Looking at supersymmetric black holes for a long time

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Strings 2022

Vienna, July 2022

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Useful discussions with Joaquin Turiaci and Vladimir Narovlansky

We, as a field, have been looking at supersymmetric extremal black holes for a long time…

Their entropies match beautifully with index computations…

Strominger-Vafa, 1996.

…

Dabholkar, Gomis, Murthy

See Iliesiu’s talk.

What more could we say?

There are many questions that remain

There is more to a black hole than its entropy!

Details of its AdS_2 near horizon geometry.

Is there a quantum mechanical dual to AdS_2 ?

There is more information in this AdS_2 theory:

Its correlation functions

Our main technical results involve computing some of them (in the disk approximation)

Haven't people been computing AdS correlators "for ever" ?

Yes, in AdS_D , D>2

In two dimensions the situation is a bit harder.
And it has been understood only within the last few years.

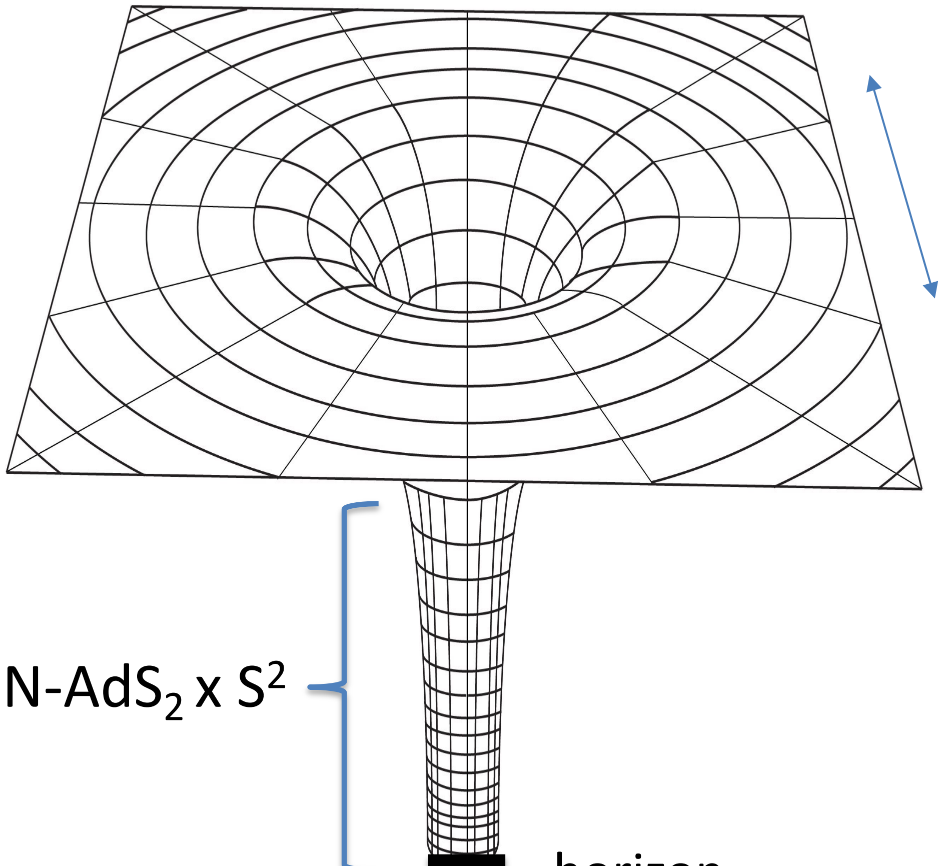
The new feature is that there is a gravitational mode that becomes strongly coupled at low energies. Even for large black holes.

