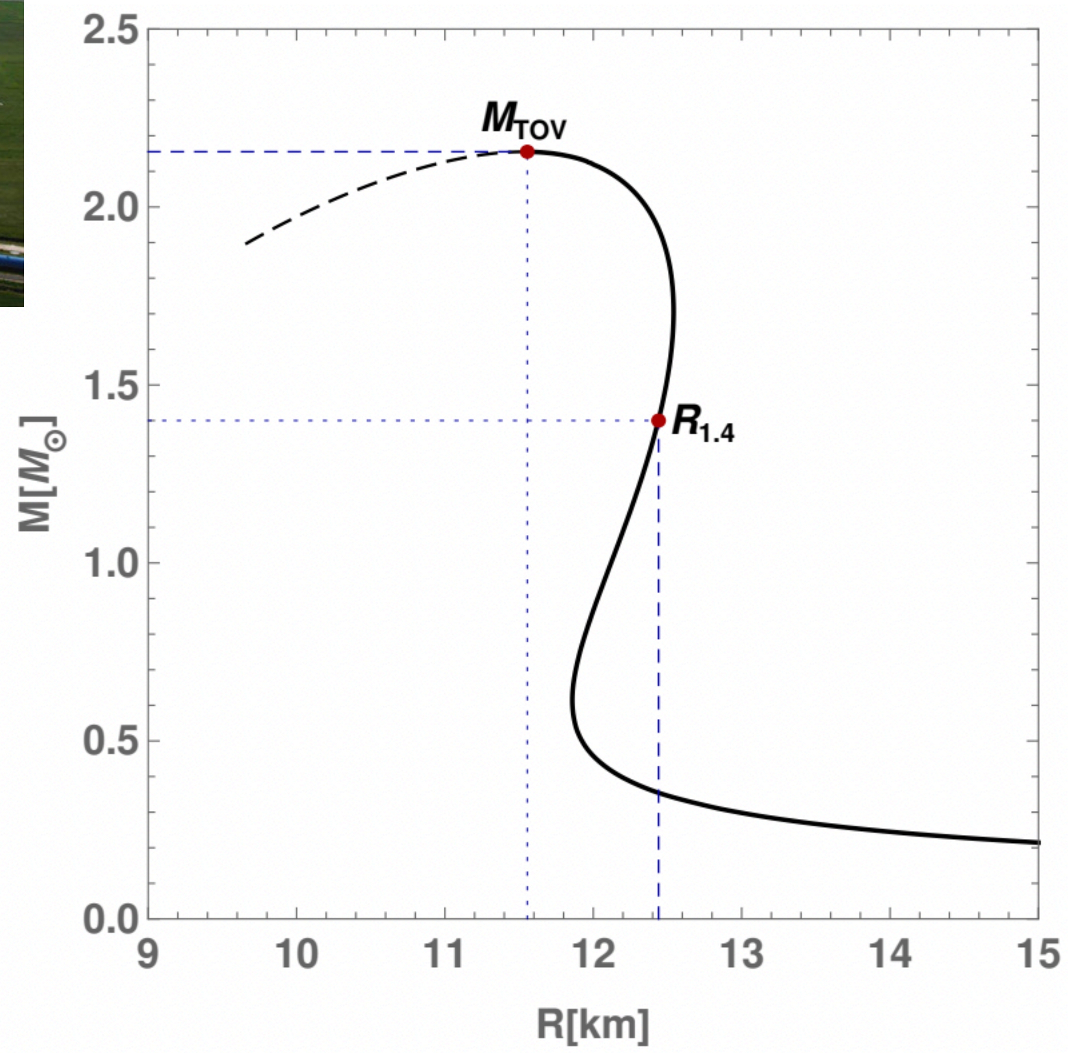
Holographic Neutron Stars



Neutron star mergers

Gravitational wave detectors

- LIGO/Virgo (ongoing)



➡ QCD EoS at finite baryon density

Conformal spin hydro

Equations of motion + constitutive relations: determine T, u and $\mu^{\alpha\beta}$

$$\mu^{ab} = 2u^{[a}\underbrace{m^{b]}}_{\text{“electric”}} + \epsilon^{abcd}u_c\underbrace{\tilde{M}_d}_{\text{“magnetic”}}$$

$$u^\alpha \mathcal{D}_\alpha T = \hat{\eta}\sigma_{\alpha\beta}\sigma^{\alpha\beta}\,,$$

$$\Delta^\nu_\beta \mathcal{D}_\alpha \sigma^{\alpha\beta} = \left(\frac{\Delta^{\nu\beta}}{3\hat{\eta}} - \frac{3\sigma^{\nu\beta}}{T}\right)\mathcal{D}_\beta T\,,$$

$$\Delta^\lambda_\beta u^\alpha \mathcal{D}_\alpha m^\beta = c_1 \Delta^\lambda_\beta \mathcal{D}_\alpha \sigma^{\alpha\beta} + c_2 \Delta^\lambda_\beta \mathcal{D}_\alpha M^{\alpha\beta} + c_4 \sigma^{\lambda\alpha} m_\alpha + c_7 M^{\lambda\alpha} m_\alpha + c_8 \Omega^{\lambda\alpha} m_\alpha\,,$$

$$\Delta^\rho_\alpha \Delta^\sigma_\beta u^\lambda \mathcal{D}_\lambda M^{\alpha\beta} = -\hat{\sigma}\Delta^\rho_\alpha \Delta^\sigma_\beta u^\lambda \mathcal{D}_\lambda \Omega^{\alpha\beta} + c_3 \Delta^{\alpha[\rho}\Delta^{\sigma]\beta}\mathcal{D}_\alpha m_\beta + c_5 \sigma^{\alpha[\rho} M^{\sigma]}{}_\alpha + c_6 \sigma^{\alpha[\rho}\Omega^{\sigma]}{}_\alpha + c_9 M^{\alpha[\rho}\Omega^{\sigma]}{}_\alpha$$

