Celestial amplitudes from flat space limits of AdS Witten diagrams

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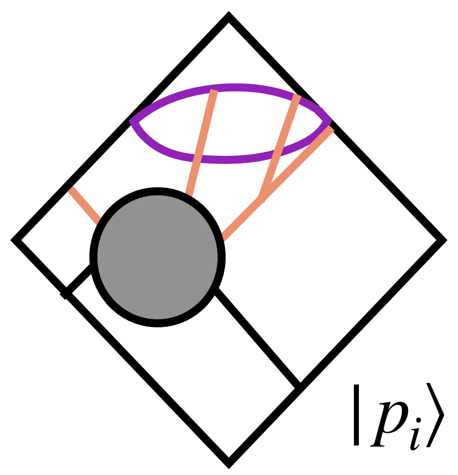
Based on 2206.10547 with Leonardo Pipolo de Gioia

Motivation

- Good observables for (quantum) gravity in 4D asymptotically flat space?

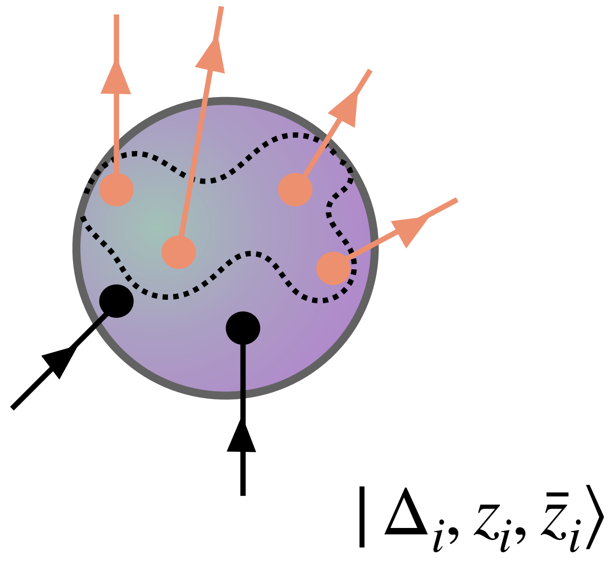


Scattering amplitudes: perturbatively calculable;

constrained by analyticity, unitarity;

IR, UV divergences?

$$|p_i\rangle = |\omega_i, z_i, \bar{z}_i\rangle \to |\Delta_i, z_i, \bar{z}_i\rangle = \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} |\omega_i, z_i, \bar{z}_i\rangle$$



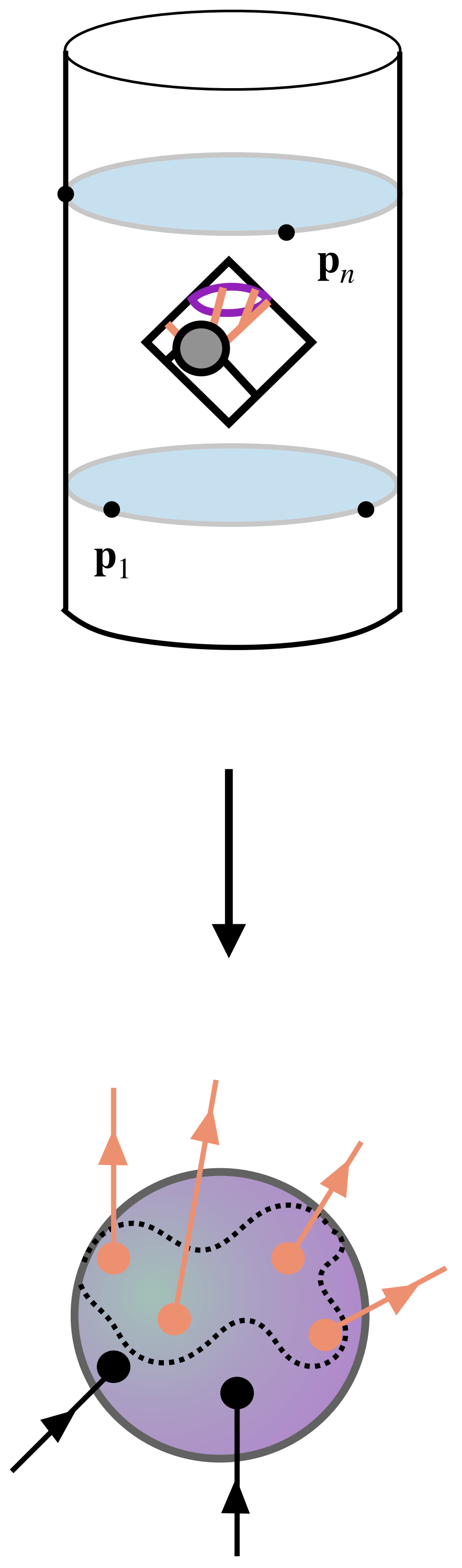
Celestial amplitudes: constrained by symmetries: BMS, Virasoro, w_∞, …;

IR divergences captured by vertex operators;

how to calculate? analyticity, unitarity?

Motivation

- Good observables for (quantum) gravity in asymptotically flat space



S-matrix from flat space limit of AdS/CFT observables:

- HKLL $$\sqrt{2\omega} a_q \propto \int_0^{\pi} d\tau e^{i\omega R(\tau - \frac{\pi}{2})} \mathcal{O}(\tau, \hat{q})$$

- Mellin correlators $$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle \propto \int [d\delta_{ij}] M(\delta_{ij}) \prod_{i<j}^{n} \Gamma(\delta_{ij}) \left(x_{ij}^2 \right)^{-\delta_{ij}}$$

[Polchinski ‘99; Susskind ’99; Giddings ’99; Penedones ’10;…; Hijano, Neuenfeld ’20]

Celestial amplitudes from flat space limit of AdS/CFT?

Outline

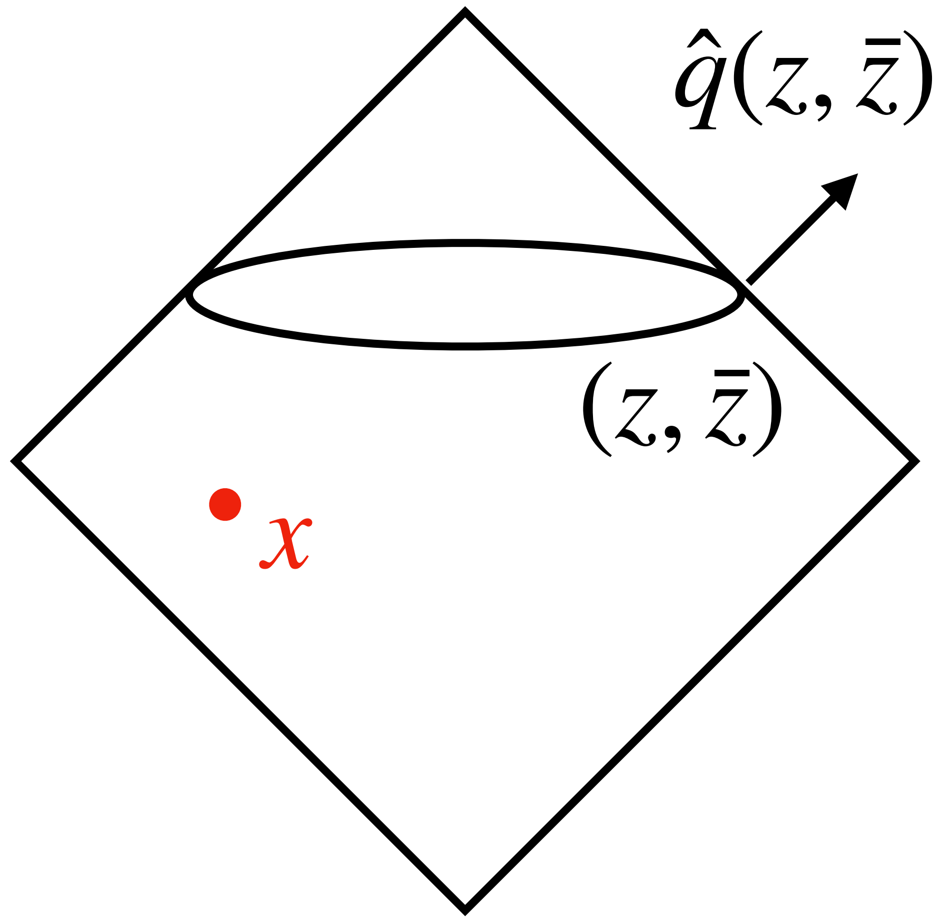
- Celestial CFT$_{d-1}$ (CCFT$_{d-1}$) amplitudes from flat space limit of AdS$_{d+1}$ Witten diagrams
- Example: Celestial two-point function in shockwave background
- Celestial eikonal amplitude for scalar 4-point scattering

Celestial amplitudes

In 4D, scalar wave equation $\nabla^2\Psi = 0$ admits conformal primary solutions with respect to $SO(1{,}3) \simeq SL(2{,}\mathbb{C})$:

[Pasterski, Shao, Strominger '17]

$$\left.\begin{aligned}(L_0+\bar{L}_0)\varphi_\Delta = \Delta\varphi_\Delta, \quad (L_0-\bar{L}_0)\varphi_\Delta = 0\\ L_1\varphi_\Delta = \bar{L}_1\varphi_\Delta = 0\end{aligned}\right\} \qquad \varphi_\Delta(\eta\hat{q};x) \equiv \frac{(i\eta)^\Delta\Gamma(\Delta)}{(-\hat{q}\cdot x \pm i\eta\epsilon)^\Delta}$$



