Dualities from dualities

sequential deconfinement & the mirror dualisation algorithm

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based on 2002.12897 and 2110.11362 with Hwang, Sacchi
2110.08001 and 2201.11090 with Bottini, Hwang, Sacchi
and work in progress with Giacomelli, Hwang, Marino, Sacchi

Dualities from dualities

Shlomo's talk: "SCFTs in all d are intimately interrelated" and we can learn a lot from these relations. In this talk we focus on d=3,4.

"Q5a: Is there a basic set d $\leq$ 4 dualities?"

No conclusive answer, but we recently learned that large families of dualities can be derived assuming only few *elementary dualities*.

We now have two new tools:

- The sequential deconfinement, for dualities with tensor matter.
- The dualisation algorithm, for mirror dualities.

Dualities from dualities I: the sequential deconfinment

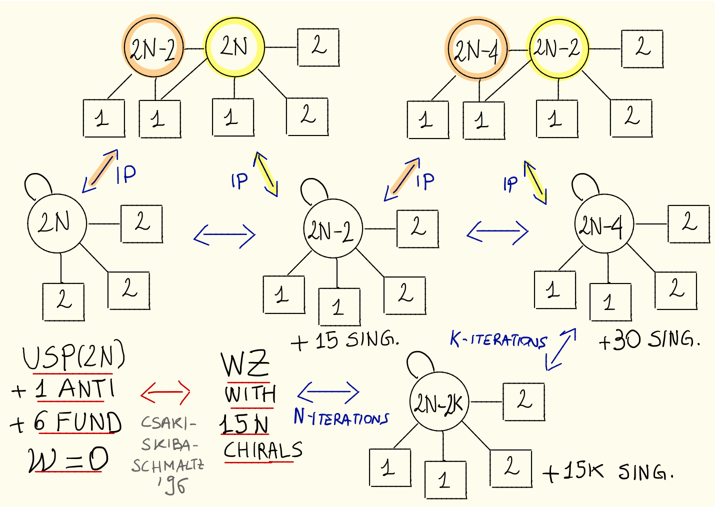
Deconfinement technique: trade a matter field in an adjoint or antisymmetric representation of the gauge group for an auxiliary gauge node by using an elementary (Seiberg-like) s-confining duality, old idea [Berkooz'96].

Recent revamp as sequential deconfinement: apply a sequence of duality moves involving deconfinement iteratively to prove more complicated dualities.

An s-confining example

[Bottini-Hwang-SP-Sacchi'22],[Bajeot-Benvenuti'22-I]

Elementary duality: $USp(2N_c)$, $2N_f$ fund, $\mathcal{W} = 0 \leftrightarrow USp(2N_f - 2N_c - 4)$, $2N_f$ fund, X_{ij} singlets $\mathcal{W} = q_i q_j X_{ij}$. [Intriligator-Pouliot'95] (IP duality)



All frames are IR dual, can match anomalies, map operators...

- All s-confining dualities (classified by [Csaki-Schmaltz-Skiba'96]) can be proved with seq. deconfinement [Bajeot-Benvenuti'22-I] and many more dualities, also for quiver theories [Bottini-Hwang-SP-Sacchi'22], [Bajeot-Benvenuti'22-II].
- Seq. deconfinement can be implemented also in 3d [SP-Sacchi'19],[Benvenuti-Garozzo-LoMonaco'20], [Amariti-Rota'22] and in 2d [Sacchi'20]. (see also [Garcia-Etxebarria-Heidenreich-Lotito-Sorout'21]).
- Implemented at the level of partition functions coincides with the strategy used by mathematicians to prove integral identities [Rains'03],[Spiridonov'04].
- Analogous manipulations appear in the study free field Dotsenko-Fateev correlators in 2d CFT [SP-Sacchi'19].

Different avatars of the same tool appear both in physics and math!

How far can we push this idea of deriving dualities from dualities? What about mirror dualities?

Dualities from dualities II: the mirror dualisation algorithm

Mirror symmetry [Intriligator-Seiberg'96] relates pairs of 3d $\mathcal{N} = 4$ theories with Higgs and Coulomb branches swapped.

