Consequences of No Global Symmetries in Quantum Gravity

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Based on 2006.10052/hep-th with Shu-Heng Shao

2012.00009/hep-th, 2104.07036/hep-th with Ben Heidenreich, Jacob McNamara, Miguel Montero, Matt Reece, and Irene Valenzuela

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The Big Picture

Global Symmetries

- Our understanding of quantum field theory has been revolutionized in recent years by the discovery and development of generalized notions of global symmetries. Examples include:
  - Higher-form global symmetries, in which charged operators are extended objects
  - Non-invertible symmetries, whose fusion algebra involves a more general category instead of a group
  - Higher-group symmetries, in which two or more higher-form symmetries are combined into a larger structure

Quantum Gravity

- Meanwhile, recent years have seen renewed interest in identifying universal features of quantum gravity. Multiple lines of evidence support:
  - The absence of global symmetries in quantum gravity
  - The completeness hypothesis, which demands the existence of states in every representation of the gauge group
  - The weak gravity conjecture, which demands a particle of charge q, mass m, with $q/m \geq 1$ in any U(1) gauge theory

Global Symmetries + Quantum Gravity

- In this talk, I will show that these two areas of research are closely related:
  - Much of our understanding of quantum gravity can be viewed as a consequence of the absence of global symmetries
  - Generalized global symmetries hold power not only for QFT but for quantum gravity as well

(Ordinary) Global Symmetries and Quantum Gravity

Global Symmetries in QG

- An evaporating black hole emits all particles in a theory, without regard to their global charges Hawking ’75
- As a result, black hole evaporation violates global symmetries and destroys global charge Zeldovich ’76, ‘77
- Continuous symmetries are always gauged in perturbative string theory Banks, Dixon ’88
- Continuous global symmetry charges lead to a violation of the Bekenstein-Hawking entropy formula Banks, Seiberg ’10
- Global symmetries are incompatible with entanglement wedge reconstruction Harlow, Ooguri ’18, Harlow, Shaghoulian ‘21
- Global symmetries are violated by Euclidean wormholes in the quantum gravity path integral Giddings, Strominger ’88, Abbott, Wise ’89, Coleman, Lee ’90, Kallosh, Linde, Linde, Susskind ’95, Chen, Lin ’21, Hsin, Iliesiu, Yang ’21, Yonekura ’21

Global Symmetries in QG

- These arguments strongly suggest that any global symmetry in a QFT must be gauged or broken upon coupling it to quantum gravity:

<u>Gauged</u>

$$\mathcal{L} = j^{\mu} A_{\mu} + \cdots$$

<u>Broken</u>

$$\partial_{\mu} j^{\mu} = \frac{1}{\Lambda^{n}} \mathcal{O}_{n}(x) + \cdots$$

- Several arguments suggest that this conclusion applies to generalized notions of global symmetries as well…

Higher-Form Global Symmetries

Higher-Form Global Symmetries

- A "q-form global symmetry" is a global symmetry for which the charged operators are q-dimensional
  - $q = 0$ corresponds to an ordinary global symmetry
- Global symmetry transformations form a group, G.
- G may be discrete or continuous. For $q = 0$, it may be nonabelian or abelian. For $q > 0$, it must abelian.
- If G is continuous, it has (under reasonable assumptions) a conserved *d-q-1*-form "Noether current" J:

$$dJ^{(d-q-1)} = 0$$

Gaiotto, Kapustin, Seiberg, Willett '14

Symmetry Generators

- Symmetry transformations are implemented by symmetry generators (a.k.a. charge operators):

$$U_g(\mathcal{M}^{(d-q-1)})$$

$g \in G$

$\mathcal{M}^{(d-q-1)} =$ Manifold of dimension $d-q-1$

- Satisfy fusion law:

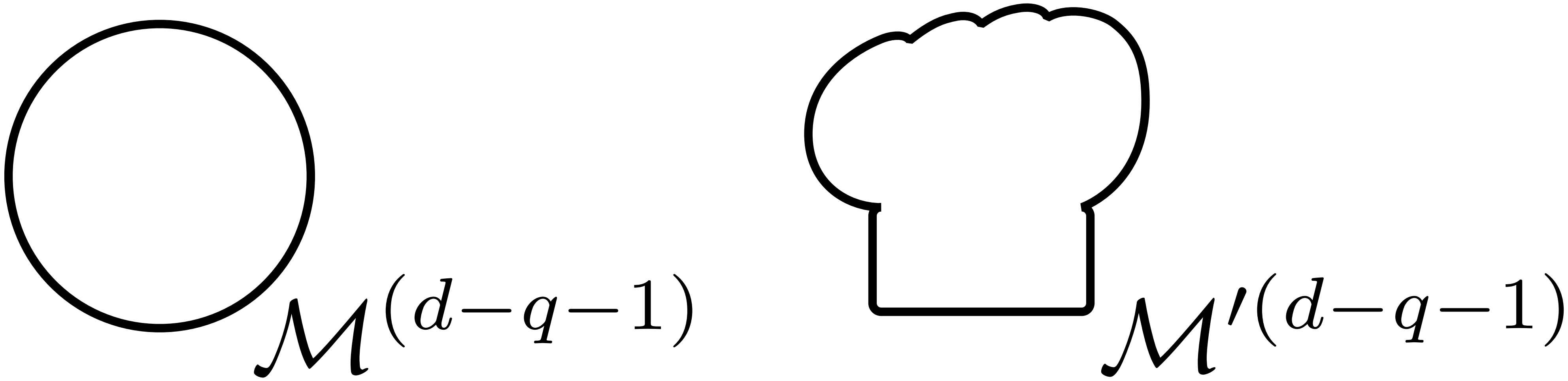
$$U_g(\mathcal{M}^{(d-q-1)})U_{g'}(\mathcal{M}^{(d-q-1)}) = U_{g''}(\mathcal{M}^{(d-q-1)})$$