Celestial amplitude: $$\widetilde{\mathscr{A}}(\Delta_j, z_j) \equiv \prod_{i=1}^{n}\left(i\int d^4x_i\varphi_{\Delta_i}(\eta_i\hat{q}_i;x_i)\right) C(x_1,\cdots,x_n)$$

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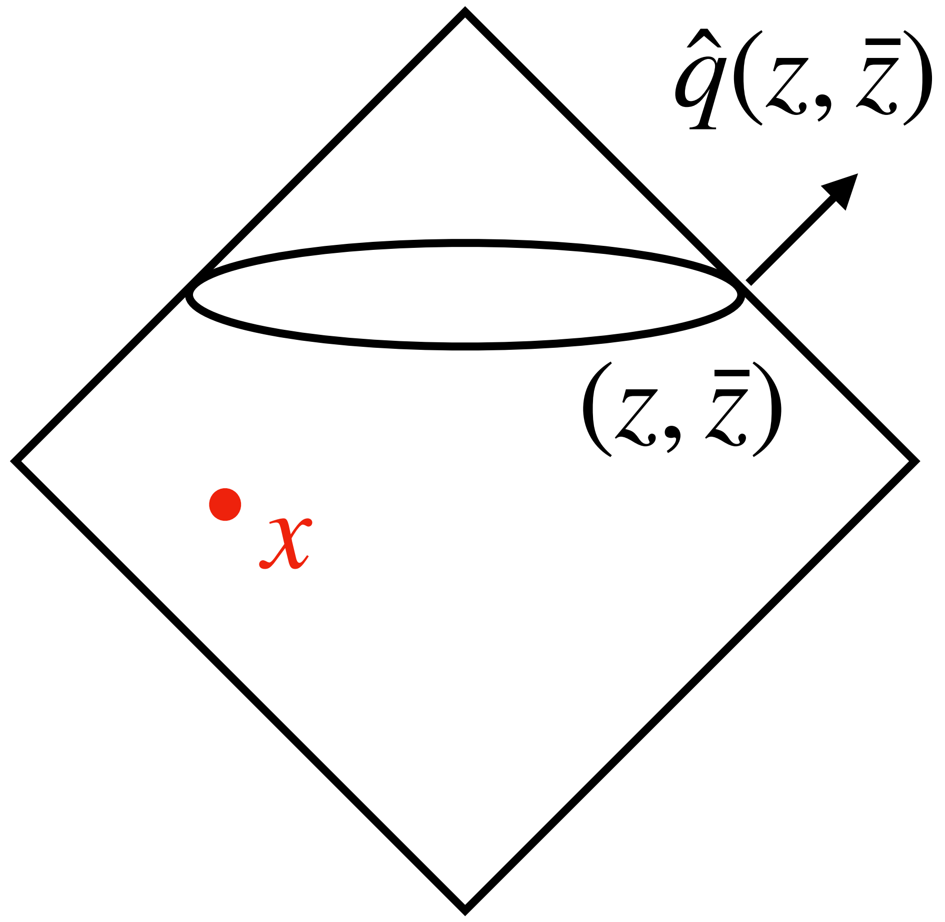

$\eta = 1 \ (\eta = -1)$

outgoing (incoming)

Celestial amplitude:

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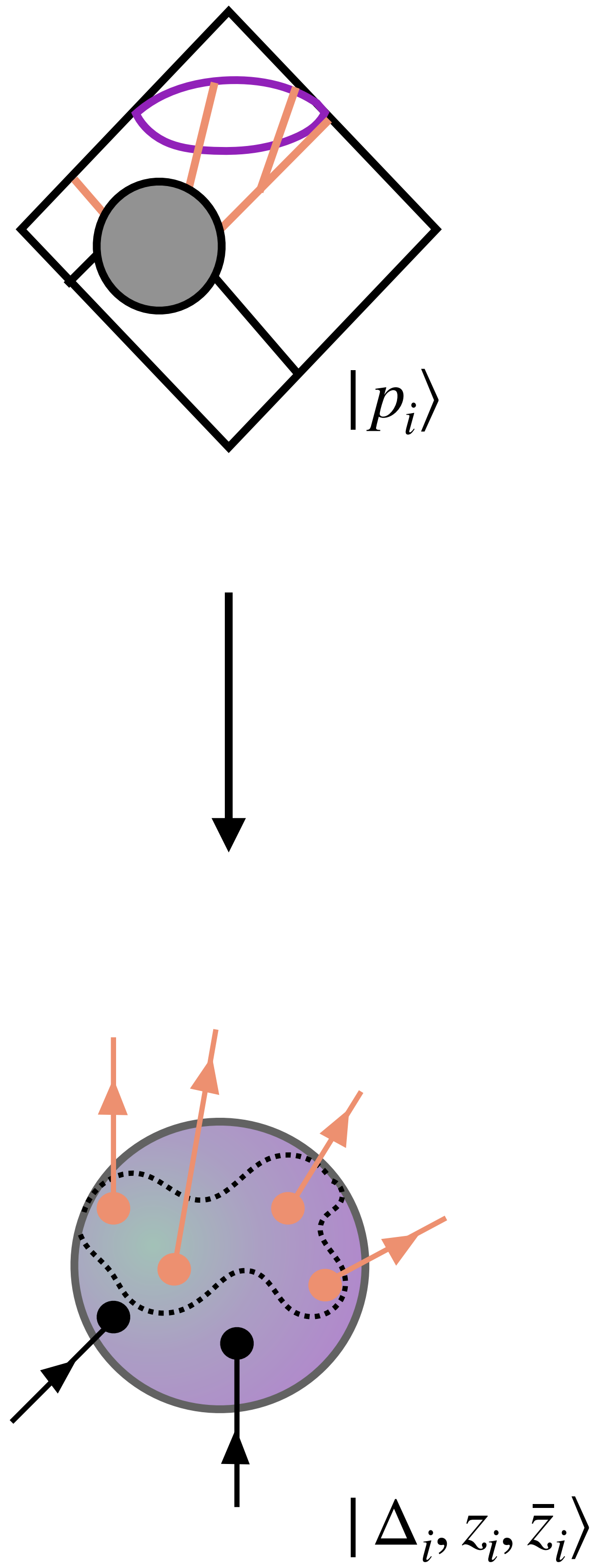
Celestial amplitudes



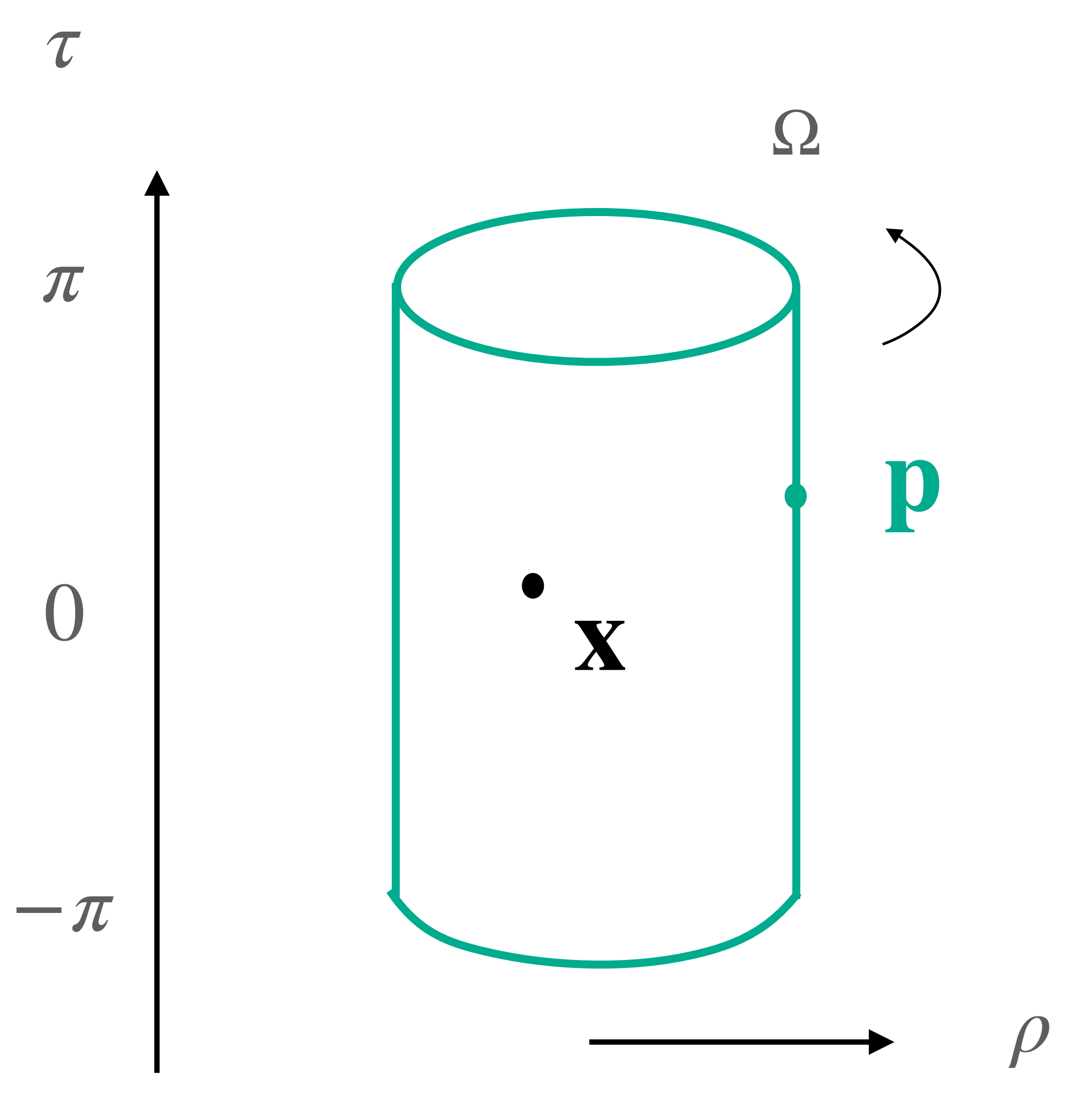
$$\mathscr{A}(p_i) \equiv \prod_{i=1}^{n} \left(i \int d^4 x_i e^{-ip_i \cdot x_i} \right) C(x_1, \cdots, x_n), \quad p_i = \eta_i \omega_i \hat{q}_i \qquad \text{(S-matrix)}$$