Two recent results:

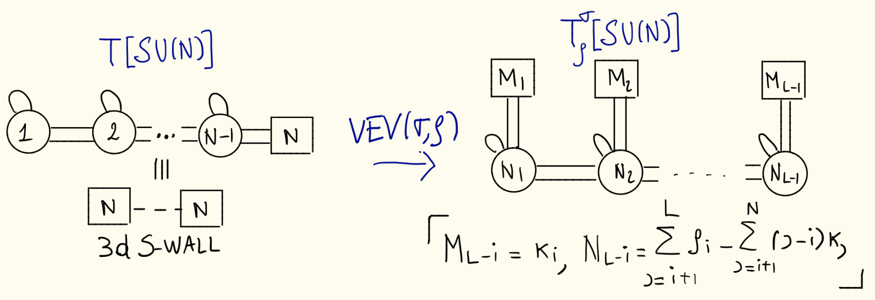
- We can construct families of 4d $\mathcal{N} = 1$ theories which enjoy mirror-like dualities and reduce to 3d $\mathcal{N} = 4$ mirror dual theories [Hwang-SP-Sacchi'20].
  So now mirror dualities fit in the huge class of dualities which can be obtained via 4d-3d reduction as in [Aharony-Razamat-Seiberg-Willett'13][2].

- Mirror dualities in 3d and 4d can be derived via a local dualization algorithm which uses two basic duality moves which can in turn be derived by iterations of Seiberg-like dualities [Hwang-SP-Sacchi'22].

I will focus on linear quivers, but our algorithm works also for circular quivers and for quivers with Chern-Simons couplings.

The 3d $\mathcal{N} = 4$ $T_\rho^\sigma[SU(N)]$ linear quivers family

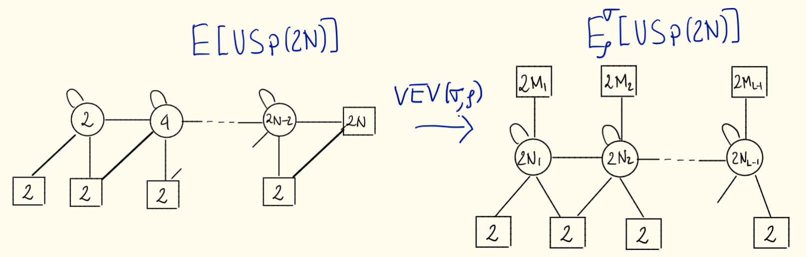
Introduced in [Gaiotto-Witten'08] are labelled by partitions of N
$\sigma = [\sigma_1, \cdots, \sigma_K] = \left[N^{k_N}, \ldots, 1^{k_1}\right]$ and $\rho = [\rho_1, \cdots, \rho_L] = \left[N^{l_N}, \ldots, 1^{l_1}\right]$:



- Emergent global symmetry group: $S(\prod_{i=1}^N U(k_i)) \times S(\prod_{i=1}^N U(l_i))$.
- $T_\rho^\sigma[SU(N)]$ is mirror dual to $T_\sigma^\rho[SU(N)]$.
- Realised on an Hanany-Witten set-up with N D3-branes suspended between K D5s and L NS5s.
- Can be obtained by turning on vevs for the moment maps in $T[SU(N)]$, self-mirror, the $\mathcal{N} = 4$ SYM S-wall [Gaiotto-Witten'08].

The 4d $\mathcal{N}=1$ $E_\rho^\sigma[USp(2N)]$ family

Introduced in [Hwang-SP-Sacchi'20]



- Defined by giving vevs to the moment maps of the $E[USp(2N)]$ theory, self-mirror, E-string tube [SP-Razamat-Sacchi-Zafrir'19].
- Global symmetry: $\prod_{i=1}^{N} USp(2k_i) \times \prod_{i=1}^{N} USp(2l_i) \times U(1)_t \times U(1)_c$ ($SU(2)$ symmetries are collected into blocks enhancing to $USp(2l_i)$)
- $E_\rho^\sigma[USp(2N)]$ is mirror dual to $E_\sigma^\rho[USp(2N)]$.
- $E_\rho^\sigma[USp(2N)]$ reduces to $T_\rho^\sigma[SU(N)]$ in 3d limit + RG flow breaking $USp(2N_i) \to U(N_i)$ and giving mass to all the fields in the *saw*.

Mirror Symmetry from (local) S-duality

If we realise 3d $\mathcal{N} = 4$ theories on Hanany-Witten brane set-ups, we can interpret mirror symmetry as a consequence of S-duality.

It has been argued that S-duality can act locally on each 5-brane creating an S-wall on its right and a S^{-1} wall on its left:

$$\overline{\mathrm{NS5}} \to S^{-1}\mathrm{D5}\,\mathrm{S}\,, \qquad \mathrm{D5} \to \mathrm{S}^{-1}\mathrm{NS5}\,\mathrm{S}\,.$$

The 3d theory on the S-wall is the $T[SU(N)]$ quiver [Gaiotto-Witten'08].

We now understand this local S-duality action in QFT and we have a dualisation algorithm for 3d and 4d quiver theories.