Let’s talk about charged extremal black holes

Towards extremality

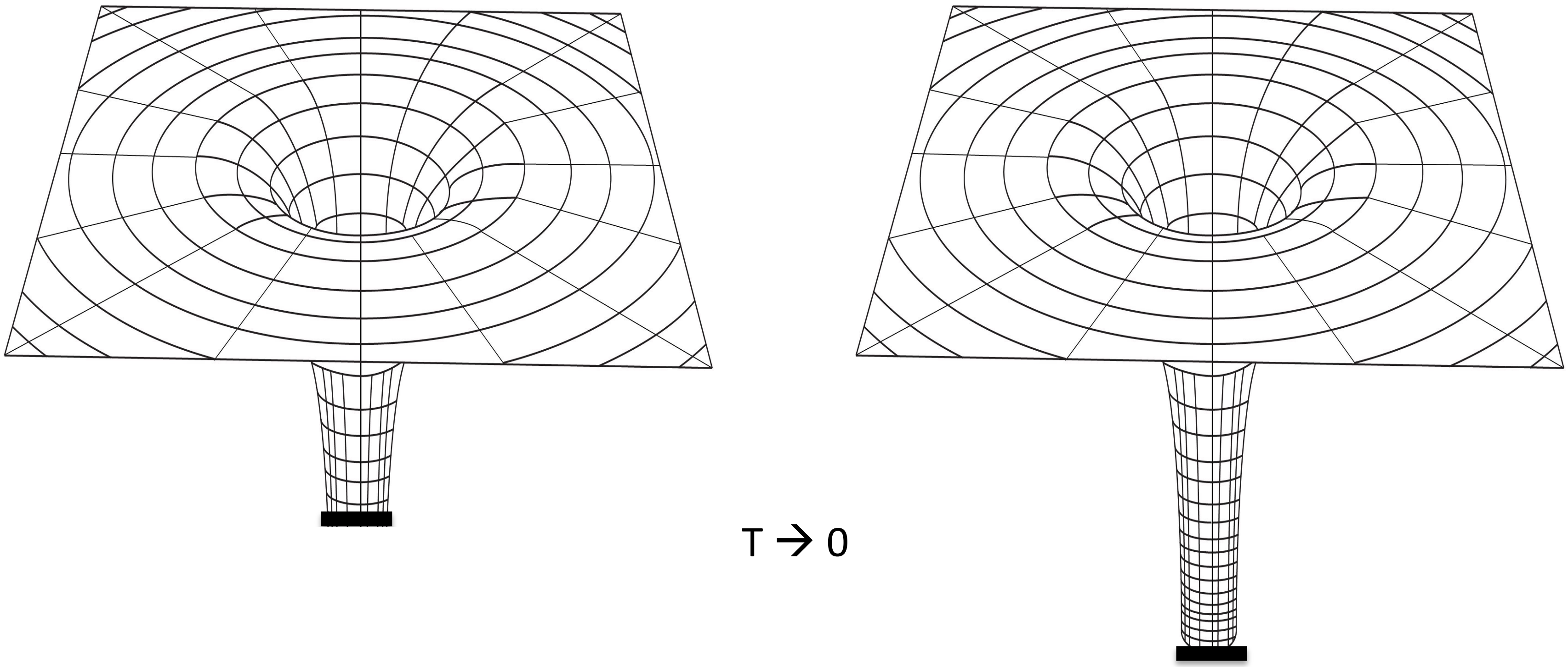
$$M \geq Q$$

$$M \sim Q$$



As we lower the temperature → The throat becomes deeper → larger redshift factor

First hint of a problem

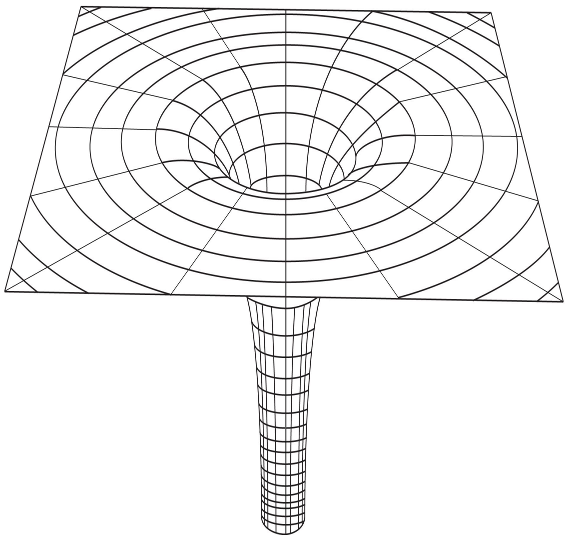


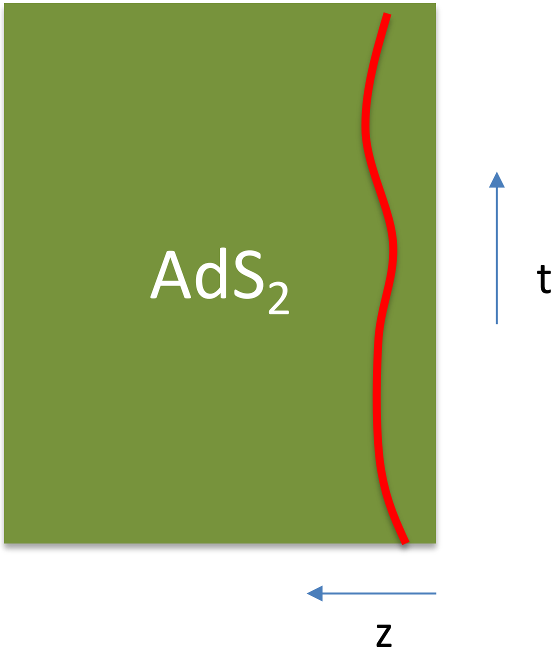
As we lower the temperature → free energy (action) becomes independent of temperature.
Classically the throat becomes longer an longer

Preskill, Schwarz, Shapere, Trivedi, Wilczek.

Soft mode that needs to be integrated → treated quantum mechanically.

Boundary graviton





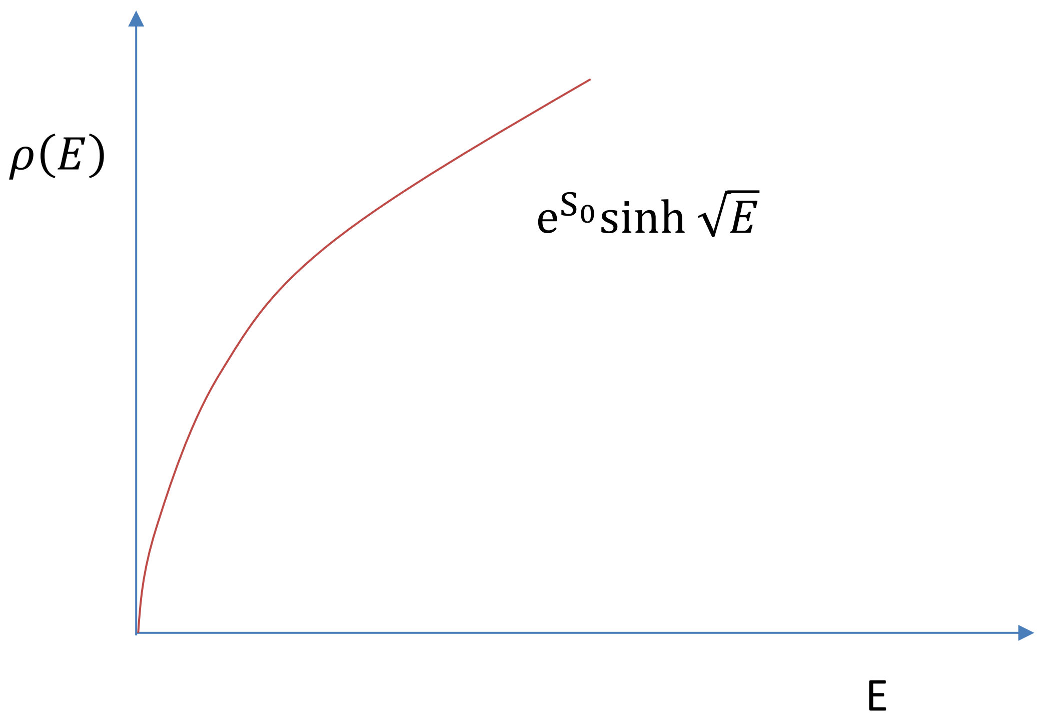
- There is a gravity mode telling us how we connect the throat region to the exterior.
- From the throat point of view, it is a ``boundary gravity mode''.
- This boundary mode becomes highly quantum mechanical → we need to treat it exactly.