$$\prod_{i=1}^{n} \left(\int_0^{\infty} d\omega_i \omega_i^{\Delta_i - 1} \right) \mathscr{A}(p_i), \qquad \varphi_\Delta(\eta \hat{q}; x) = \int_0^{\infty} d\omega \omega^{\Delta - 1} e^{-i\omega\eta\hat{q}\cdot x}$$

$$\widetilde{\mathscr{A}}(\Delta_j, z_j) \equiv \prod_{i=1}^{n} \left(i \int d^4 x_i \varphi_{\Delta_i}(\eta_i \hat{q}_i; x_i) \right) C(x_1, \cdots, x_n) \qquad \text{(Celestial amplitude)}$$

- Can be generalized to higher dimensions

Flat space from AdS observables



Embedding space

$$\begin{cases} \mathbf{x} = (X^+, X^-, X^i), \quad i = 1, \cdots, d \\ \mathbf{p} = \lim_{\rho \to \frac{\pi}{2}} \frac{1}{2R} \cos\rho \ \mathbf{x} \end{cases}$$

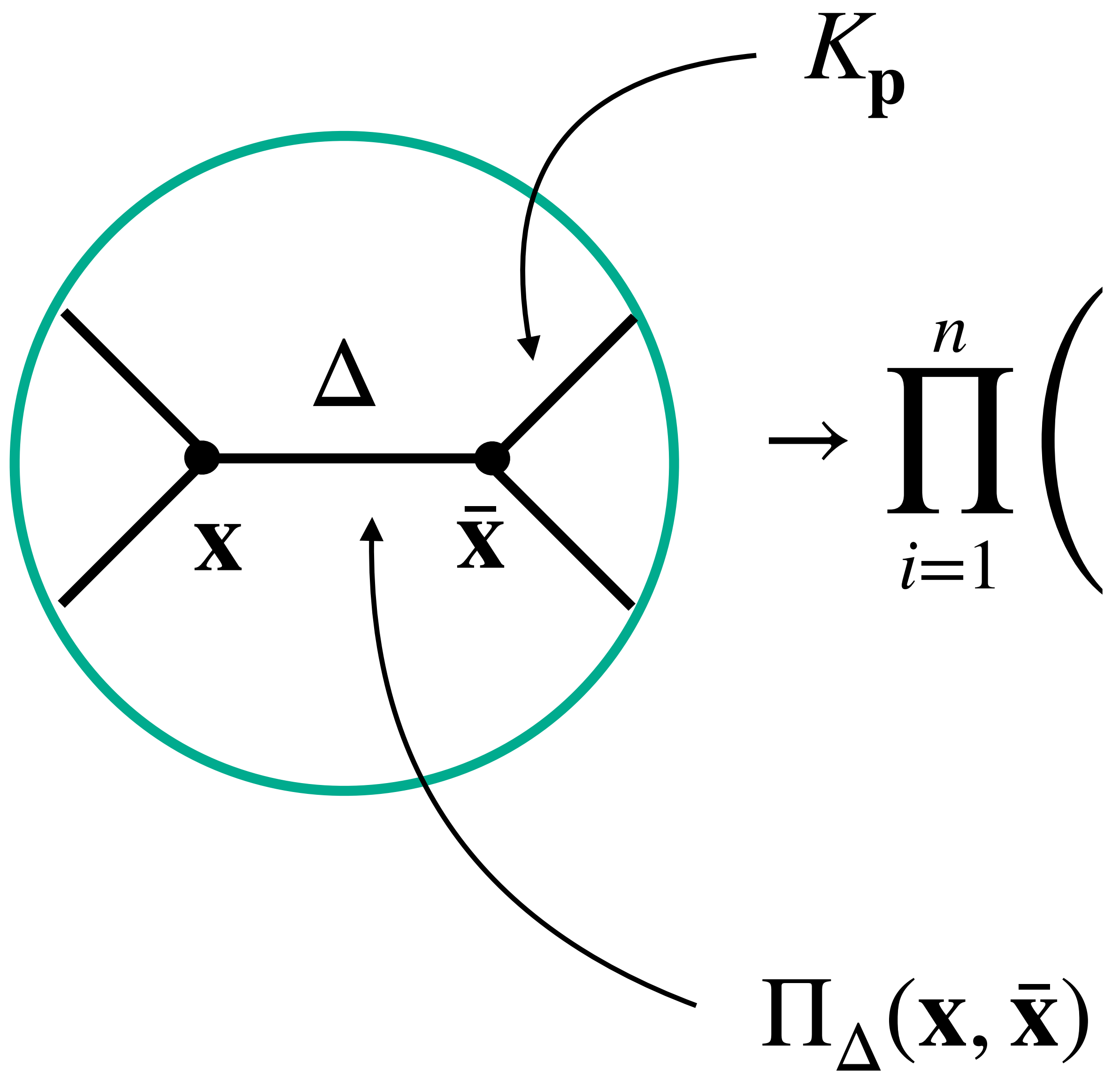
$$X^+ = -R\frac{\cos\tau - \sin\rho\Omega_{d+1}}{\cos\rho},$$

$$X^- = -R\frac{\cos\tau + \sin\rho\Omega_{d+1}}{\cos\rho},$$

$$X^1 = -R\frac{\sin\tau}{\cos\rho},$$

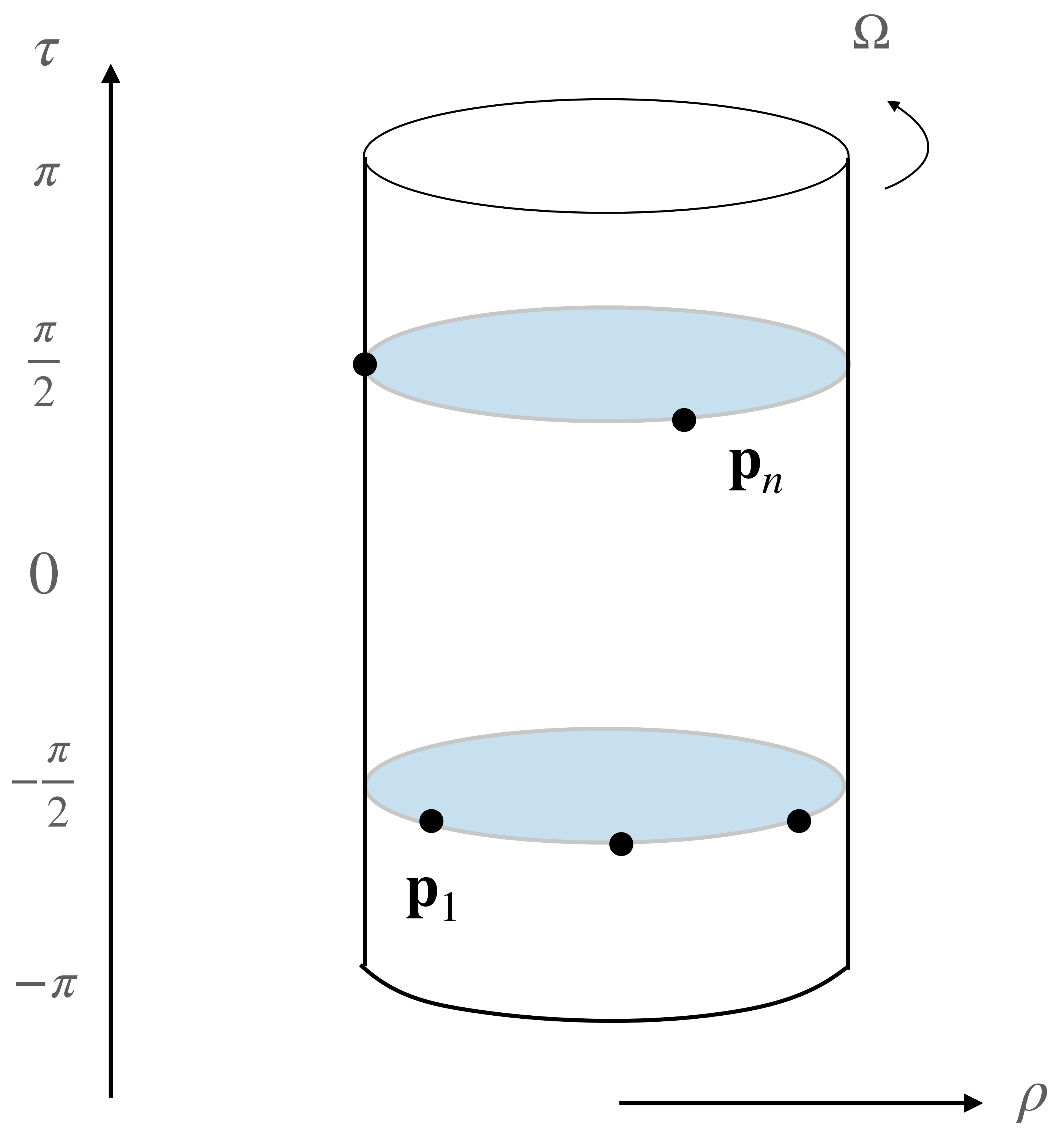
$$X^i = R\tan\rho\Omega_i, \quad \Omega_i \in S^{d-1}$$