Need: initial conditions + transport coefficients

First order hydrostatics

In this talk: Conformal and parity invariant fluid

Weyl invariance: $\delta S = 0\,, \qquad e^{a}{}_{\mu} \to e^{\phi} e^{a}{}_{\mu}$

$\Longrightarrow$ Conformal Ward identity of spin fluid: $\quad T^{\mu}{}_{\mu} = \mathring{\nabla}_{\mu} S_{\lambda}{}^{\lambda\mu}$

$$\epsilon = \epsilon_0 T^4 + 3\rho_0 M^2 T^2 + \cdots\,, \quad P = \frac{1}{3}\epsilon_0 T^4 + \rho_0 M^2 T^2 + \cdots\,, \quad \rho_{ab} = 8\rho_0 T^2 M_{ab} + \cdots$$

Most general correction to ideal fluid:

$$W_h = \int d^4x |e| \left(\chi^{(1)} T^3 \kappa + 2\chi_1^{(2)} T^2 \kappa_A^{\mu\nu} M_{\mu\nu} + 2\chi_2^{(2)} T^2 K^{\mu\nu} M_{\mu\nu} \right)$$

linear in torsion

$$\Rightarrow \quad S^{\lambda}_{ab} = u^{\lambda}\rho_{ab} + 2T^3\chi^{(1)}\Delta^{\lambda}{}_{[a}u_{b]} - 4T^2\chi_1^{(2)} M^{\lambda}{}_{[a}u_{b]} + 4T^2\chi_2^{(2)} u^{\lambda} M_{ab}$$

Hydrostatics in action formalism

Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom ’12;
Banerjee, Bhattacharyya, Jain, Minwalla, Sharma ’12

- Example: charged fluid in presence of external sources

$$g_{\mu\nu}(x) \qquad A_\mu(x)$$

- Most general scalar $S_{hydro} = \int d^4x \sqrt{g}\, W[g, A]$

- Diffeomorphism and gauge invariance: hydro equations

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\mu \qquad \partial_\mu J^\mu = 0$$

- Thermal equilibrium: timelike Killing vector ξ

$$|\xi| = 1/T \qquad \xi/|\xi| = u \qquad u \cdot A = \mu_E$$

- Expand W in T, u, μ_E and derivatives: constitutive relations

⟹ Spin hydrodynamics from W[e,ω]