Results from quantizing the boundary mode and computing the partition function

The results depend on whether we have pure gravity or supergravity (supersymmetry)

Non-SUSY

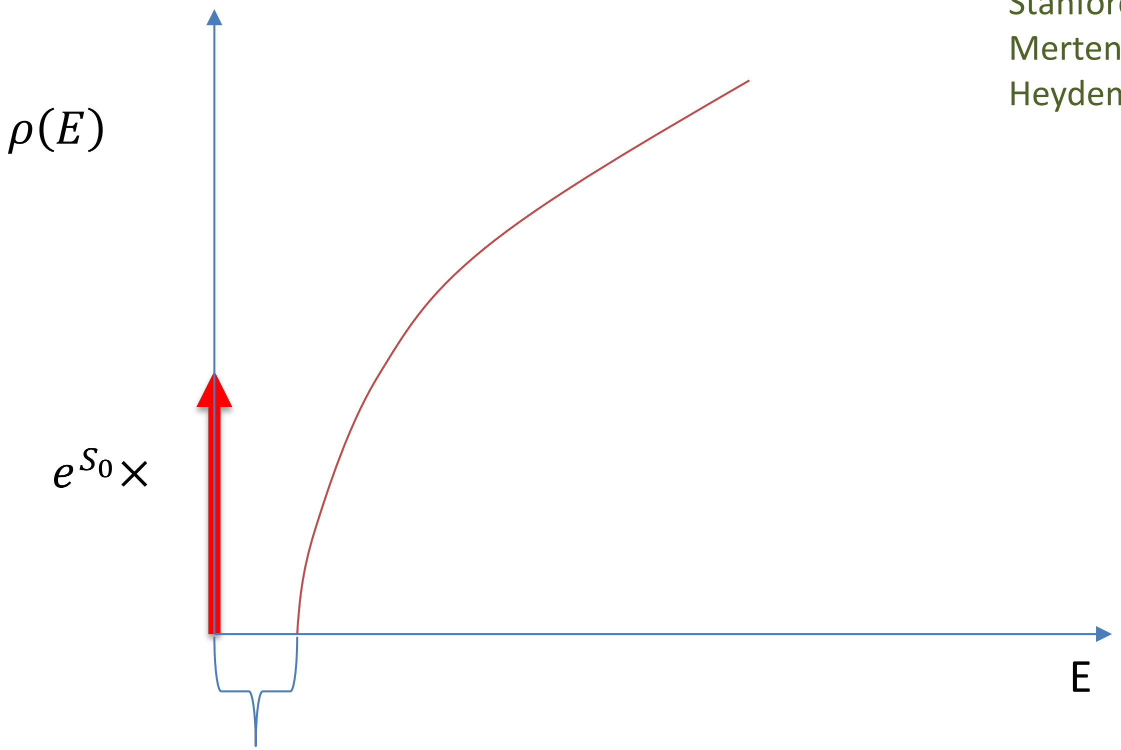
Bagrets, Altland, Kamenev
Stanford, Witten
Kitaev, Suh
Mertens Turiaci Verlinde



Entropy goes to zero at low energies.

$\mathcal{N}$=2, 4 SUSY

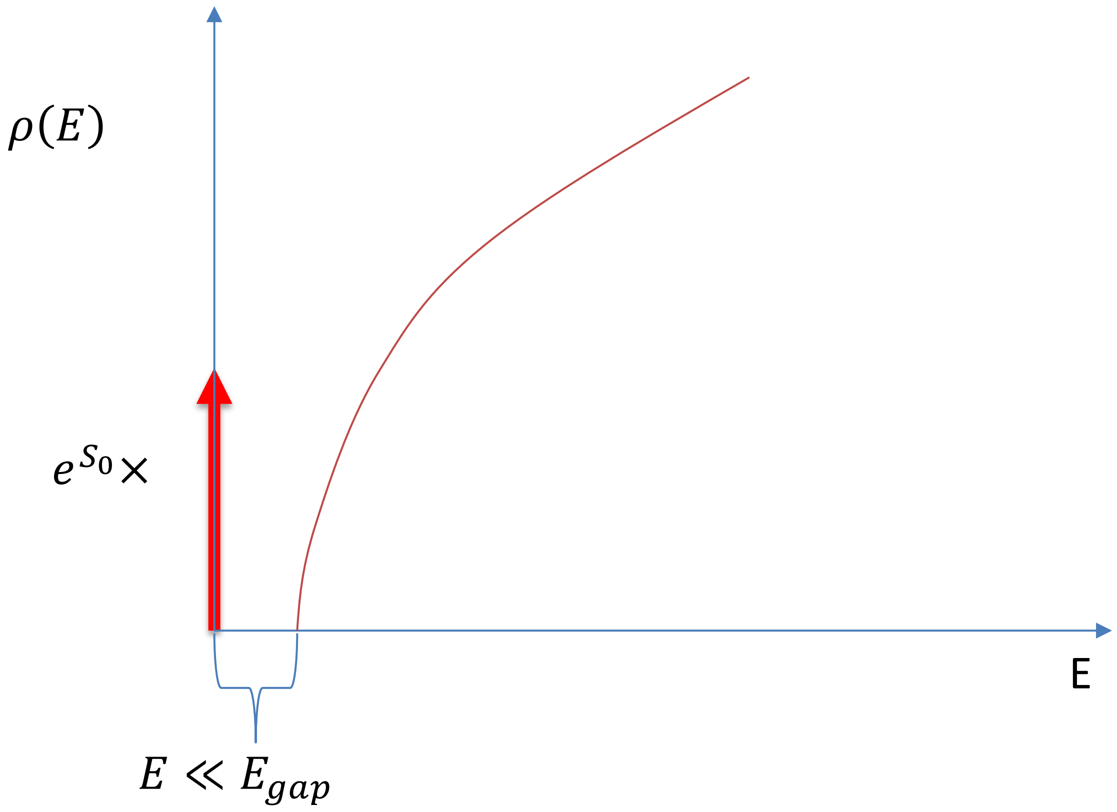
Stanford, Witten
Mertens Turiaci Verlinde
Heydeman, Iliesiu, Turiaci, Zhao



$$E_{gap} \sim \frac{E_{UV}}{S_e} \sim \frac{1}{r_e\, S_e} \sim \frac{1}{\phi_r} \sim T_0$$

$$S = -\phi_r \int du\{t(u), u\} + \text{partners}$$

A clean low energy limit



Look at the system at low energies, smaller than this gap. → Only ground states survive.