The algorithm uses the properties of the S-wall and two basic duality moves which essentially dualise a fundamental into a bifundamental and viceversa.

The basic duality moves are genuine IR dualities and can be proved by iterative applications of Seiberg-like dualities.

A key ingredient of our algorithm is the fact that we gauge emergent symmetries so it is an IR procedure.

Earlier works on S^3 matrix models [Gulotta-Herzog-Pufu'11,Assel'14].

In the abelian case a piecewise dualisation is given by a generalised Fourier transform [Kapustin-Strassler'99].

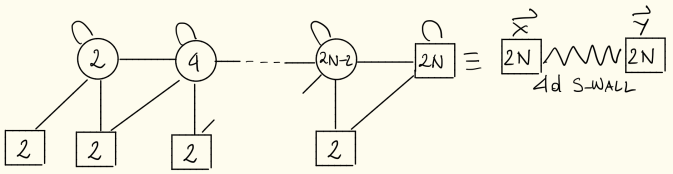
3d mirror symmetry can be realised as T-duality [Hori-Ooguri-Vafa'97].

I will present the 4d algorithm, the 3d one is completely analogous.

Let's begin with the ingredients:

- The S-wall
- The Identity wall
- The QFT blocks
- The basic duality moves

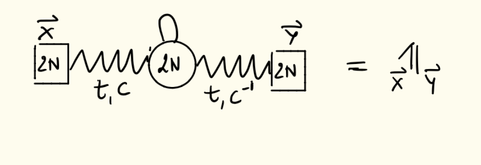
The 4d S-wall



- Global IR symmetry $USp(2N)_x \times USp(2N)_y \times U(1)_t \times U(1)_c$ with $SU(2)^N \to USp(2N)_y$ enhancement .
- Reduces to the 3d $\mathcal{N} = 4$ $T[SU(N)]$ S-wall theory.
- Is the building block for E-string compactifications [SP-Razamat-Sacchi-Zafrir'19].
- $\mathcal{I}_{FE[USp(2N)]}$ coincides with the interpolation kernel [Rains'14].

The Identity-wall

Gluing two S-walls we get the Identity-wall which identifies the two remaining $USp(2N)$ symmetries:



At the level of the index we have:

$$\oint \mathrm{d}\vec{z}_N \, \Delta_N(\vec{z};t) \mathcal{I}_{FE}^{(N)}(\vec{x};\vec{z};t;c) \mathcal{I}_{FE}^{(N)}(\vec{z};\vec{y},t;c^{-1}) = {}_{\vec{x}}\hat{\mathbb{I}}_{\vec{y}}(t) \,,$$

where

$${}_{\vec{x}}\hat{\mathbb{I}}_{\vec{y}}(t) = \frac{\prod_{i=1}^N 2\pi i x_i}{\Delta_N(\vec{x};t)} \sum_{\sigma \in S_N} \sum_{\pm} \prod_{i=1}^N \delta\left(x_i - y_{\sigma(i)}^{\pm 1}\right) \,,$$

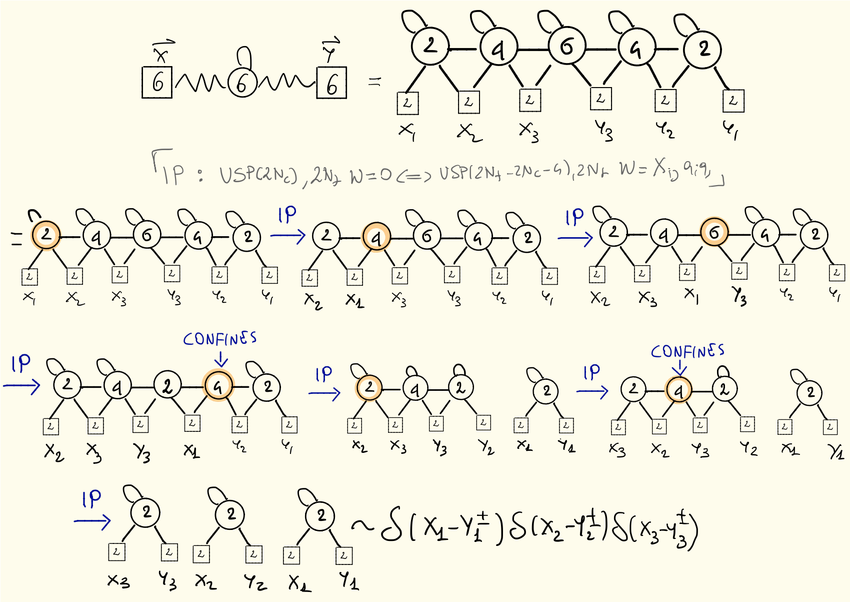
with $\sum_{\sigma \in S_N} \sum_{\pm}$ spanning the Weyl group of $USp(2N)$.

