Line Defects in QFT: an Entropy Function, Conformality, and Symmetries

Zohar Komargodski (SCGP SimonsCenter FOR GEOMETRY AND PHYSICS)

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Part I

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- Entropy Function and a Theorem about RG Flows.
- Lines in Landau-Ginzburg Theories.

Gabriel Cuomo

Mark Mezei

Avia Raviv-Moshe

Part II

Work to Appear!

- Wilson Lines in Conformal Gauge Theories.

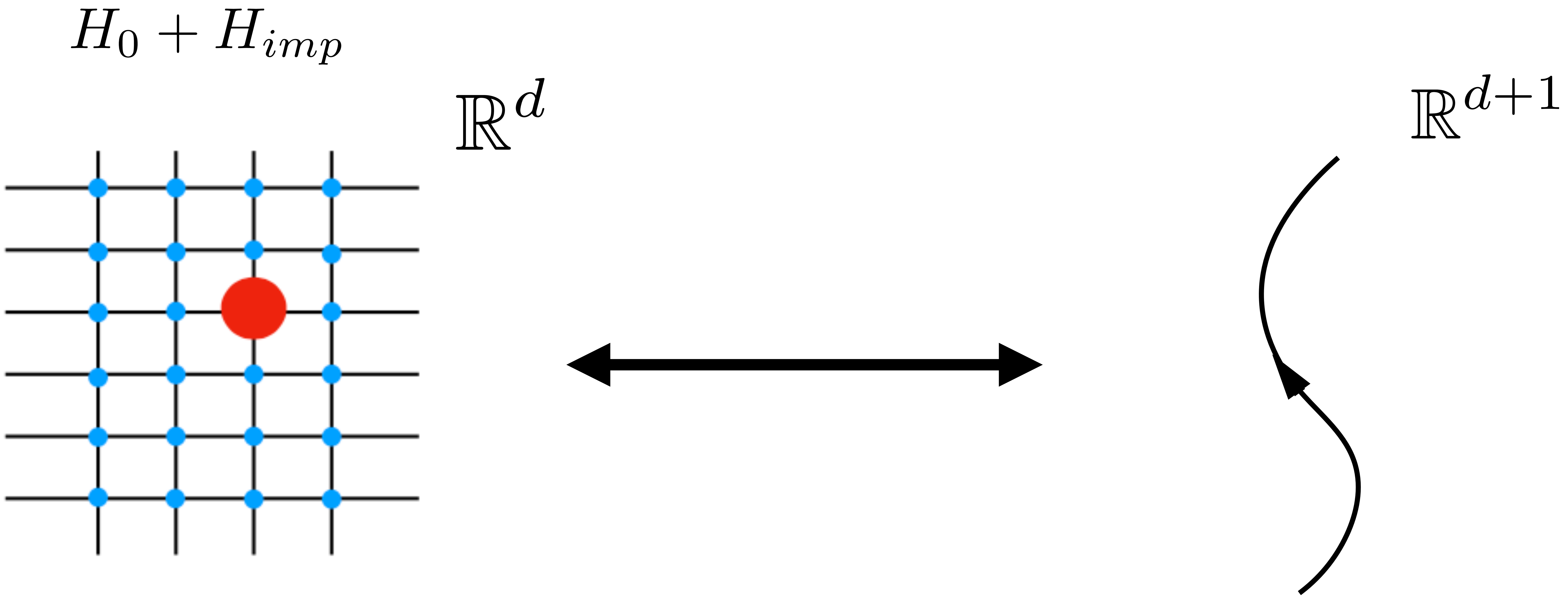
Ofer Aharony

Gabriel Cuomo

Mark Mezei

Avia Raviv-Moshe

Point-like impurities in space are line defects in space-time. Historically the Kondo problem led to the RG, and to progress on integrability, phase transitions etc. Line defects have also played a pivotal role in high-energy physics, especially Wilson lines.



Assume: Bulk d+1 dimensional theory is CONFORMAL (CFT).

The possible **infrared** fates of line defects in CFTs:

- A nontrivial conformal line defect preserving

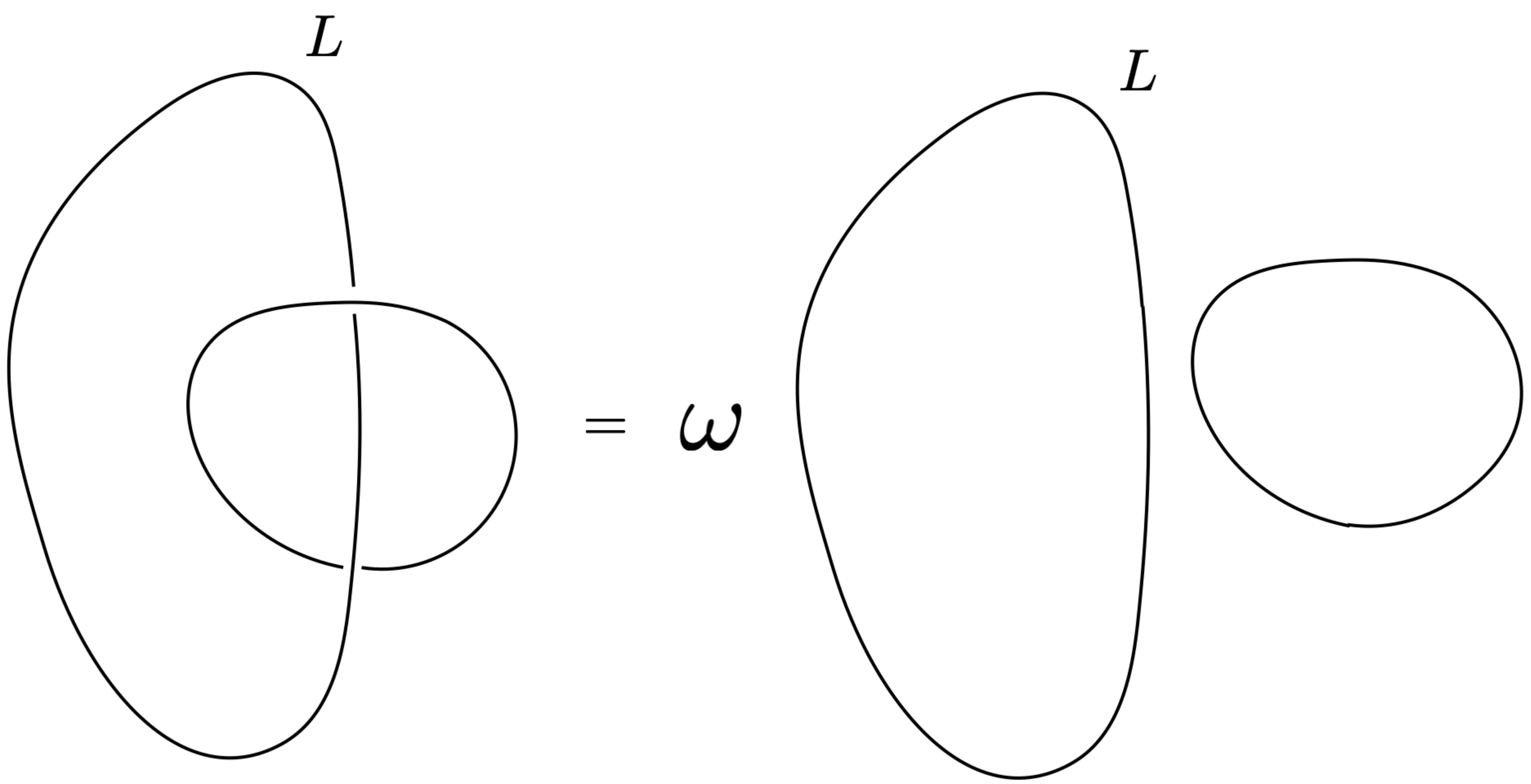
$$sl(2,\mathbb{R}) \subset so(d+2,1)$$

- A topological or trivial line defect.
- More exotic infrared scenarios?!

General considerations

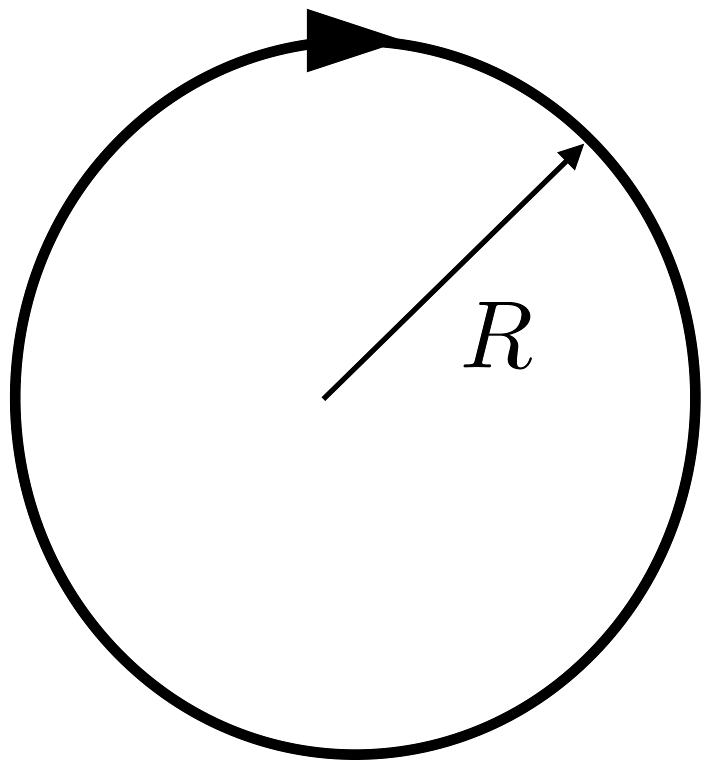
- If we have a 1-form symmetry, a charged line operator cannot flow in the infrared to a trivial line. And if the one-form symmetry surfaces can be cut open, it cannot flow to a topological line either.

$$\omega^N = 1$$



- For special bulk CFTs, one might be able to prove that no nontrivial conformal line defects exist whatsoever. See [Lauria-Liendo-Van Rees-Zhao ; Herzog-Shrestha] for results of this type for free bulk theories.