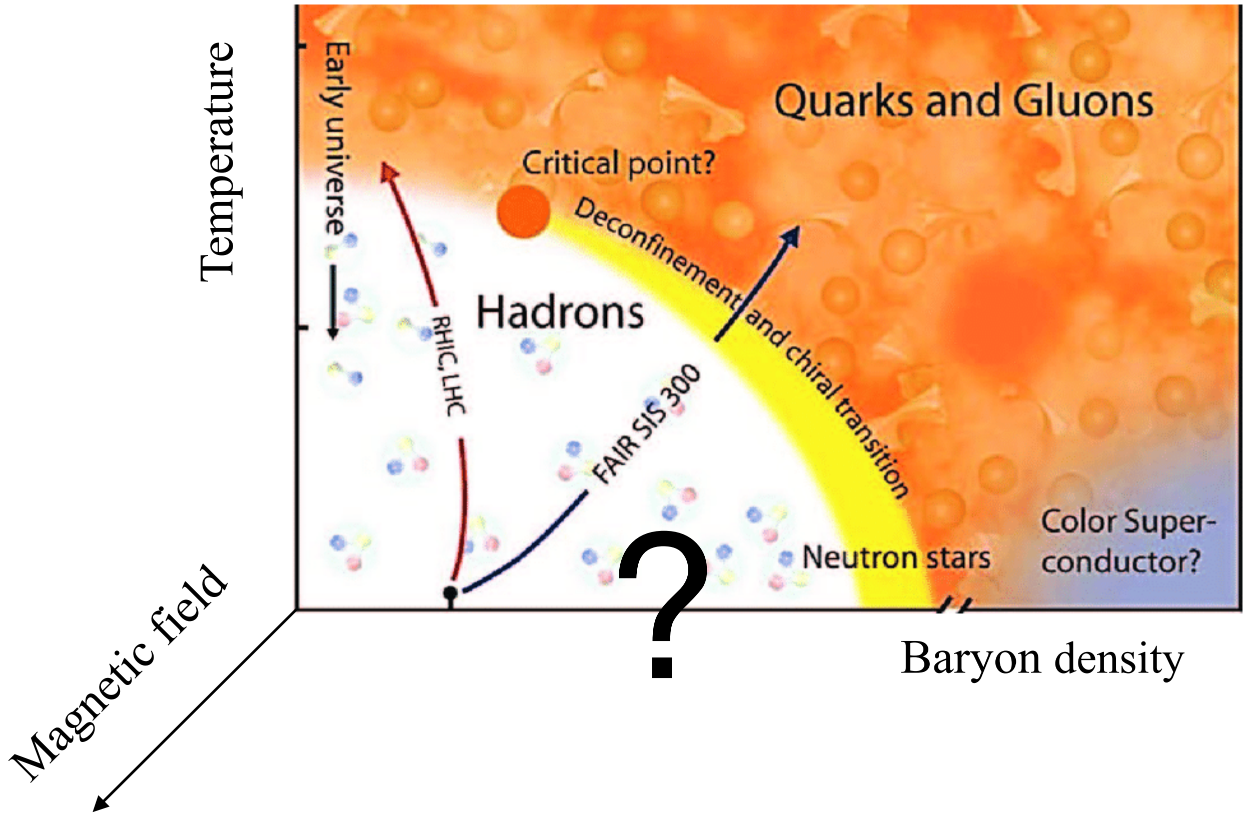
vielbein

spin connection

Open questions

Ground state: dependence of $\langle \bar{q}q \rangle$ on B inverse magnetic catalysis

Thermodynamics: phase diagram of QCD at finite B



Hydrodynamics and transport: 2 conductivities, 2 shear, 3 bulk viscosities
B dependence of η, ζ

Hernandez, Kovtun ’17
Grozdanov, Hofman, Iqbal ‘17

Anomalous transport: chiral magnetic and vortical effects, Chern-Simons diffusion rate

Out of equilibrium: initial conditions for hydro, generation of chiral imbalance

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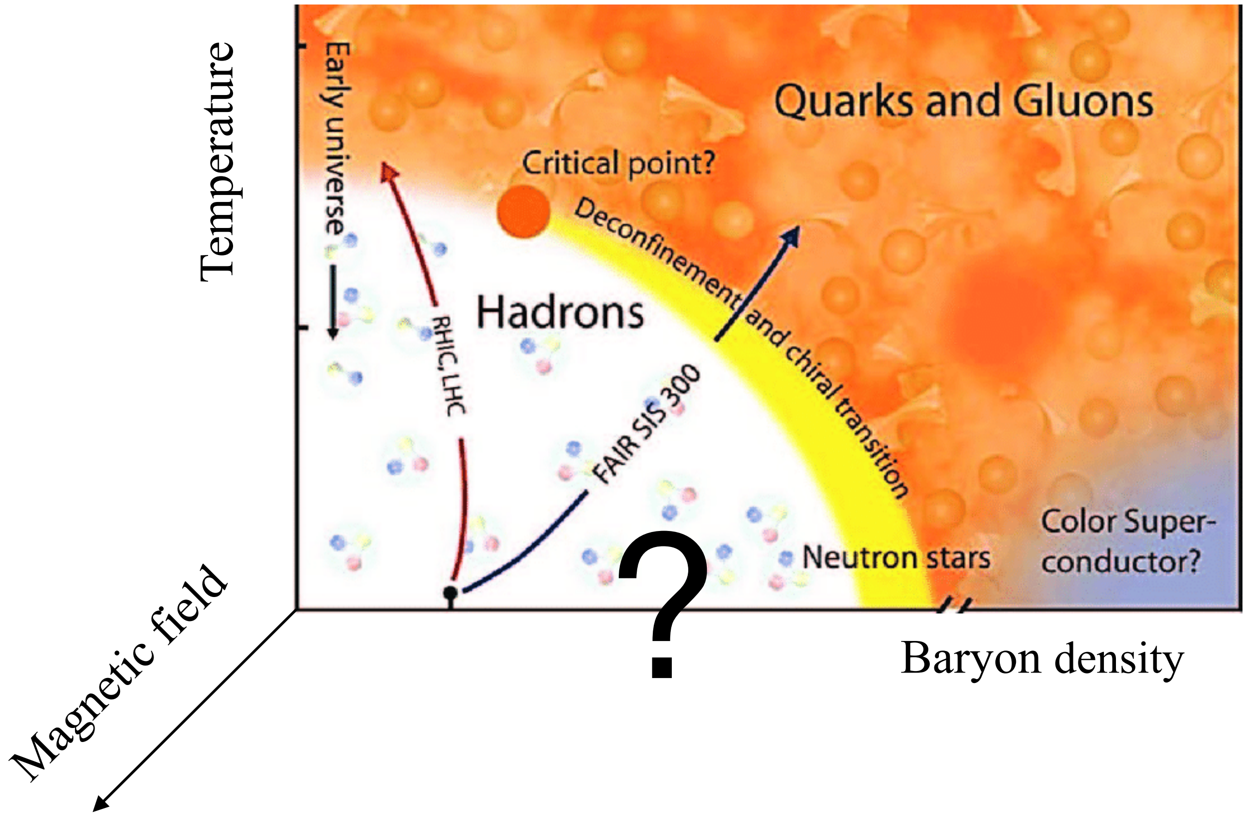

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Improved holographic QCD in the Veneziano limit

Kiritsis, Nitti, UG ’07; Kiritsis, Nitti, Mazzanti, UG ’08 ’09
Jarvinen, Kiritsis ’11; Alho et al ’12

$$S_g = M^3 N_c^2 \int d^5x \, \sqrt{-g} \left(R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right)$$

Glue sector

$T_{\mu\nu}$ $\mathrm{tr}\, G^2$

$$S_f = -\frac{1}{2} M^3 N_c \mathbb{T}r \int d^4x \, dr \, \left(V_f(\lambda, T^\dagger T) \sqrt{-\det \mathbf{A}_L} + V_f(\lambda, T T^\dagger) \sqrt{-\det \mathbf{A}_R} \right)$$