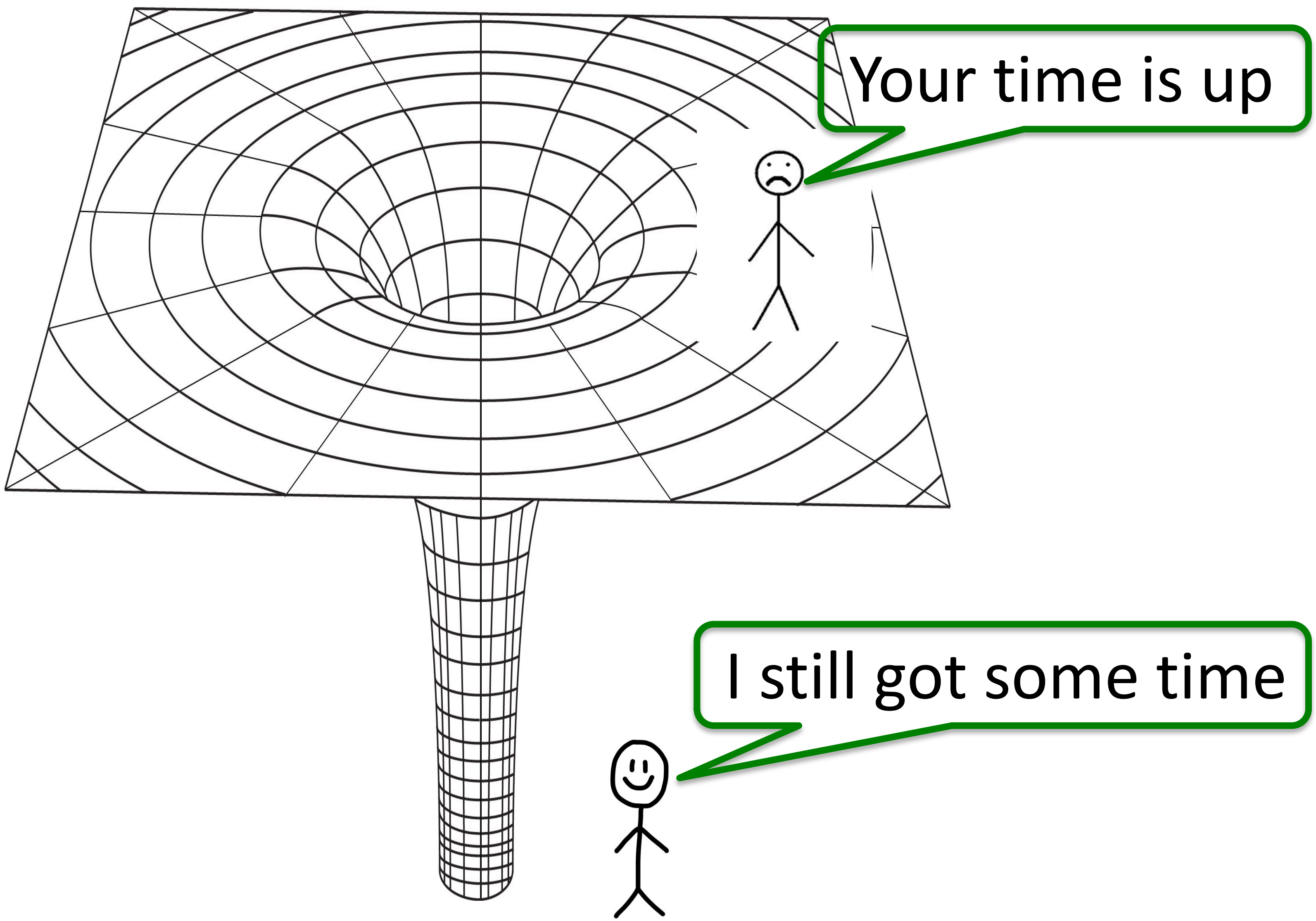
If you look at supersymmetric black holes for a long (Euclidean) time

→

Only the ground states survive.

There is no boundary time:
H=0.

What happens with the bulk?


Your time is up
I still got some time

Why do we study supersymmetric extremal black holes?

- This limit is conceptually clearer with supersymmetry.
- But we will see that we have similar features without supersymmetry.

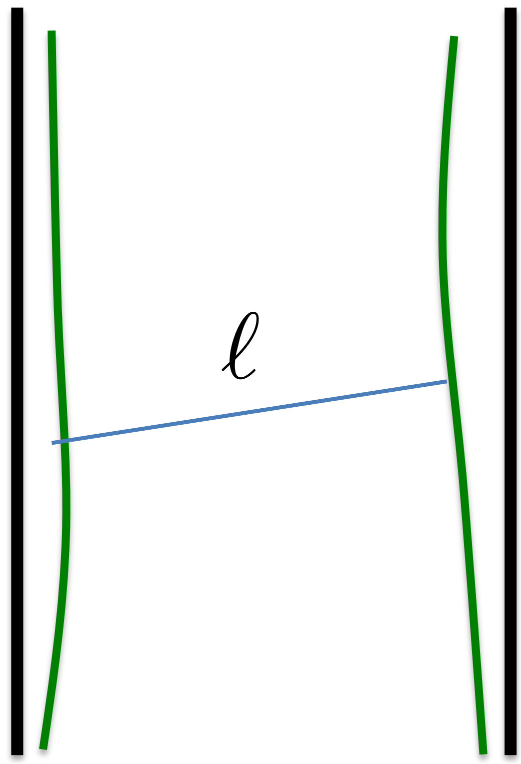
Supersymmetric black hole examples

- 4 or 5 dimensional black holes in flat space supergravity has N=4 supersymmetry.
- The 1/16 BPS black hole in $AdS_5 x S^5$ has N=2 supersymmetry.

Boruch, Heydeman, Iliesiu, Turiaci

We start by looking at the two sided black hole

Wormhole dynamics



Basic variable of the two sided problem:
the distance = length of the wormhole.

It turns out that its action is a Liouville-like action

$$\int du[\dot{\ell}^2 + e^{-\ell}]$$

Harlow-Jafferis, Henry Lin

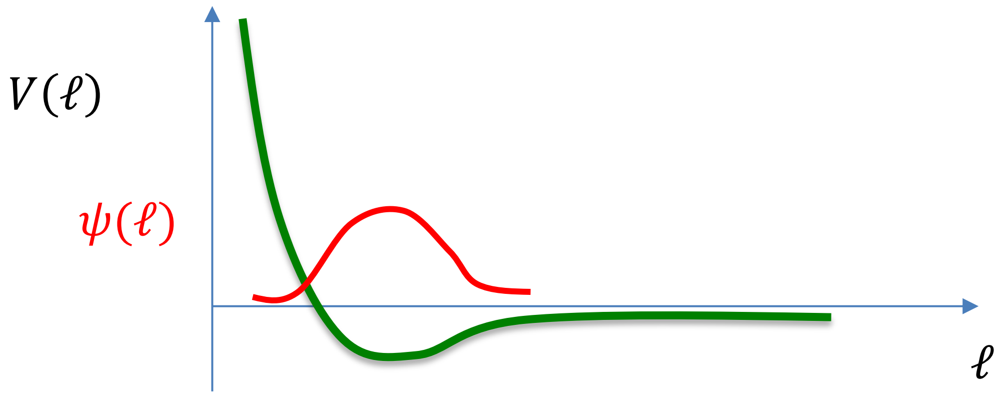
With supersymmetry → Super Liouville theory.