- An interesting observable is the defect entropy:

$W(R) =$ 

This expectation value is ambiguous; even as the cutoff is removed, $\log W(R)$ can have an arbitrary linear in R real term and an arbitrary R independent imaginary term (only in d=1). They arise from the c.c. and extrinsic curvature on the line defect.

Since the extrinsic curvature term exists only in d=1, we will ignore it. We therefore see that

$$s(R) \equiv \left(1 - R\frac{d}{dR}\right)\log W(R)$$

is well defined. We call this the defect entropy. This is a sort of zero-temperature measure of the degrees of freedom on the defect. It is R independent for conformal defects. It is possible that $s < 0$.

The central property of the defect entropy is that (unitarity and locality are assumed)

$$\frac{ds}{dR} \leq 0$$

Increasing the radius is tantamount to flowing to the infrared of the defect. The defect entropy is independent of exactly marginal operators and when there is a flow triggered by a relevant operator

$$s_{uv} > s_{ir}$$

We will see some nice applications of this theorem. This extends the result of [Affleck-Ludwig; Friedan-Konechny, Casini-Salazar Landea-Torroba] from d=1 to any d.

Screening

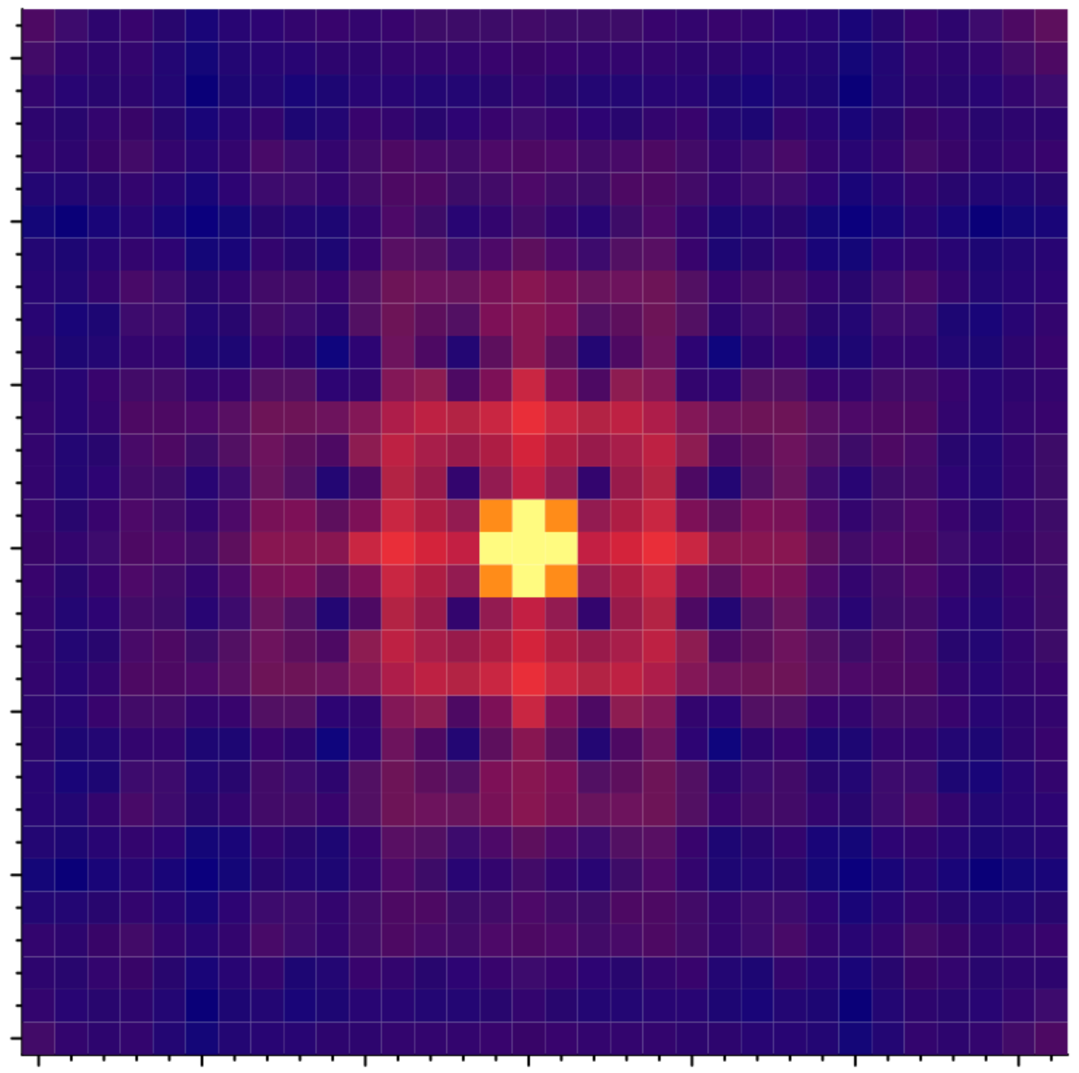
We say that a line defect is screened if the infrared is trivial. Then

$$s(R \to \infty) \to 0$$

This notion of screening is more refined than the familiar perimeter/area criterion. (Line defects in CFTs always obey a perimeter law.)

