UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

The large charge expansion

Susanne Reffert
University of Bern

based on arXiv:1505.01537, 1610.04495, 1707.00710, 1809.06371, 1902.09542, 1905.00026, 1909.08642,1909.02571, 2008.03308, 2010.07942, 2102.12488, 2110.07617, 2110.07616, 2203.12624 + work in progress

L. Alvarez-Gaume (SCGP), D. Banerjee (Kolkata),
Sh. Chandrasekharan (Duke), N. Dondi (Bern), S. Hellerman (IPMU),
I. Kalogerakis (Bern), R. Moser (Bern), O. Loukas,
D. Orlando (INFN Torino), V. Pellizzani (Bern), F. Sannino (Odense/Napoli),
T. Schmidt (Bern), M. Watanabe (Kyoto)

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Large charge Q becomes controlling parameter in a perturbative expansion!

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many degrees of freedom

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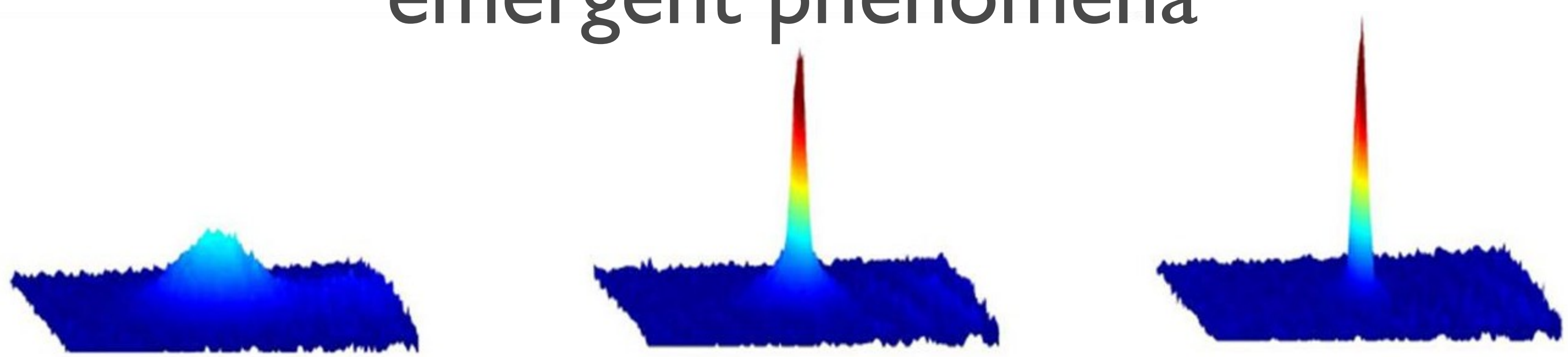
emergent phenomena



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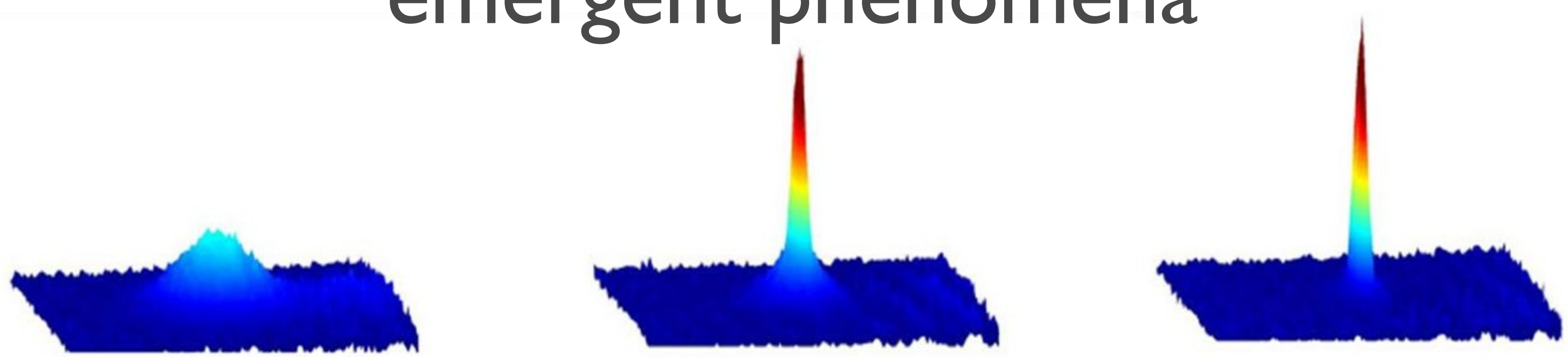

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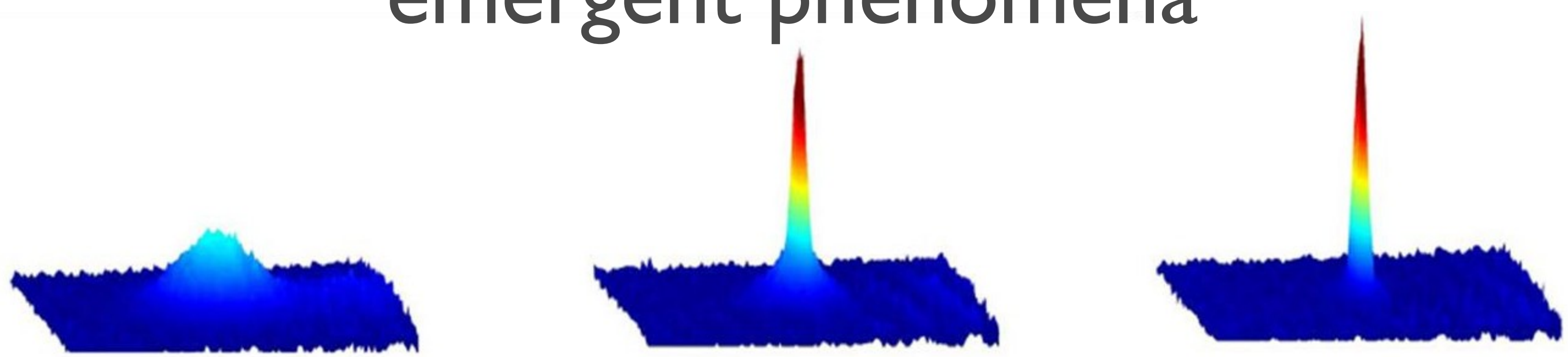
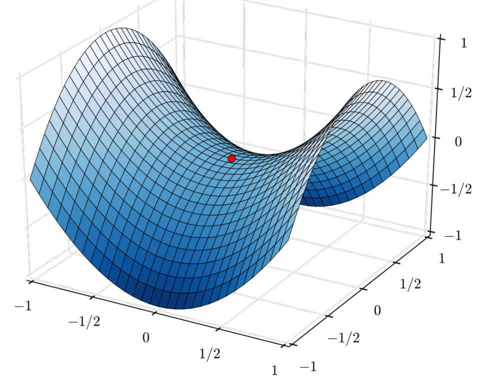


semiclassical description



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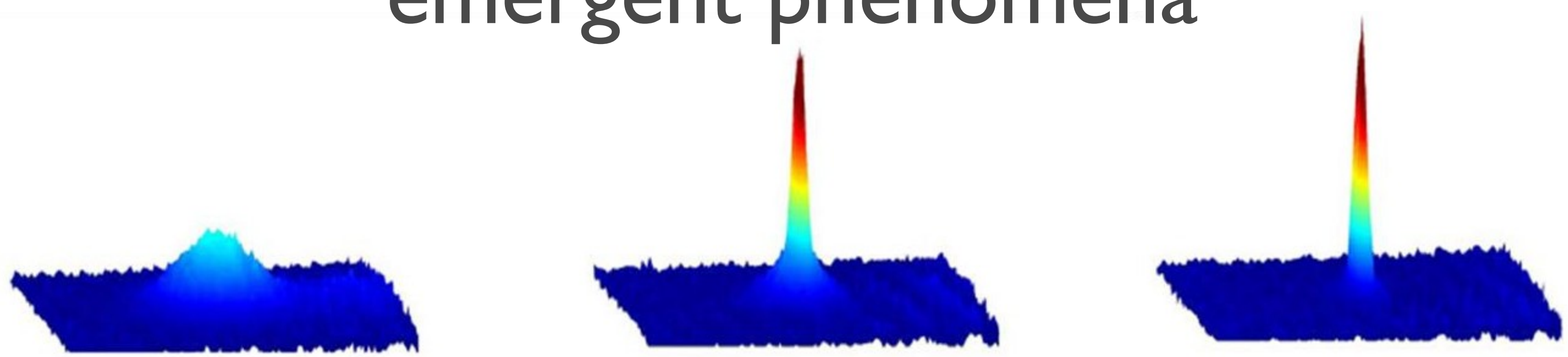

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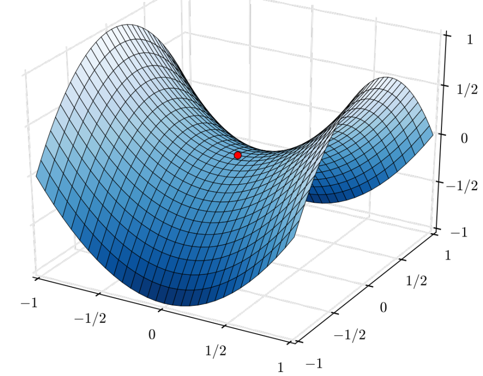

e.g. superfluid

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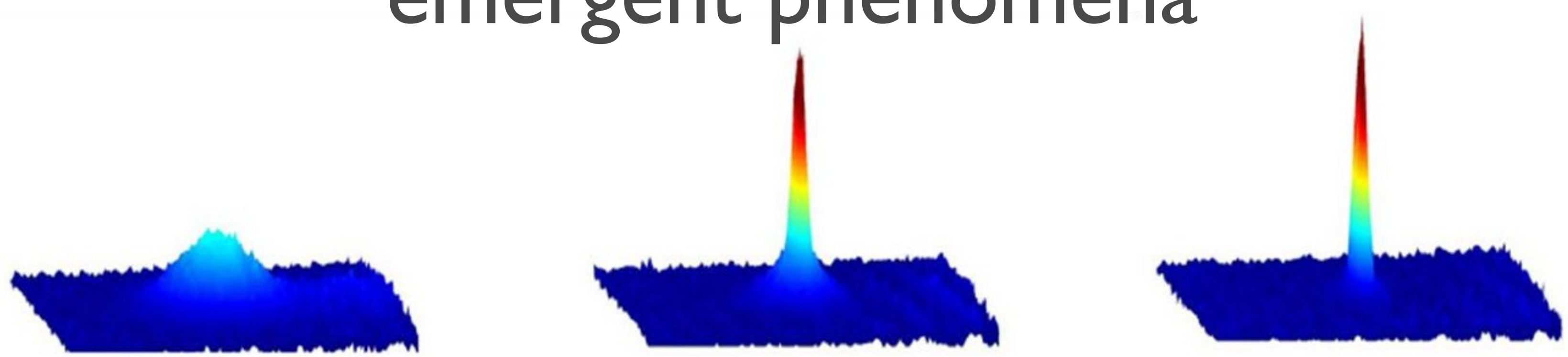

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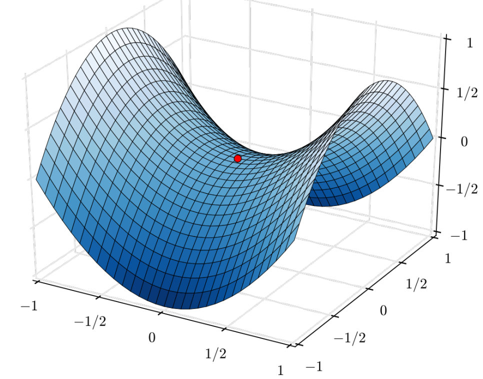

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works also for strongly coupled systems!

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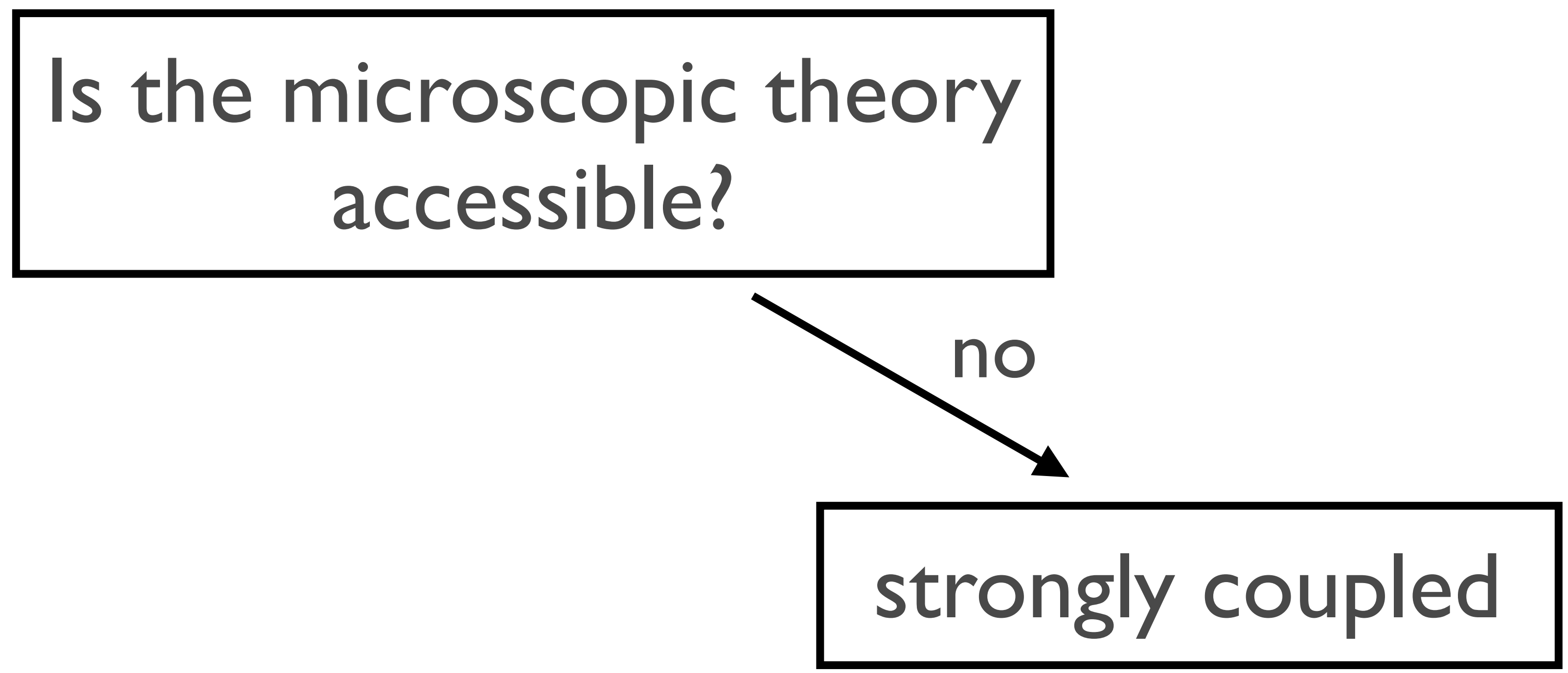

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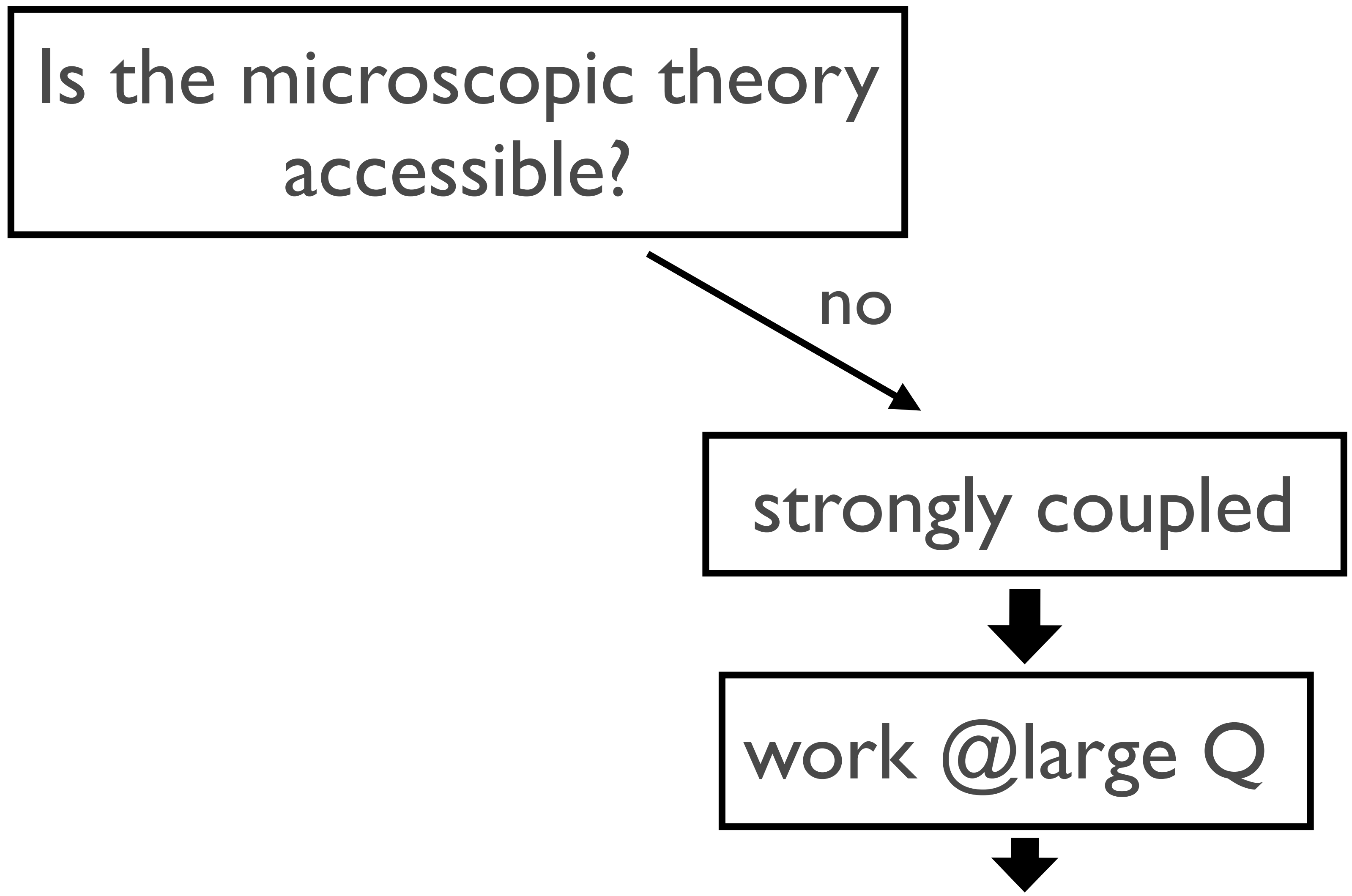

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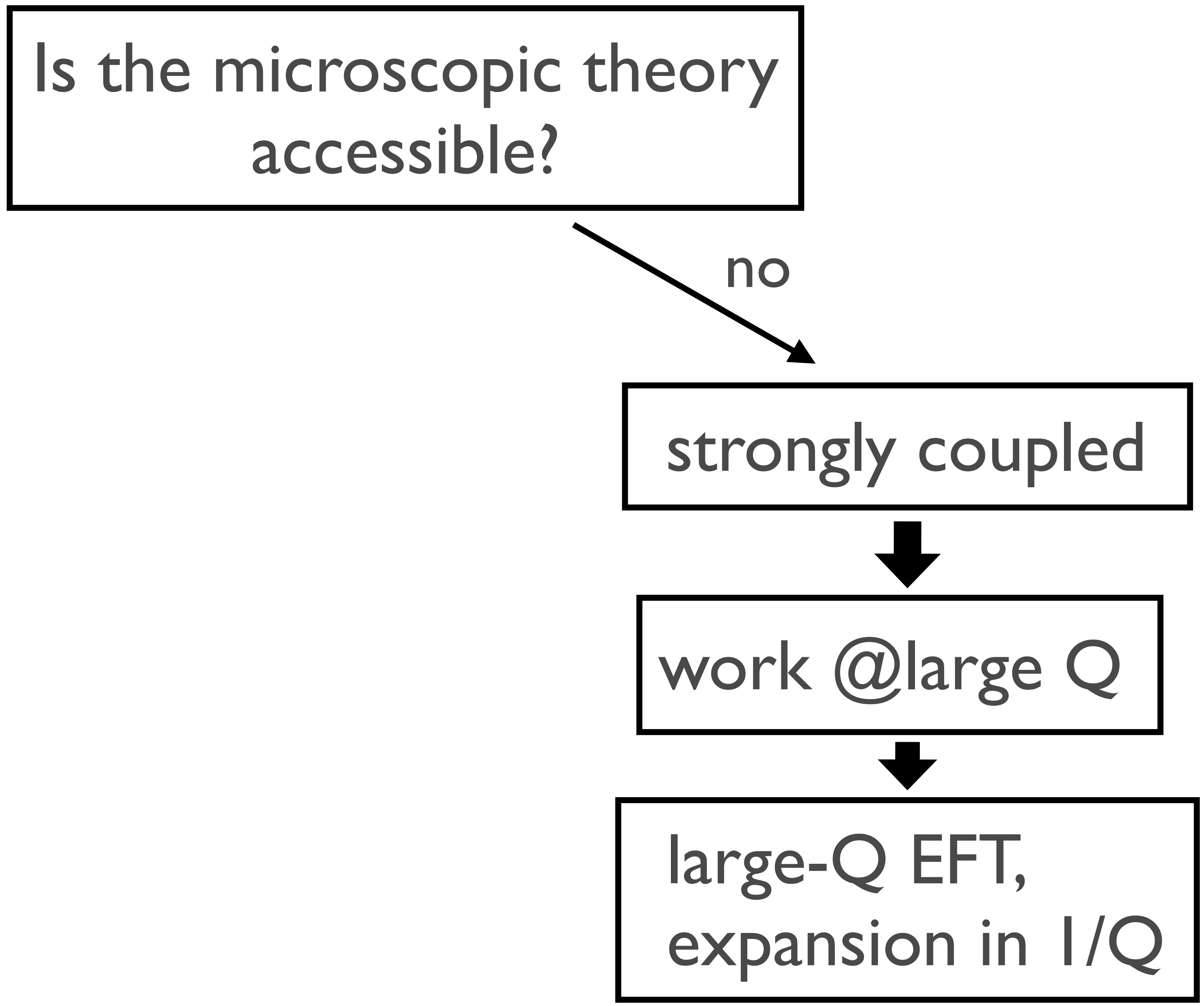