Naively we would try to consider an N=2 super-Liouville theory. However, we need an N=4 one because we have 2 SUSYs on the left and 2 SUSYs on the right.

$$S = \int du \left[\frac{1}{4}\dot{\ell}^2 + \dot{a}^2 + \bar{\psi}_\pm \dot{\psi}_\pm + e^{-\ell/2 - ia}\psi_+\psi_- + e^{-\ell/2+ia}\bar{\psi}_+\bar{\psi}_- + e^{-\ell} \right]$$

$$S = \int du \left[\frac{1}{4}\dot{\ell}^2 - \# e^{-\ell/2} + e^{-\ell} \right]$$

There is a zero energy normalizable ground state



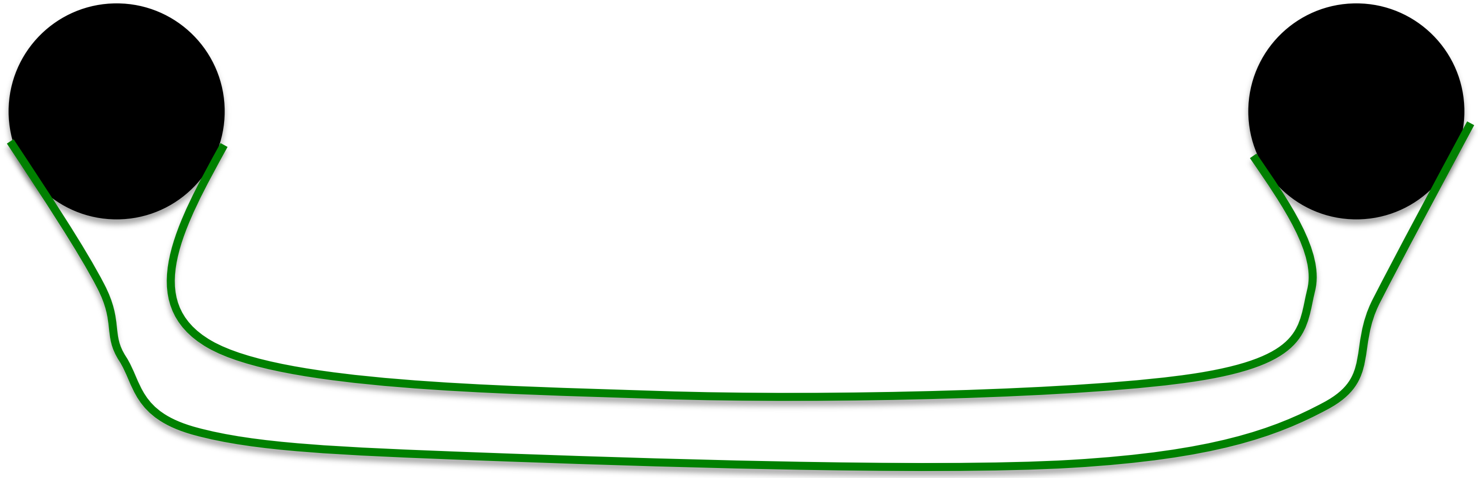
For this state the length of the wormhole is bounded, and time independent.

This is different from the naïve classical picture of an infinitely long throat, or

$$\mathrm{ds}^2 = -r^2 dt^2 + \frac{dr^2}{r^2}$$

Let us emphasize the last point by asking:

Are there supersymmetric wormhole configurations?



Yes!

The normalizable ground state

2 point functions at long times

$$\langle OO \rangle \propto \int d\ell |\psi_0(\ell)|^2 e^{-\Delta \ell}$$

$$\langle OO \rangle = Tr[POPO] = e^{S_0} c_\Delta \ , \qquad P = e^{-\infty H}$$

Calculable once we know the
UV normalization of the operator

An explicit check of the c_Δ

Long time two point functions in the $\mathcal{N}$=2 SUSY SYK model

$$H = \{Q^\dagger, Q\} \ , \qquad \text{with} \qquad Q = \sum_{ijk} C_{ijk}\psi_i\psi_j\psi_k$$