The proof of the Identity wall property has two main ingredients:

- The iterative application of the IP duality which splits the Identity-wall into a product of $SU(2)$ theories with 2 flavors ($+$ singlets).

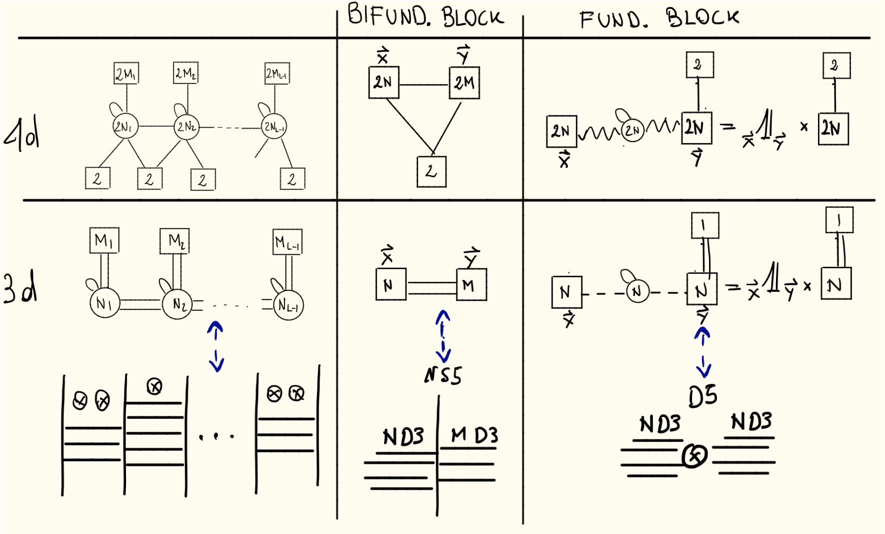
- The fact that $SU(2)$ with 2 flavors has a quantum deformed moduli space with chiral symmetry breaking, $Pff\left(TrQ_{[i}Q_{j]}\right) = \Lambda^4$.
  Its SCIndex seems to identically vanishes but poles appear for specific values of the fugacities. It behaves as a Dirac-δ distribution acting on *test* theories [Spiridonov-Vartanov'14].

$N = 3$ Identity wall



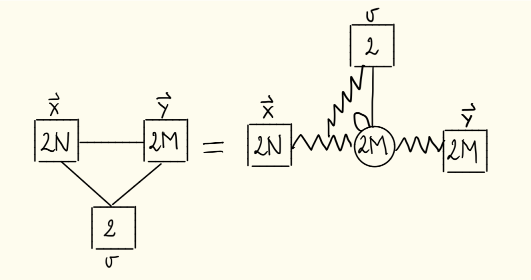
QFT blocks

We chop linear quivers into building blocks:

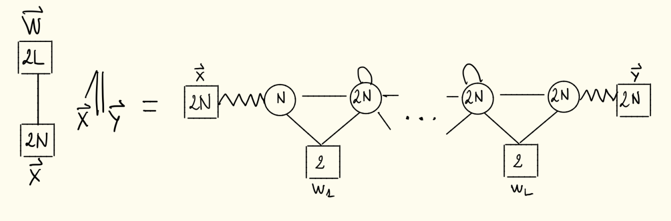


Basic duality moves

- Bifundamental block dualisation $N > M$:



- $2L$ fundamentals block dualisation:

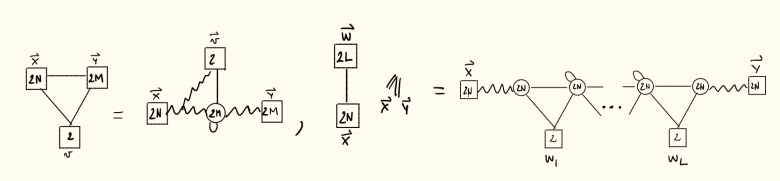


These are IR dualities which can be derived by iterative applications of the elementary IP duality! (Aharony duality in the 3d case)

The mirror dualisation algorithm

The local S-duality action is implemented as follows:

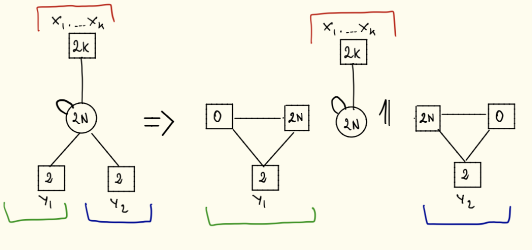
1. Chop the quiver by ungauging the gauge nodes into either triangle or fundamental blocks.
2. Dualise each block using the basic duality moves