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large-Q EFT, expansion in 1/Q

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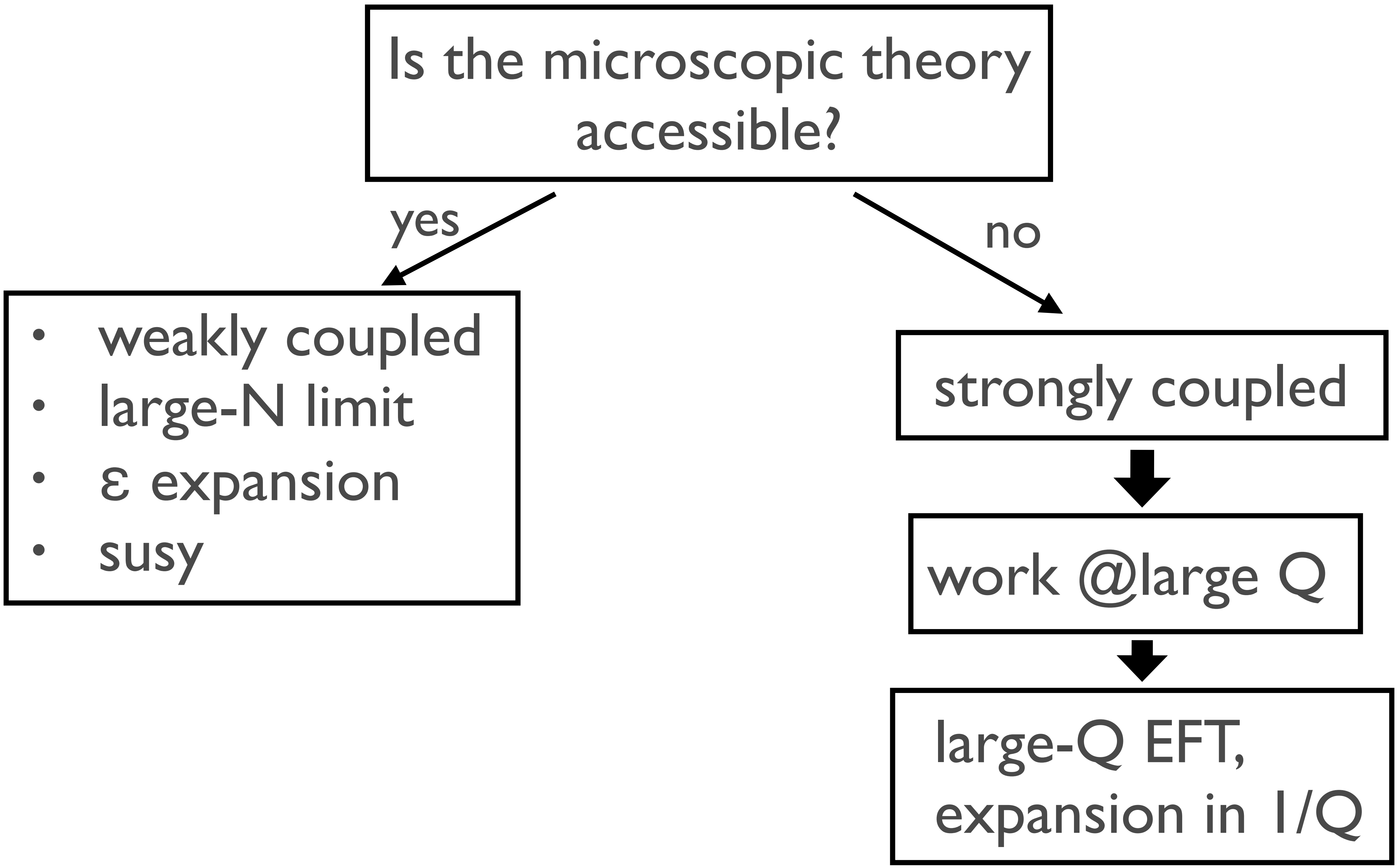

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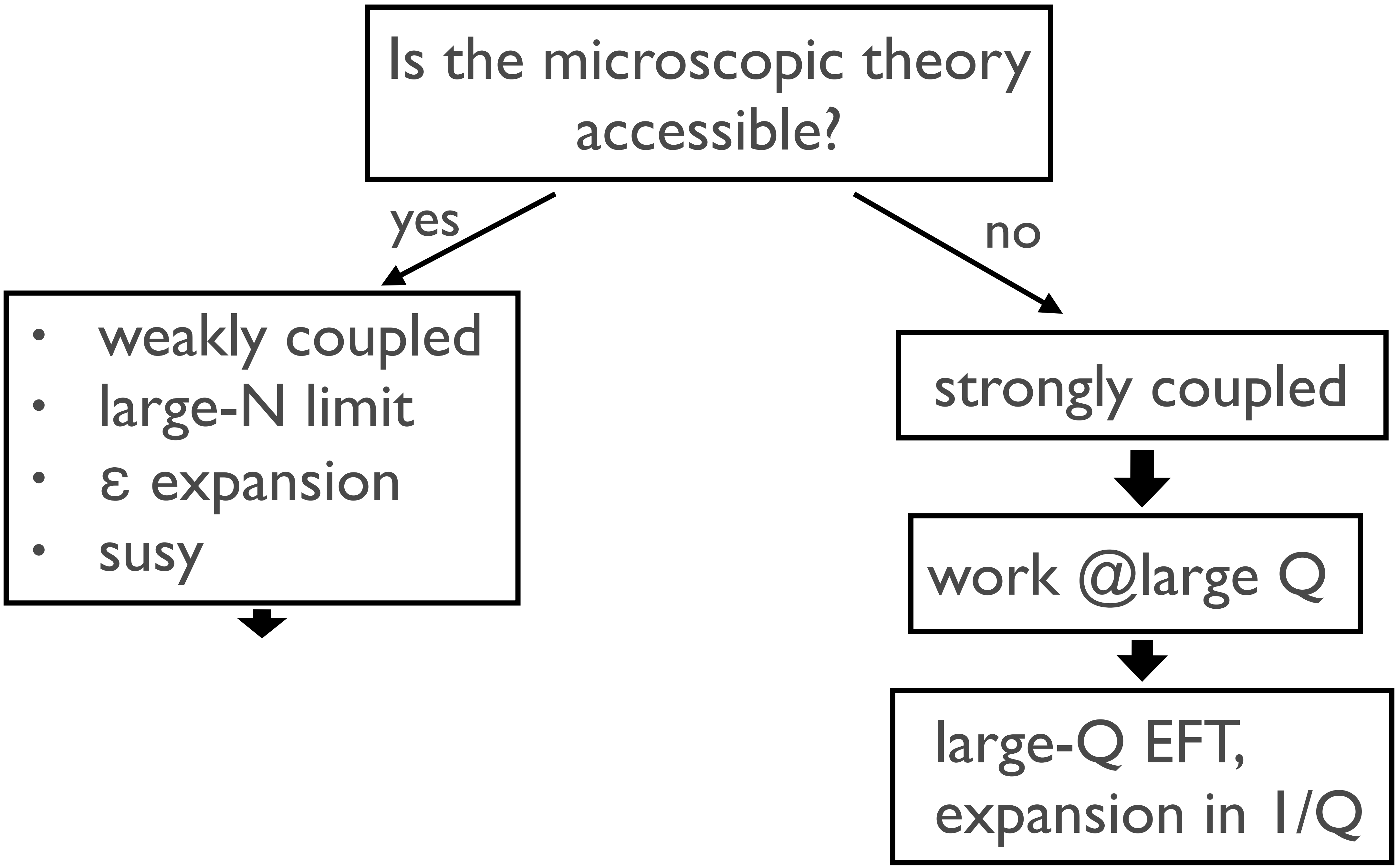

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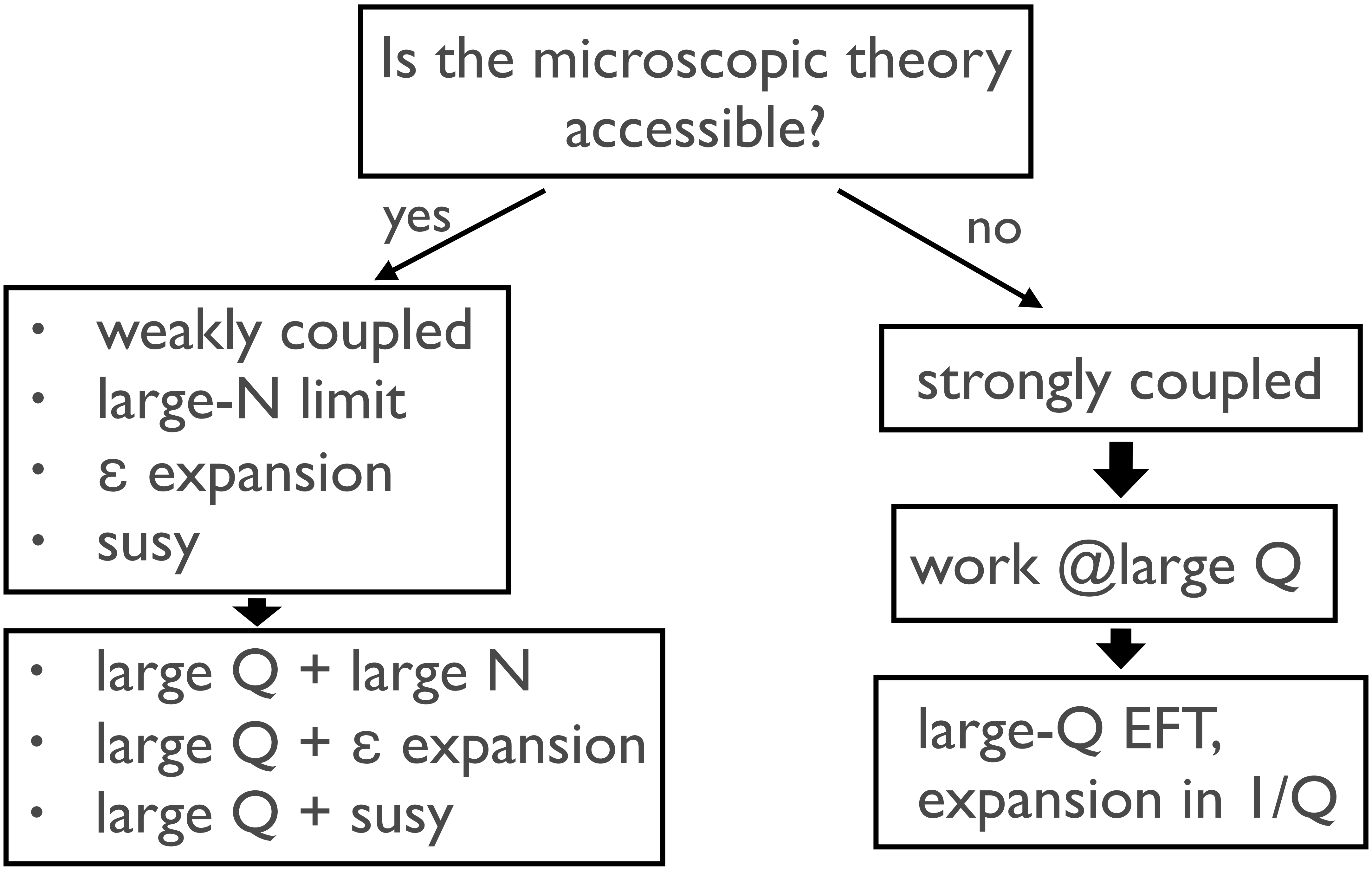

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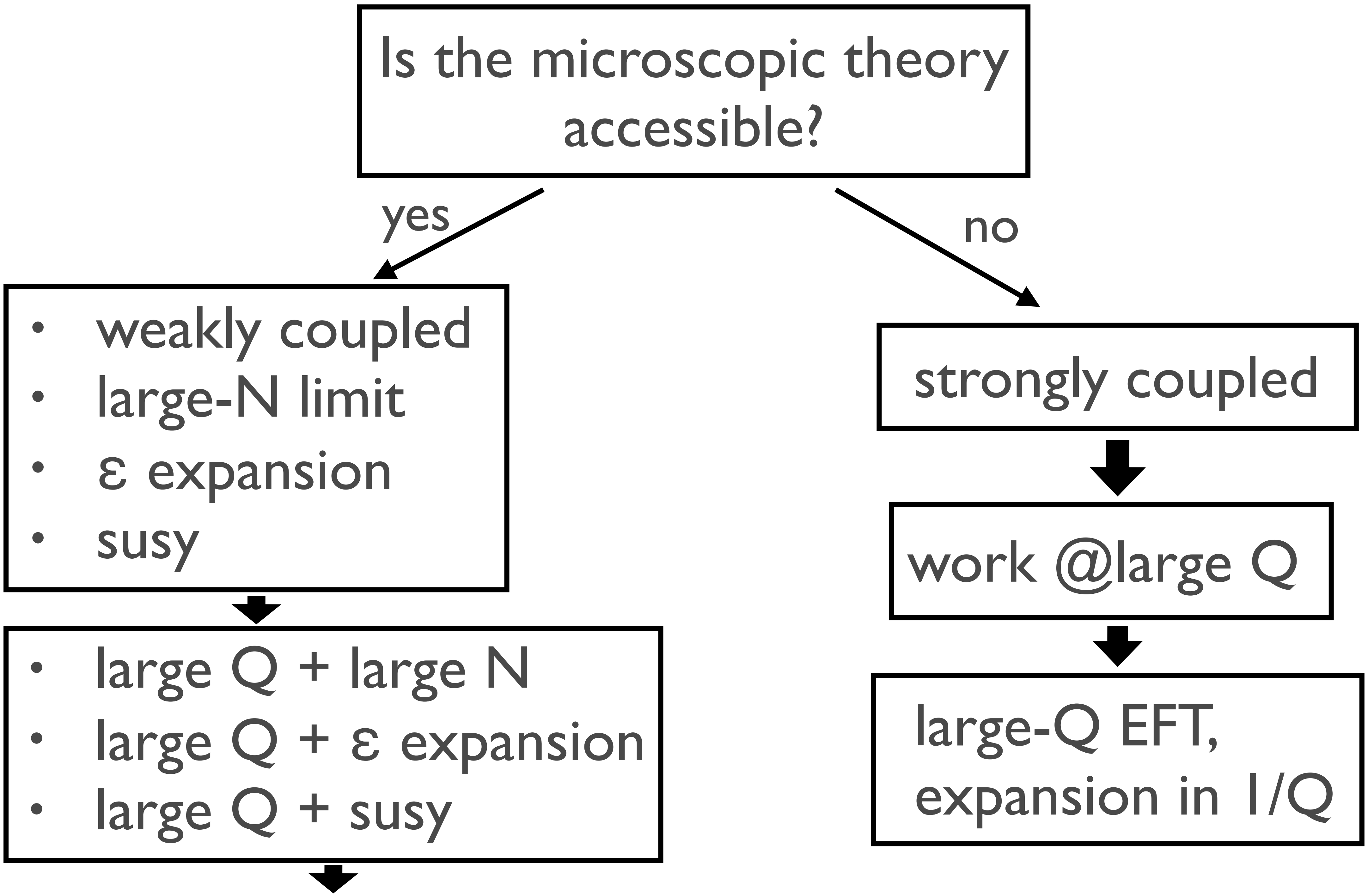

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- weakly coupled
- large-N limit
- ε expansion
- susy

- large Q + large N
- large Q + ε expansion
- large Q + susy

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work @large Q

large-Q EFT, expansion in 1/Q

go beyond perturbation theory in 1/Q, calculate non-perturbative (exponential) corrections!

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<u>Superfluid</u>

isolated vacuum

- Wilson-Fisher CFT
- NRCFT (unitary Fermi gas)
- N=2 SCFT in 3d
- asymptotically safe model in 4d
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<u>EFT of the moduli space</u>

moduli space of vacua

- free boson
- N=2 theories in 4d

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Example: Scalar field theories in $2<D<4$ have a strongly-coupled interacting fixed point, the Wilson-Fisher FP.

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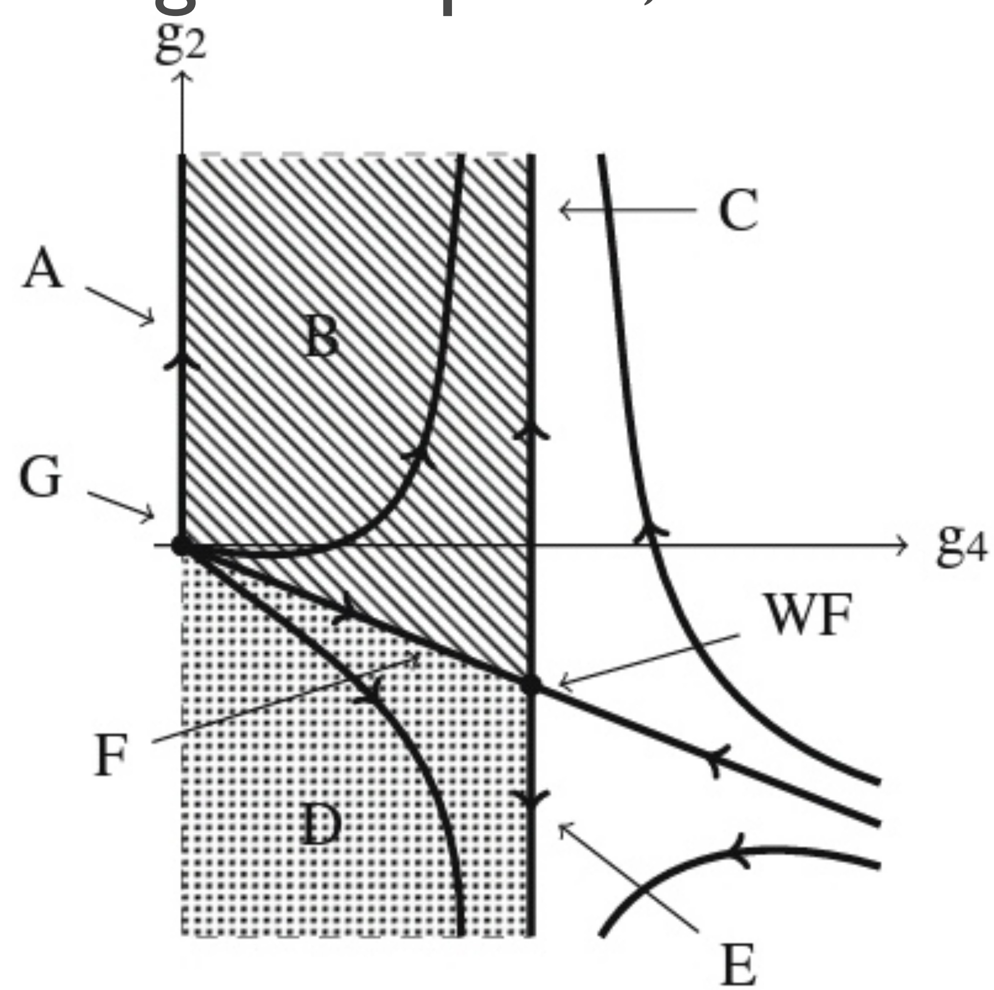