Quark sector

$$\mathbf{A}_{L\,MN} = g_{MN} + w(\lambda, T) F^{(L)}_{MN} + \frac{\kappa(\lambda, T)}{2} \left[(D_M T)^\dagger (D_N T) + (D_N T)^\dagger (D_M T) \right]$$

$$\mathbf{A}_{R\,MN} = g_{MN} + w(\lambda, T) F^{(R)}_{MN} + \frac{\kappa(\lambda, T)}{2} \left[(D_M T)(D_N T)^\dagger + (D_N T)(D_M T)^\dagger \right]$$

$$V_M = \frac{A^L_M + A^R_M}{2} \;, \; A_M = \frac{A^L_M - A^R_M}{2} \,. \qquad D_M T = \partial_M T + i T A^L_M - i A^R_M T \,.$$

$U(1)_B$ ⇔ magnetic field $U(1)_A$ $\overline{q}q$

$$S_a = -\frac{M^3 N_c^2}{2} \int d^5x \sqrt{g} \, Z(\lambda) \left[da - x \left(2V_a(\lambda, T) \, A - \xi \, dV_a(\lambda, T) \right) \right]^2$$

$\mathrm{tr}\, G \wedge G$

CP-odd sector

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$U(1)_B$ ⇔ magnetic field $\quad$ $U(1)_A$ $\quad$ $\overline{q}q$

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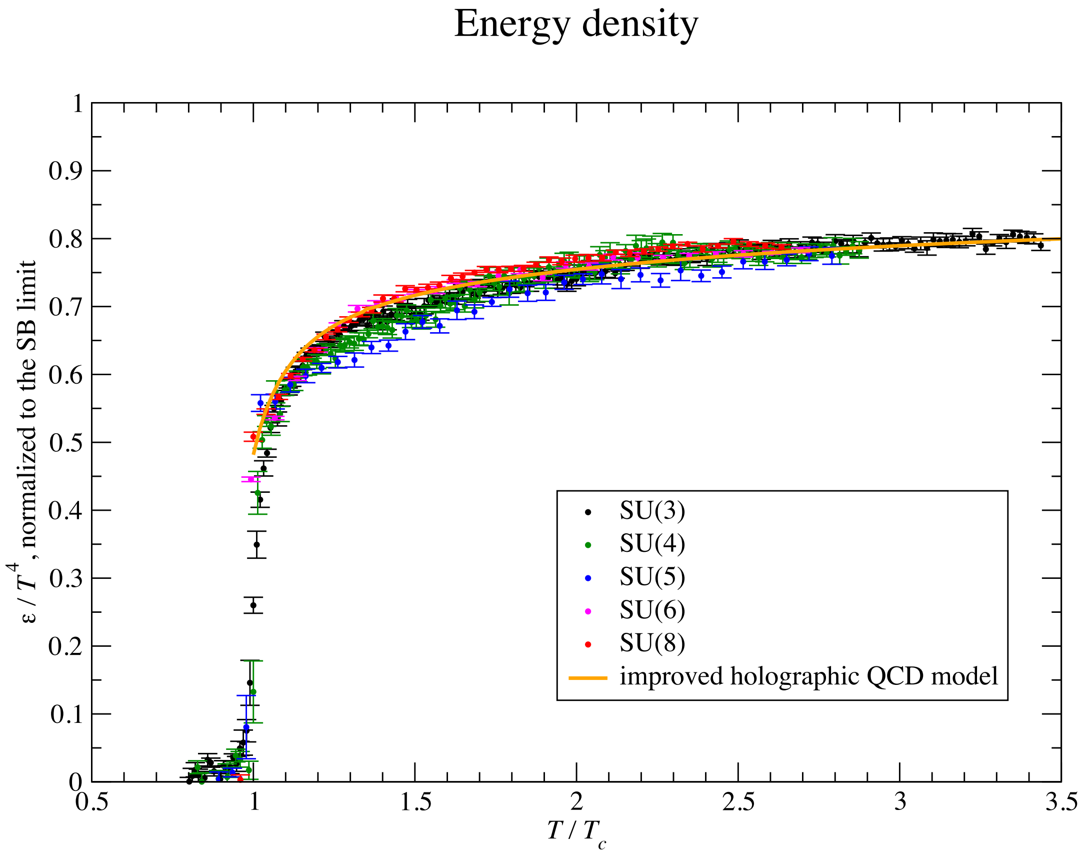
$\mathrm{tr}\, G \wedge G$

CP-odd sector

Fixing the potentials

- Fix V_g by non-singular IR, linear confinement, linear mass spectrum, lowest glueball mass, $\Delta S(T_c)$



Panero ’09

- Fix V_f , $\kappa(\lambda)$ by non-singular IR, qualitative features of the phase diagram in μ and x, condensate anomalous dimension, chiral anomaly, meson mass spectrum

Jarvinen, Kiritsis ’11; Alho et al ’12 ’13

- Choose $w(\lambda) = \kappa(c\lambda)$ by conductivity, diffusion const. of the plasma

Iatrakis, Zahed ‘12; Alho et al ’13

$$w(\lambda) = \kappa(c\lambda) = \frac{(1+\log(1+c\,\lambda))^{-\frac{1}{2}}}{\left(1+\frac{3}{4}\left(\frac{115-16x}{27}-\frac{1}{2}\right)c\,\lambda\right)^{4/3}}$$