Scalar Witten diagram



$$\to \prod_{i=1}^{n} \left(\int_{AdS_{d+1}} d^{d+1}\mathbf{x}_i K_{\Delta_i}(\mathbf{p}_i, \mathbf{x}_i) \right) \mathbf{\Pi}_\Delta$$

Flat space from AdS observables



Spheres antipodally matched

Flat space limit: $\tau = \frac{t}{R}, \quad \rho = \frac{r}{R}$

$R \to \infty, \quad t, r \;\; \text{fixed} \implies$ AdS $\to$ Minkowski

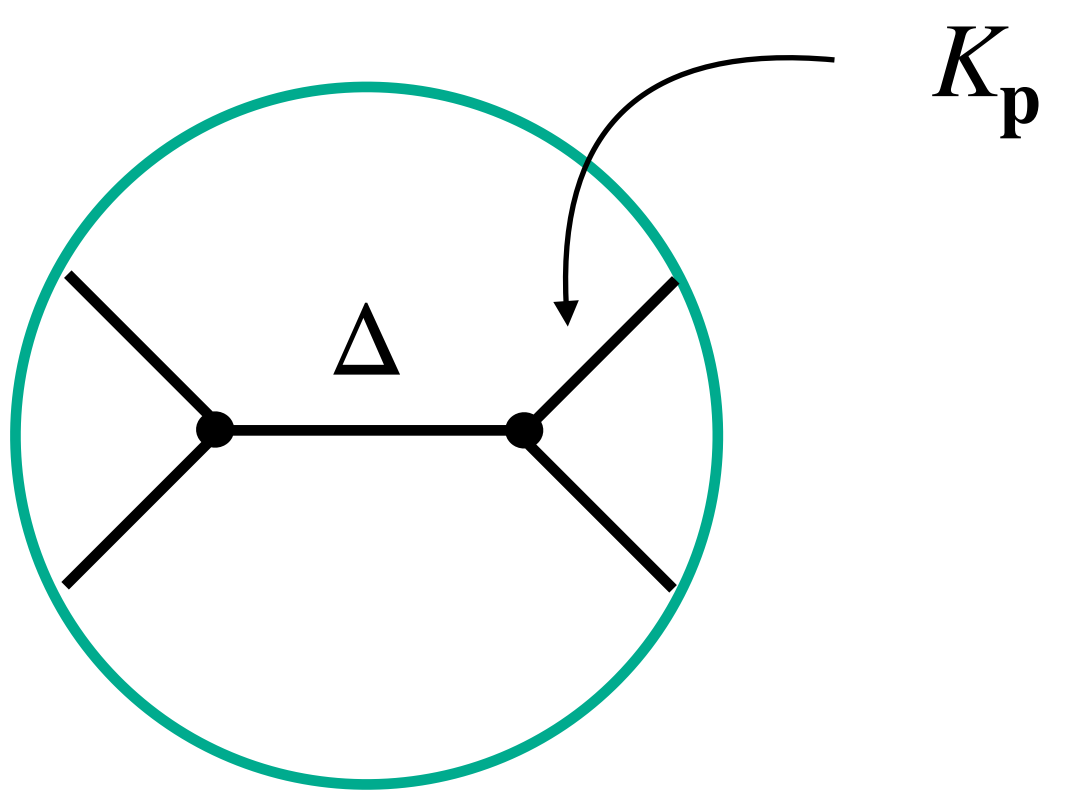
(Scalar) AdS$_{d+1}$ Witten diagrams with boundary operator insertions separated by $\Delta\tau = \pi$ reduce to celestial amplitudes to leading order as $R \to \infty$

Flat space from AdS observables

- Bulk-to-boundary propagators:

$$K_\Delta(\mathbf{p};\mathbf{x}) = \frac{C^d_\Delta}{(-2\mathbf{x}\cdot\mathbf{p}+i\epsilon)^\Delta}$$

$$\tau = \frac{t}{R}, \quad \rho = \frac{r}{R}, \quad R \to \infty$$



$$K_\Delta(\mathbf{p},\mathbf{x}) = C^d_\Delta\left[\frac{1}{(R\cos\tau_p + t\sin\tau_p - r\Omega_p\cdot\Omega + O(R^{-1}) + i\epsilon)^\Delta}\right],$$

$$\tau_p = \frac{\pi}{2}: \quad K_\Delta(\mathbf{p},\mathbf{x}) = C^d_\Delta\left[\frac{1}{(-\hat{p}\cdot x + i\epsilon)^\Delta} + O(R^{-1})\right], \ \hat{p} = (1,\Omega_p)\in\mathbb{R}^{1,d}$$

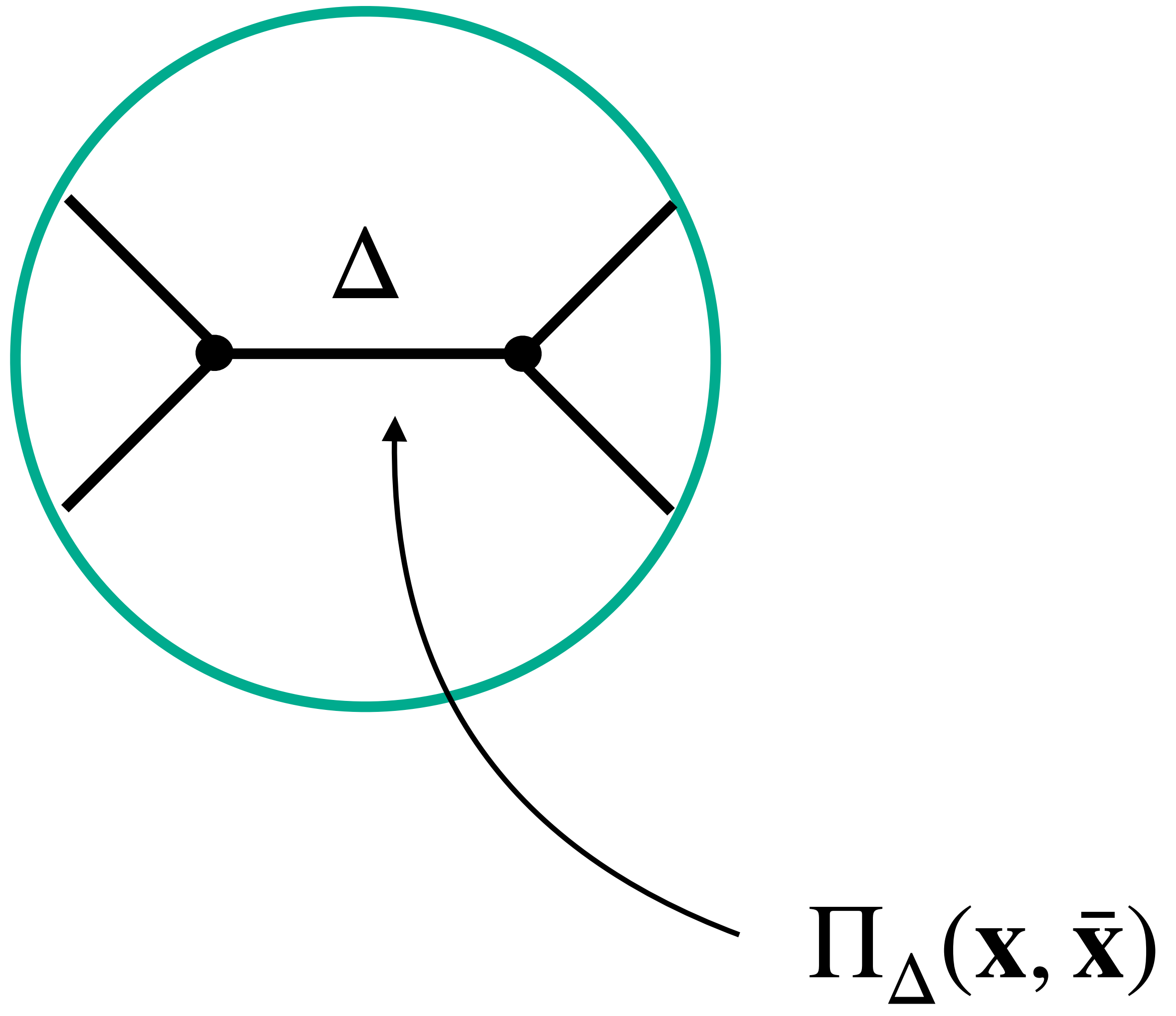
$$\tau_p = -\frac{\pi}{2}: \quad K_\Delta(\mathbf{p},\mathbf{x}) = C^d_\Delta\left[\frac{1}{(\hat{p}\cdot x + i\epsilon)^\Delta} + O(R^{-1})\right], \ \hat{p} = (1,-\Omega_p)\in\mathbb{R}^{1,d}$$