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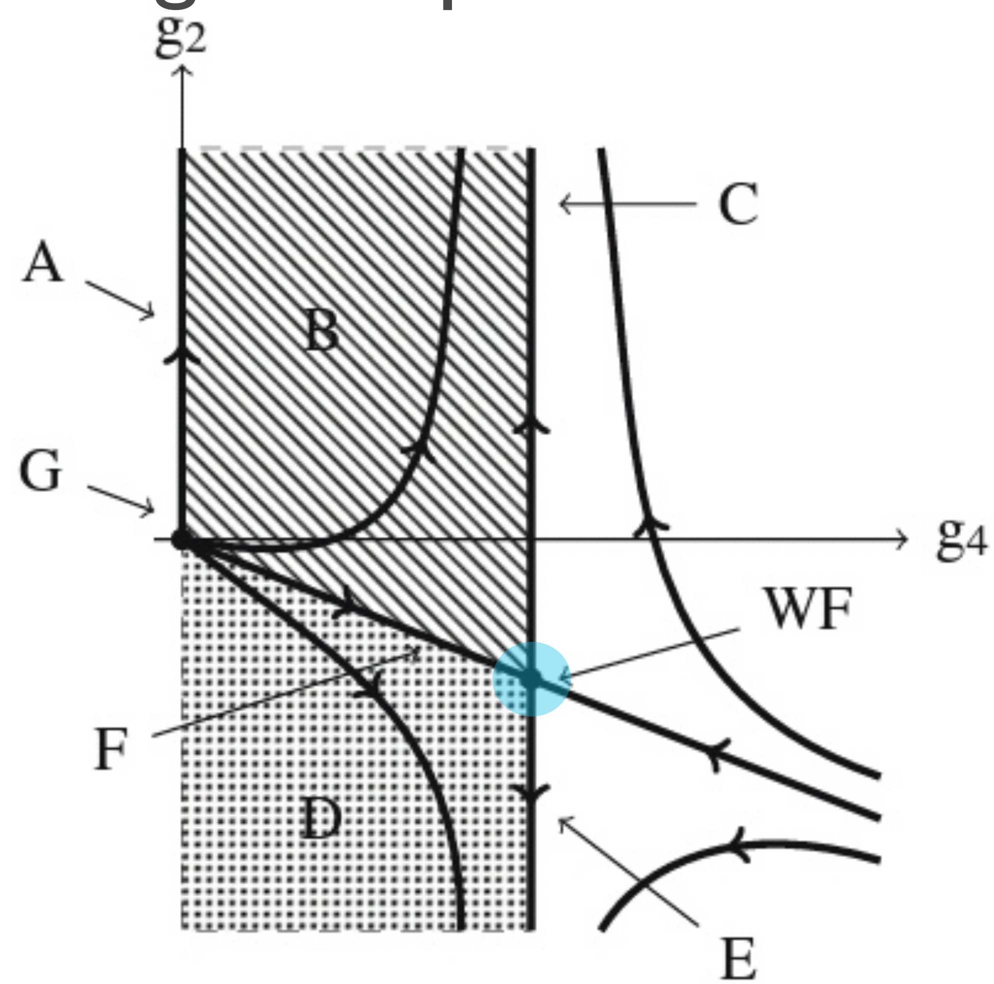

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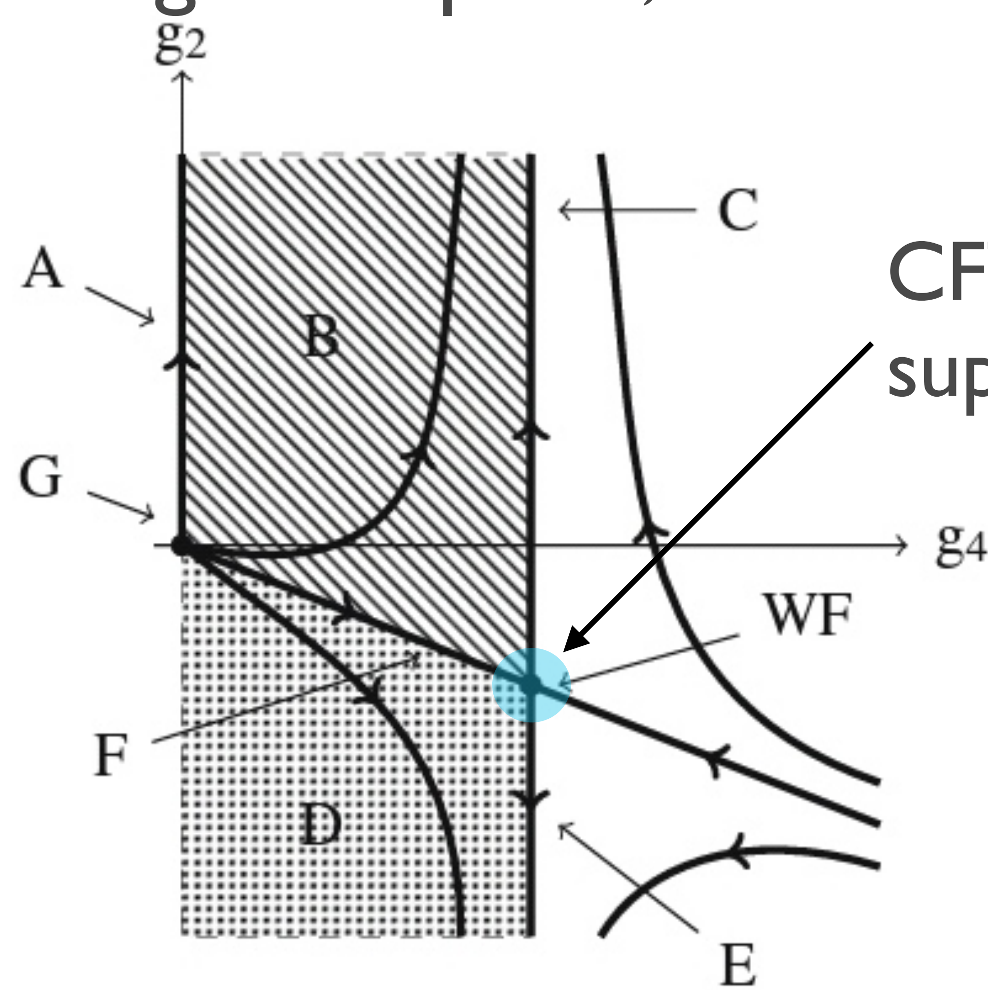

CFT, strongly coupled superfluid at large Q

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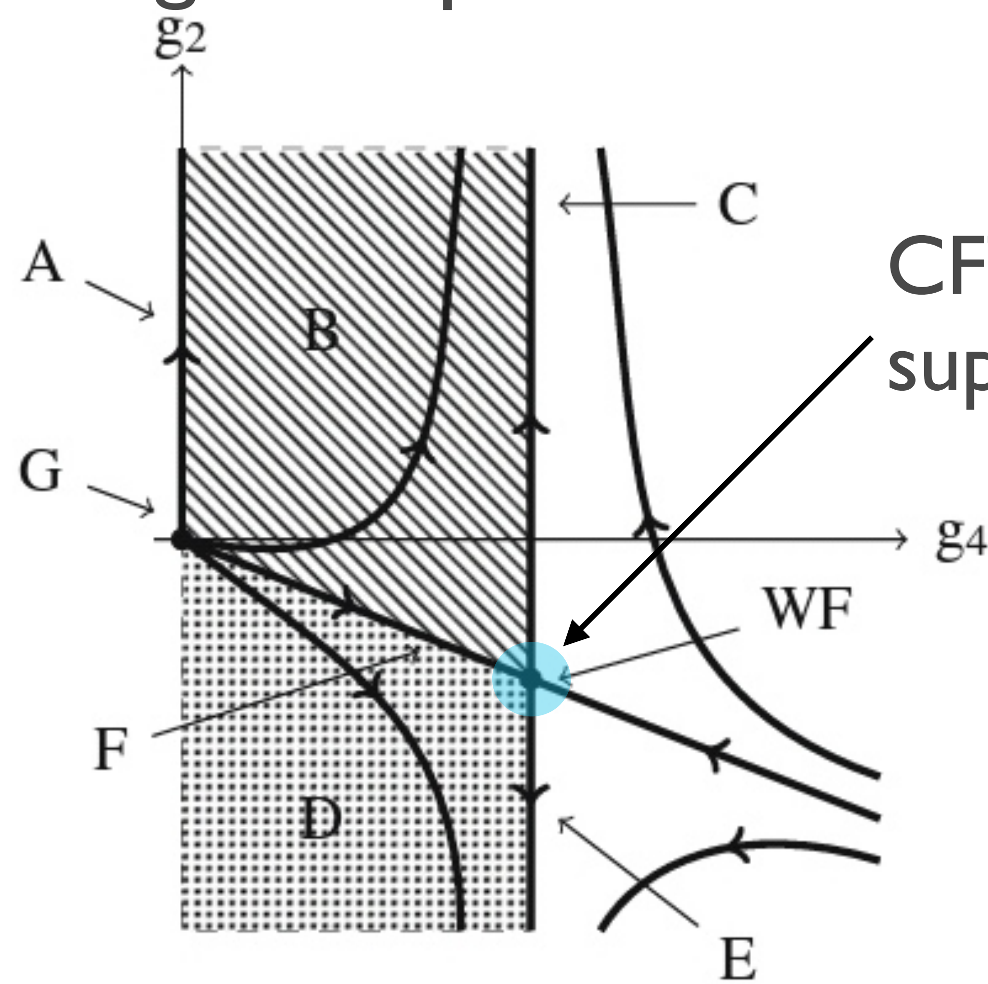

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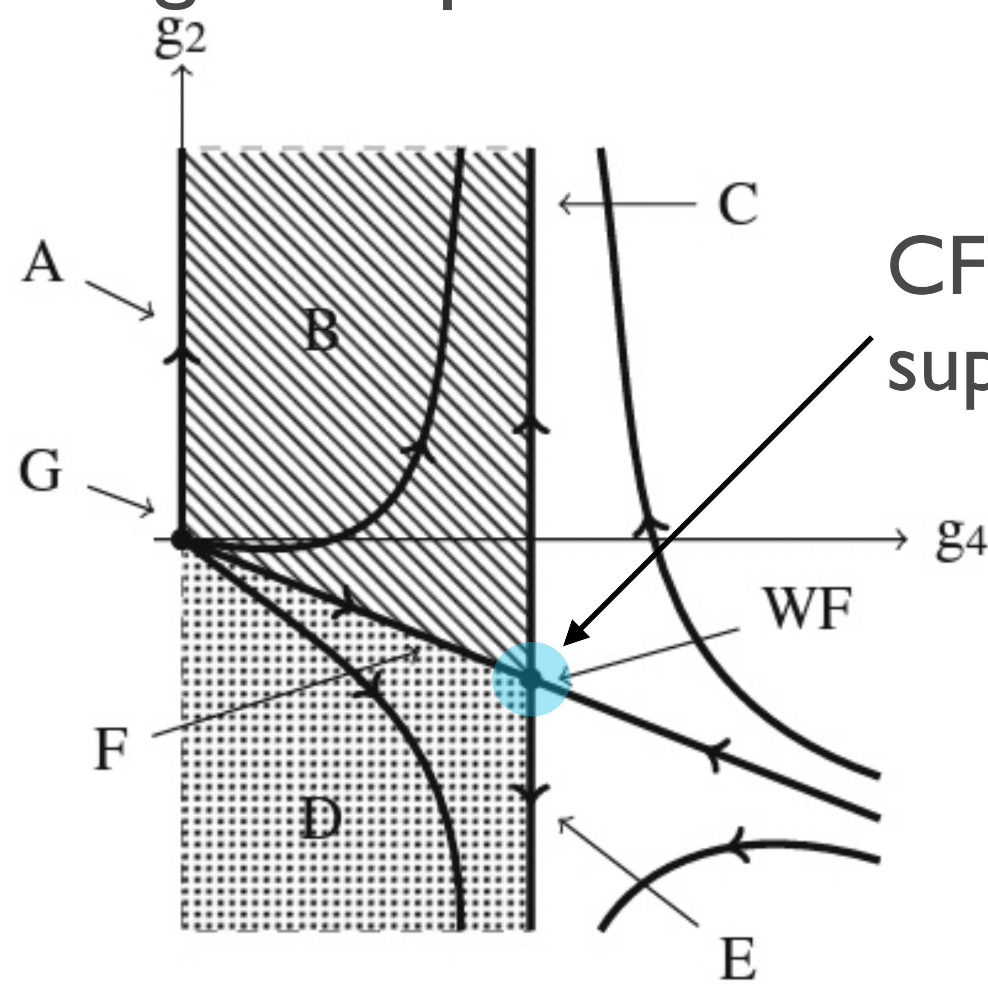

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O(2N) vector model in D=3:

$$S[\phi_i] = \sum_{i=1}^{N} \int \mathrm{d}t\mathrm{d}\Sigma \left[g^{\mu\nu} (\partial^i_\mu \phi_i)^\dagger (\partial^i_\nu \phi_i) + r(\phi_i^\dagger \phi_i) + \frac{u}{2N} (\phi_i^\dagger \phi_i)^2 \right]$$

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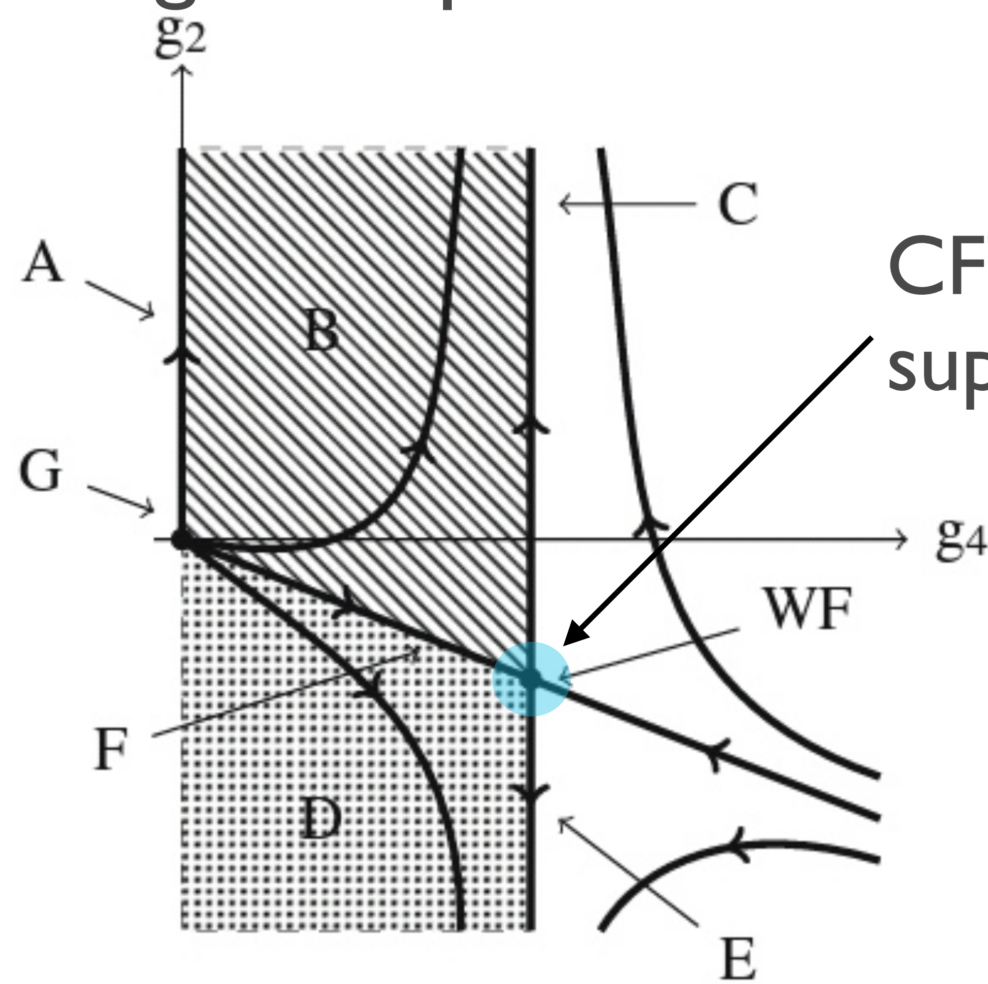

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For r=R/8, this flows to the WF fixed pt in the IR, $u \to \infty$

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Simplest example: O(2) model in (2+1) dimensions

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Beyond LO: use dimensional analysis, parity and scale invariance to determine (tree-level) operators in effective action (Lorentz scalars of scaling dimension 3, including couplings to geometric invariants)

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Energy of classical ground state at fixed charge:

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$$\mathcal{L} = k_{3/2}(\partial_\mu\chi\partial^\mu\chi)^{3/2} + k_{1/2}R(\partial_\mu\chi\partial^\mu\chi)^{1/2} + \mathcal{O}(Q^{-1/2})$$

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Energy of classical ground state at fixed charge:

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Effective theory at large Q:

vacuum + Goldstone + 1/Q-suppressed corrections

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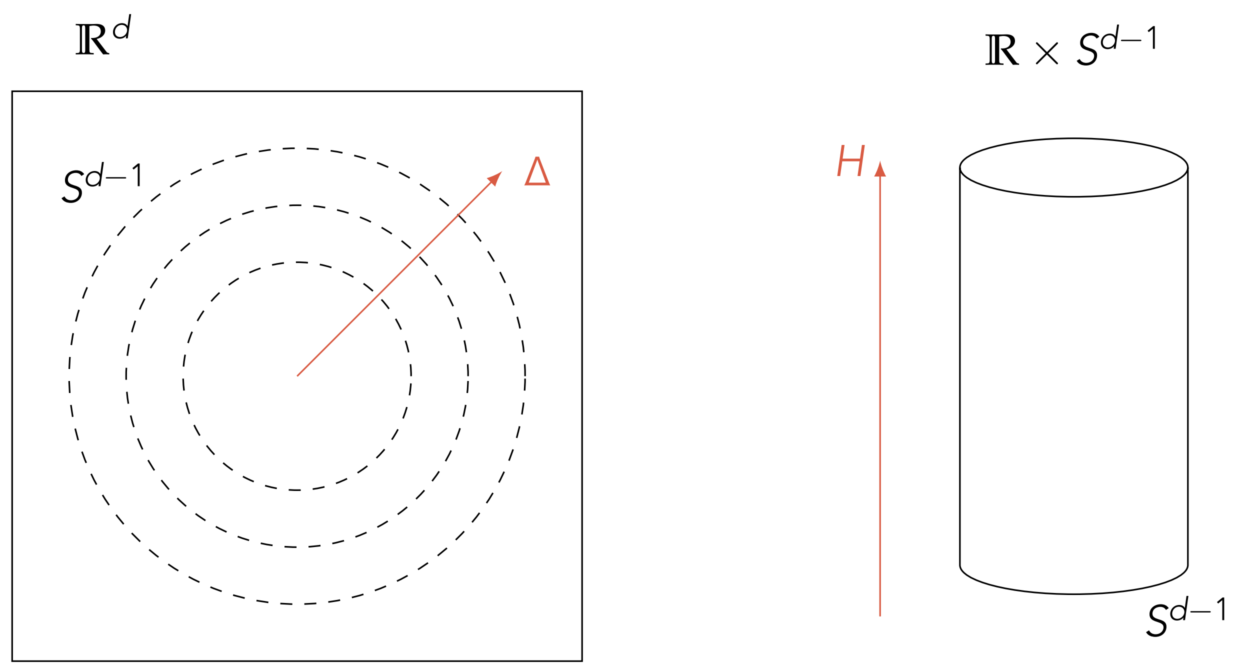

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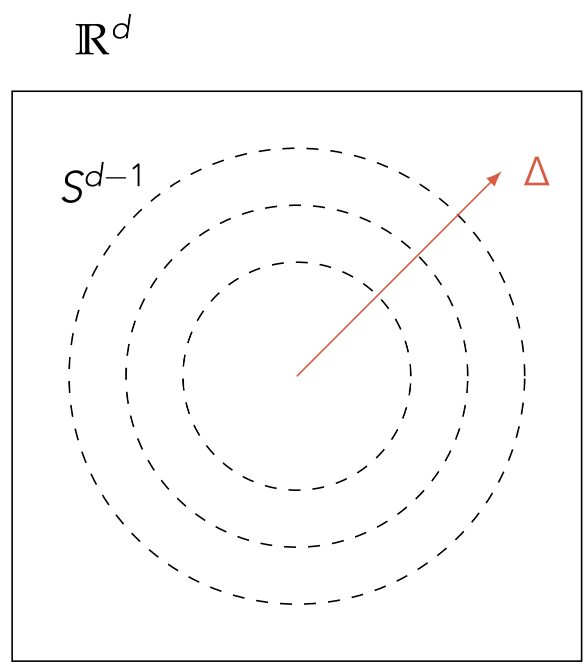