Cond-Mat

Apply localized staggered external field in an anti-ferromagnet (pinning field defect)



[Polkovnikov-Vojta-Sachdev]

O(N) model

$$S = \int d^{d+1}x \left[\frac{1}{2}(\partial\vec{\phi})^2 + g_*(\vec{\phi}^2)^2\right]$$

Consider a relevant deformation of the trivial line defect

$$S_{new} = \int dt\ \vec{h}\cdot\vec{\phi}$$

From the general theorem, $s_{ir} < 0$ and hence the pinning field cannot be screened!

For the ordinary Wilson-Fisher fixed point with $g_* > 0$ there is indeed a nontrivial conformal line defect, and we solved it exactly in the large N limit. We found that for the lowest neutral nontrivial defect operator

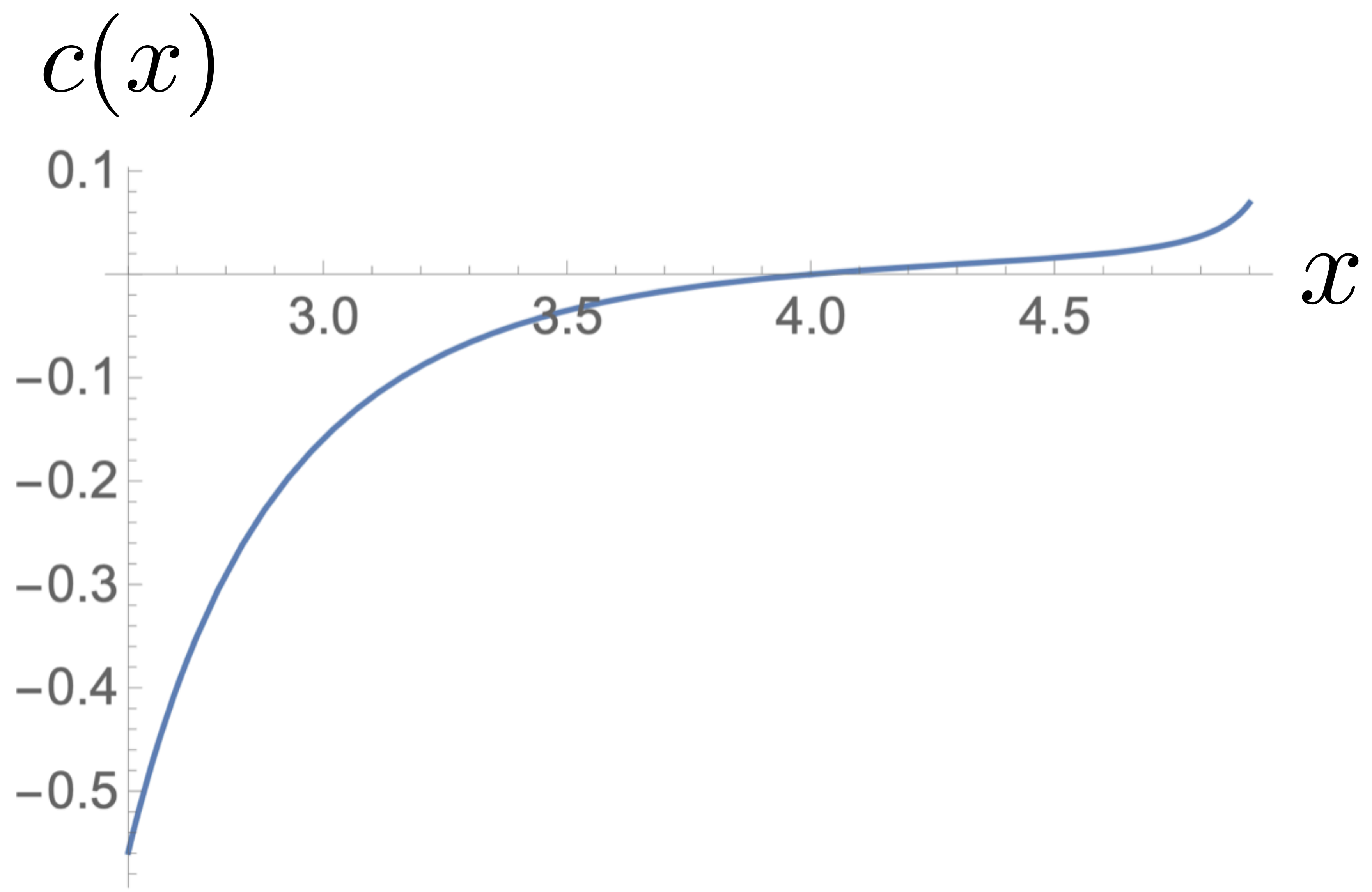
$$\Delta(\hat{\phi}) = 1.542...$$

which agrees very nicely with various simulations by [Parisen Toldin-Assaad-Wessel; Allais..] which also suggest $\Delta(\hat{\phi}) \sim 1.5 \pm 0.1$.

A similar exactly solvable line defect in tensor models was considered by [Popov-Wang].