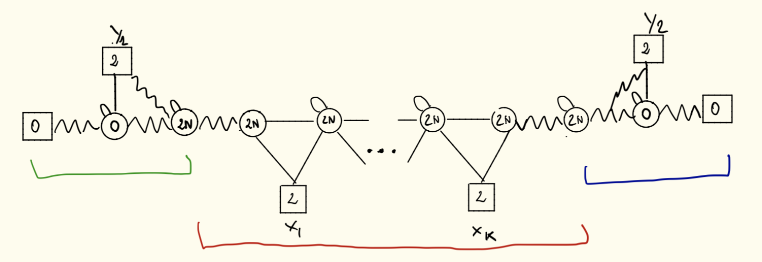
3. Glue back the dualised blocks producing Identity-walls to arrive at a quiver with no S-walls lefts.
4. If some operators acquired a VEVs follow the RG flow to the IR final configuration, which coincides with the expected mirror of the original theory.

The algorithm at work: SQCD mirror dualisation

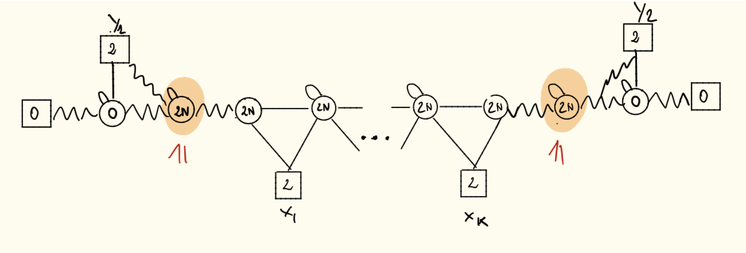
Starting from SQCD we decompose it into blocks



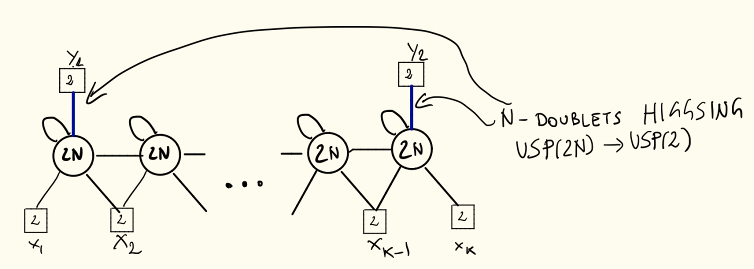
we dualise each block using the two basic duality moves:



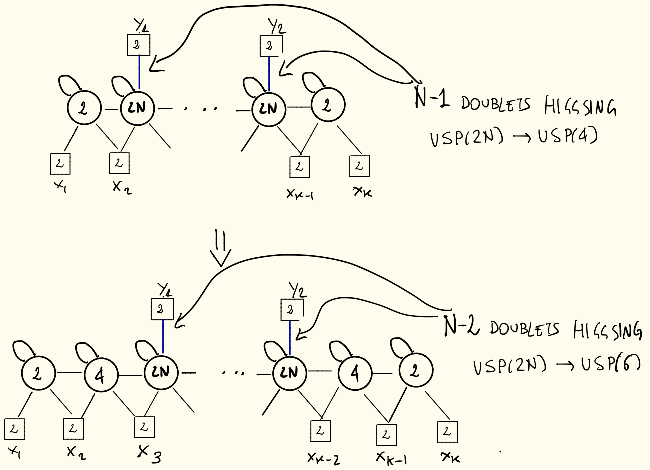
Integrating over the original nodes yields Identity-walls:



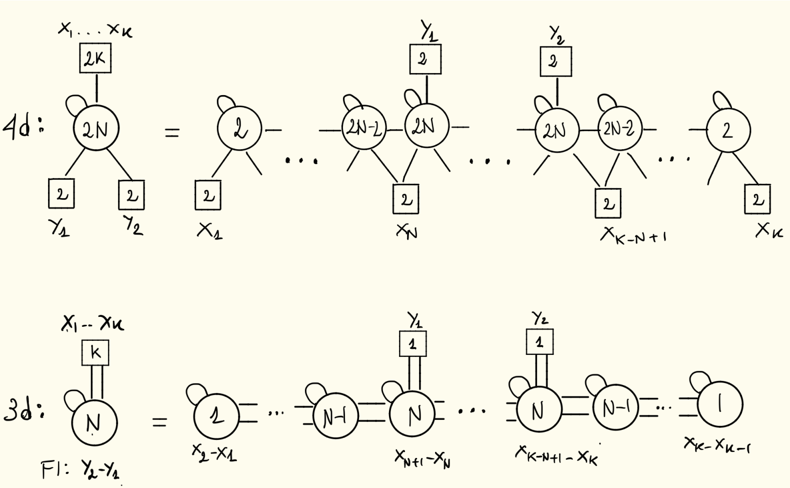
Implementing the identifications some operators acquire a VEV:



after partially higgsing the two external nodes the VEV propagates



The VEV propagates until we reach the mirror dual frame.