Flat space from AdS observables

- Bulk-to-bulk propagators: $$\left(\Box_{AdS_{d+1}} - \frac{\Delta(\Delta-d)}{R^2}\right)\mathbf{\Pi}_\Delta(\mathbf{x},\bar{\mathbf{x}}) = i\delta_{AdS_{d+1}}(\mathbf{x},\bar{\mathbf{x}})$$

$$\Box_{\mathbb{R}^{1,d}} - m^2 + O(R^{-2}) \qquad\qquad \delta_{\mathbb{R}^{1,d}}(x,\bar{x}) + O(R^{-2})$$

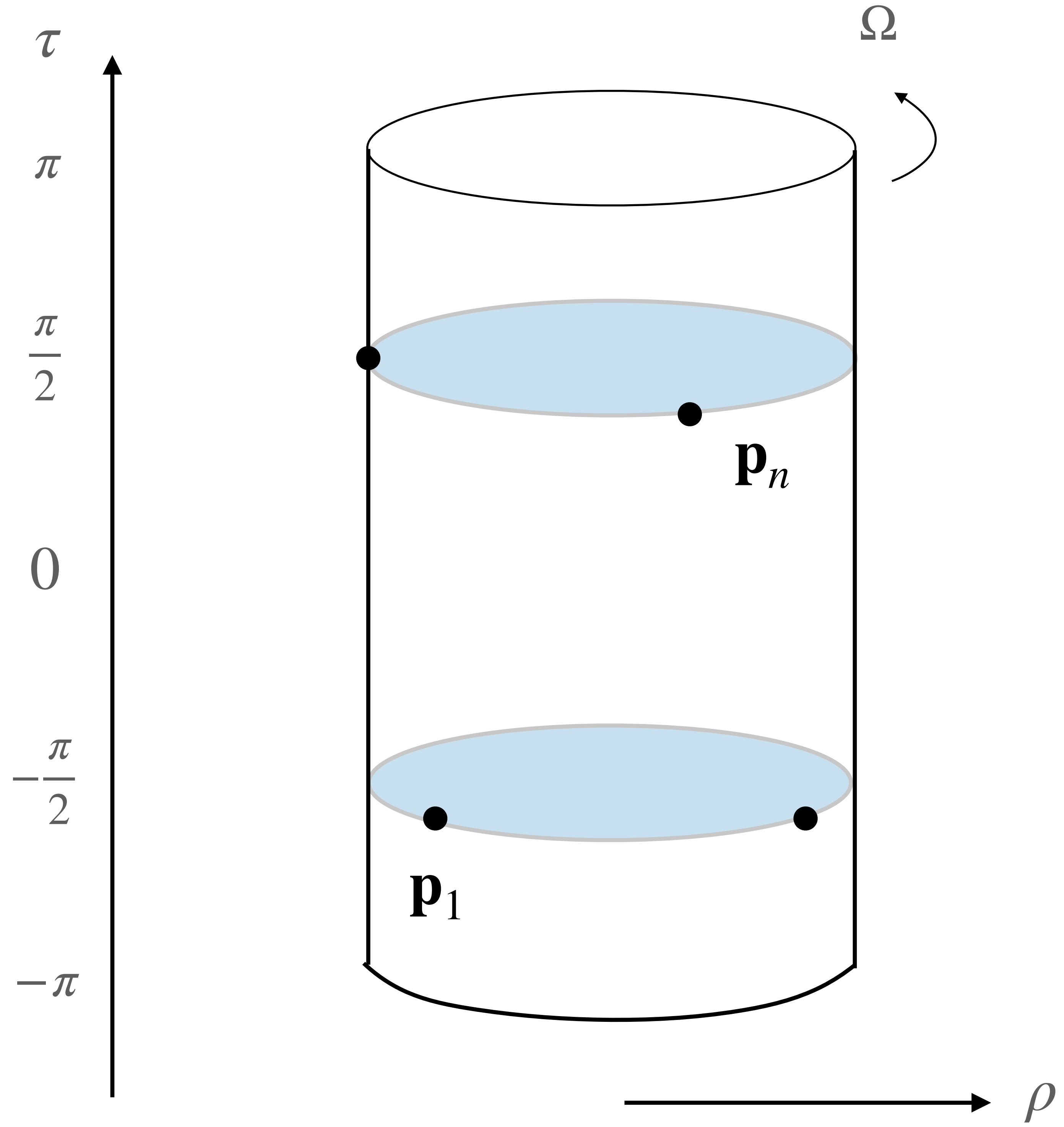
$$\mathbf{\Pi}_\Delta(\mathbf{x},\bar{\mathbf{x}}) = G(x,\bar{x}) + O(R^{-2})$$



where $$(\Box_{\mathbb{R}^{1,d}} - m^2)G(x,\bar{x}) = i\delta_{\mathbb{R}^{1,d}}(x,\bar{x}), \quad m \equiv \lim_{R\to\infty}\frac{\Delta}{R}$$

- Vertices: Integral over AdS$_{d+1}$ $\rightarrow$ integral over $\mathbb{R}^{1,d}$

CCFT$_{d-1}$ amplitudes from AdS$_{d+1}$ Witten diagrams



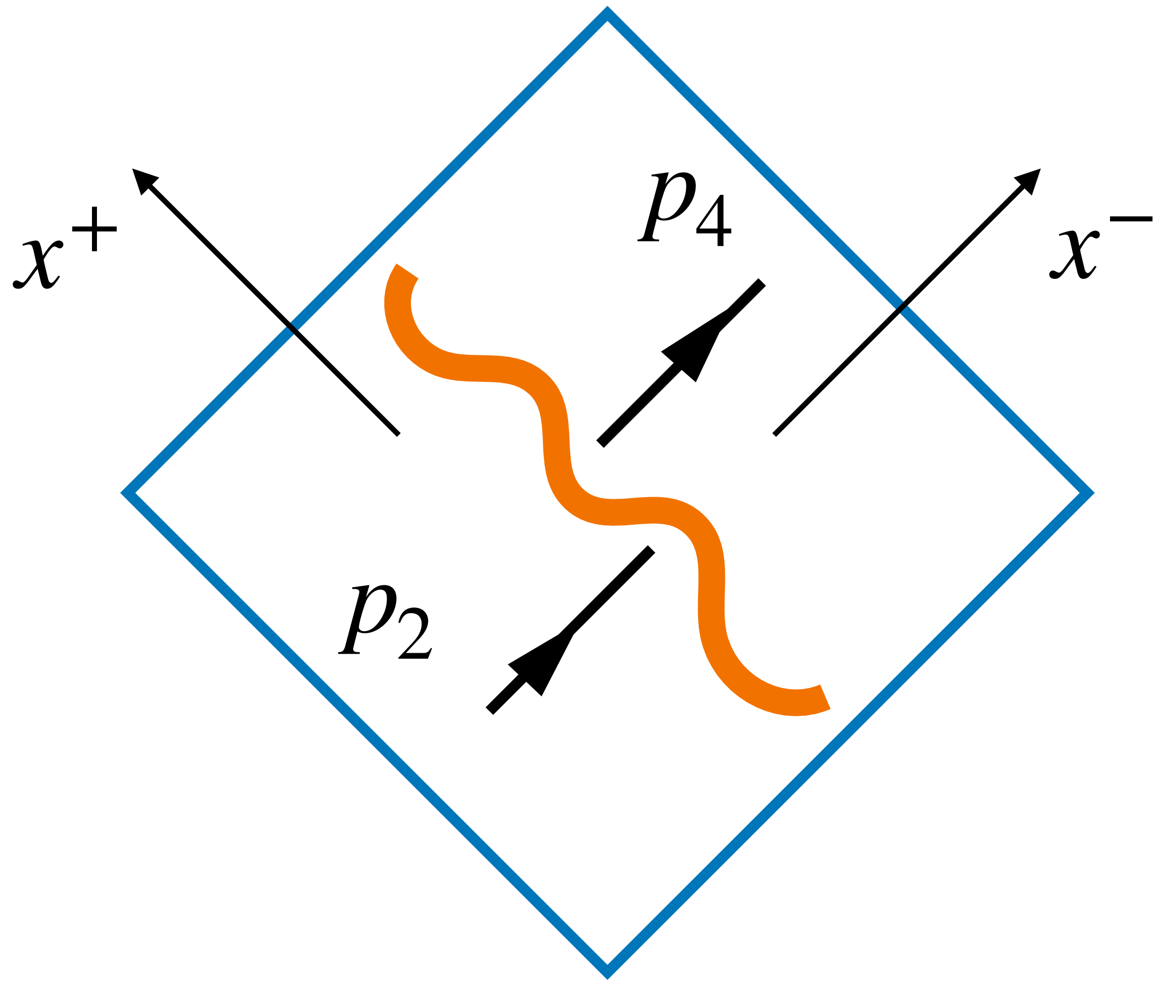
$$K_\Delta(\mathbf{p},\mathbf{x}) = \frac{C^d_\Delta}{(-2\mathbf{p}\cdot\mathbf{x}+i\epsilon)^\Delta} \xrightarrow[R\to\infty]{} \varphi_\Delta(x;\eta\hat{q}) = \frac{(i\eta)^\Delta\Gamma(\Delta)}{(-\hat{q}\cdot x+i\eta\epsilon)^\Delta}$$