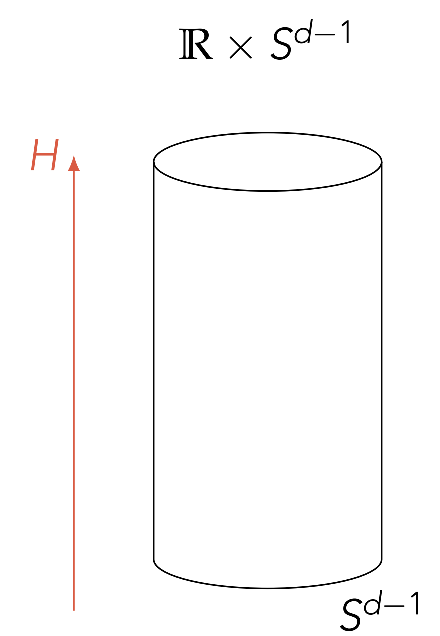

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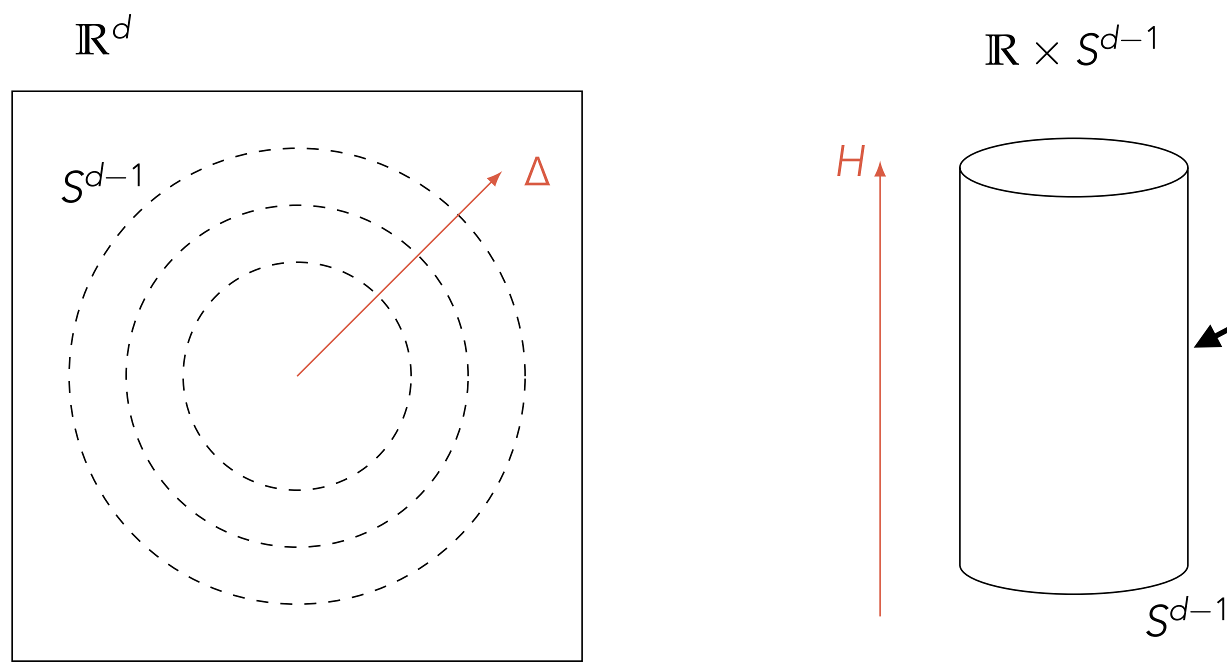

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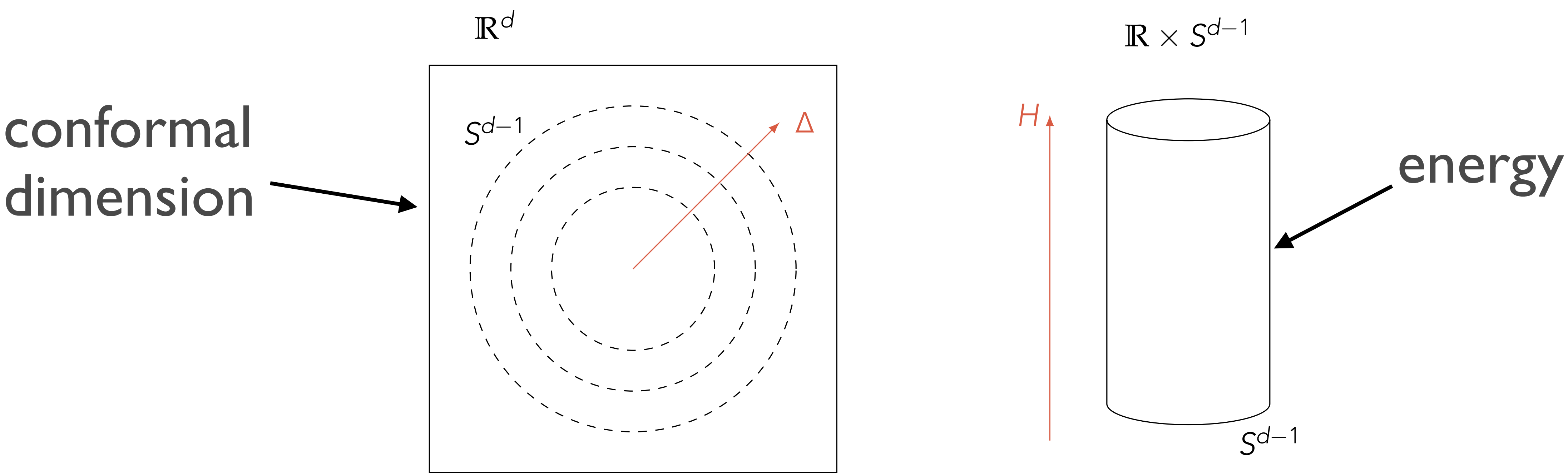

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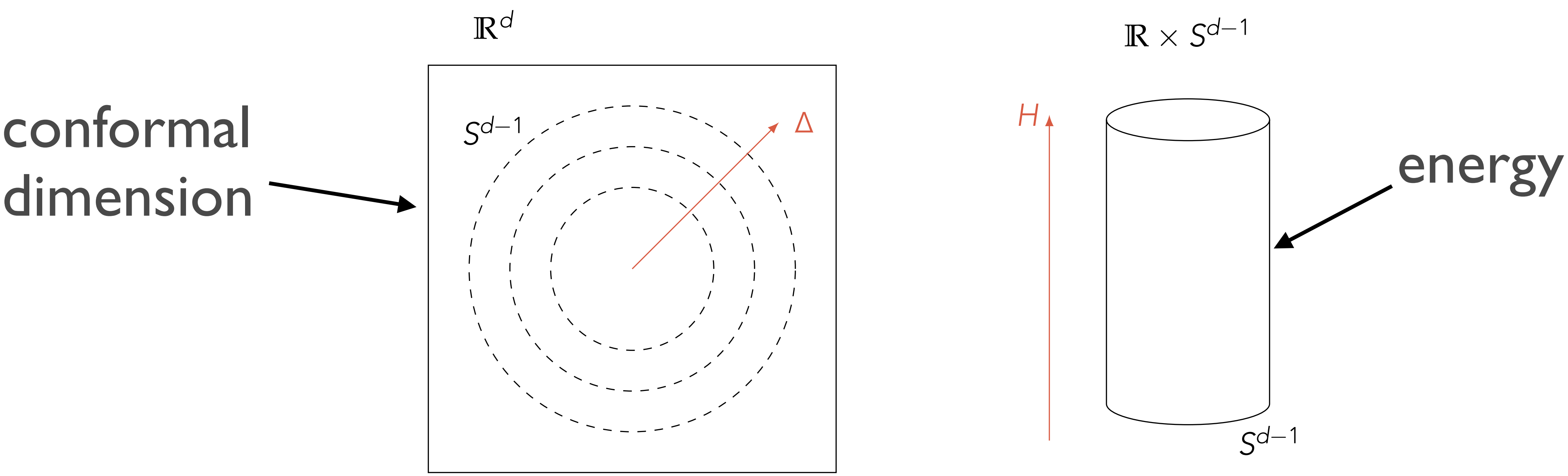

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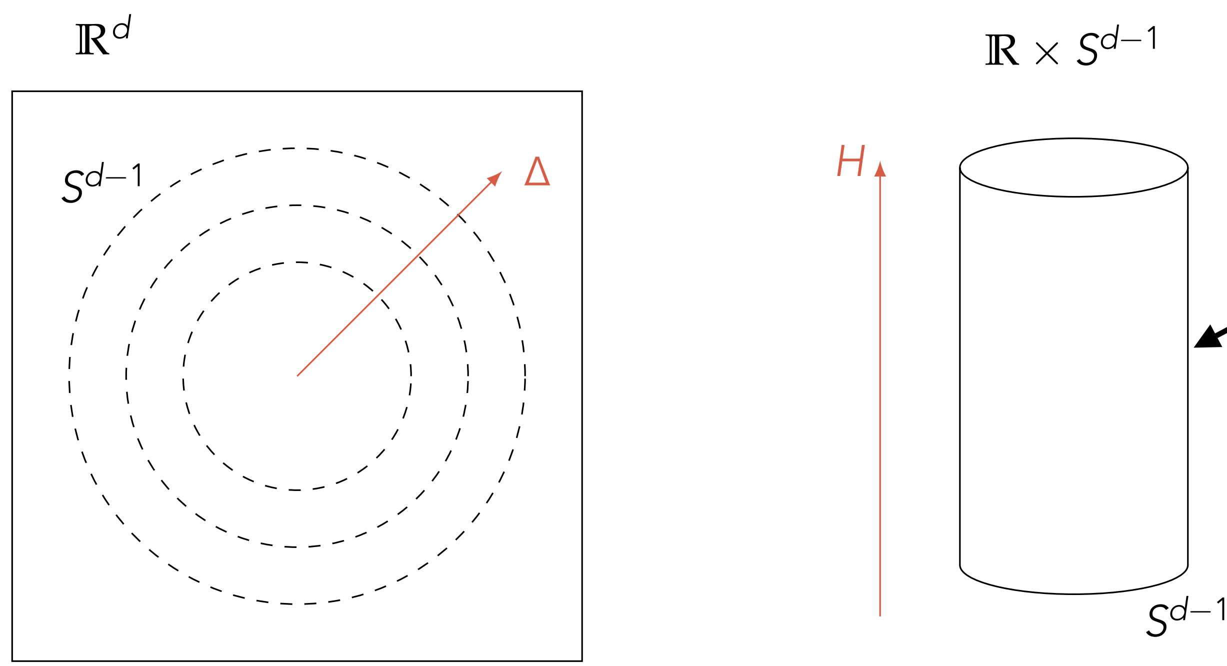

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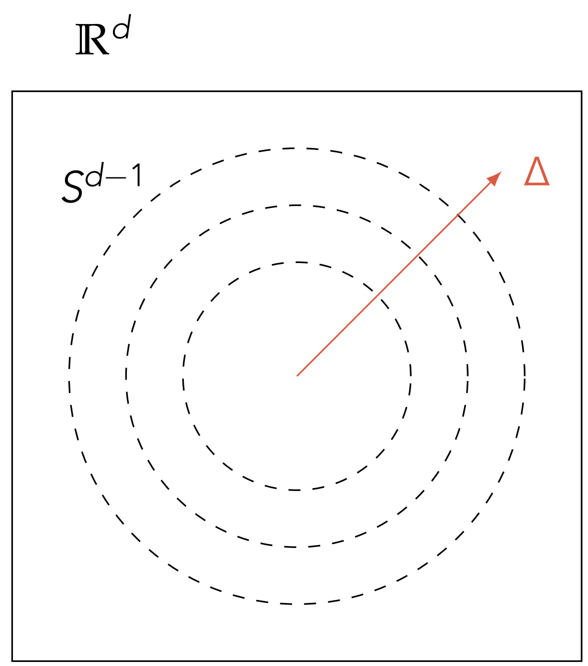

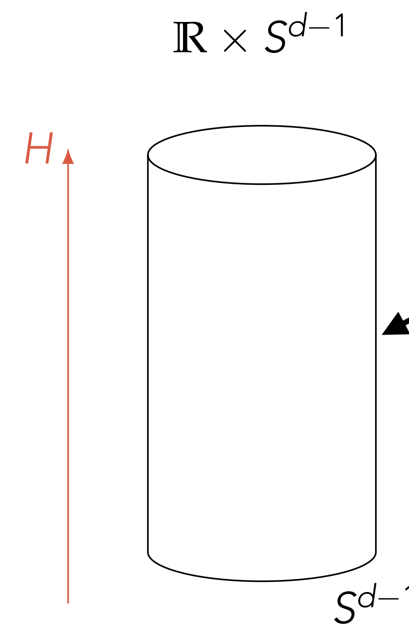

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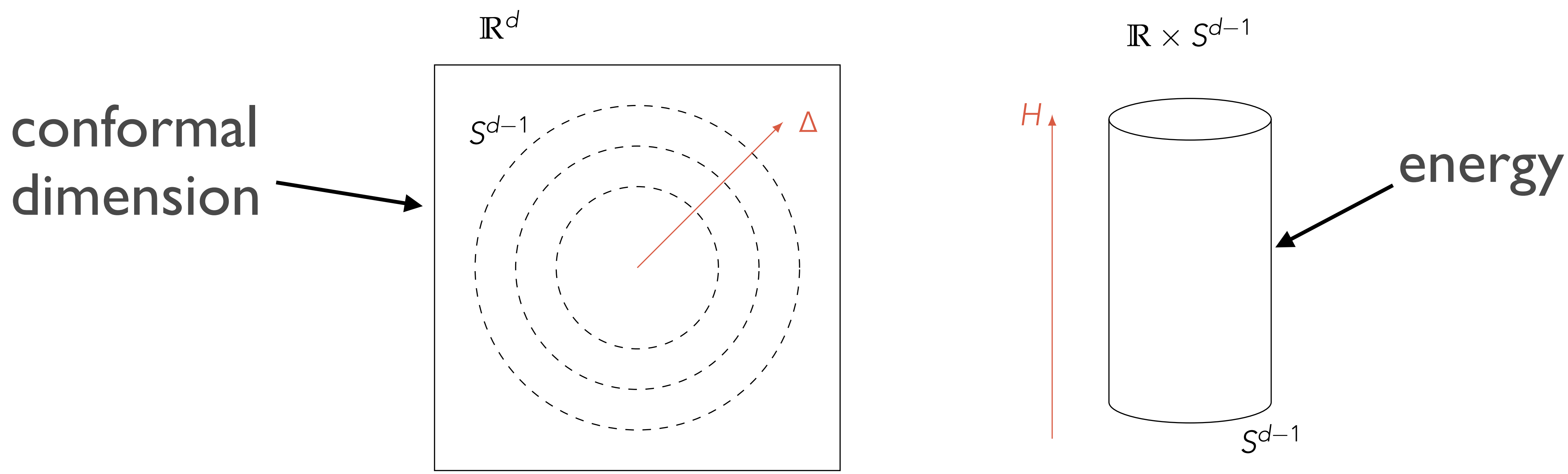

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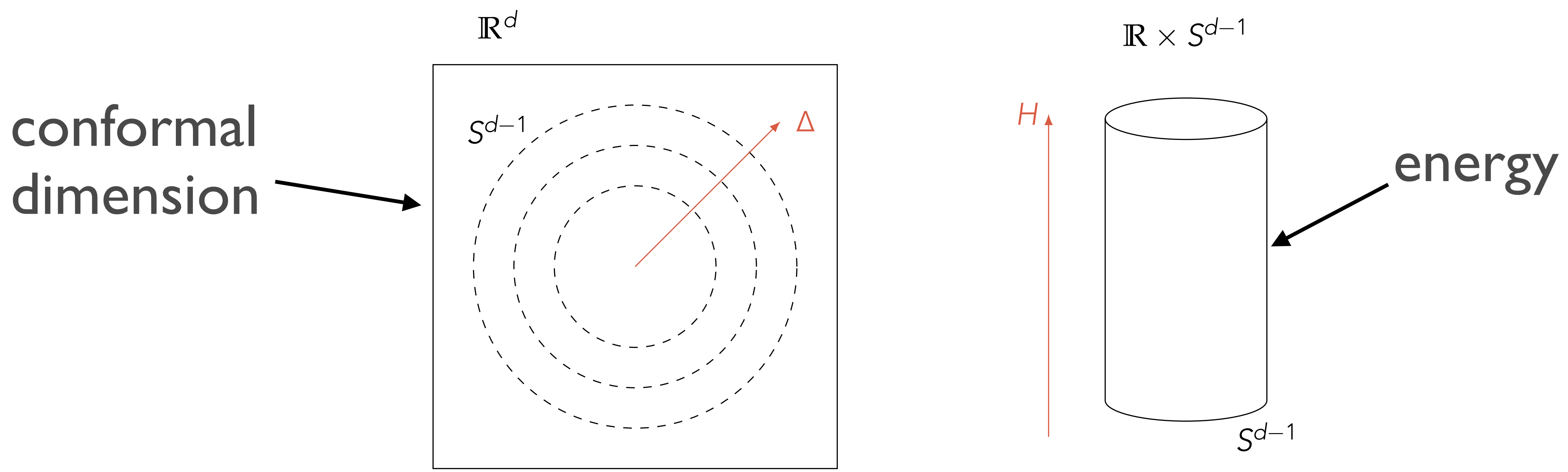

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S. Hellerman, D. Orlando, S. R., M. Watanabe, arXiv:1505.01537 [hep-th]

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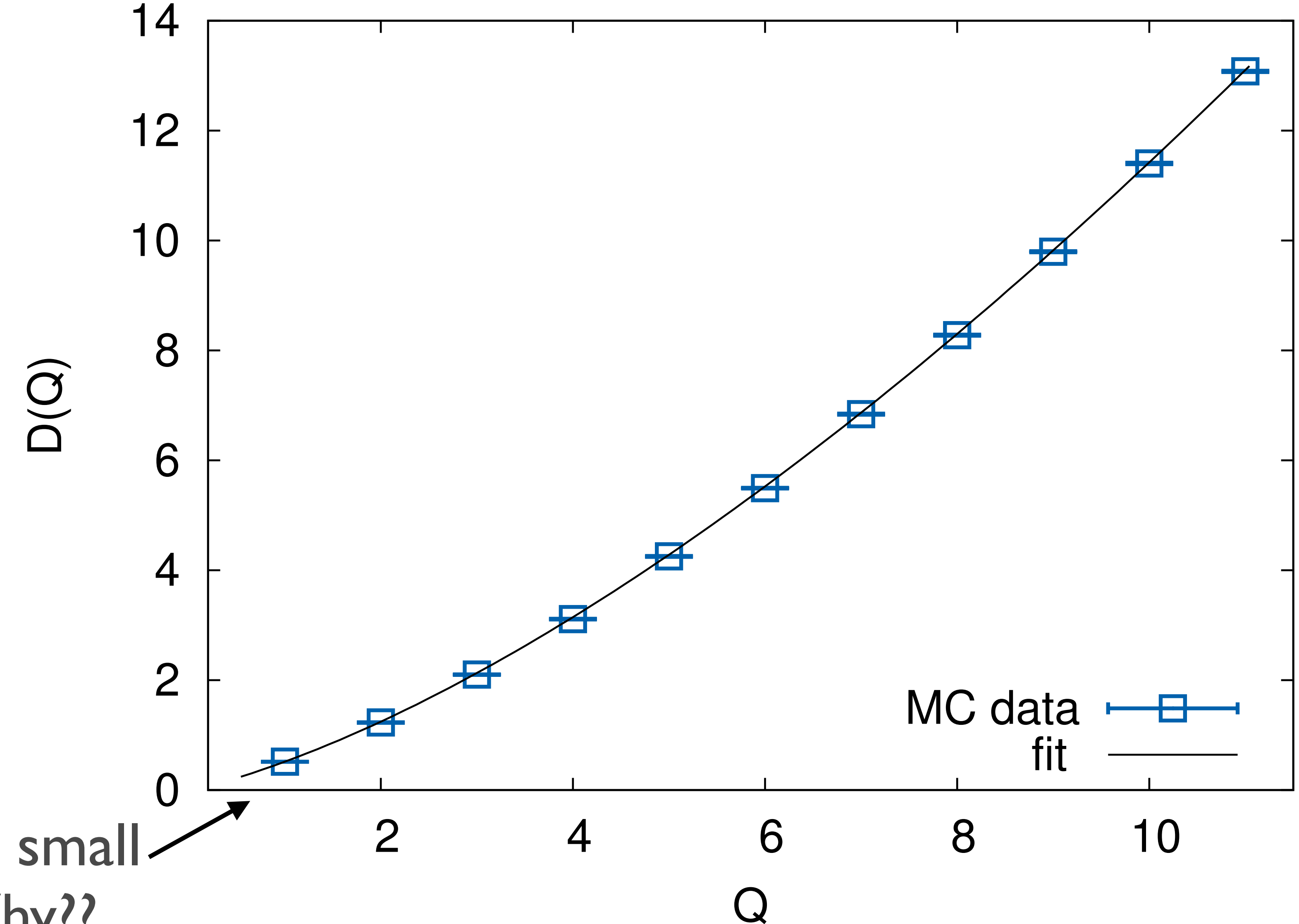

works for small charge. Why??