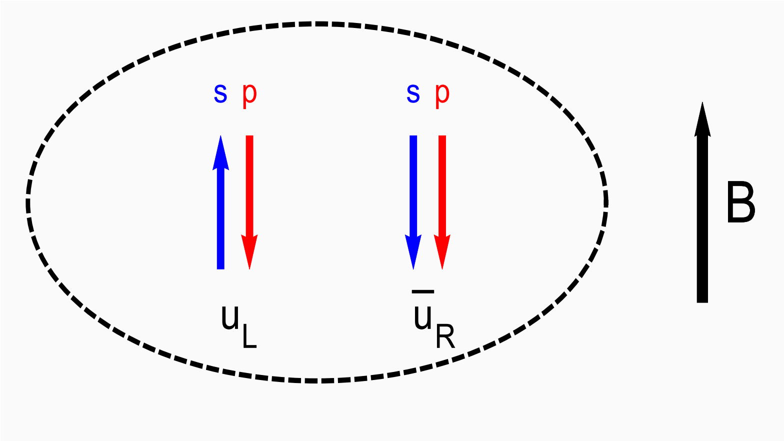
- Fix Z by topological susceptibility, axial glueball spectrum

$$Z(\lambda) = Z_0\left(1+c_4\lambda^4\right)$$

$$0 \lesssim c_1 \lesssim 5, \quad 0.06 \lesssim c_4 \lesssim 50.$$

Magnetic catalysis

Klevansky, Lemmer ’89; Suganuma, Tatsumi ’91; Gusynin, Miransky, Shovkovy ‘94



- B catalyses chiral symmetry breaking
- Generic: QED, NJL, free(!) … 2+1, 3+1
- B aligns spins, effectively reduces 3+1⇒ 1+1
- Stronger correlation between opposite chiralities

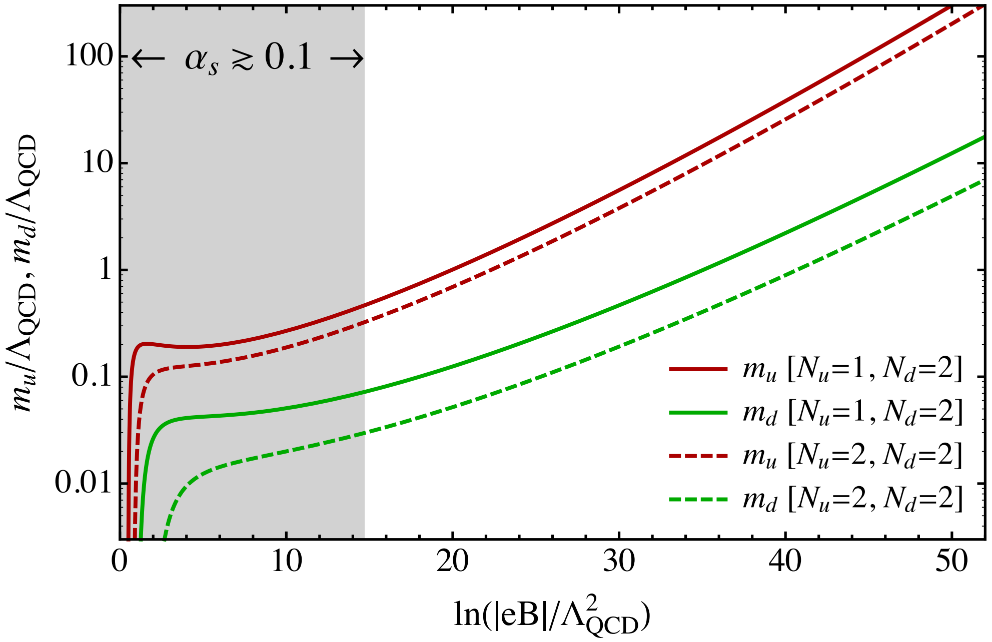
Free chiral fermions in 2+1:

$$\langle \bar{q}q \rangle = \frac{|eB|}{2\pi}$$

NJL in 3+1(supercritical):

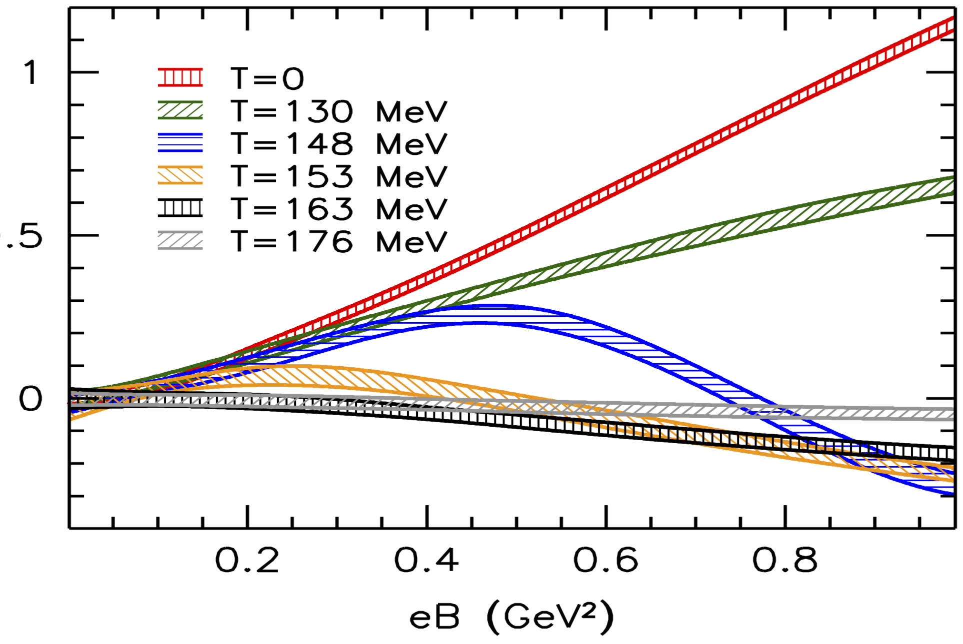
$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 \left(1 + \frac{|eB|^2}{3G^4 (\langle \bar{q}q \rangle_0)^4 \log(\Lambda/G\langle \bar{q}q \rangle_0)^2} \right)^{\frac{1}{2}}$$