Fu, Gaiotto, Sachdev, JM

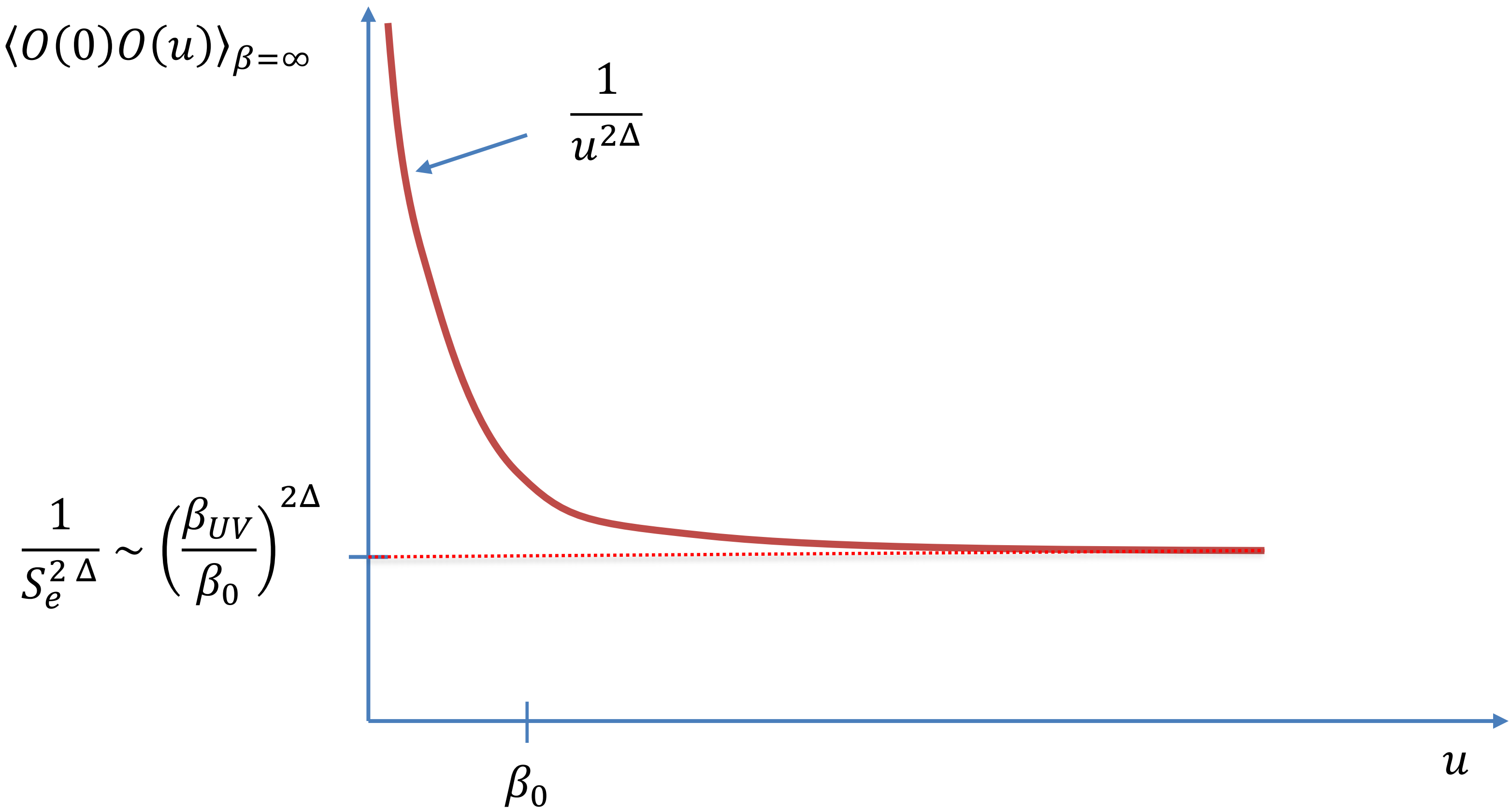
Zero energy states have R charges 0, 1/3 , -1/3.

Operator	R-charge	Schwarzian prediction	Numerical answer (N=16)
ψ_i	0	0.103	0.110 ± 0.005
	$-1/3$	0.103	0.110 ± 0.005
$\psi_i\psi_j$	$-1/3$	0.0213	0.024 ± 0.003
$\bar{\psi}_i\psi_j$	$-1/3$	0.0243	0.027 ± 0.001
	0	0.0754	0.079 ± 0.001
	$+1/3$	0.0243	0.027 ± 0.001

Used a result from Heydeman, Turiaci, Wenli Zhao.

It is also possible to compute the two point functions for all times…

The two point function at zero temperature



$\beta_{UV} \sim r_e \sim \frac{1}{J}$

$\beta_0 \sim 1/E_{gap}$

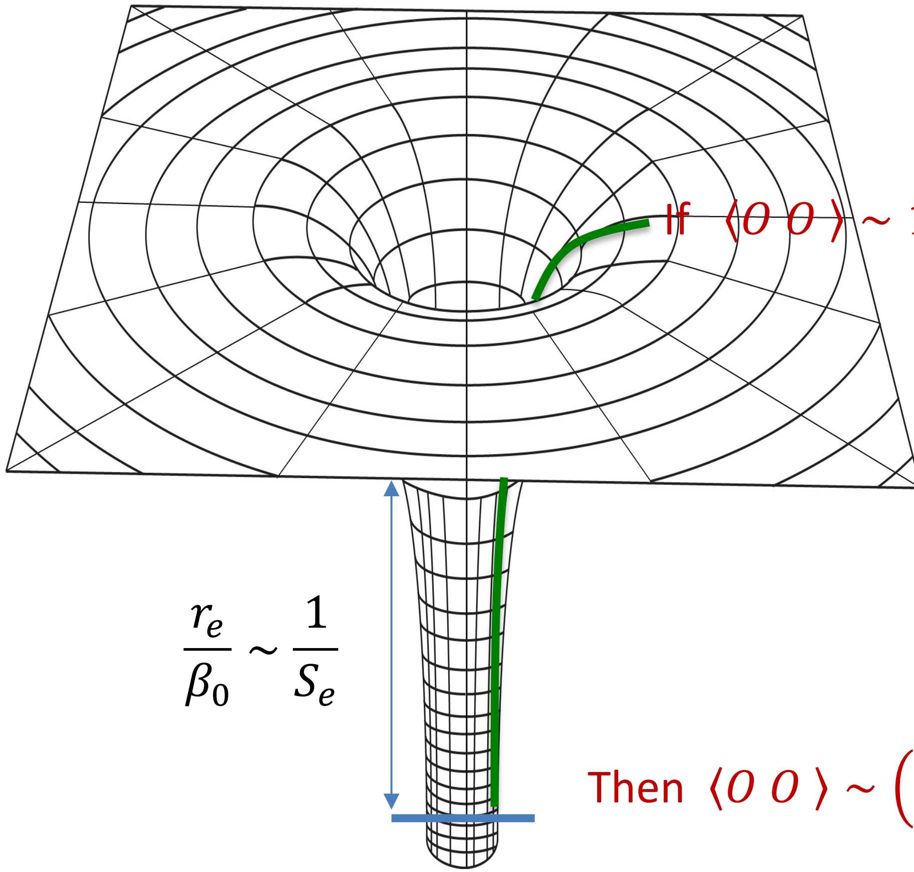
An implication of this constant value

Typical values of eigenvalues

We could diagonalize the (Hermitian) operator O, $O_{\alpha,\beta} \sim o_\alpha \delta_{\alpha,\beta}$

$$\langle OO \rangle_{IR} = e^{-S_0} Tr[\hat{O}\hat{O}] = e^{-S_0} \sum_\alpha o_\alpha^2$$

The (normalized) two point function is giving us size of the typical eigenvalues of the operator.



If $\langle O\, O \rangle \sim 1$

$$\frac{r_e}{\beta_0} \sim \frac{1}{S_e}$$

Universal factor coming from the propagation of the particle in AdS_2

Then $\langle O\, O \rangle \sim \left(\frac{r_e}{\beta_0}\right)^{2\Delta} \sim \frac{1}{S_e^{2\Delta}}$

Implications for microstates in this basis

Fuzzballs are supposed to be the gravity description of microstates, which ought to diagonalize the operators that we have been talking about.