For the multi-critical point (free field theory) with $g_* = 0$ the infrared is again not screened, but it does not end up in a conformal line defect. Instead we have $s(R \to \infty) \to -\infty$, which is some sort of never-ending flow for d<4.

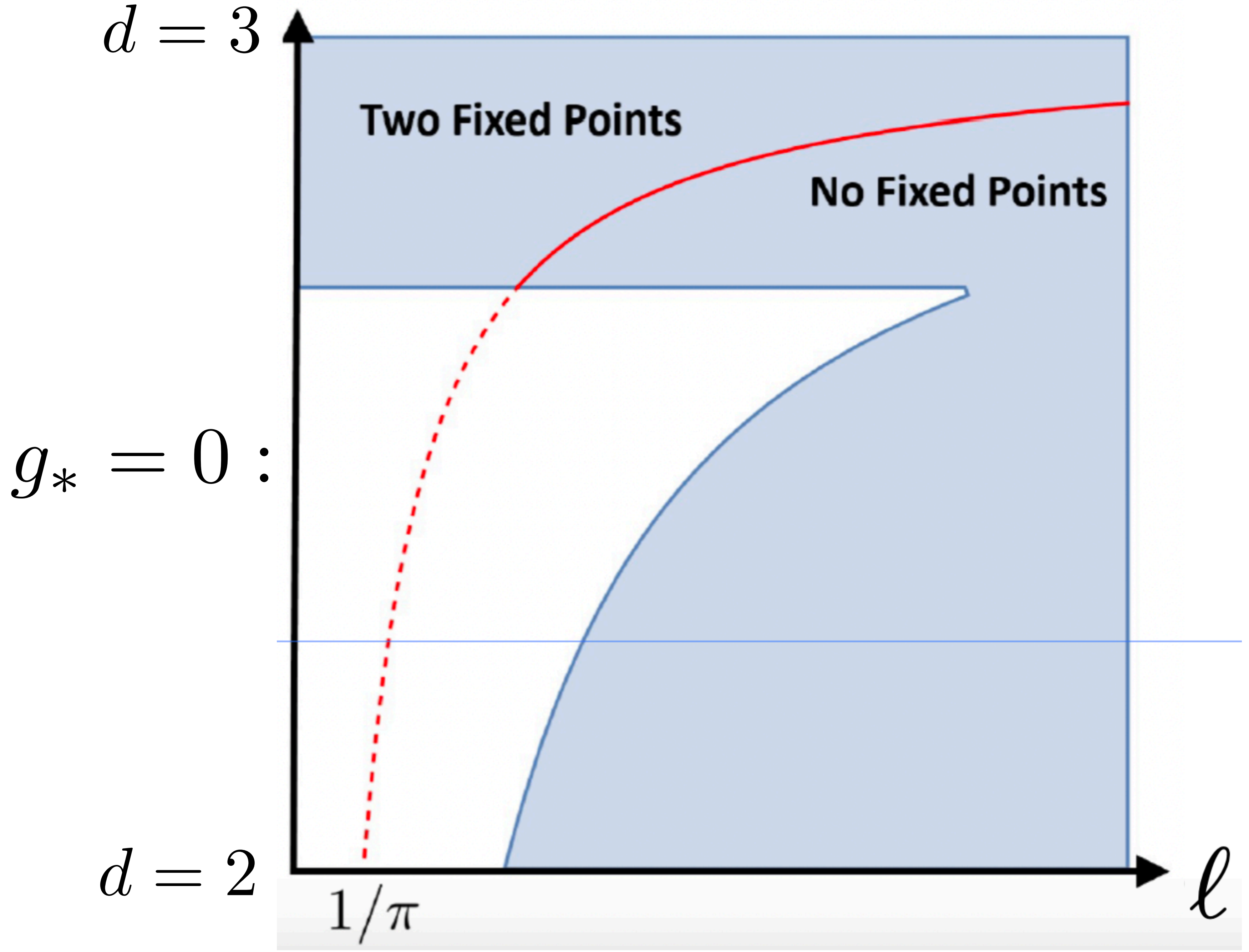
$$s = R^{3-d} c(d+1)$$



Another important class of line operators in such Landau-Giznburg models are the spin impurities. These are natural generalizations of the Kondo problem to higher d. Consider the O(3) model with field ϕ_a , $a = 1, 2, 3$.