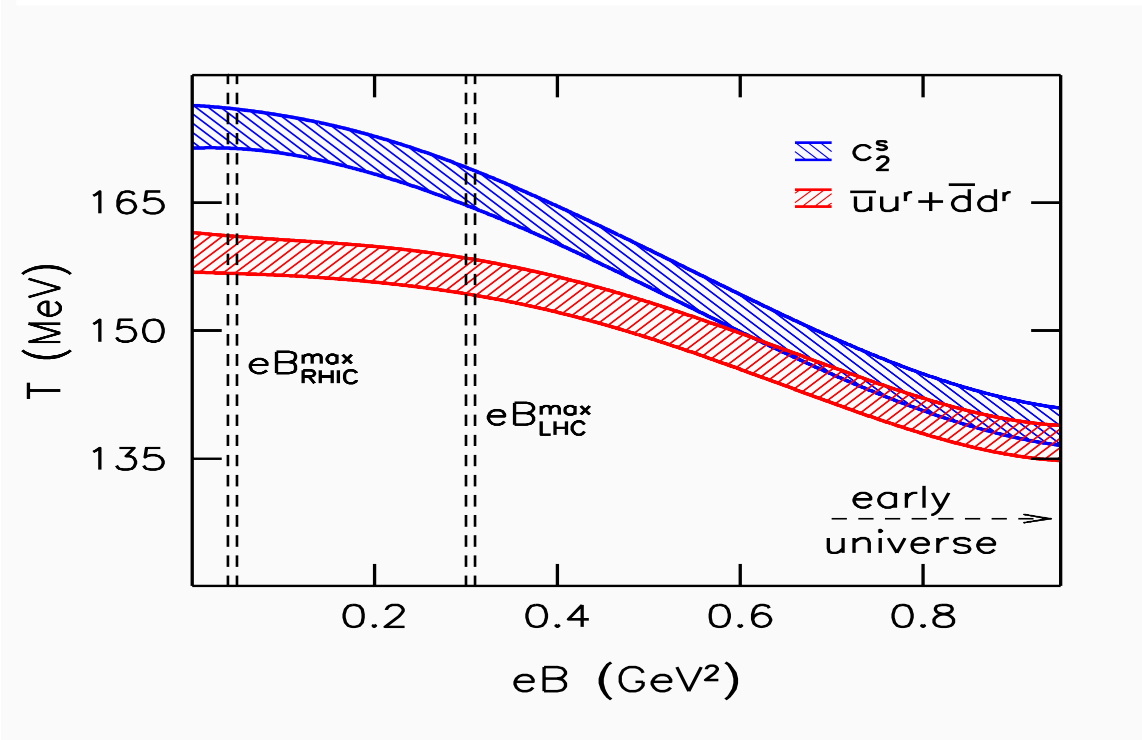
Gap equation in resummed pQCD



Magnetic catalysis on the lattice

Bali, Schafer et al ’11 ’12

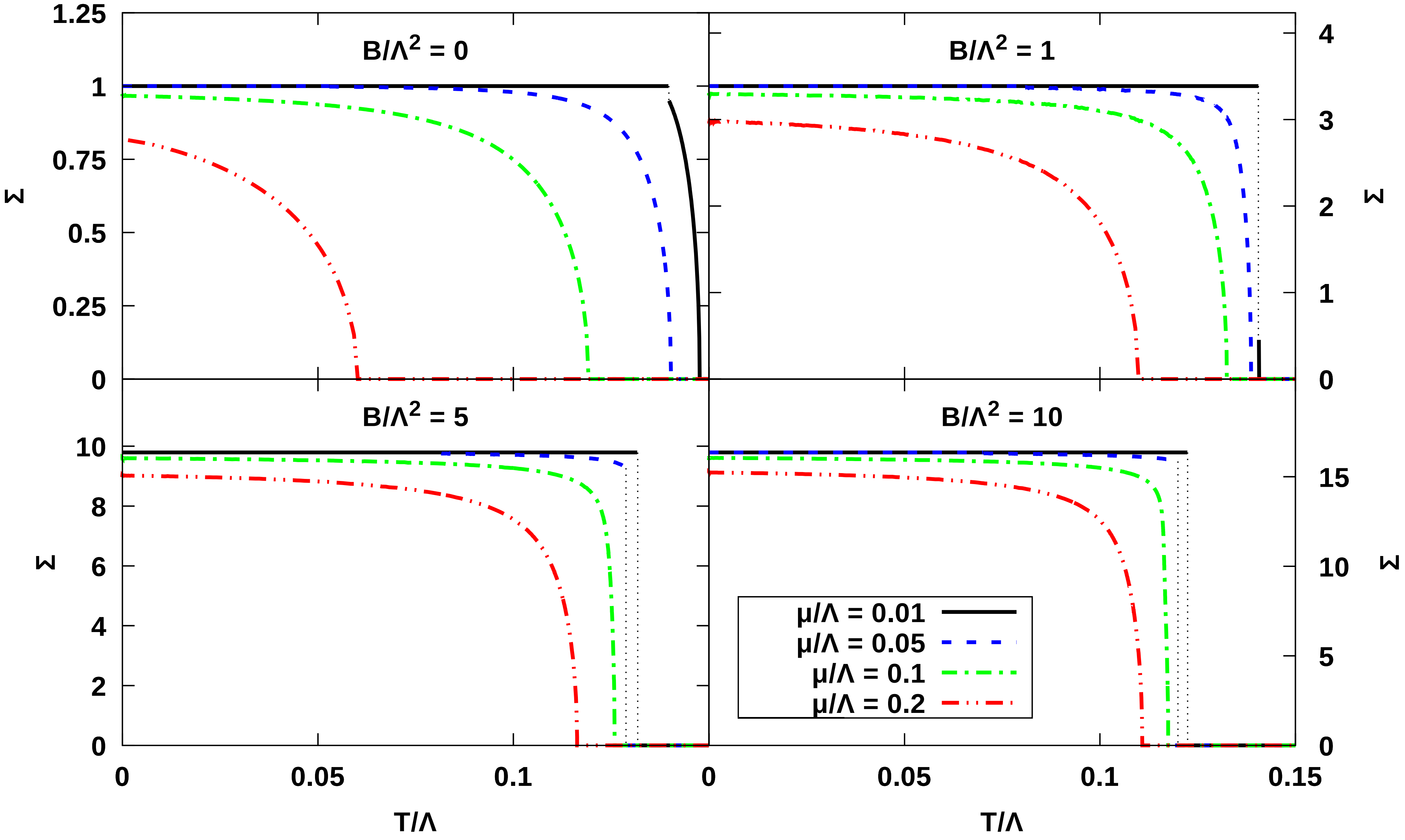




- B acts destructively for T ≳ T_c
- Inverse effect missed in earlier studies with large m & coarse lattices

D’Elia et al ‘11

Chiral condensate at finite μ



- μ decreases the condensate at fixed B
- B generically increases the condensate, except around T_χ and for μ < 0.1
- No T dependence in the confined phase, due to $1/N^2$ suppression

A heuristic discussion

Jarvinen, Nijs, Pedraza, UG ‘18