Excellent agreement!!

$$c_{3/2} = 1.195(10)$$
$$c_{1/2} = 0.075(10)$$

D. Banerjee, Sh. Chandrasekharan, D. Orlando [hep-th/1707.00711]

Large-charge expansion works extremely well for O(2).

Beyond O(2)

Where else can we apply the large-charge expansion?

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On top of the conformal Goldstone of O(2), a new sector with N-1 non-relativistic type II Goldstones and N-1 massive modes with m=2μ appears.

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 L. Alvarez-Gaume, O. Loukas, D. Orlando and S. R., arXiv:1610.04495 [hep-th]

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verified at large N for CP(N-1) model de la Fuente

 L. Alvarez-Gaume, O. Loukas, D. Orlando and S. R., arXiv:1610.04495 [hep-th]

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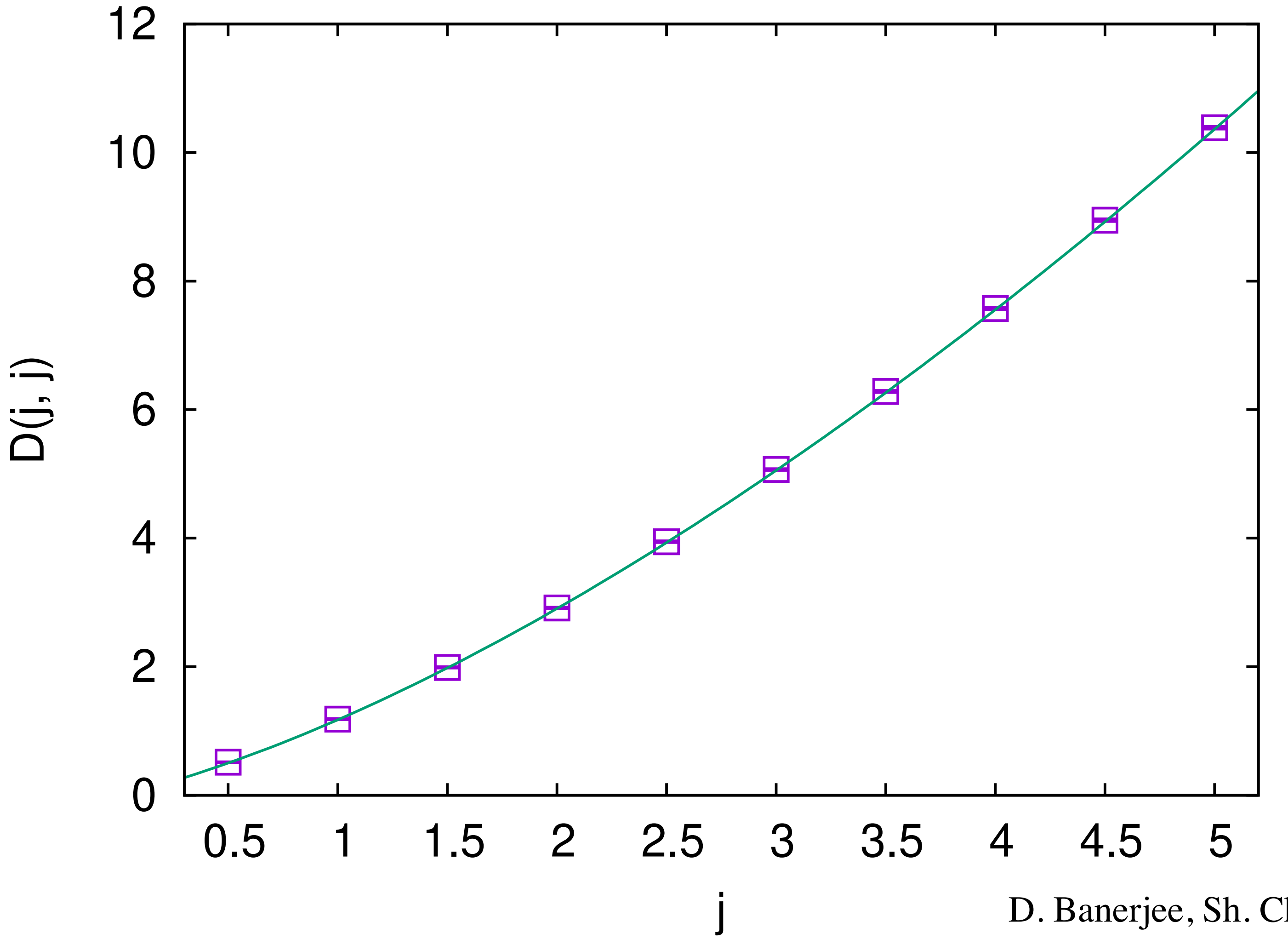
New lattice data for O(4) model:



$c_{3/2} = 1.068(4)$

$c_{1/2} = 0.083(3)$

D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. 1902.09542

Again excellent agreement with large-Q prediction!

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Standard large-N methods, expand path integral around saddle point (no EFT!)

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For r=R/8, this flows to the WF fixed pt in the IR, $u \to \infty$

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Adding the chemical potential gives us more structure to work with!

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L is the Legendre transform of V in Φ^2

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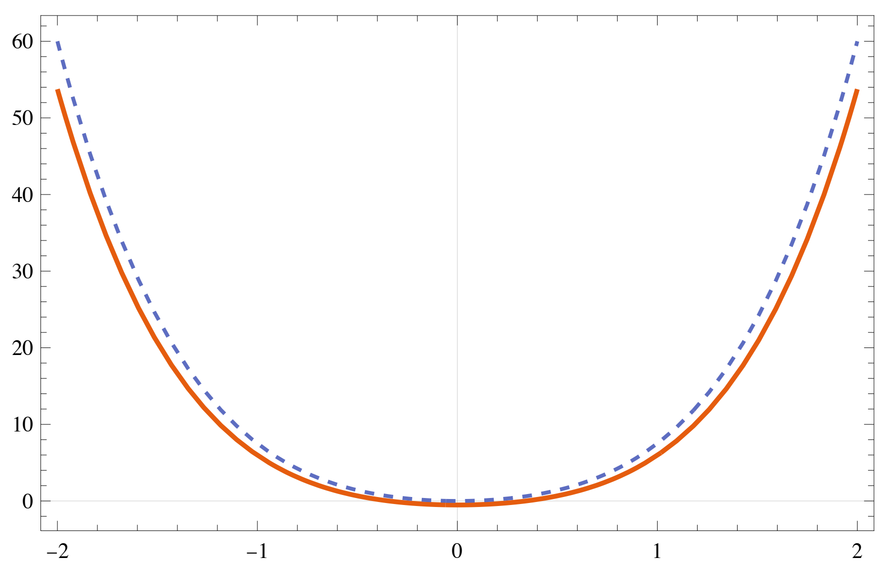

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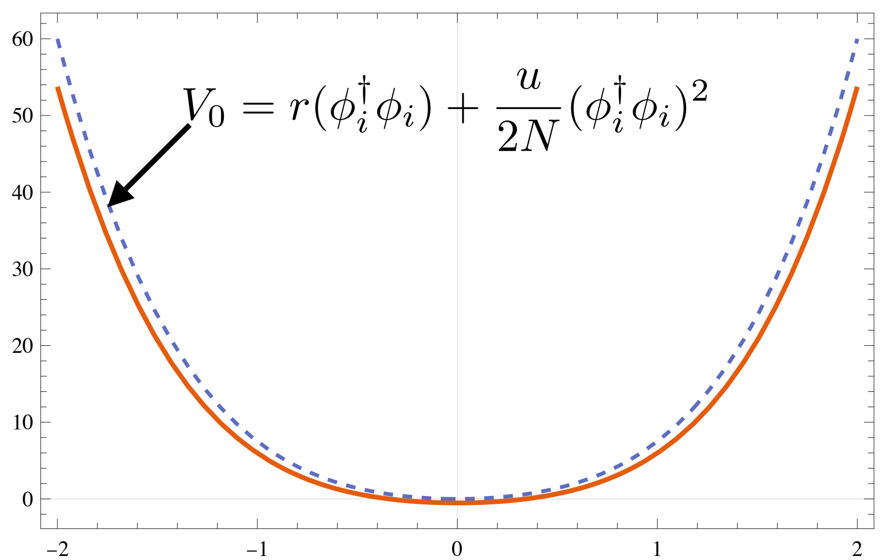

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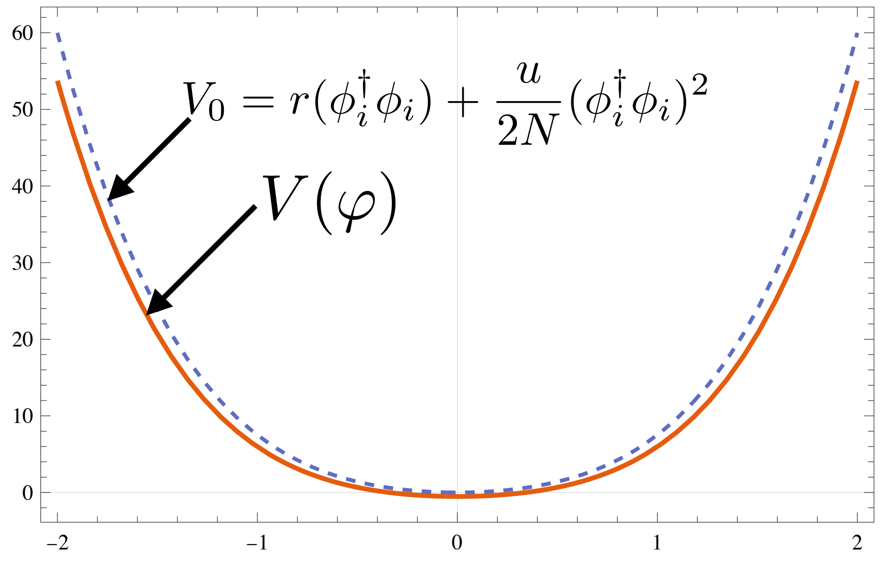

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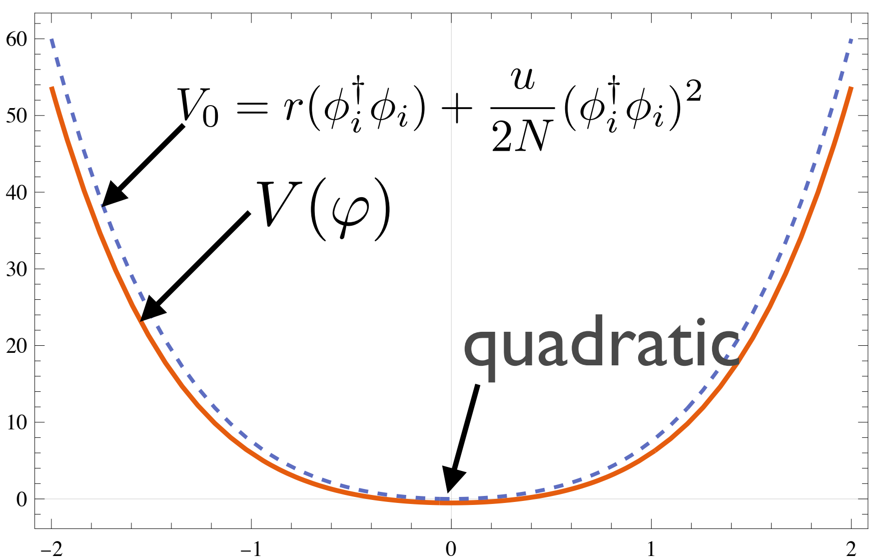

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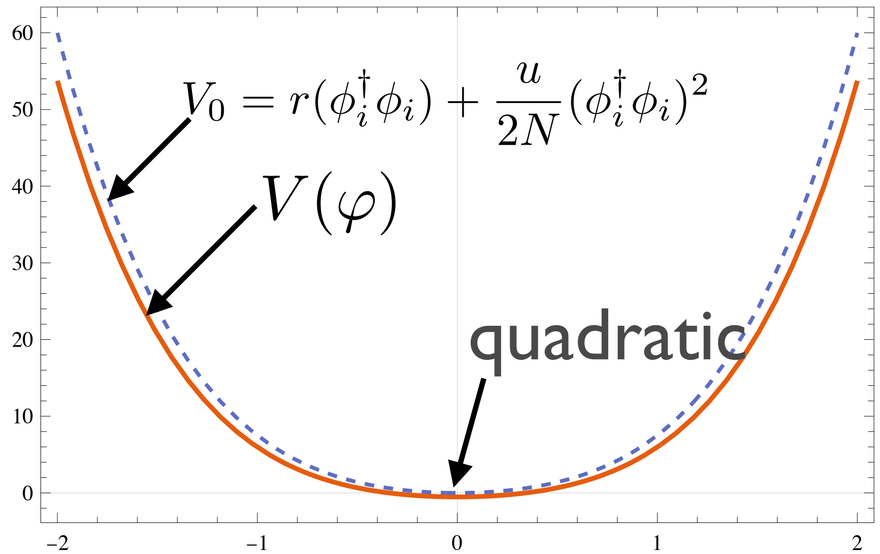

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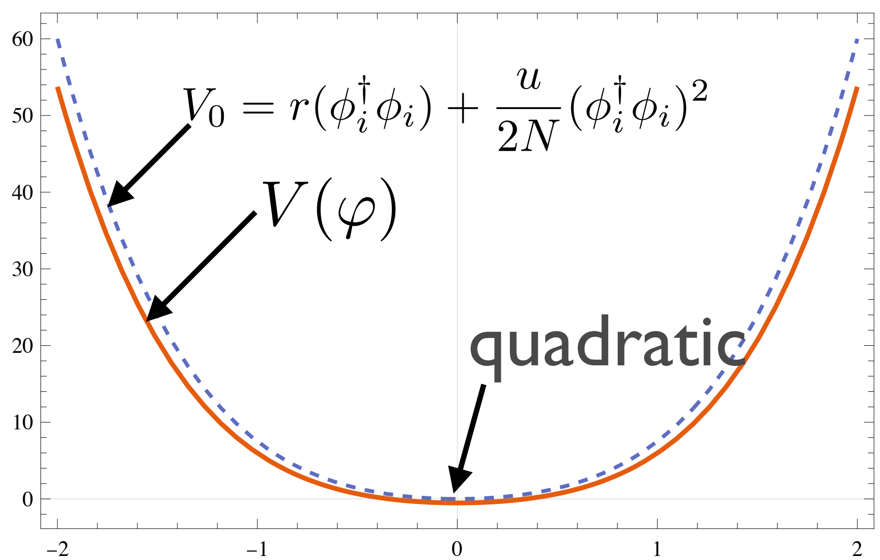

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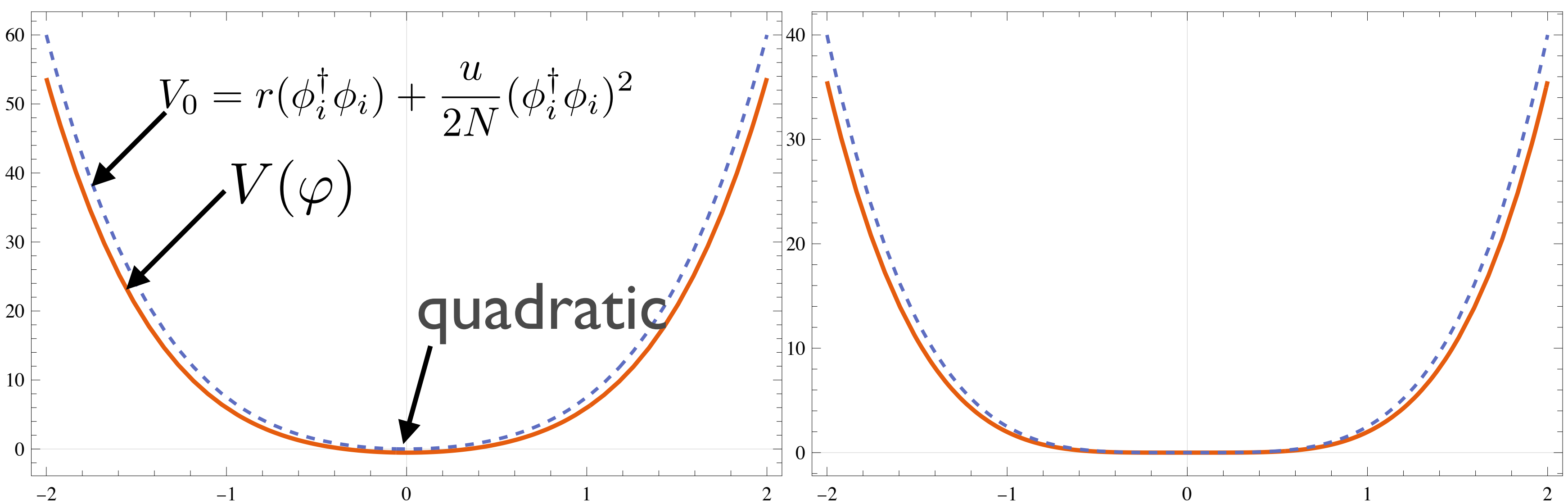

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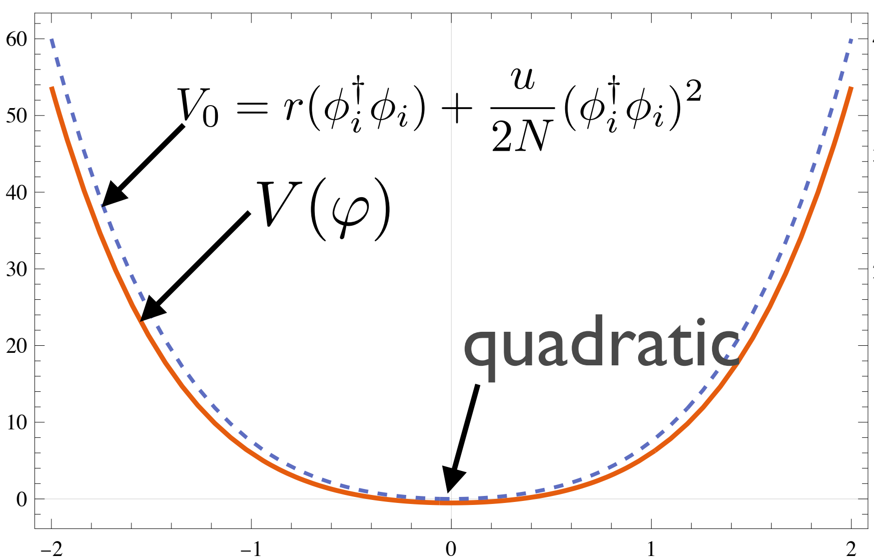

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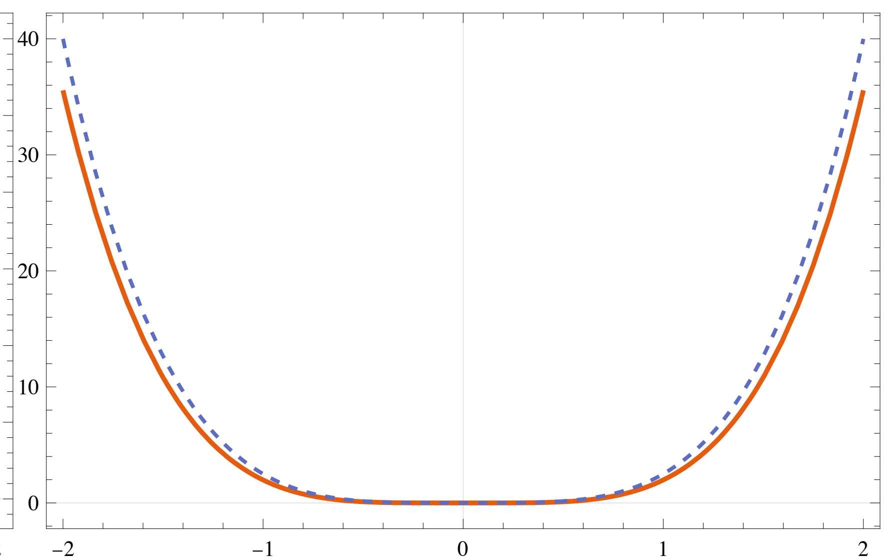