$$Tr_{2\ell+1}[P \exp \left(\gamma \int dt \ \phi_a T^a \right)]$$

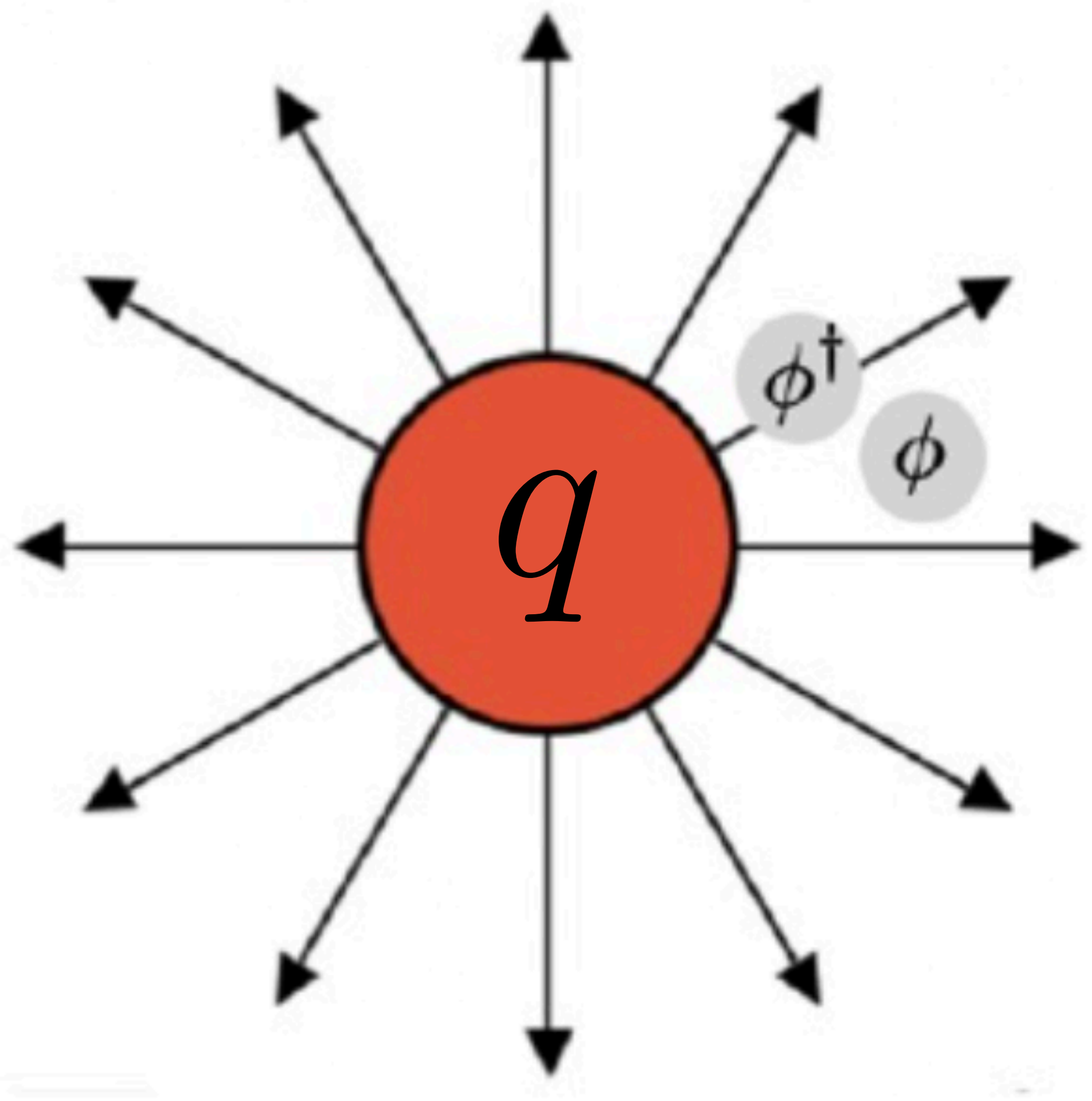
For odd ℓ we have projective rep of SO(3). See also [Liu-Shapourian-Vishwanath-Metlitski].



Recently observed in MC [Weber-Vojta]

Also: a non-perturbative prediction for $\ell \to \infty$ for any d both for $g_* = 0$ and $g_* \neq 0$
— yet to be confirmed in MC or experiment.

Wilson Lines



$$A_0 = \frac{e^2 q}{4\pi r}$$

Wilson Lines describe point probe charges in a representation R of the gauge group

$$W = Tr_R \left[P\exp \left(i \int dx^\mu A_\mu^a T^a \right) \right]$$

There is no free parameter in the defect action. But that DOES NOT mean that it is a conformal line operator for all reps!

Wilson lines are labeled by representation in the sense of their ultraviolet definition. The infrared behavior of such probe charges is highly nontrivial. Let us demonstrate that in massless scalar QED in 3+1 dimensions.

$$W = P\exp\left(iq\int dt A_0\right)$$

One can think of it as a massive external nucleus in massless QED.

$$S = \frac{1}{e^2}\int d^4x\left[-\frac{1}{4}F_{\mu\nu}^2 + |D\phi|^2 - \frac{\lambda}{4e^2}|\phi|^4\right] + q\int dt A_0$$

Charge 1 scalar field

Charge $q > 0$ Wilson line

The bulk now is not strictly conformal, but we take e^2 to be small enough so that this is not an issue. Formally, one can take a double scaling limit in which the bulk becomes conformal and the dynamics remains nontrivial

$$e^2 \sim \lambda \to 0\,, \quad q \to \infty \quad \text{with} \quad e^2 q \sim \lambda q = \text{fixed}$$