$\eta = \pm 1$ **depending on** $\tau_p = \pm\frac{\pi}{2}$

- Bulk-to-bulk propagator and metric $\xrightarrow[R\to\infty]{}$ flat versions

$$\mathscr{W}(\mathbf{p}_j) = \prod_{i=1}^{n}\left(\int_{AdS_{d+1}} d^{d+1}\mathbf{x}_i K_{\Delta_i}(\mathbf{p}_i,\mathbf{x}_i)\right)\mathbf{\Pi}_\Delta(\mathbf{x}_j) \xrightarrow[R\to\infty]{} \widetilde{\mathscr{A}}(\Delta_j,z_j,\bar{z}_j) = \prod_{i=1}^{n}\left(\int_{\mathbb{R}^{1,d}} d^{d+1}x_i\varphi_{\Delta_i}(\eta_i\hat{q}_i;x_i)\right)\mathrm{C}(x_j)$$

Example: shockwave two-point function



- Shockwave in $\mathbb{R}^{1,3}$ $\qquad ds^2 = -dx^- dx^+ + ds_\perp^2 + h(x_\perp)\delta(x^-)(dx^-)^2$

- Massless scalar probe in this background

$$-4\partial_-\partial_+\phi - 4\delta(x^-)h(x_\perp)\partial_+^2\phi + \partial_\perp^2\phi = 0$$

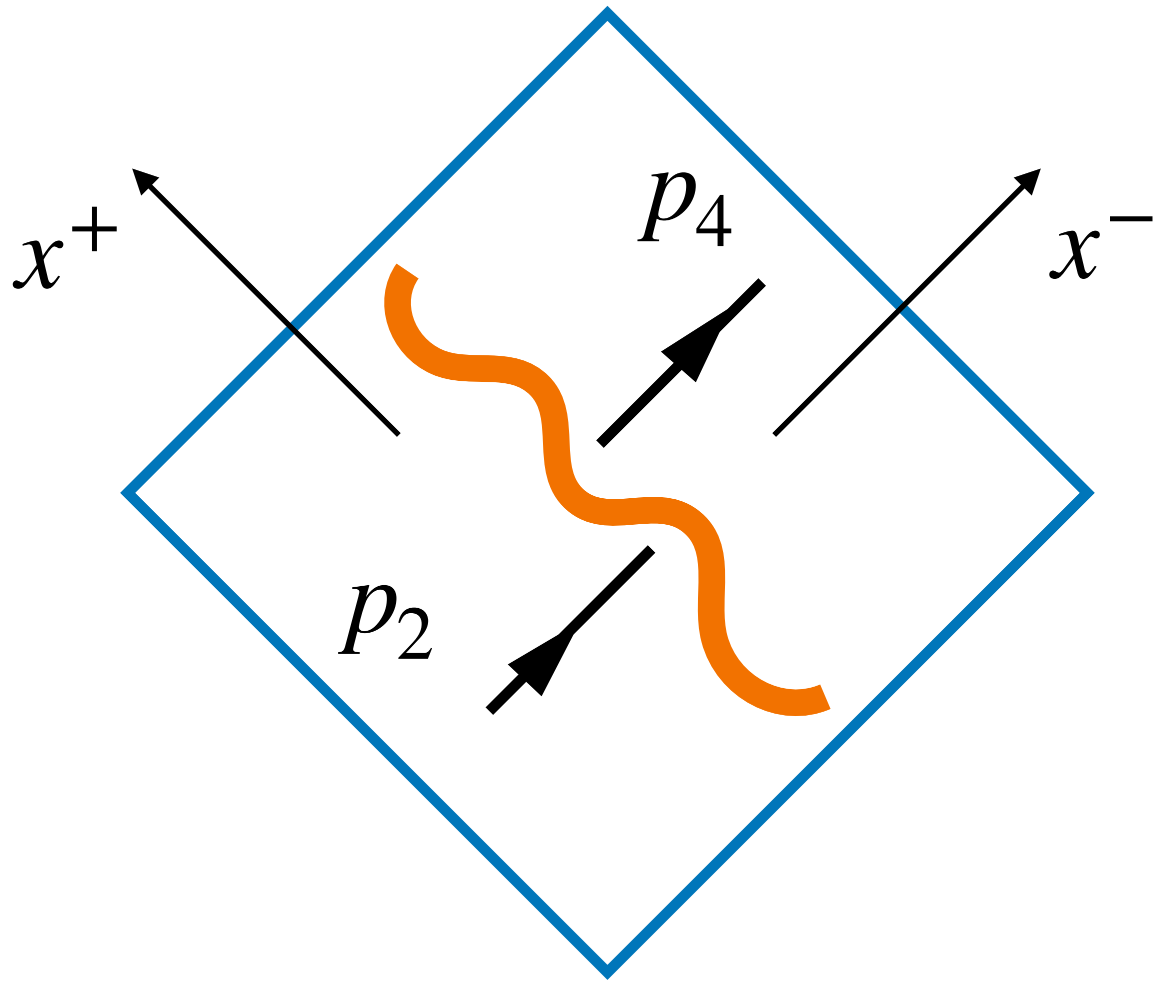
- Matching near $x^- = 0$: $\quad \phi(\epsilon, x^+, x_\perp) = \phi(-\epsilon, x^+ - h(x_\perp), x_\perp)$

Two-point function:

$$A_{\text{shock}}(p_2, p_4) = 4\pi\omega_4\delta(\omega_4 - \omega_2)\int d^2x_\perp e^{i(\omega_4 q_{4,\perp} - \omega_2 q_{2,\perp})\cdot x_\perp} e^{-i\omega_2 h(x_\perp)}$$

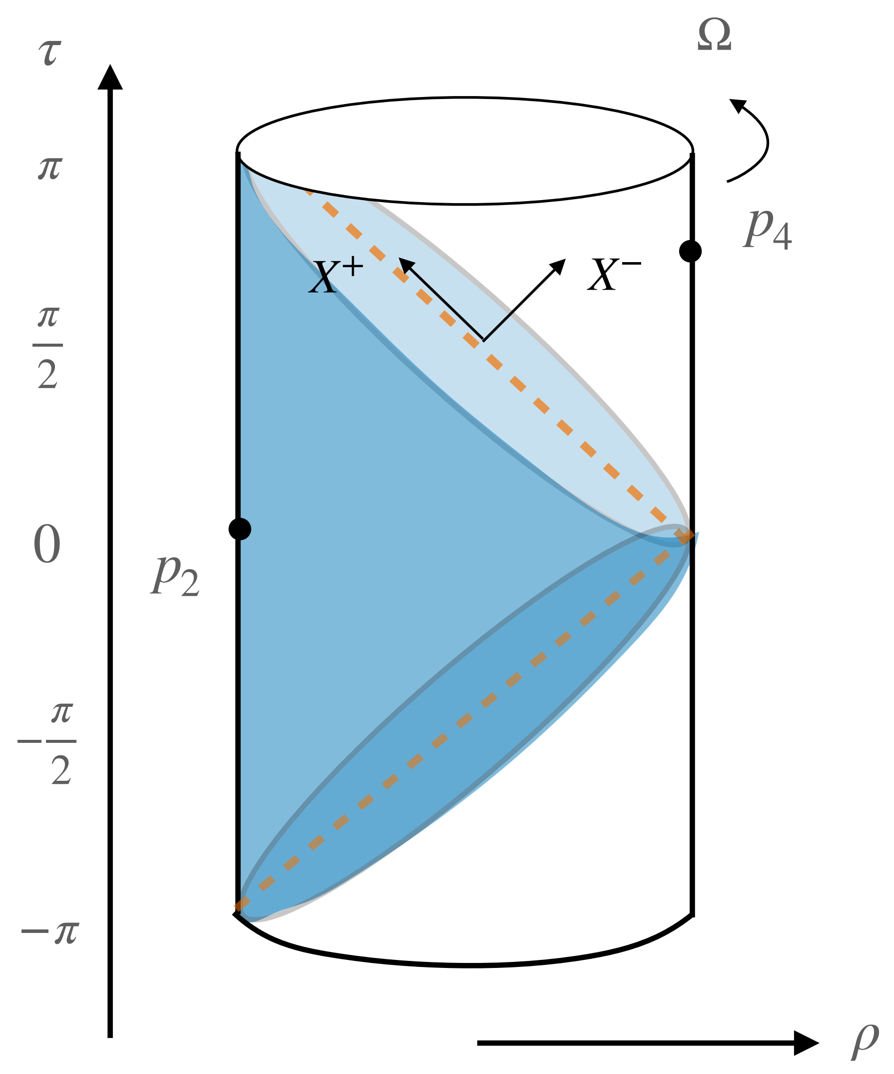
[t'Hooft '87]