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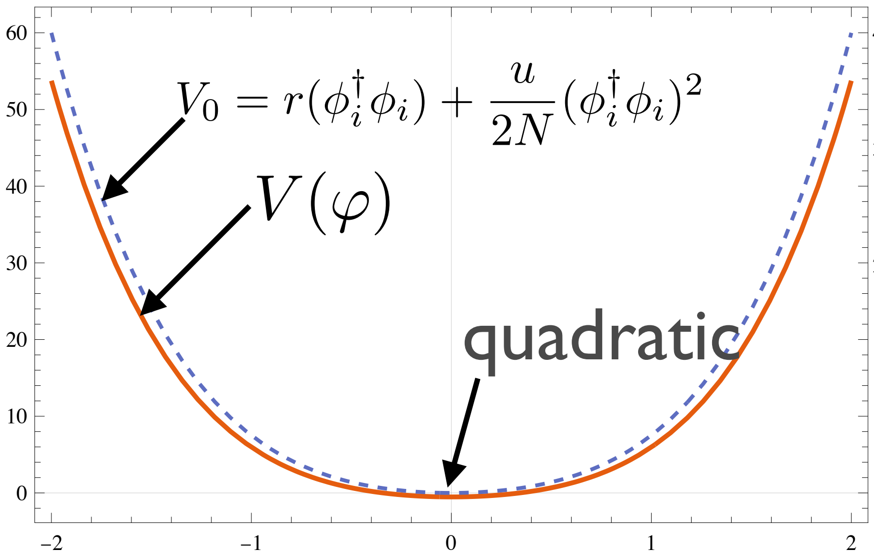

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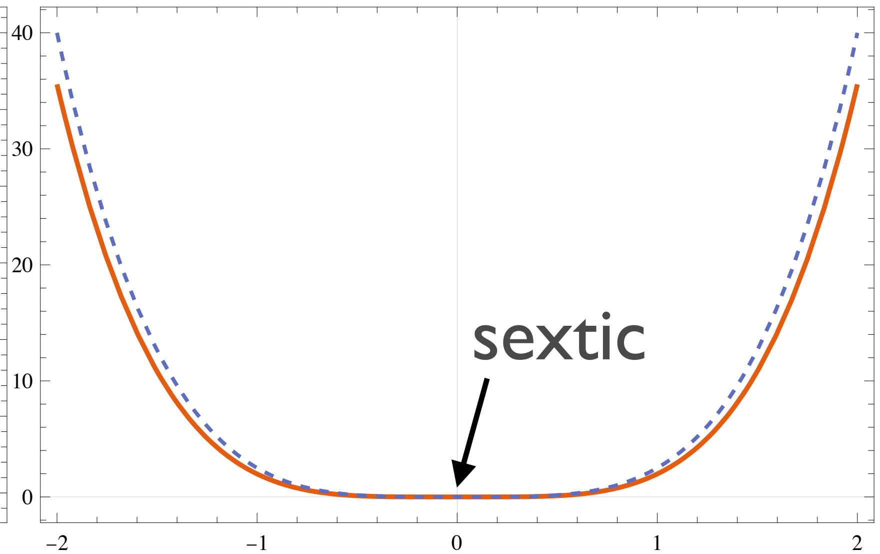

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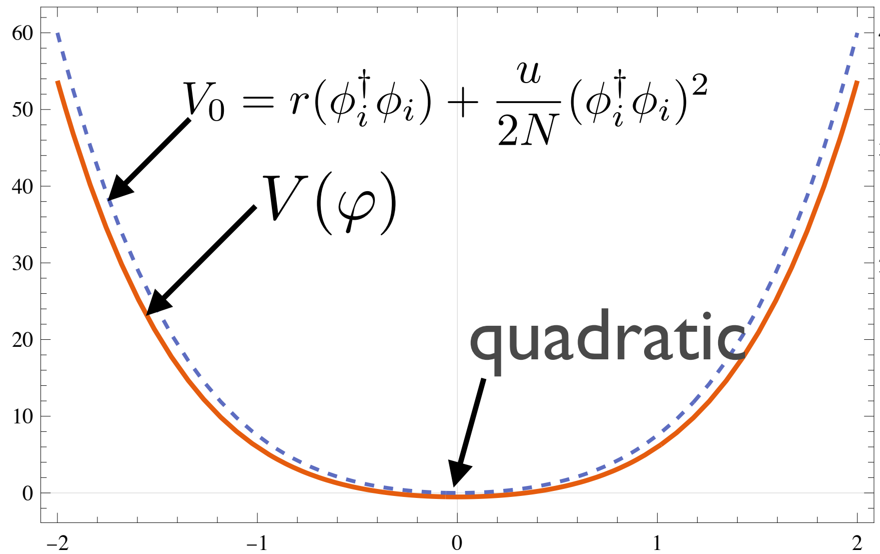

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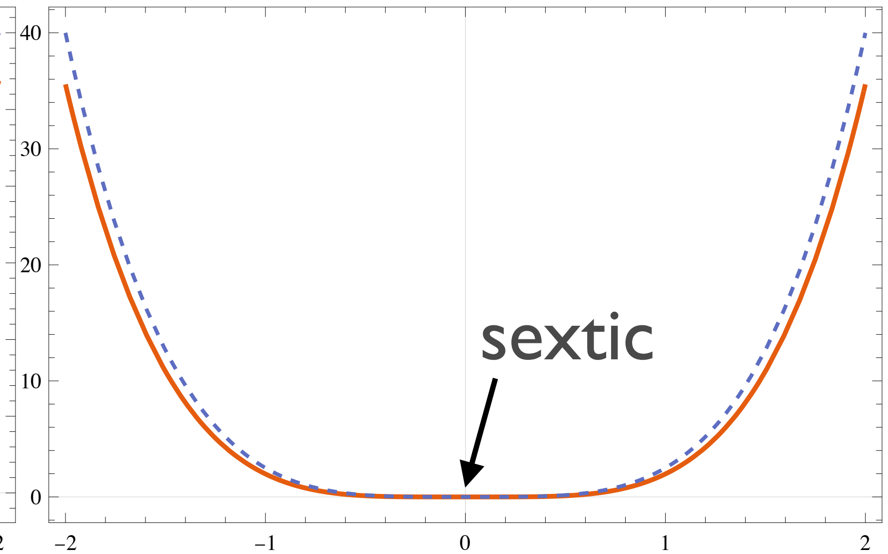

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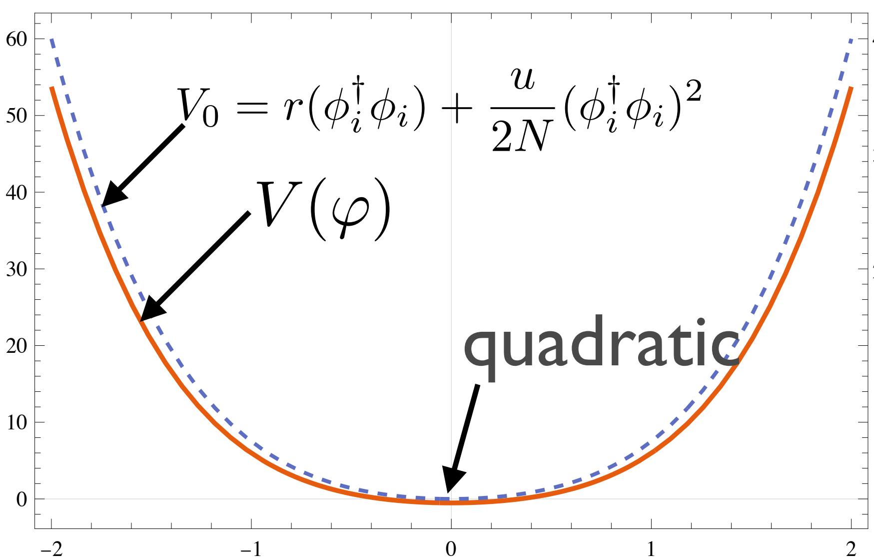

unbroken phase

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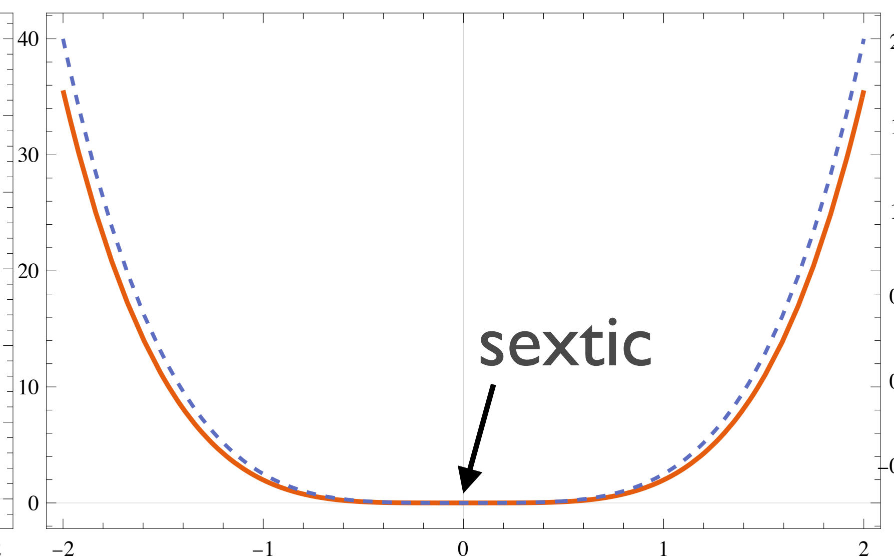

critical point

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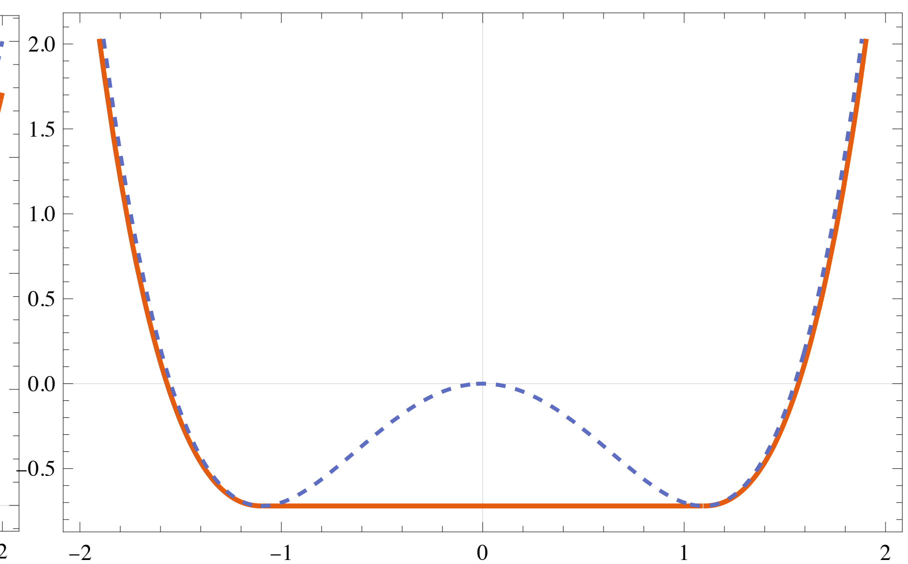

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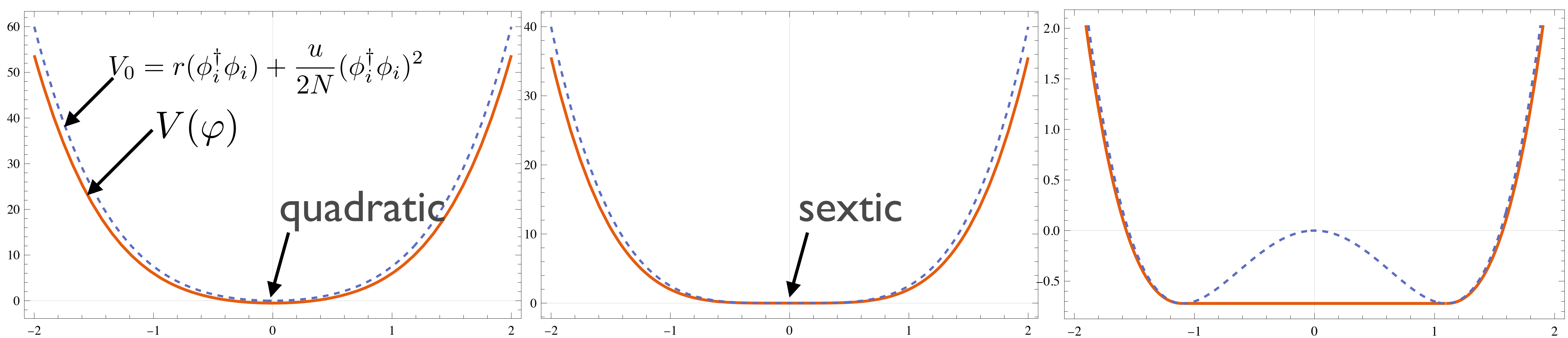

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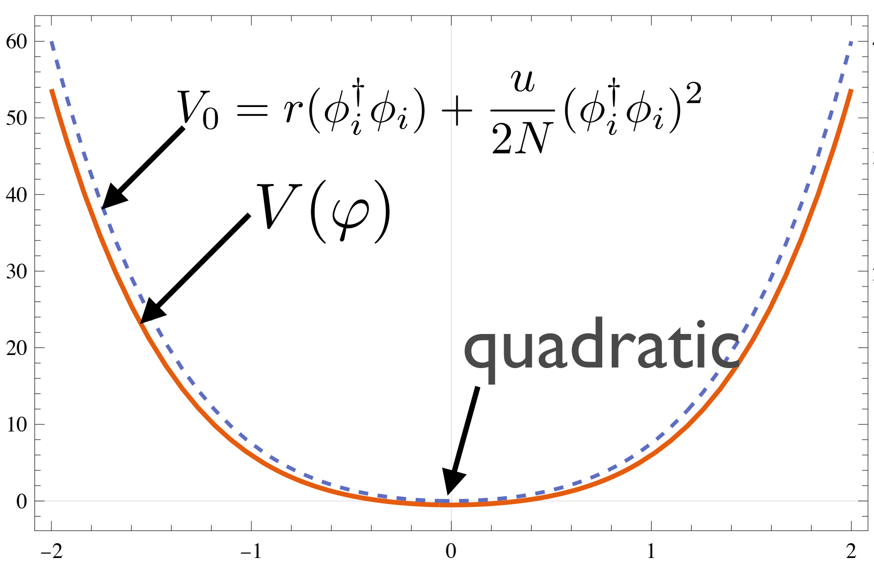

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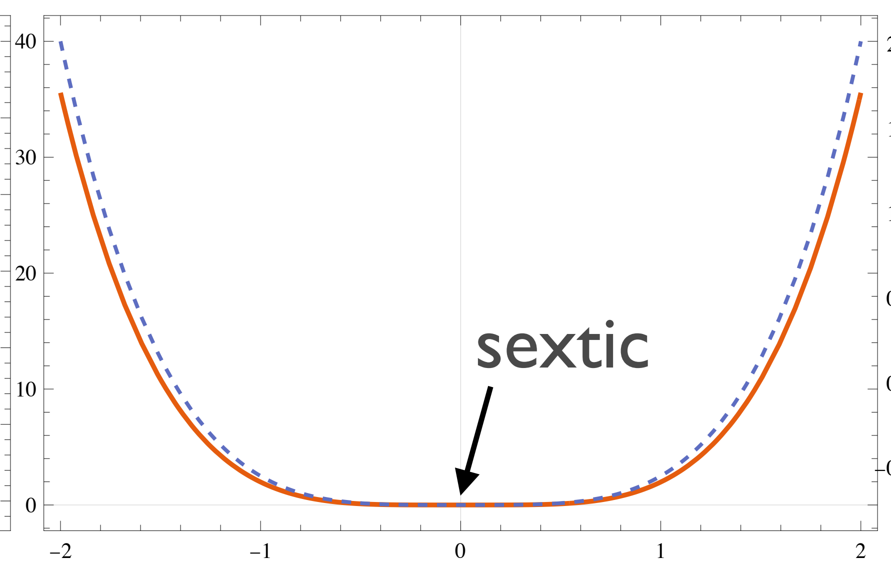

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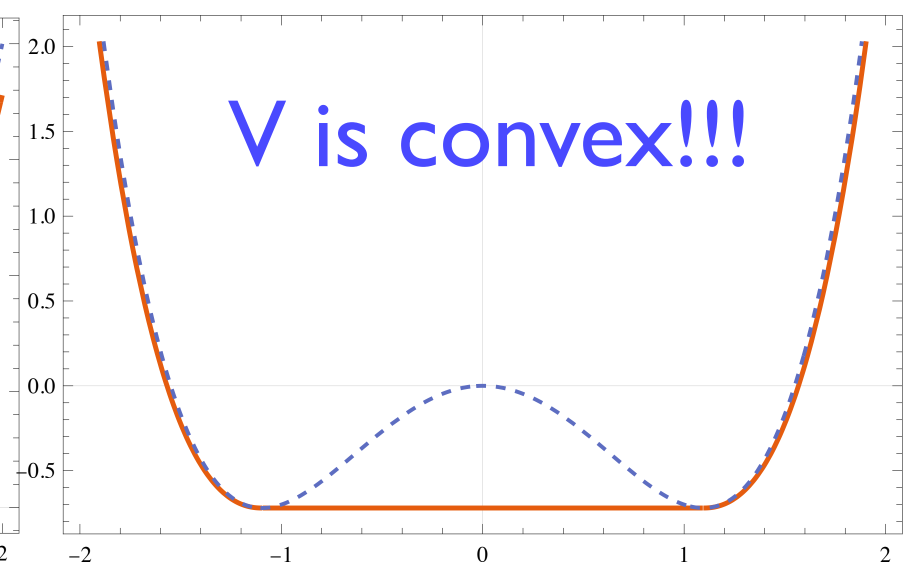

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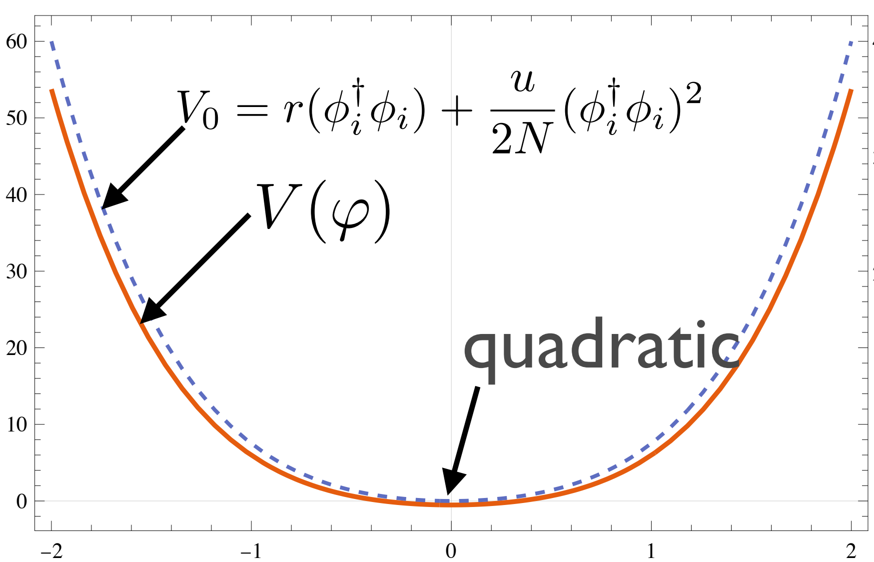

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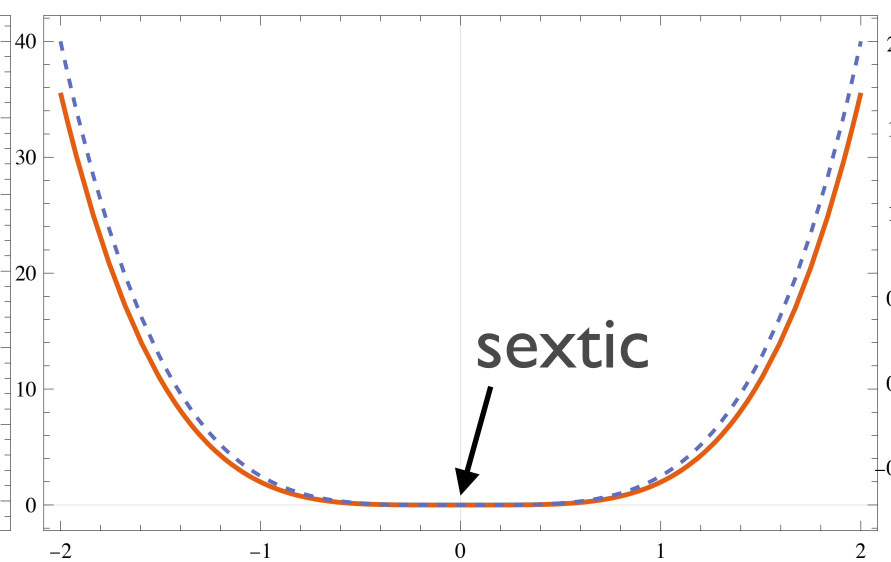