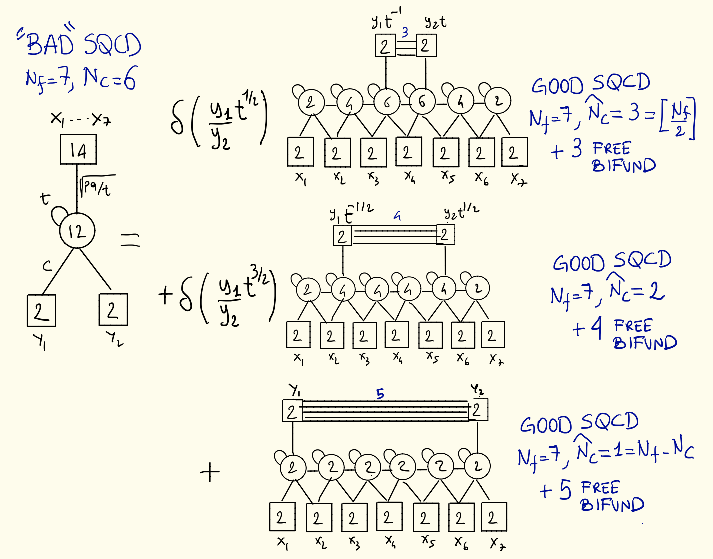


We have obtained Mirror dualities from Seiberg dualities!

For $K < 2N$ (3d *bad* SQCD) the propagation of VEVs becomes trickier, the index develops singularities at distinct loci in the moduli space each associated to a magnetic dual frame.

[Giacomelli-Hwang-Marino-SP-Sacchi, to appear]

Multiple magnetic dual frames for Bad SQCD



Taking the 3d limit we recover known results for bad $U(N_c)$ $\mathcal{N}=4$ SQCD, but with a new insight...

Mirror of 3d $\mathcal{N} = 4$ Bad SCQD

- The theory develops singularities when the FI is tuned to particular values. At each of these loci we find a given dual magnetic frame with an interacting sector: the dual of $U(r)$ SQCD and $N_c - r$ free hypers with $r = N_f - N_c + 1, \cdots [N_f/2]$. (These are the frames predicted by [Assel-Cremonesi'17]).

- Implementing the constraint on the FI on the electric side corresponds to turning on a VEV for a dressed monopole which Higgses $U(N) \to U(N-k)$ to recover the corresponding dual frame, including the free sector.

- When the FI is turned on we have a single non-singular dual frame: the dual of $U(N_f - N_c)$ SQCD with $2N_c - N_f$ free hypers (as in [Kim-Kim-Kim-Lee'12],[Yaakov'13],[Gaiotto-Koroteev'13]).

Conclusions

- Many dualities with tensor matter can be demonstrated by iterations of Seiberg-like dualities via the sequential deconfinment technique.
- Mirror dualities for 4d and 3d linear quivers can be demonstrated via the local dualisation algorithm assuming only Seiberg-like dualities.
- We can extend this construction to circular quivers and to more general $SL(2,\mathbb{Z})$ dualities [Comi-Hwang-Marino-SP-Sacchi, in progress].
- The local dualisation approach can clarify the global structure of the moduli space of bad theories [Giacomelli-Hwang-Marino-SP-Sacchi, in progress] and provide mirror duals of 3d theories with $\mathcal{N} < 4$ susy [Benvenuti-Comi-SP, in progress].
- Are there dualities which cannot be proved in terms of elementary dualities? Are there 3d dualities which cannot be uplifted to 4d? What about non-susy dualitities?
- Shlomo's Q5: "Do all dualities in d $\leq$ 4 have a geometric explanation?"

THANK YOU!