Introduce anisotropy through space dependent θ-term: θ=a z

$$Z[A_5,\theta] = \int \mathcal{D}q\,\mathcal{D}A^a e^{-\int L[A^a,q]+A_5\cdot J^5+\theta \mathrm{Tr}\star F\wedge F}$$

invariant under $A_5 \to A_5 + d\lambda_5, \qquad \theta \to \theta - c_a\lambda_5\,.$

because of the anomaly $d\star J_5 = c_a\,\mathrm{Tr}F\wedge F\,.$

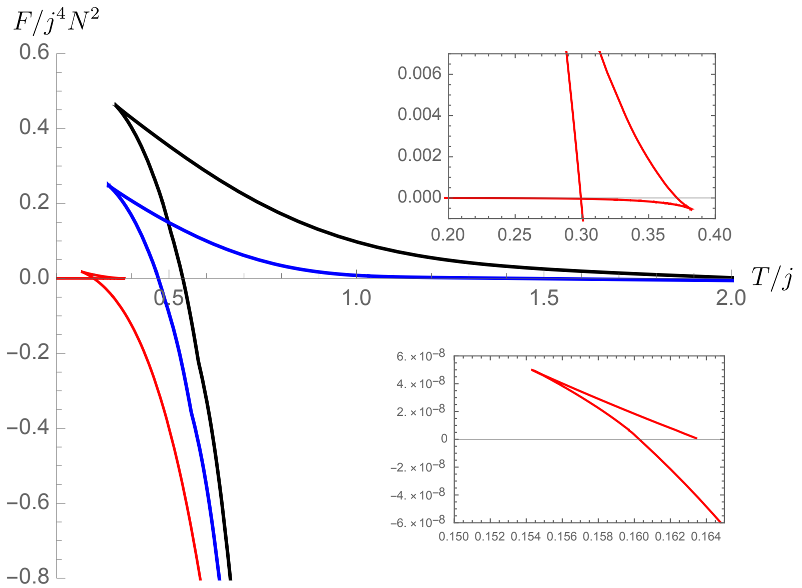
Rotate θ into the quark propagator:

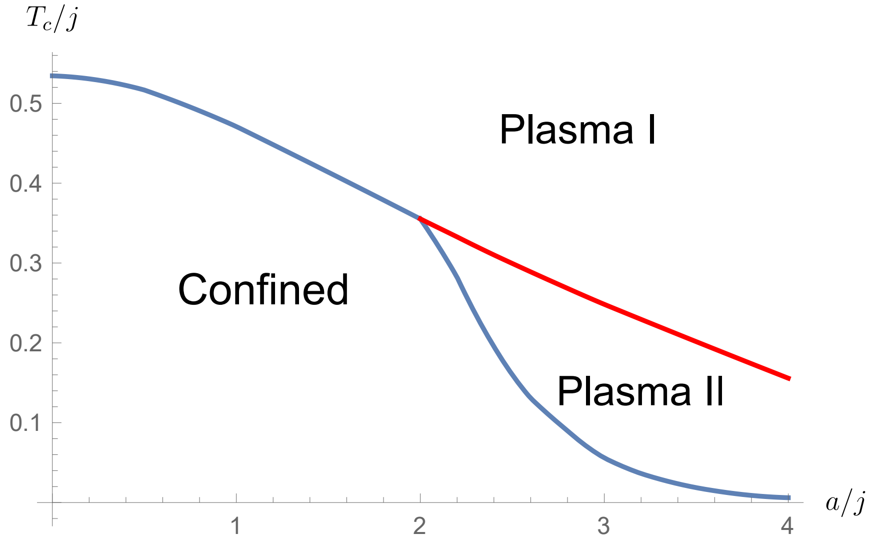
$$\langle \bar{q}q\rangle_a = \frac{1}{\mathcal{Z}(a)}\int \mathcal{D}A^a_\mu e^{-S_g}\,\det(\not{D}(a))\mathrm{Tr}\,(\not{D}(a))^{-1}\,,$$