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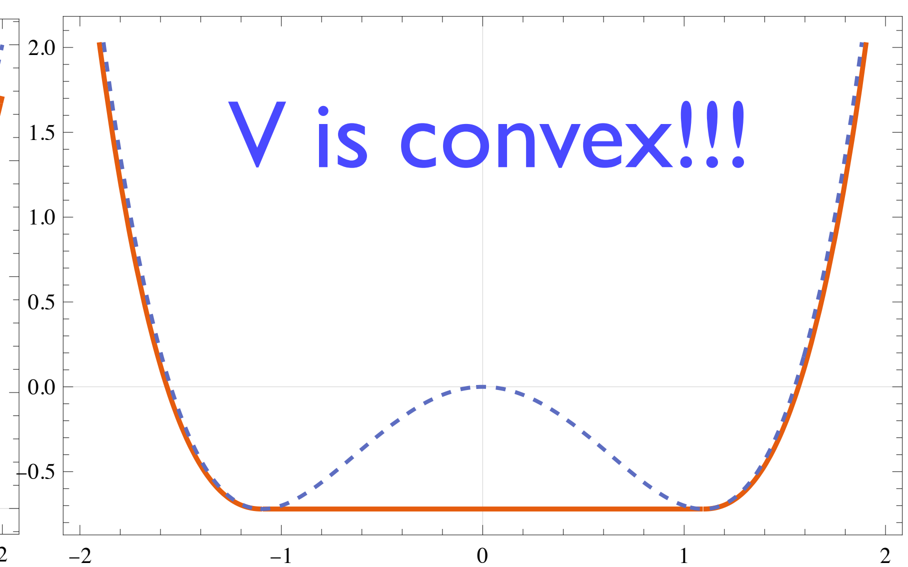

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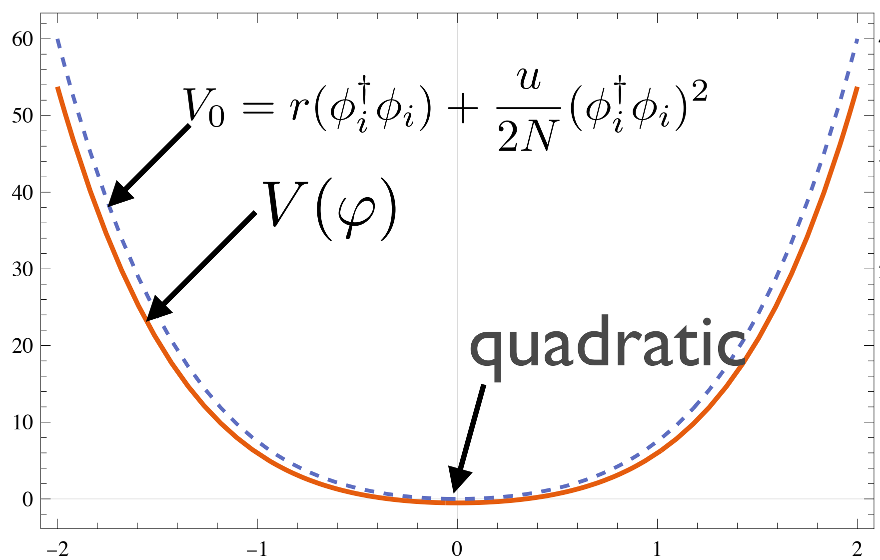

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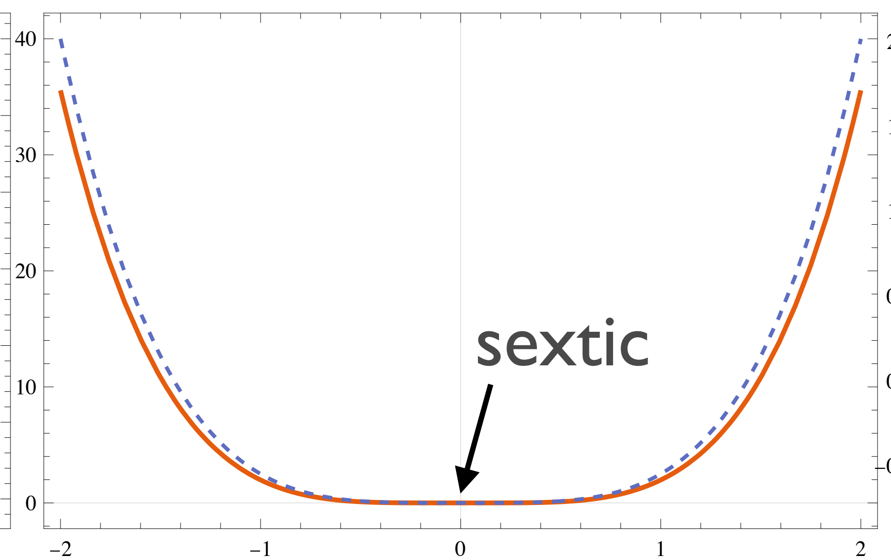

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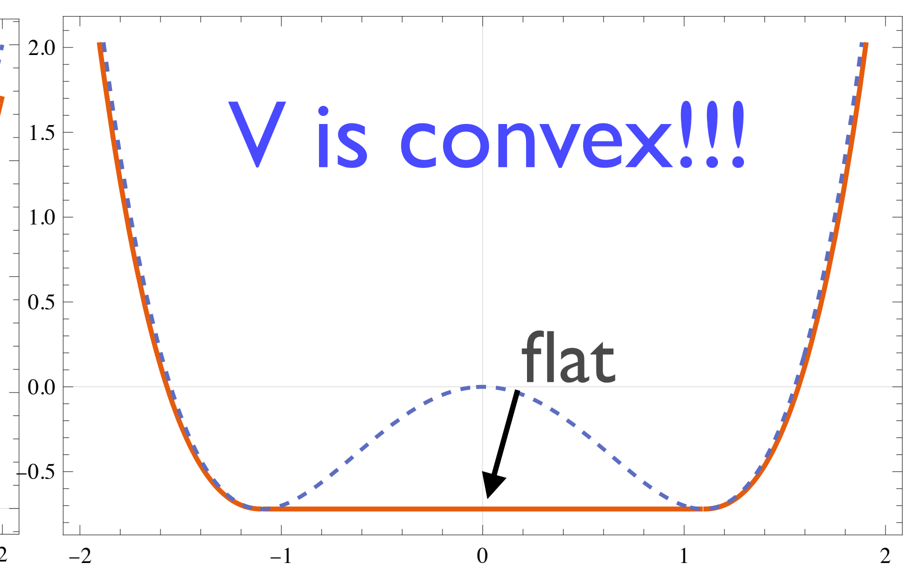

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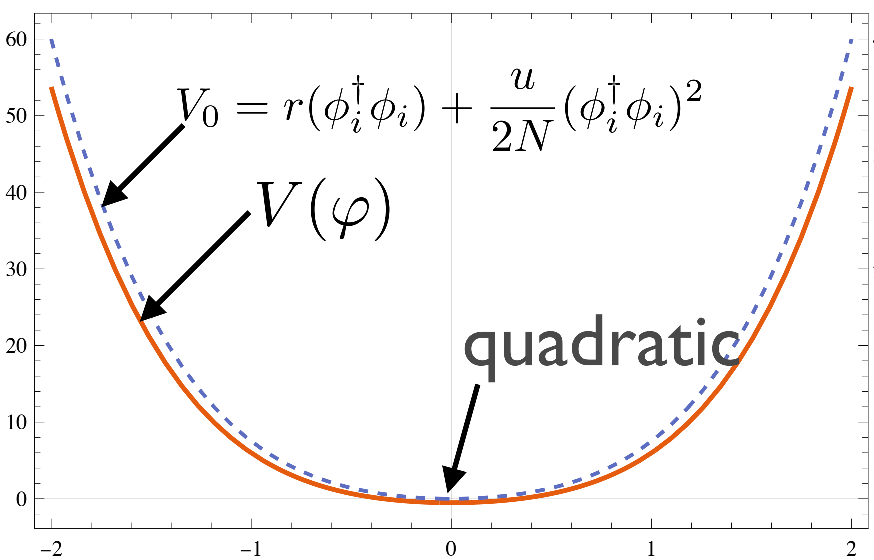

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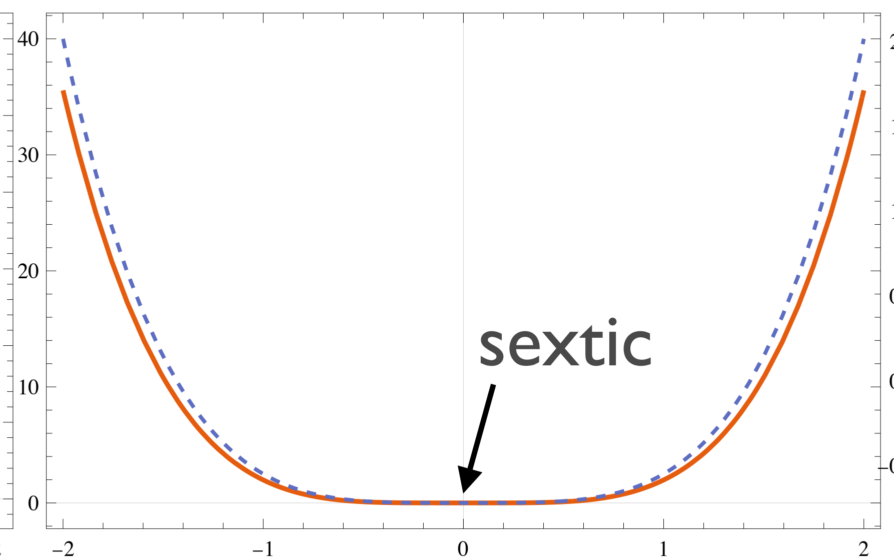

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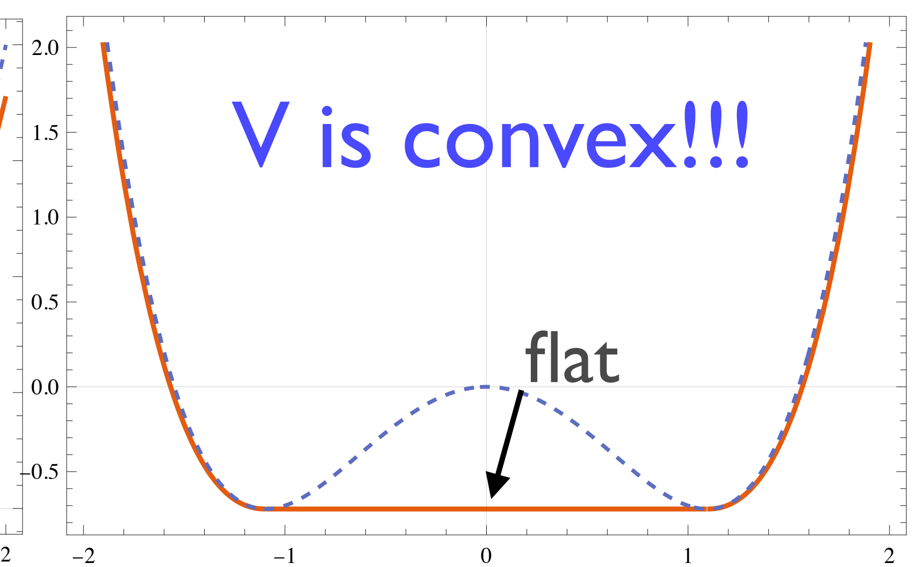

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Orlando, Reffert, Schmidt 2110.07616

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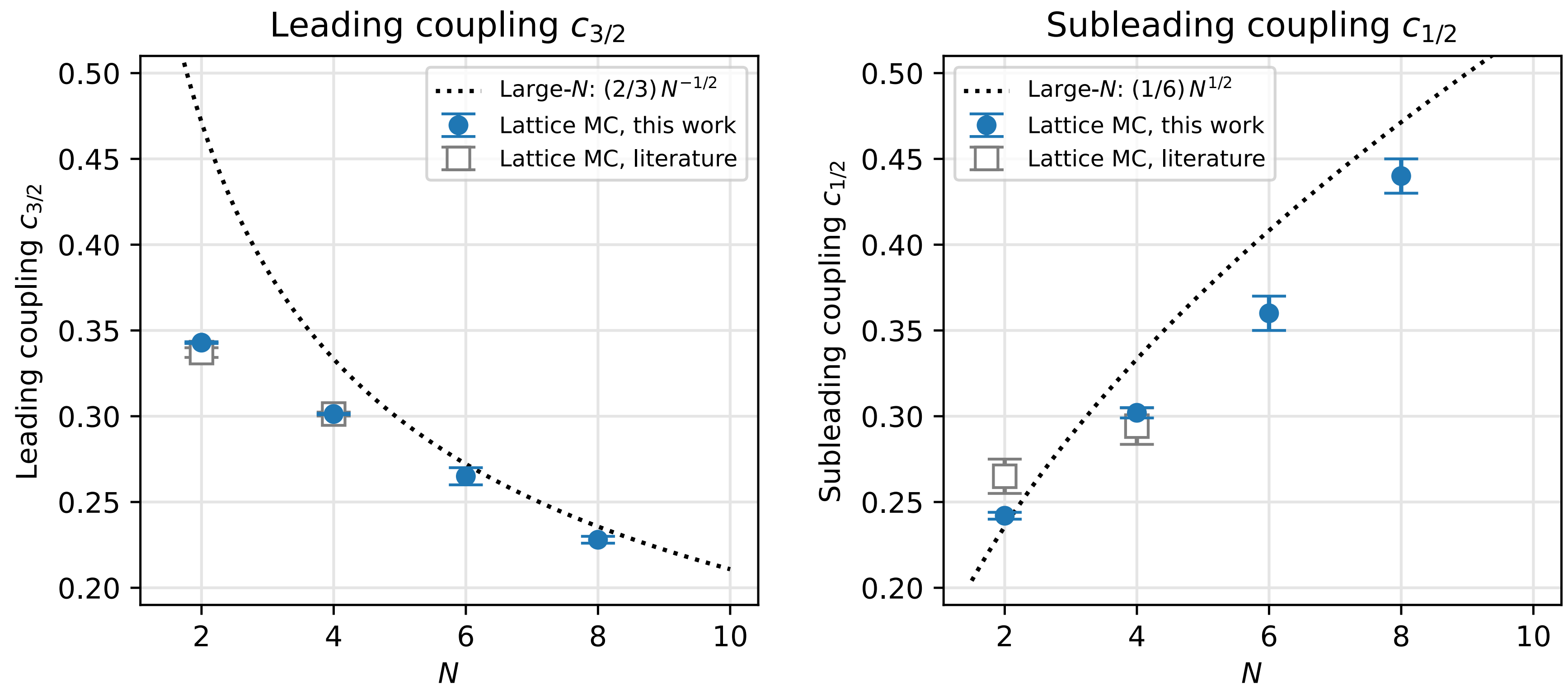

Singh, arXiv:2203.00059 [hep-lat]

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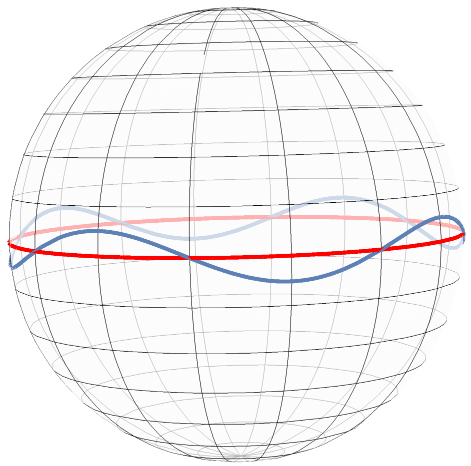

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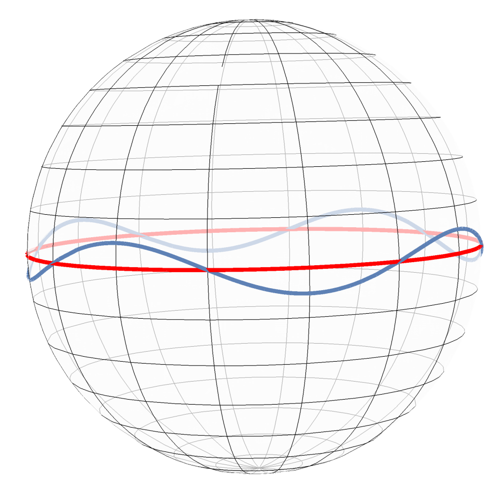

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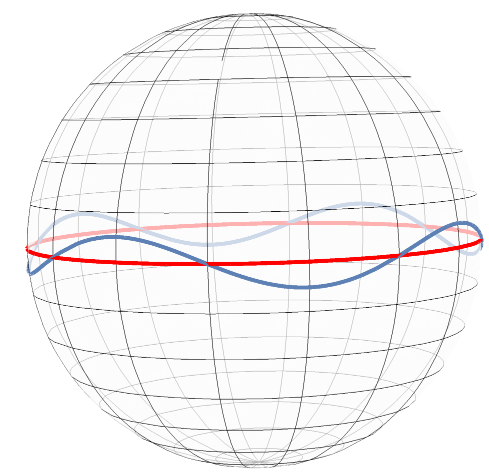

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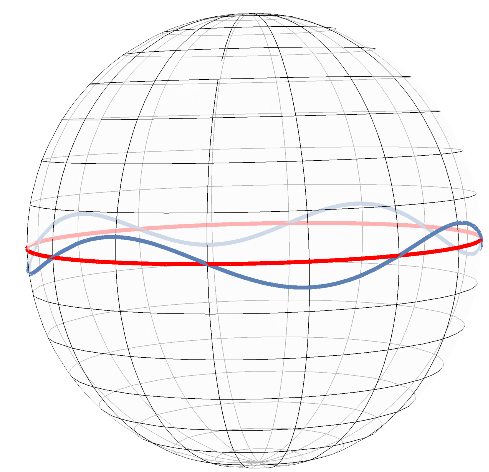

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A. Dondi, I. Kalogerakis, D.Orlando, S.R, arXiv: 2102.12488 [hep-th]

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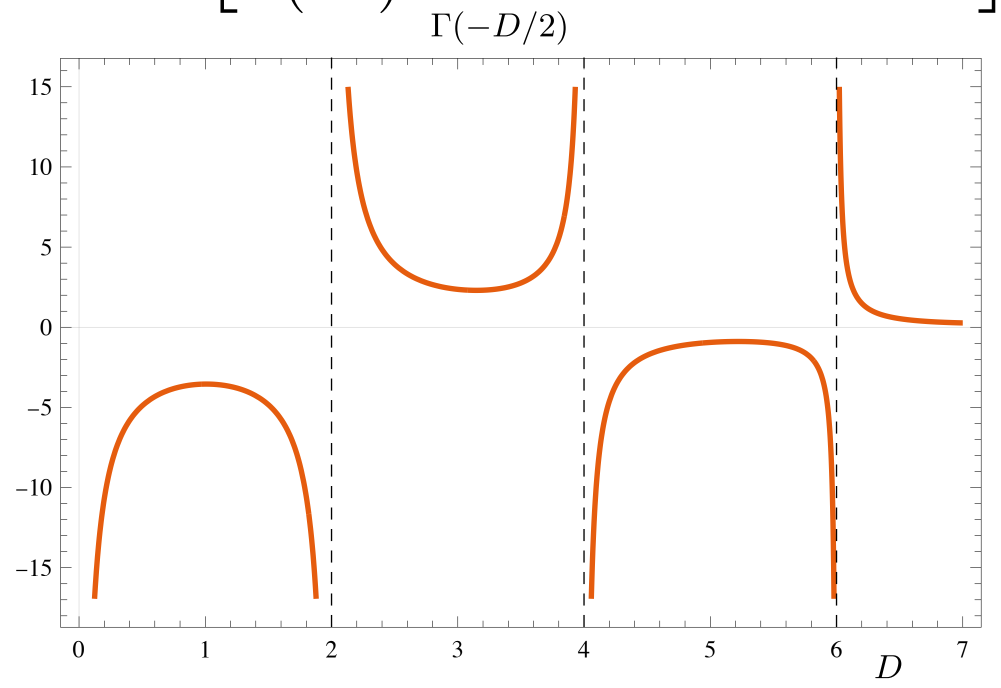

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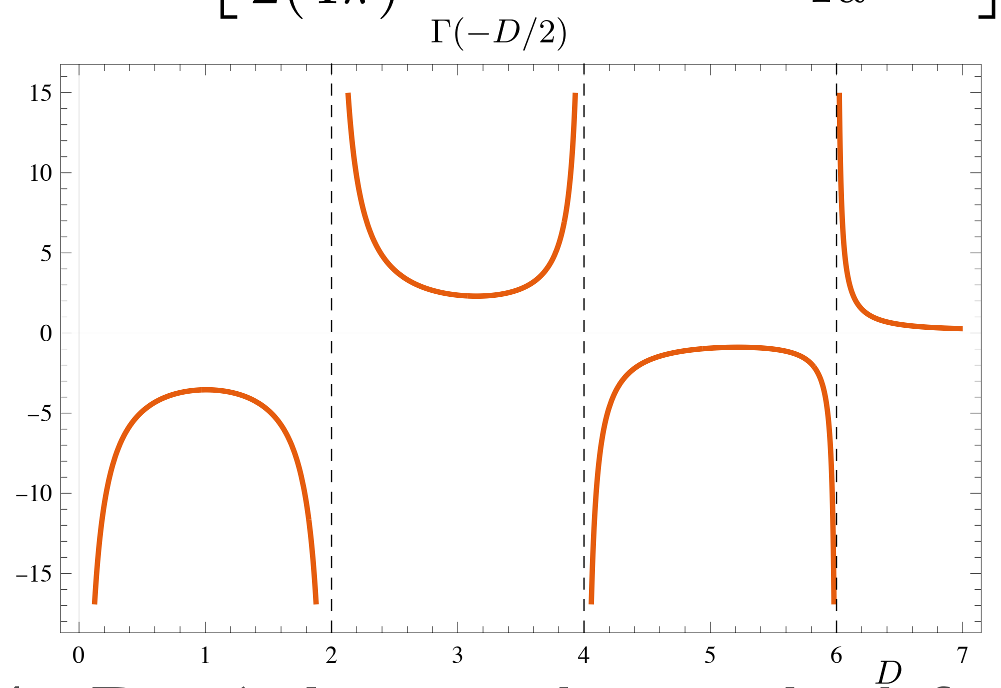