We now turn to higher point functions

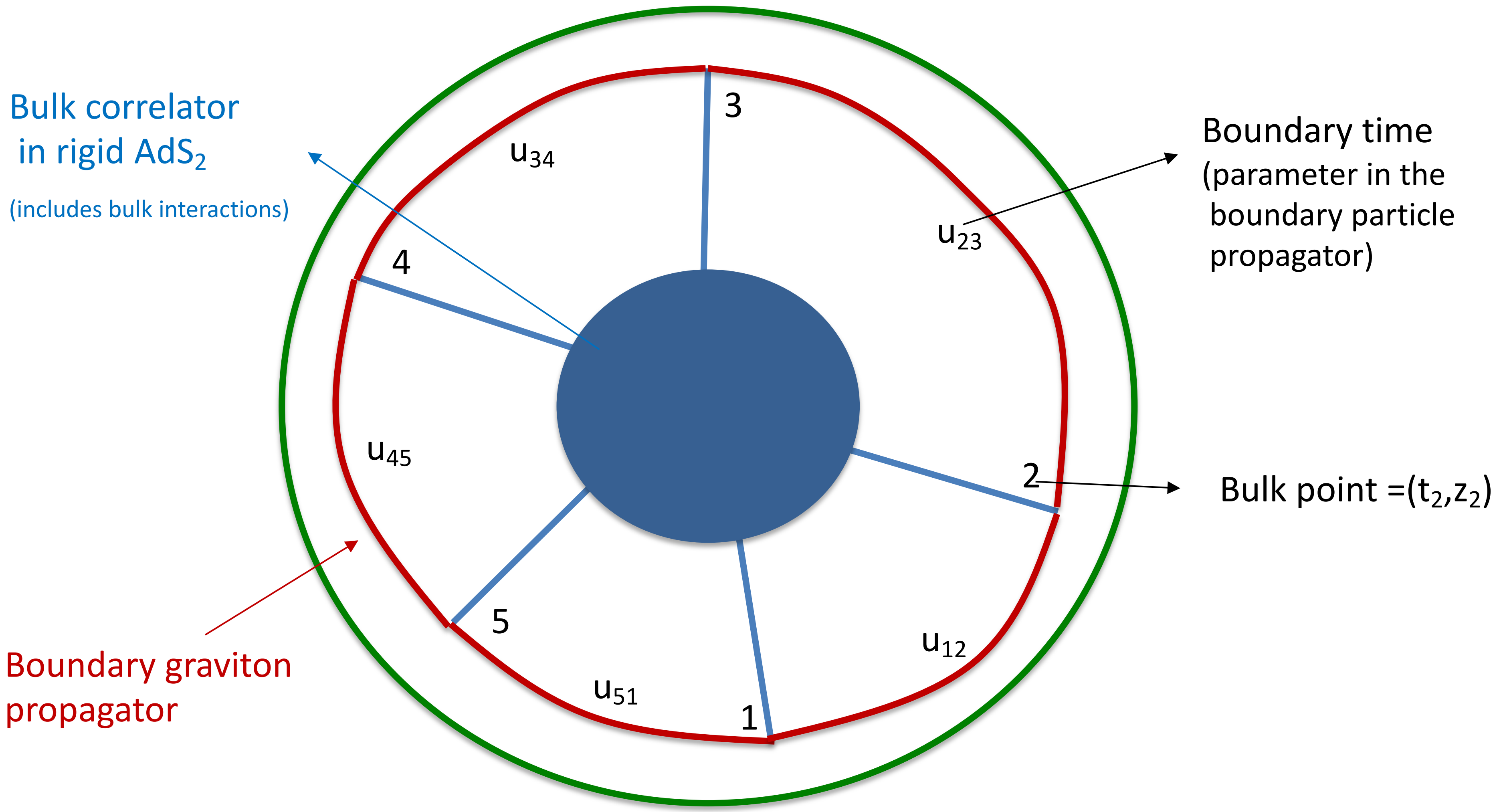
We will explain the final answer and give more details later

AdS_2 Witten diagram + boundary particle propagator.

Recall the non SUSY case

Quantum gravity from Witten-like diagrams

Z. Yang, Kitaev and Suh



Treat the quantum mechanics of the boundary graviton exactly

SUSY case is the same, but with some Grassmann variables

The N=2 case, at long times

Schematically:

$$\langle O(u_1)\cdots O(u_n)\rangle = \int \frac{\prod_i dx_i dz_i d^2\theta_i/z_i}{\mathrm{Vol}(SU(1,1|1))} P(\vec{x}_i,\vec{x}_{i+1}) \prod_i z_i^{\Delta_i} \langle O(x_1,\theta_1,\bar\theta_1)\cdots O(x_n,\theta_n,\bar\theta_n)\rangle$$

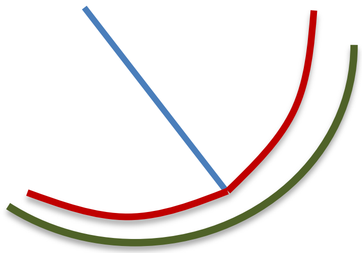
Independent of the boundary (long) times.

Depends on the order.

$$\langle O(u_1)\cdots O(u_n)\rangle = \text{number} = e^{S_0} F(\Delta_i, g_i)$$

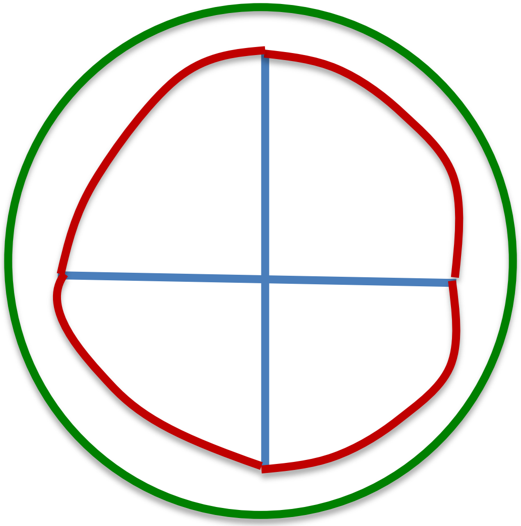
$$\langle O(u_1)\cdots O(u_n)\rangle = Tr[\hat O_1 \cdots \hat O_n] \ ,$$