$$\not{D}(a) = \gamma^\mu\left(\partial_\mu + A^a_\mu T^a\right) + \frac{a}{c_a}\gamma^3\gamma^5\,.$$

Do valence and sea also have opposite effects?

Thermodynamics of the anisotropic theory





- T_c decreases with anisotropy
- A new plasma phase and two phase boundaries

⟹ **inverse anisotropic catalysis?**

Holographic, anisotropic, non-conformal, neutral plasma

Giataganas, Pedraza, UG ‘17

Nonconformality ⇔ a scalar ϕ, anisotropy ⇔ another scalar χ

$$S = \frac{1}{2\kappa^2}\int d^5x\,\sqrt{-g}\,[R + \mathcal{L}_M]\,,$$

$$\mathcal{L}_M = -\frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{1}{2}Z(\phi)(\partial\chi)^2,$$

$$V(\phi) = 12\cosh(\sigma\phi) + b\,\phi^2, \qquad Z(\phi) = e^{2\gamma\phi},$$

$$ds^2 = e^{2A(r)}\left[-f(r)dt^2 + d\vec{x}_\perp^2 + e^{2h(r)}dx_3^2 + \frac{dr^2}{f(r)}\right],$$

$$\phi = \phi(r), \qquad \chi = a\,x_3. \qquad \phi \to jr^{4-\Delta}$$

IR geometry is hyper scaling violating:

$$ds^2 = \tilde{L}^2(ar)^{2\theta/3z}\left[\frac{-dt^2 + d\vec{x}_\perp^2 + dr^2}{a^2r^2} + \frac{c_1\,dx_3^2}{(ar)^{2/z}}\right],$$

$$\phi = c_2\log(ar) + \phi_0.$$

$$ds \to \lambda^{\theta/3z}ds\,.$$

$$t \to \lambda t, \quad \vec{x}_\perp \to \lambda\vec{x}_\perp, \quad r \to \lambda r, \quad x_3 \to \lambda^{\frac{1}{z}}x_3\,.$$

Magnetic QCD

Anisotropy

B reduces original Lorentz : $SO(3,1) \Longrightarrow \underbrace{SO(1,1)}_{\text{boost // B}} \times \underbrace{SO(2)}_{\text{rotation } \perp \text{ B}}$

propagators, transport coefficients decomposed using projectors

$$\Delta_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu - \frac{B_\mu B_\nu}{B^2} \qquad \text{etc.}$$

Anisotropic confinement: $\sigma_\perp > \sigma_{||}$

Bonati, D’Elia et al. ‘14

Chiral symmetry breaking

B reduces original chiral symmetry: u +2/3, d -1/3:

$$SU(N_u)_L \times SU(N_u)_R \times SU(N_d)_L \times SU(N_d)_L \times U(1)_{A-} \Longrightarrow SU(N_u)_V \times SU(N_d)_V$$

IR effective theory: $\chi PT \text{ of } N_u^2 + N_d^2 - 1 \text{ NG bosons}$

Magnetic QCD

Fundamental scales at vanishing temperature and density

$1/\sqrt{eB}$	Magnetic screening length
$\Lambda_{QCD}(B)$	Confinement scale
$m_{dyn}(B)$	Dynamically generated quark mass

Separation of scales:

$m_{dyn} \ll k \ll \sqrt{eB}$ χSB

$k \ll m_{dyn}$ confinement

Additional scales T, μ

$T \neq 0,\ \mu = 0$ pQCD + χPT + lattice QCD

$T \neq 0,\ \mu \neq 0$ Holographic models

Various regimes

I. $eB \gg \Lambda_{QCD}^2$ Perturbative QCD $\frac{1}{\alpha_s} \approx b \log \frac{|eB|}{\Lambda_{QCD}^2}$ Kabat, Lee, Weingerg '02

II. $eB \approx \Lambda_{QCD}^2$ Lattice QCD, NJL effective theory, holography

III. $eB \ll \Lambda_{QCD}^2$ Perturbative EM

- **Testing the valence vs. sea idea:** Bruckmann, Endrodi, Kovacs '13

$$\bar{\psi}\psi^{\text{val}}(B) = \frac{1}{\mathcal{Z}(0)} \int \mathcal{D}U\, e^{-S_g} \det(\not{D}(0)+m)\, \mathrm{Tr}(\not{D}(B)+m)^{-1},$$

$$\bar{\psi}\psi^{\text{sea}}(B) = \frac{1}{\mathcal{Z}(B)} \int \mathcal{D}U\, e^{-S_g} \det(\not{D}(B)+m)\, \mathrm{Tr}(\not{D}(0)+m)^{-1}.$$