Celestial shockwave two-point function



- Mellin transform $\Longrightarrow$ celestial two-point function:

$$\widetilde{A}_{\text{shock}}(\Delta_2, z_2, \bar{z}_2; \Delta_4, z_4, \bar{z}_4) = 4\pi \int d^2x_\perp \frac{i^{\Delta_2+\Delta_4}\Gamma(\Delta_2+\Delta_4)}{\left[-q_{24,\perp}\cdot x_\perp - h(x_\perp) + i\epsilon\right]^{\Delta_2+\Delta_4}}$$

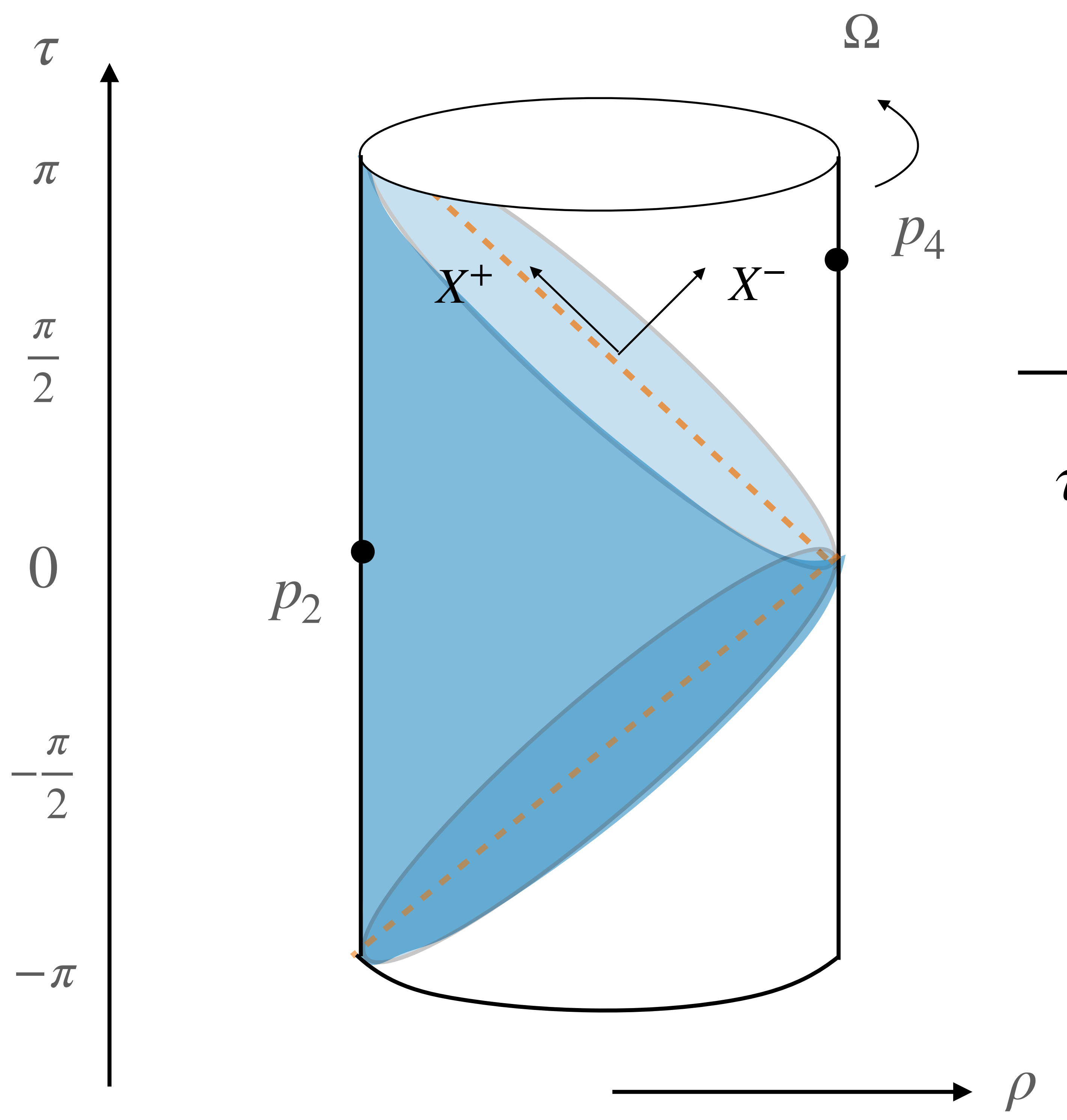


- AdS_4 shockwave two-point function:

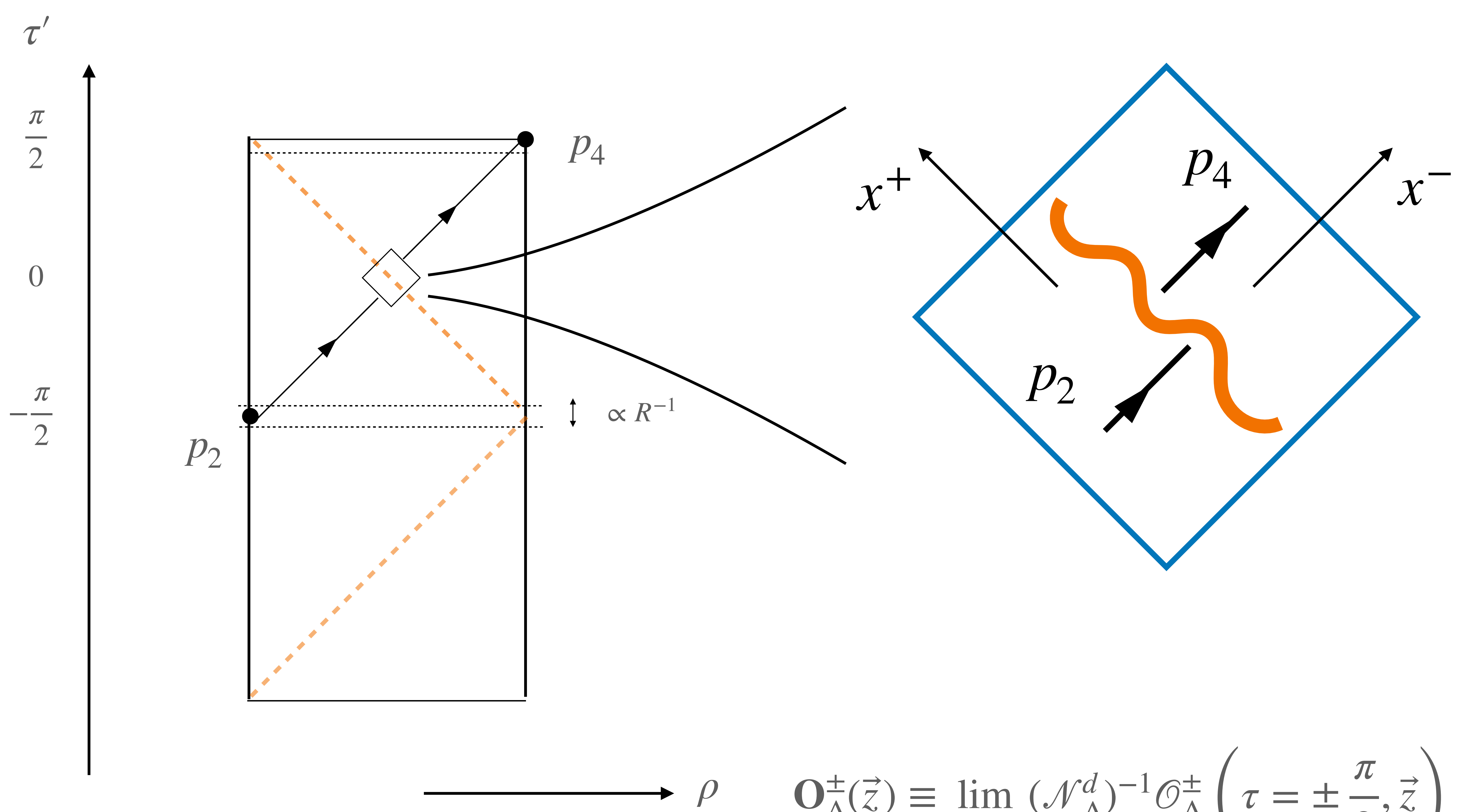
$$\langle \mathcal{O}_\Delta(\mathbf{p}_2)\mathcal{O}_\Delta(\mathbf{p}_4)\rangle_{\text{shock}} = C_\Delta \int_{H_2} d^2\mathbf{x}_\perp \frac{i^{2\Delta}\Gamma(2\Delta)}{\left[-\mathbf{q}_{24,\perp}\cdot\mathbf{x}_\perp - \mathbf{h}(\mathbf{x}_\perp) + i\epsilon\right]^{2\Delta}}$$

[Cornalba, Costa, Penedones ’07]