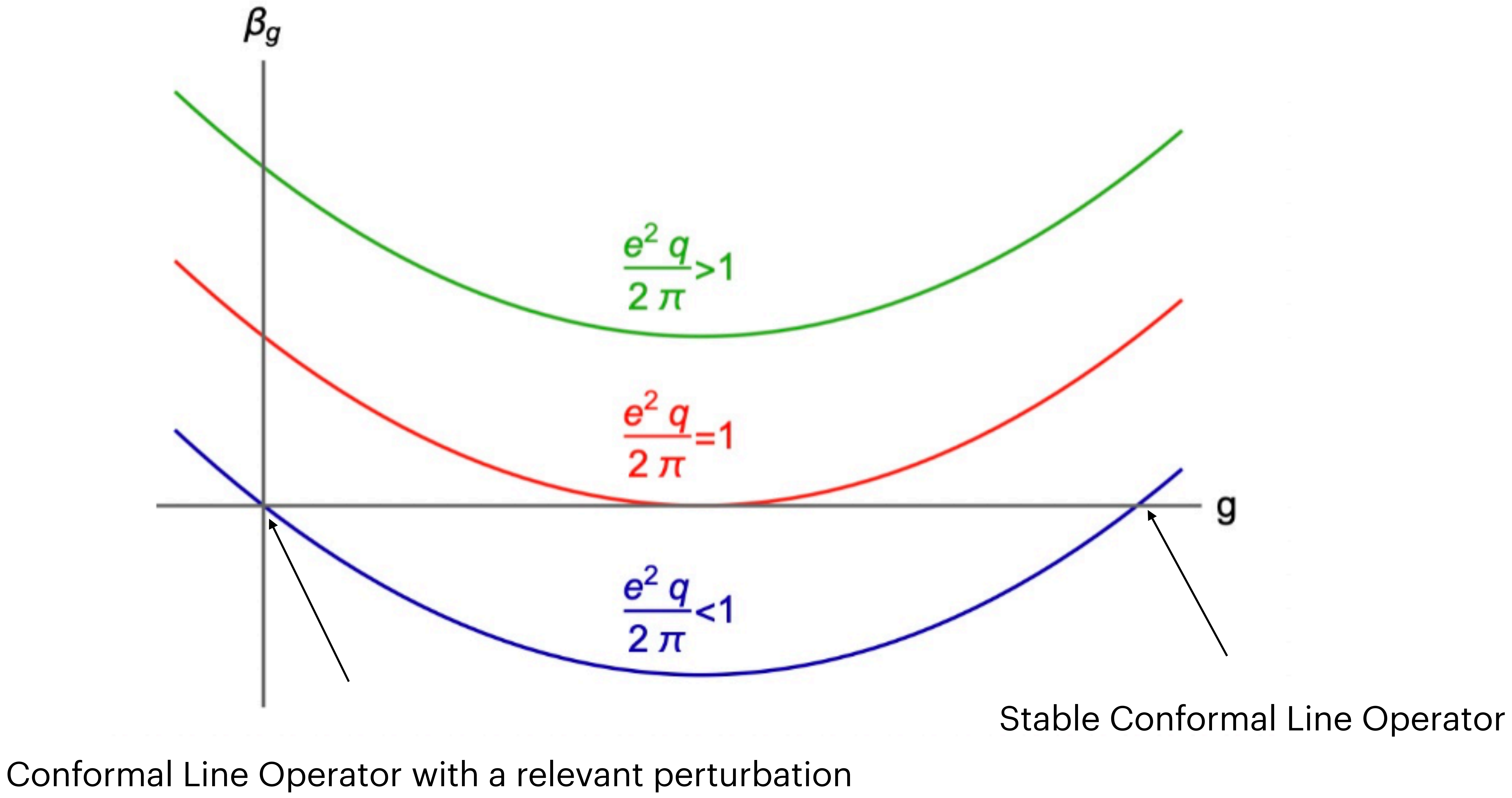
(Similar double-scaling limits exist for spin impurities and also extended defects, see also [Rodriguez Gomez ; Nahum; Rogriduez Gomez - Russo] and even for bulk CFTs [Badel-Cuomo-Monin-Rattazzi...].)

The operator $\phi^\dagger\phi$ has bulk dimension 2, but as a defect operator it receives corrections