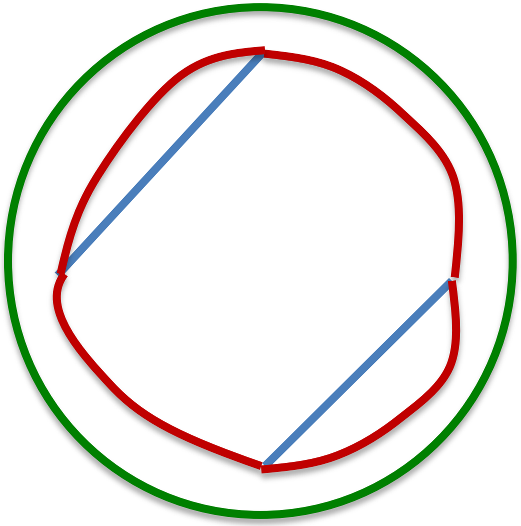
$$\hat O = POP \ , \qquad P = e^{-\infty H}$$



Applications

4 pt function

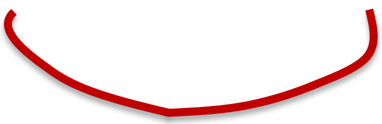




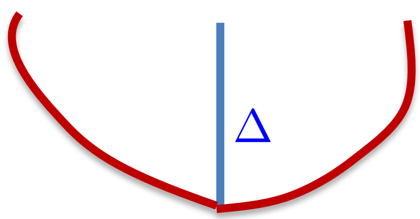
OTOC is similar to TOC for $\Delta \sim 1$.

OTOC << TOC for $\Delta \gg 1$

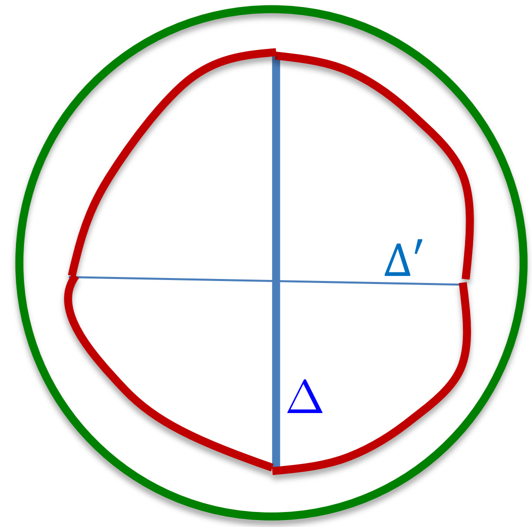
Wormhole with matter



Empty wormhole



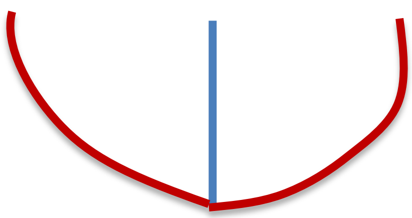
Add matter to the wormhole



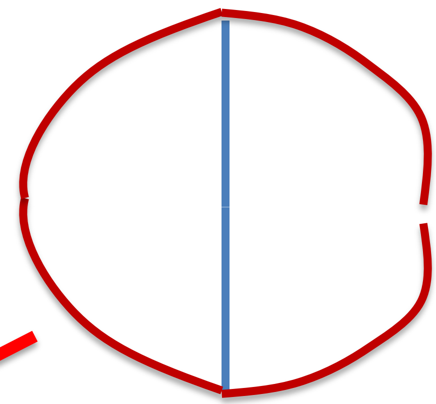
Calculate new length

Length increases when we add matter, extra length $\sim 2\log\Delta$

Density matrix of one side

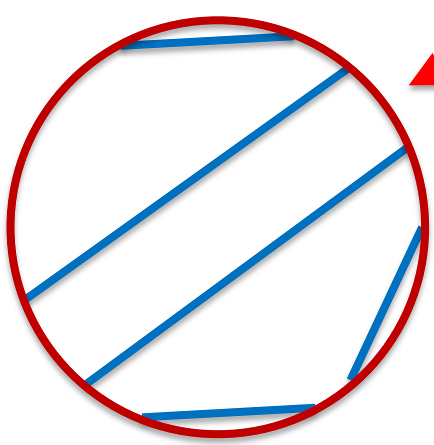


Add matter to the wormhole



Density matrix ρ

Replica trick, $Tr[\rho^n]$



2n point functions

For Δ>>1 planar diagrams dominate → gaussian random matrix

(See also Jafferis, Kolchmeyer, Mukhametzhanov, Sonner, to appear)

Entropy

$S = S_0$ $-$ (order one number, independent of Δ)

Entropy less than maximal for all states.

Type II_1 algebra in the $S_0 \to \infty$ limit.

See Witten's talk

Eigenvalues of $\widehat{O}$ are distributed as a random gaussian matrix for large Δ.

Also some evidence for eigenvalue repulsion from the cylinder diagram…

Comments on the bulk time

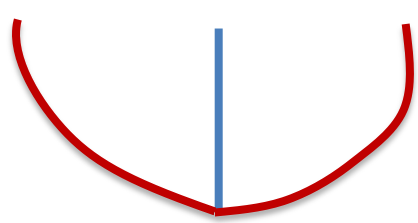
One can define a set of physical SL(2) generators moving the bulk matter relative to the boundaries

Henry Lin, JM, Zhao
Harlow, Jie-Quiang Wu

They do not commute with the Hamiltonian.

Their Casimir commutes with the Hamiltonian → Casimir of the matter inside the wormhole.

Should be a special two sided operator (probably defined only in the $S_0 \to \infty$ limit)



We now comment on the non-supersymmetric case

There is a somewhat similar structure for the correlators

At long, but not exponentially long times.

Non SUSY case

Correlators = (Simple time dependence) x (Details of the bulk theory)

$$\langle O(u_1)\cdots O(u_n)\rangle_{\text{connected}} = \underbrace{\left[\frac{1}{\prod_i (u_i - u_{i+1})^{3/2}}\right]}_{\text{Simple time dependent factor}} F(\Delta_i, g_i)$$

Obtained by setting E=0 in the boundary particle propagator,

Conclusions

- Extremal black holes offer us an interesting laboratory to study the emergence of the bulk theory.
- Correlation functions are interesting observables. A first step.
- It is crucial to take into account the quantum mechanics of the boundary mode.

Conclusions (continued)

- We obtained the full 2 pt function.
- Integral expression for the n point function.
- Explored the eigenvalue distributions of IR operators at low energies.
- All of these are constraints for any candidate description of the explicit microstates.

Future

- Understand better the emergence of the bulk time.

Thank you