Flat space-limit of AdS_4 formula



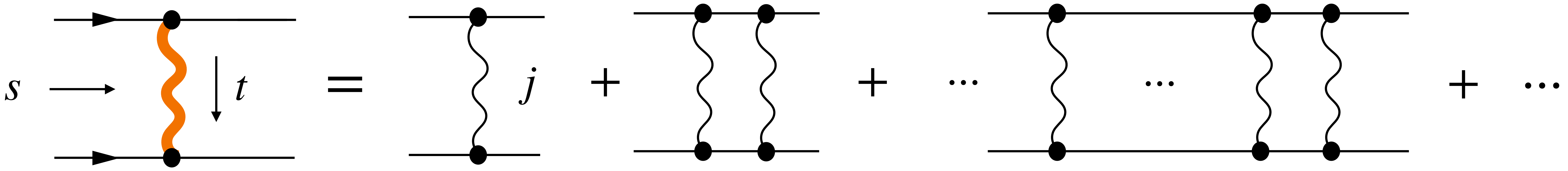
$$\tau' = \tau - \frac{\pi}{2}$$



$$\mathbf{O}^{\pm}_{\Delta}(\vec{z}) \equiv \lim_{R\to\infty} (\mathcal{N}^{d}_{\Delta})^{-1} \mathcal{O}^{\pm}_{\Delta}\left(\tau = \pm\frac{\pi}{2}, \vec{z}\right)$$

$$\langle \mathcal{O}_{\Delta}(\mathbf{p}_2)\mathcal{O}_{\Delta}(\mathbf{p}_4)\rangle_{\text{shock}} \xrightarrow[R\to\infty]{\tau' = \frac{t}{R},\quad \rho = \frac{r}{R}} \widetilde{A}_{\text{shock}}(\Delta, z_2, \bar{z}_2; \Delta, z_4, \bar{z}_4)$$

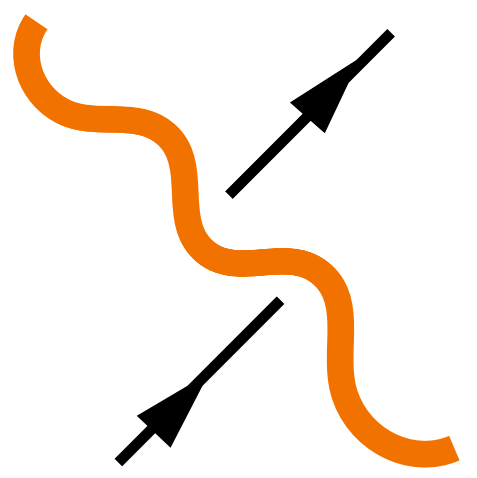
Eikonal amplitudes



[M. Levy and J. Sucher ’69; Kabat, Ortiz ’92; Cornalba, Costa, Penedones ’06,’07]

$$\mathscr{A}_{\text{eik}}(s, t = -p_\perp^2) \simeq 2s \int_{\mathbb{R}^2} d^2x_\perp e^{ip_\perp \cdot x_\perp} \left(e^{\frac{ig^2}{2} s^{j-1} G_\perp(x_\perp)} - 1 \right)$$

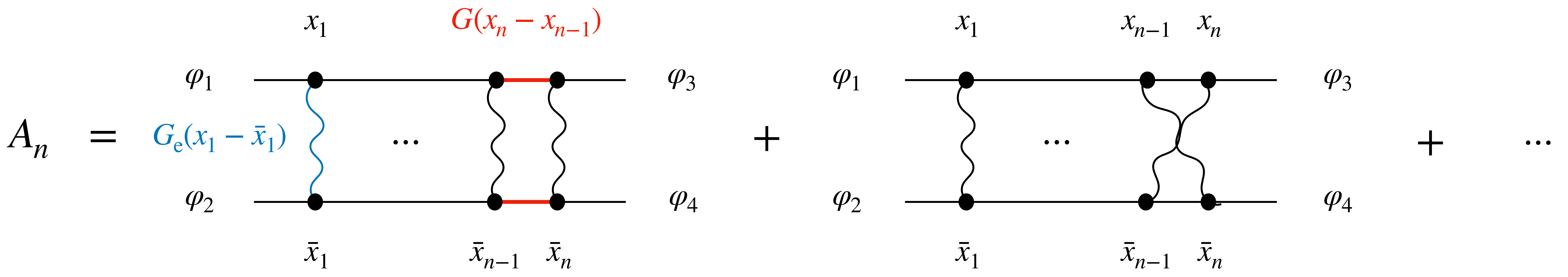
= **Propagation through shockwave** $(j = 2)$

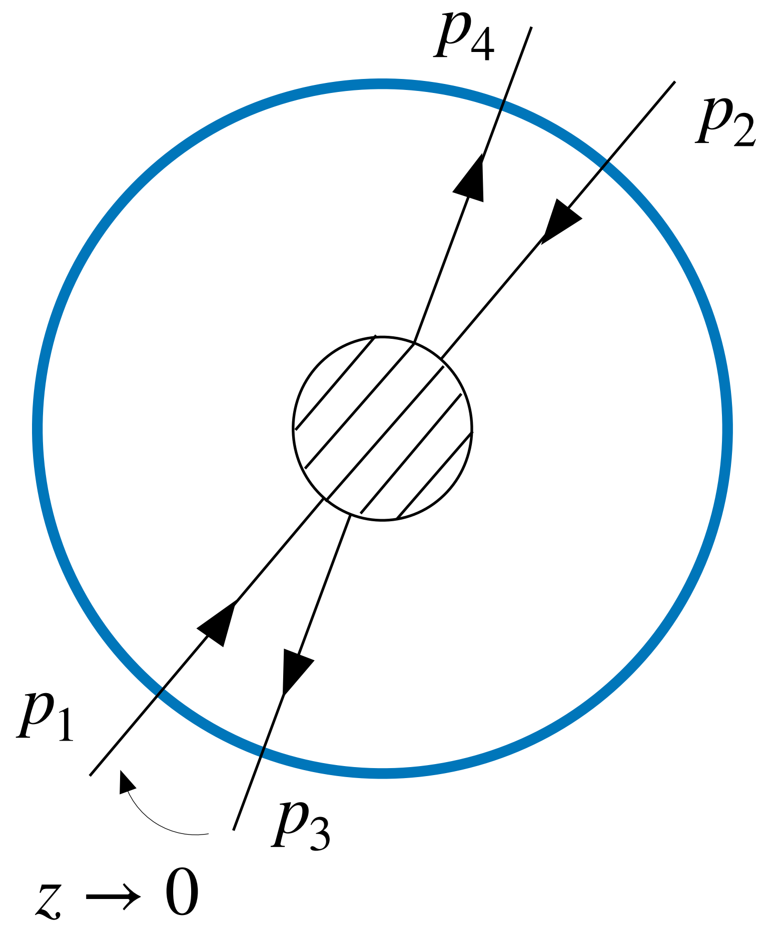


[Dray, t’Hooft ’85]

Eikonal amplitude in CCFT

Eikonal amplitude: $\mathbb{A}_{\text{eik}}(p_1, \cdots, p_4) = \sum_{n=1}^{\infty} A_n(p_1, \cdots, p_4);$ $\varphi_i = \frac{(i\eta_i)^{\Delta_i}\Gamma(\Delta_i)}{(-\hat{q}_i \cdot x_i + i\eta_i\epsilon)^{\Delta_i}}$





$$\begin{cases} s \gg t \iff z \ll 1 \\ |\Delta_1|, |\Delta_2| \gg 1 \end{cases} \implies$$ **Operator valued propagators** $G_{13}(x_i, x_j)$, $G_{24}(\bar{x}_i, \bar{x}_j)$

Eikonal amplitude in CCFT

Result: $$\lim_{z\to 0} \widetilde{\mathcal{A}}\Big|_{|\Delta_1|,|\Delta_2|\gg 1} \equiv \widetilde{\mathcal{A}}_{\text{eik}} = 4(2\pi)^2 \int d^2x_\perp d^2\bar{x}_\perp \left(e^{i\hat{\chi}_j} - 1\right) \frac{i^{\Delta_1+\Delta_3}\Gamma(\Delta_1+\Delta_3)}{(-q_{13,\perp}\cdot x_\perp)^{\Delta_1+\Delta_3}} \frac{i^{\Delta_2+\Delta_4}\Gamma(\Delta_2+\Delta_4)}{(-q_{24,\perp}\cdot \bar{x}_\perp)^{\Delta_2+\Delta_4}}$$

Eikonal phase: $$\hat{\chi}_j \equiv \frac{g^2(4e^{\partial_{\Delta_1}}e^{\partial_{\Delta_2}})^{j-1}}{2} G_\perp(x_\perp, \bar{x}_\perp)$$

Transverse propagator: $$G_\perp(x_\perp, \bar{x}_\perp) \equiv \int \frac{d^2k_\perp}{(2\pi)^2} \frac{e^{ik_\perp\cdot(x_\perp - \bar{x}_\perp)}}{k_\perp^2 + m^2 - i\epsilon}$$

- Operator valued eikonal phase for arbitrary spin exchanges
- Perturbative expansion leads to expected disconnected and t-channel results
- Similar to AdS; related to shockwave two-point function with certain source

Summary

- Celestial amplitudes in CCFT_{d-1} from flat space limit of AdS_{d+1} Witten diagrams
- Celestial shockwave 2-point function
- Eikonal regime in CCFT: $|\beta| \gg 1, \quad z \ll 1$

Outlook

- CCFT_{d-1} from CFT_d? Top down CCFT constructions? [Andy's talk]
- BMS symmetries & more from AdS_4 flat space limit; matching condition?
- Scattering in other backgrounds, chaos? [Pasterski, Verlinde '22]
- Causality signatures in CCFT, relation to memory effects?
- Leading eikonal amplitude vs. Weinberg IR-divergent phase?

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