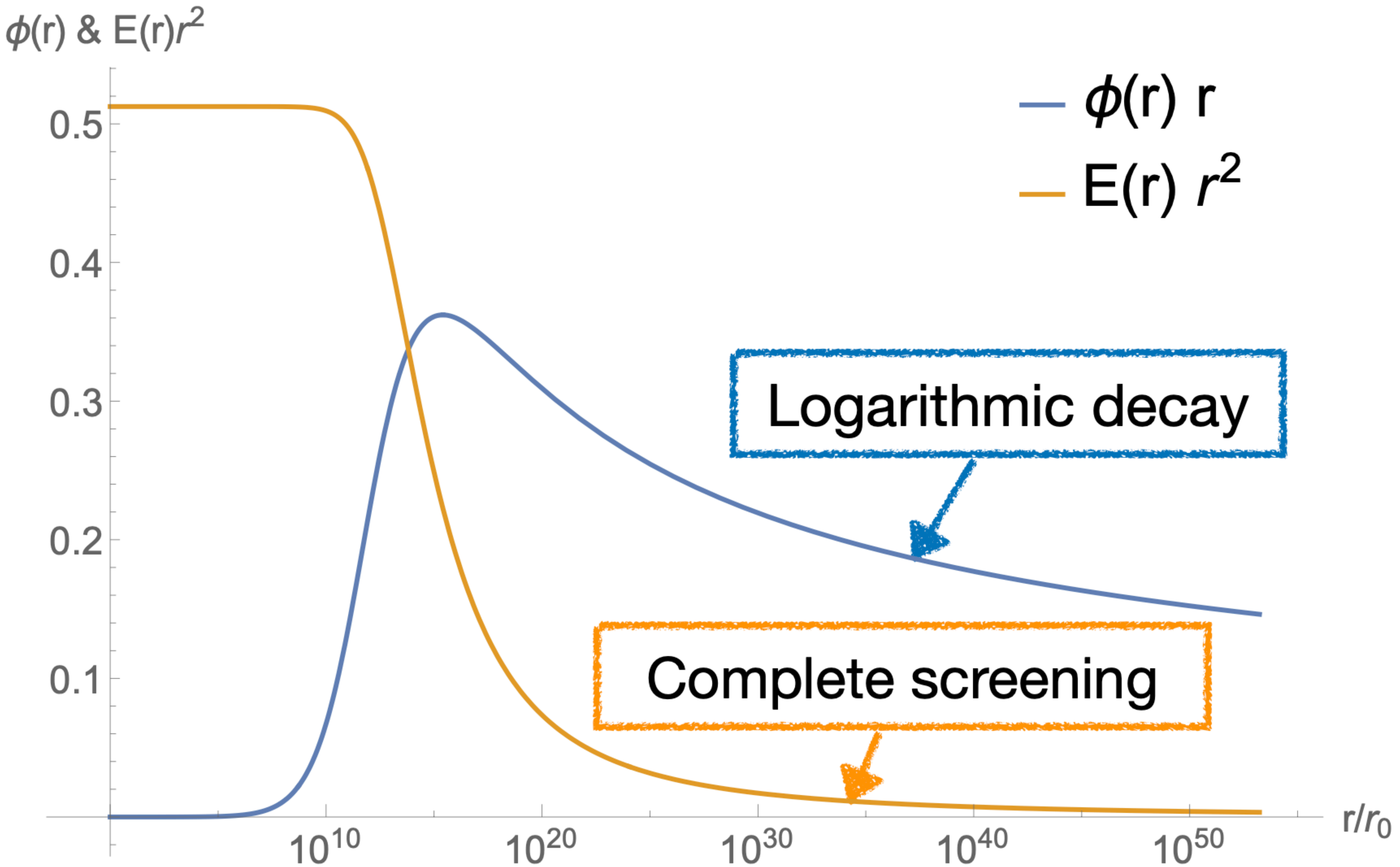
$$\Delta(\hat{\phi}^\dagger\hat{\phi}) = 1+\sqrt{1-\frac{e^4q^2}{4\pi^2}} = 2-\frac{e^4q^2}{8\pi^2}-\frac{e^8q^4}{128\pi^4}+\cdots$$

The defect operator $\hat{\phi}^\dagger\hat{\phi}$ therefore becomes marginal at $e^2q=2\pi$. Since it is neutral under all the symmetries one cannot consistently neglect it! One must therefore consider the more general line operator (this is an avatar of the [Pomeranchuk-Smorodinsky] effect for large nuclei.)

$$W = P\exp\left(i\int dt(qA_0+g|\phi|^2)\right)$$


β_g
$\frac{e^2 q}{2\pi} > 1$
$\frac{e^2 q}{2\pi} = 1$
$\frac{e^2 q}{2\pi} < 1$
g
Stable Conformal Line Operator
Conformal Line Operator with a relevant perturbation

One can also ask about the $g \to -\infty$ limit which arises in the super-critical regime as well as by deforming the unstable fixed point. One finds a boson condensate with an exponentially large scale!
This is similar to the phenomenon of dimensional transmutation in QCD.



This discussion for massless QED is actually very general and applies with technical differences (particularly with fermions) to all weakly coupled conformal gauge theories, including $\mathcal{N}=4$ SYM theory (see also [Beccaria-Giombi-Tseytlin]).

Similar issues arise for Wilson lines in 2+1 dimensions as well. We have considered QED in 2+1 dimensions with N_f flavors. The Wilson lines which are unscreened have

$$q \lesssim N_f/2$$

For Chern-Simons theories, [Gabai-Sever-Zhong] have found a similar picture, too. We also found 2+1 dimensional theories which admit no unscreened Wilson lines altogether.

- Is there a minimal possible s given a bulk theory with no moduli space? ([Friedan-Konechny-Schmidt Colinet, Collier-Mazac-Wang] in 1+1 dimensions)
- 't Hooft lines and electric-magnetic duality.
- Testable predictions in 2+1 dimensional de-confined critical points.
- Interplay with the planar limit and holography?
- In theories with one-form symmetries, large rep Wilson lines are not always completely screened. How does this come about?
- The unstable conformal fixed point has more than one relevant perturbation as e^2q is decreased. What is the full phase diagram in QED?
- What is the relation between EE and s and the monotonicity theorem?
- Progress from the bootstrap and Integrability? See [Giombi et al, Komatsu et al., Agmon-Wang, Barrat-Grau-Liendo]

Thank you!