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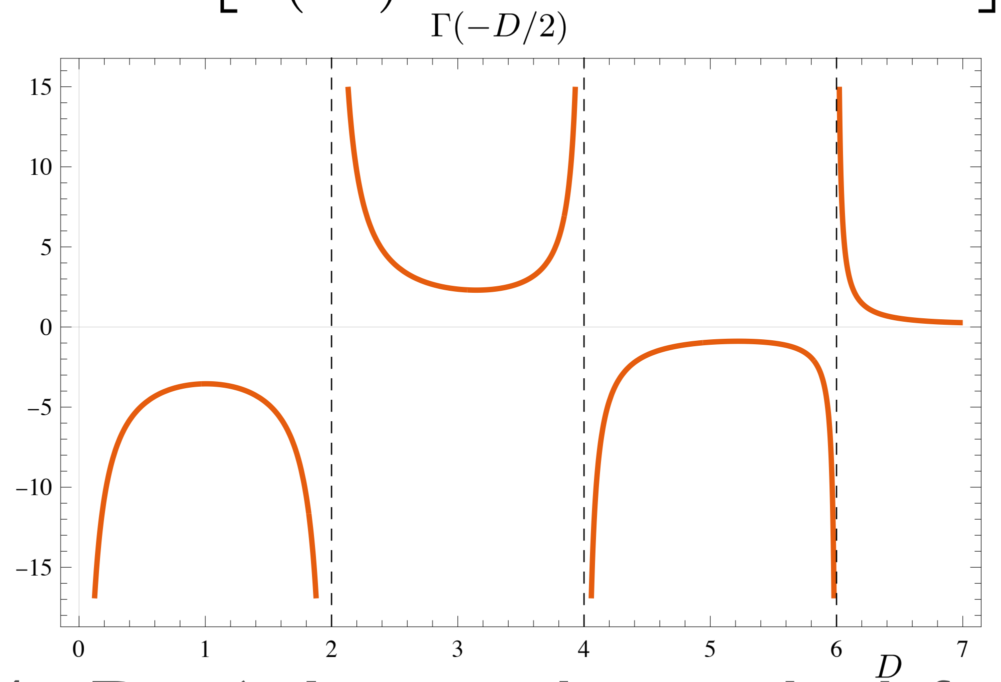

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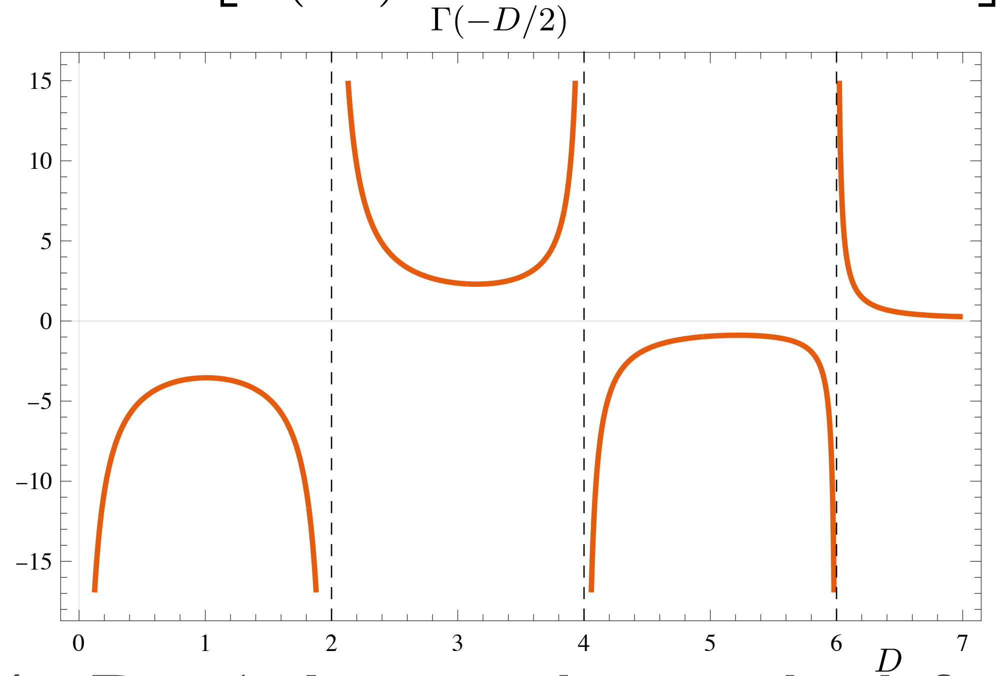

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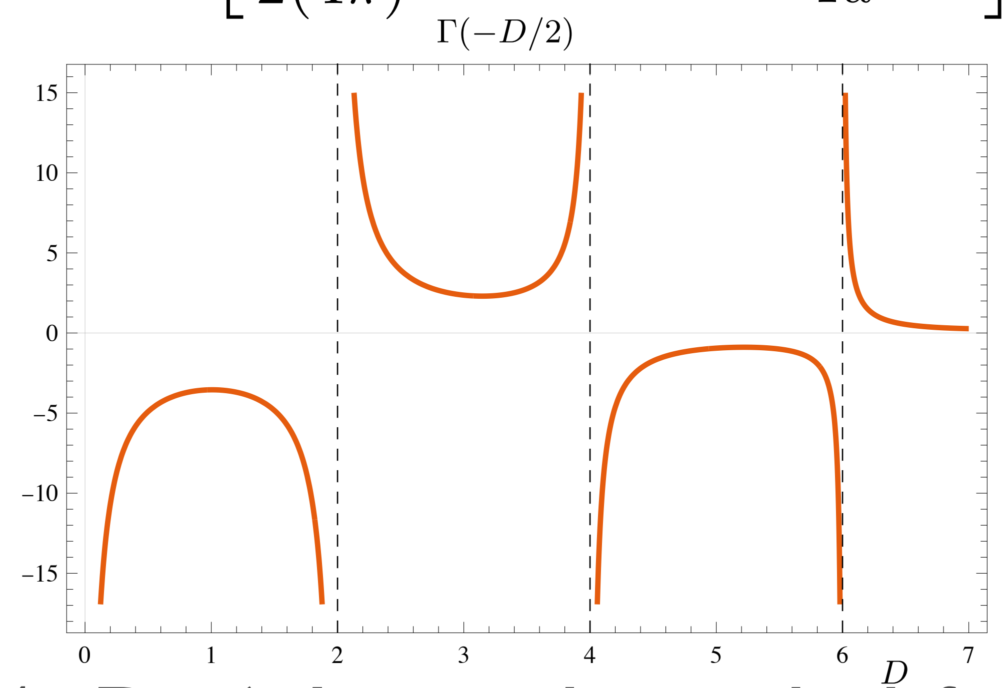

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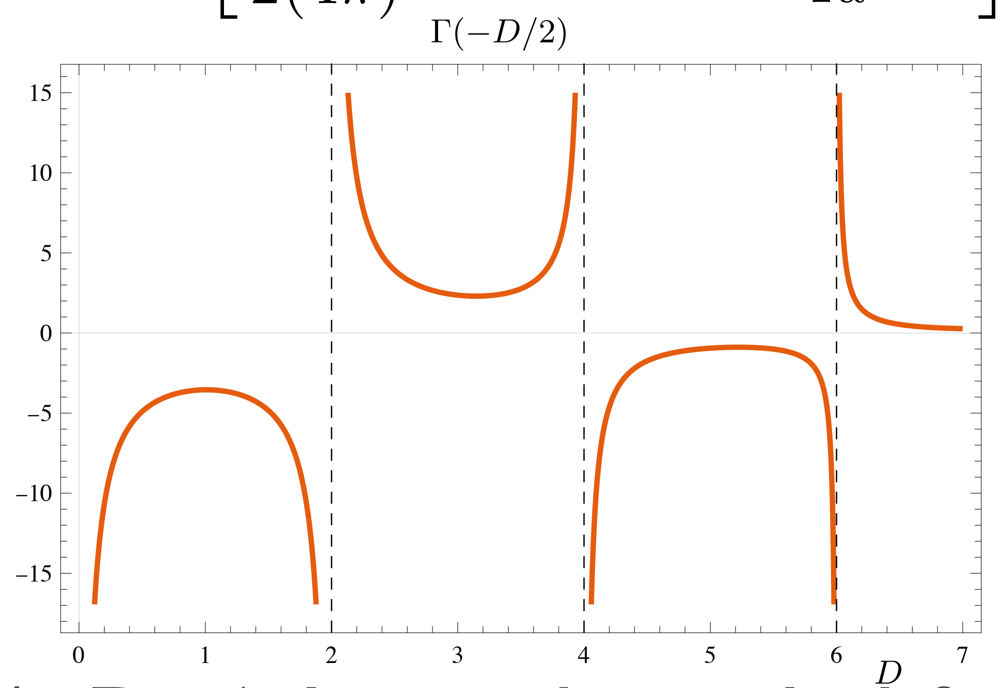

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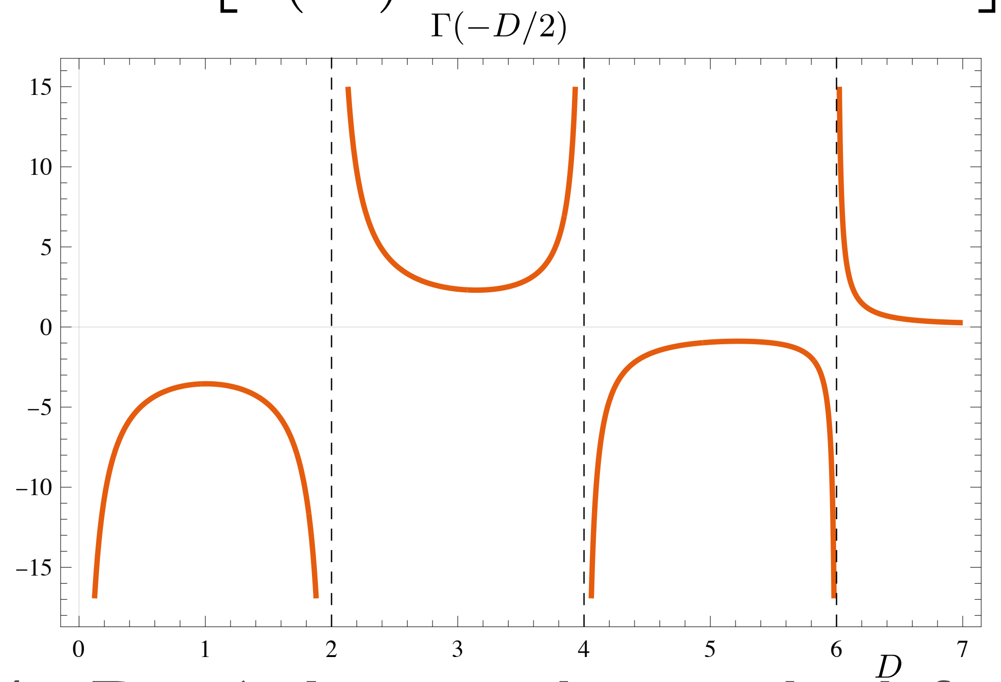

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Giombi, Hyman;
Moser, Orlando, Reffert 2110.07617

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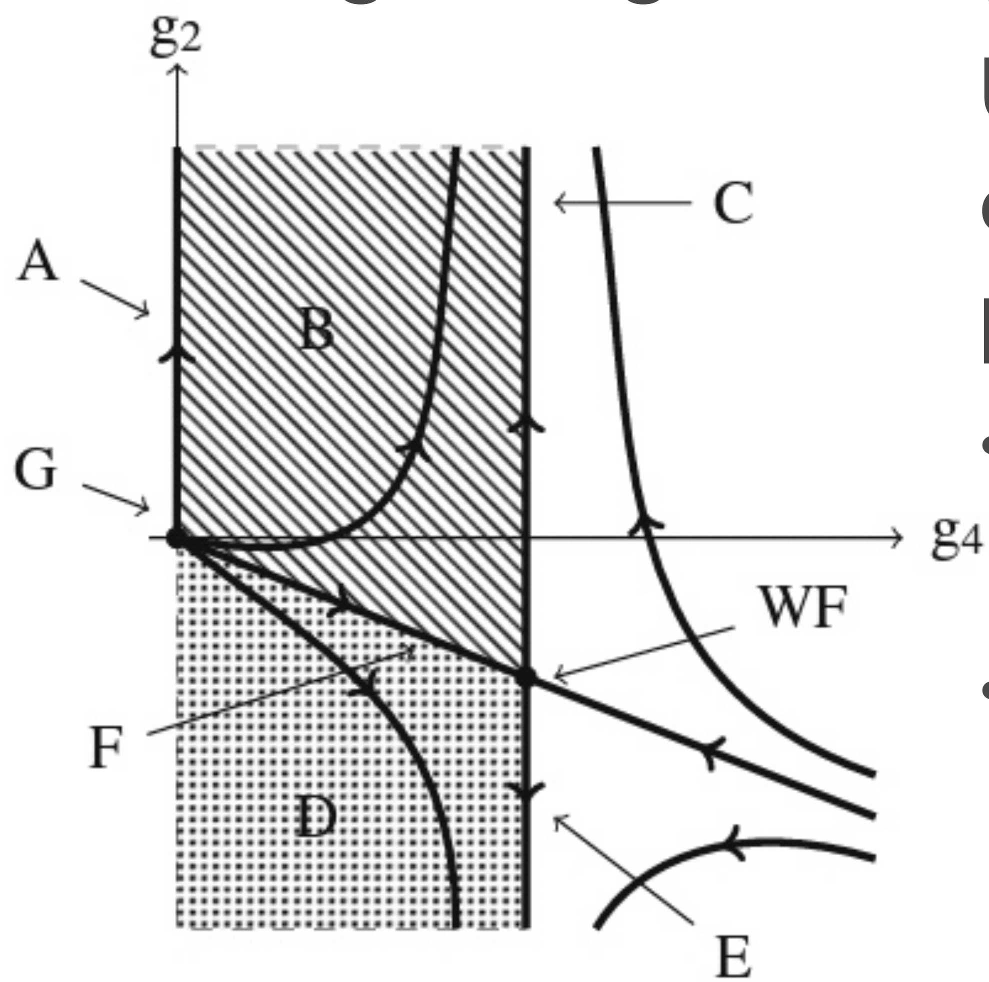
Summary

Concrete examples where a strongly-coupled CFT simplifies at large charge.



O(2N) model in 3d: in the limit of large U(1) charge Q, we computed the conformal dimensions in a controlled perturbative expansion:

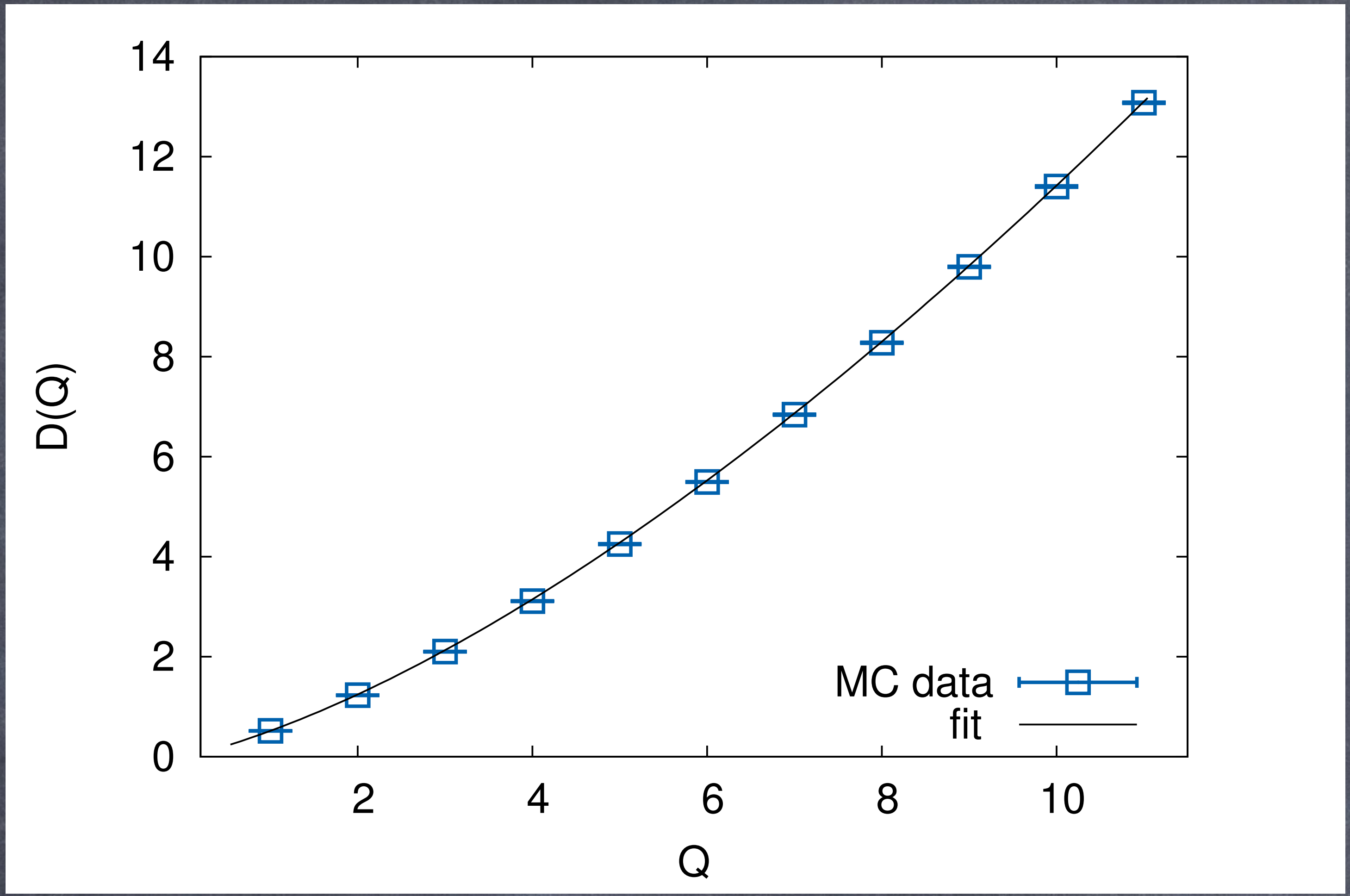
- Excellent agreement with lattice results for O(2), O(4)
- large Q and large N: path integral at saddle pt., more control than in EFT, can calculate coefficients
- can follow the flow away from conformal point
- find the full effective potential

Further directions

- Further study of supersymmetric models at large R-charge (higher-dim. moduli spaces) Hellerman, Maeda, Orlando, Reffert, Watanabe; Argyres et al.
- Connection to holography (gravity duals) Loukas, Orlando, Reffert, Sarkar; De la Fuente, Zosso; Giombi, Komatsu, Offertaler.
- Operators with spin; connection to large-spin results Cuomo, de la Fuente, Monin, Pirtskhalava, Rattazzi; Cuomo
- Understanding dualities semi-classically at large charge
- Use/check large-charge results in conformal bootstrap Jafferis and Zhiboedov
- Further lattice simulations: inhomogeneous sector, general O(N) Chandrasekharan et al.
- CFTs in other dimensions (2, 5, 6) Komargodski, Mezei, Pal, Raviv-Moshe; Araujo, Celikbas, Reffert, Orlando; Moser, Orlando, Reffert

Further directions

- Chern-Simons matter theories @large charge Watanabe
- 4-ε expansion @large charge Arias-Tamargo, Rodriguez-Gomez, Russo; Badel, Cuomo, Monin, Rattazzi; Watanabe; Antipin et al.
- going away from the conformal point Orlando, Reffert, Sannino; Orlando, Pellizzani, Reffert
- non-relativistic CFTs Favrod, Orlando, Reffert; Kravec, Pal; Orlando, Pellizzani, Reffert; Hellerman, Swanson; Pellizzani
- Boundary CFTs at large Q Cuomo, Mezei, Raviv-Moshe
- Weak gravity conjecture Aharony, Palti; Antipin et al.
- Study fermionic theories. Can large-charge approach be used for QCD (e.g. large baryon number)? Komargodski, Mezei, Pal, Raviv-Moshe



Thank you for your attention!