$SU(2)$ with 2 flavors

Start from the $SU(2)$ theory with $3F$ which has a WZ dual:

$$I^{3F} = \frac{(p;p)(q;q)}{2} \oint \frac{dz}{2\pi i z} \prod_{i=1}^{6} \frac{\Gamma_e\left(s_i z^{\pm 1}\right)}{\Gamma_e\left(z^{\pm 2}\right)} = \prod_{i<j} \Gamma_e\left(s_i^{\pm} s_j^{\pm}\right)$$

$\prod_{i=1}^{6} s_i = pq$. We integrate out one flavor:

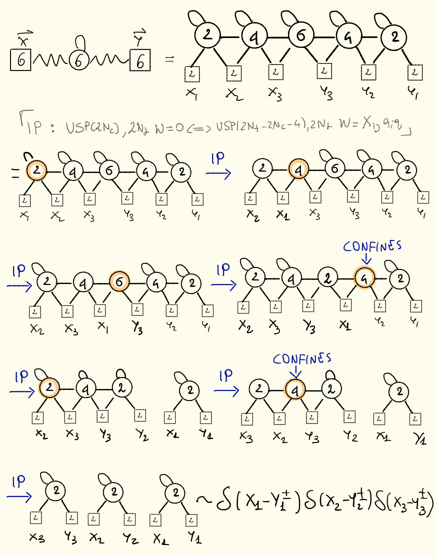
$$\lim_{s_5 s_6 \to pq} I^{3F} = I^{2F} \,,$$

I^{2F} identically vanishes but poles appear for specific values of the fugacities. The key fact is that

$$\lim_{s\to 1} \Gamma_e\left(pqs^2\right)\Gamma_e\left(s^{-1}x^{\pm 1}y^{\pm 1}\right) = \frac{\Gamma_e\left(x^{\pm 2}\right)}{(p;p)_\infty (q;q)_\infty} \left[\delta\left(X+Y\right) + \delta\left(X-Y\right)\right] .$$

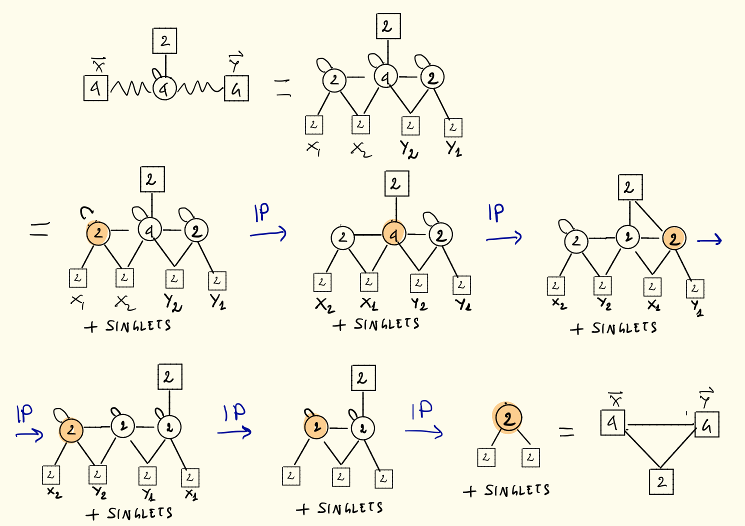
The index behaves as a Dirac delta, its support are the points where chiral symmetry breaking occurs and the proper confining phase of free chirals appears.

Intriligator-Pouliot iterations



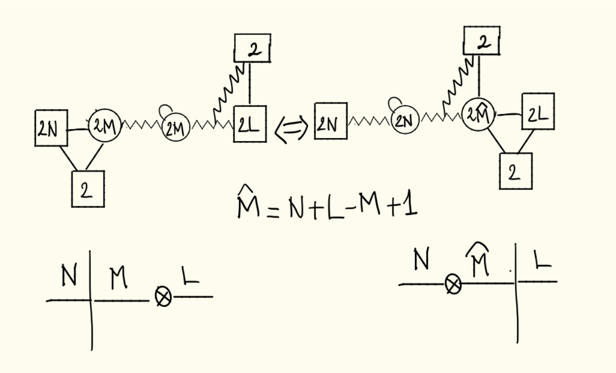
Bifundamental block dualisation

IR duality derived by iterative applications of the elementary IP duality:



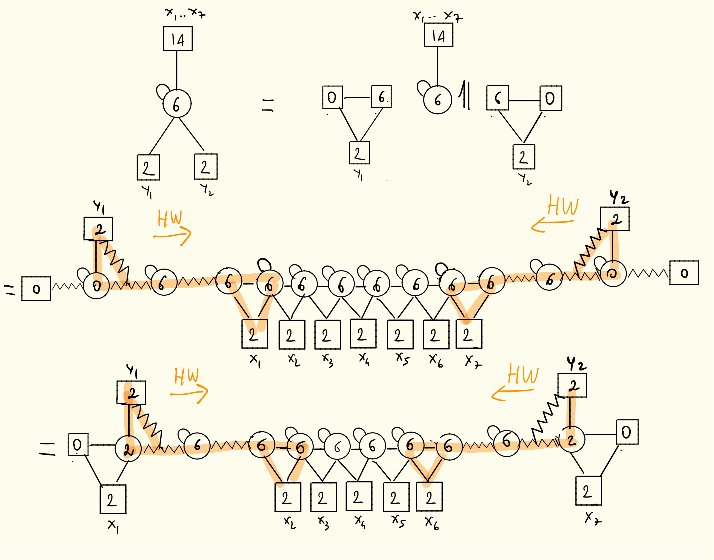
The Hanany-Witten move duality

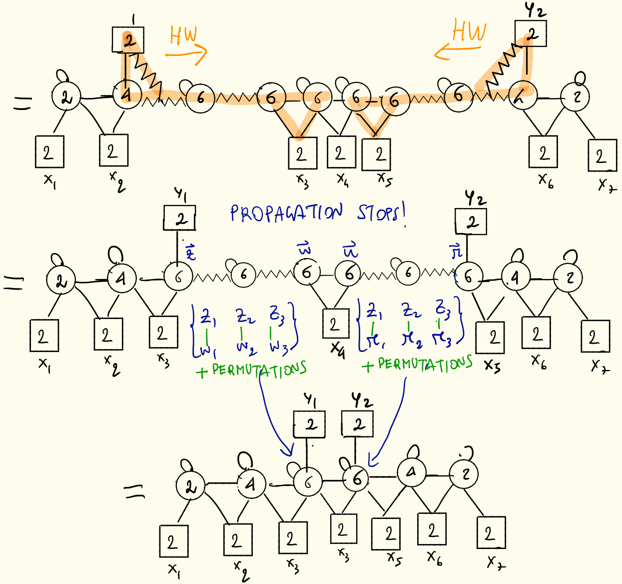
We actually have a duality move corresponding to the Hanany-Witten move $NS5$ and $D5$ passing through each other:



we can use this move to propagate the vev and get rid of S-walls.

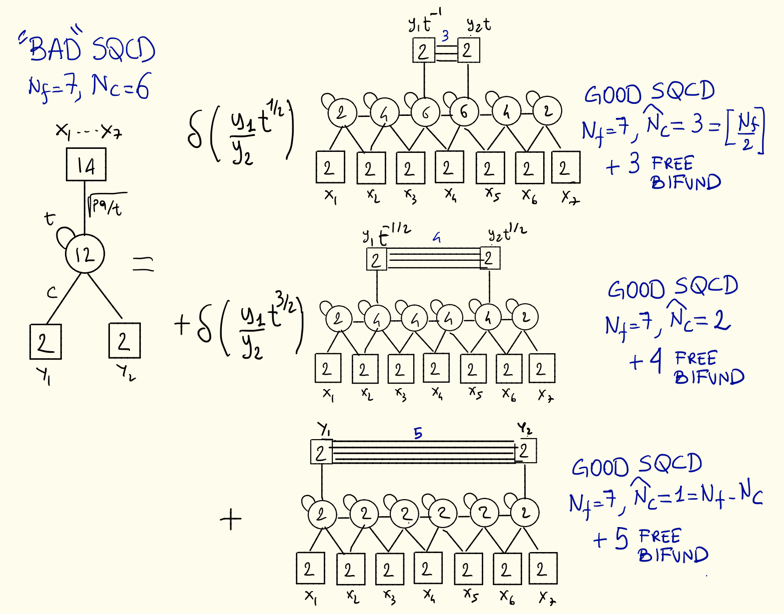
Good SQCD $N_f = 7, N_c = 3$





HW
HW
PROPAGATION STOPS!
+ PERMUTATIONS
+ PERMUTATIONS

Bad SQCD $N_f = 7, N_c = 6$



Mirror of 4d $\mathcal{N} = 1$ Bad SQCD

- The index develops singularities when the fugacities are tuned to particular values, at each of these loci we find a given dual magnetic frame. Each frame has an interacting part given by the dual of $USp(2r)$ SQCD and $N_c - r$ free bifund. with $r = N_f - N_c + 1, \cdots [N_f/2]$.

- The constraint on the fugacities on the electric side corresponds to turning on a VEV for the dressed meson $PA^{k-1}Q$ which Higgses $USp(2N) \to USp(2N - 2k)$ to recover the corresponding dual frame, including the free sector.

- When the fugacities are all distinct we have a single non-singular frame; the dual of $USp(2N_f - 2N_c)$ SQCD with $2N_c - N_f$ free bifund.