$$gg' = g''$$

Symmetry Operators

- Operators are topological

$$U_g(\mathcal{M}^{(d-q-1)}) = U_g(\mathcal{M}'^{(d-q-1)})$$



- When G = U(1),

$$U_{g=e^{i\alpha}}(\mathcal{M}^{(d-q-1)}) = e^{i\alpha \int_{\mathcal{M}^{(d-q-1)}} J}$$

Ward Identities

- Symmetry generators act on charged operators, giving Ward identities:

$$U_g(S^{d-q-1})V(\mathcal{C}^{(q)}) = \omega_g(V)V(\mathcal{C}^{(q)})$$

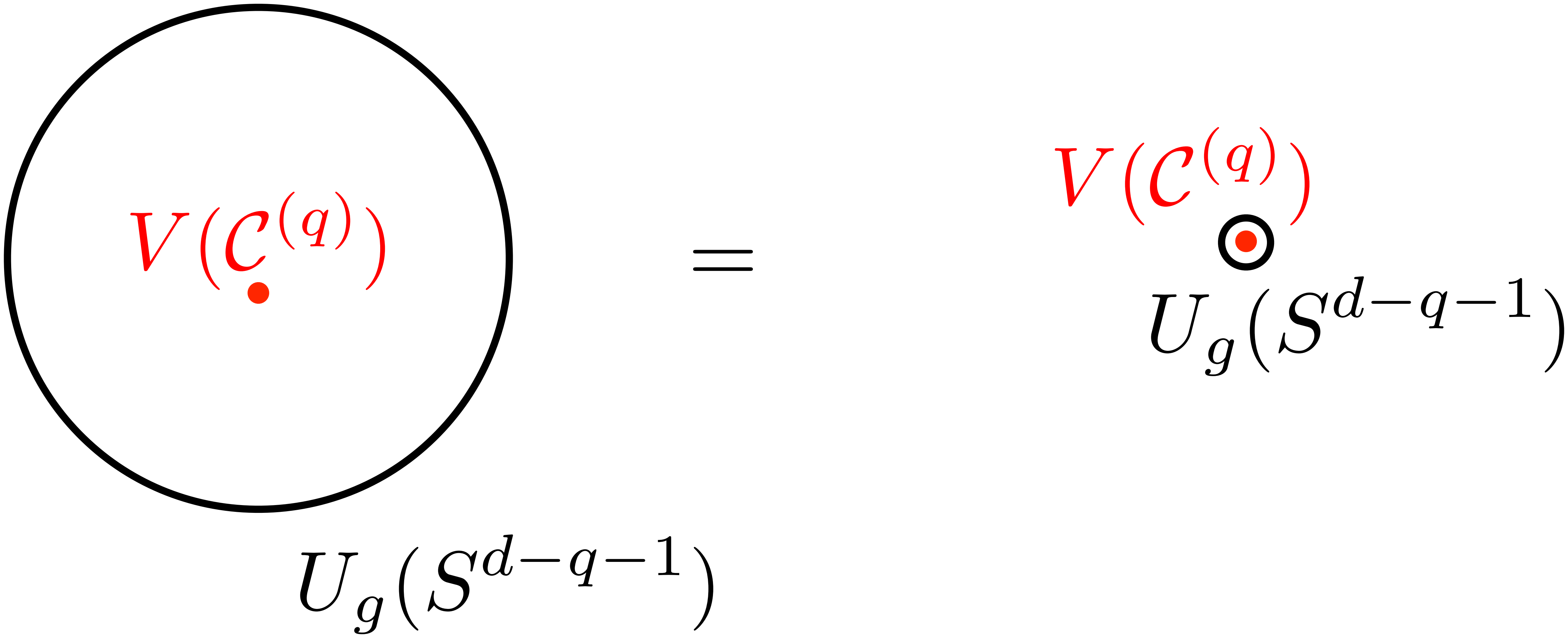
$V =$ charged operator

$\mathcal{C}^{(q)}=$ Manifold of dimension q linked by S^{d-q-1}

$\omega_g =$ representation of g

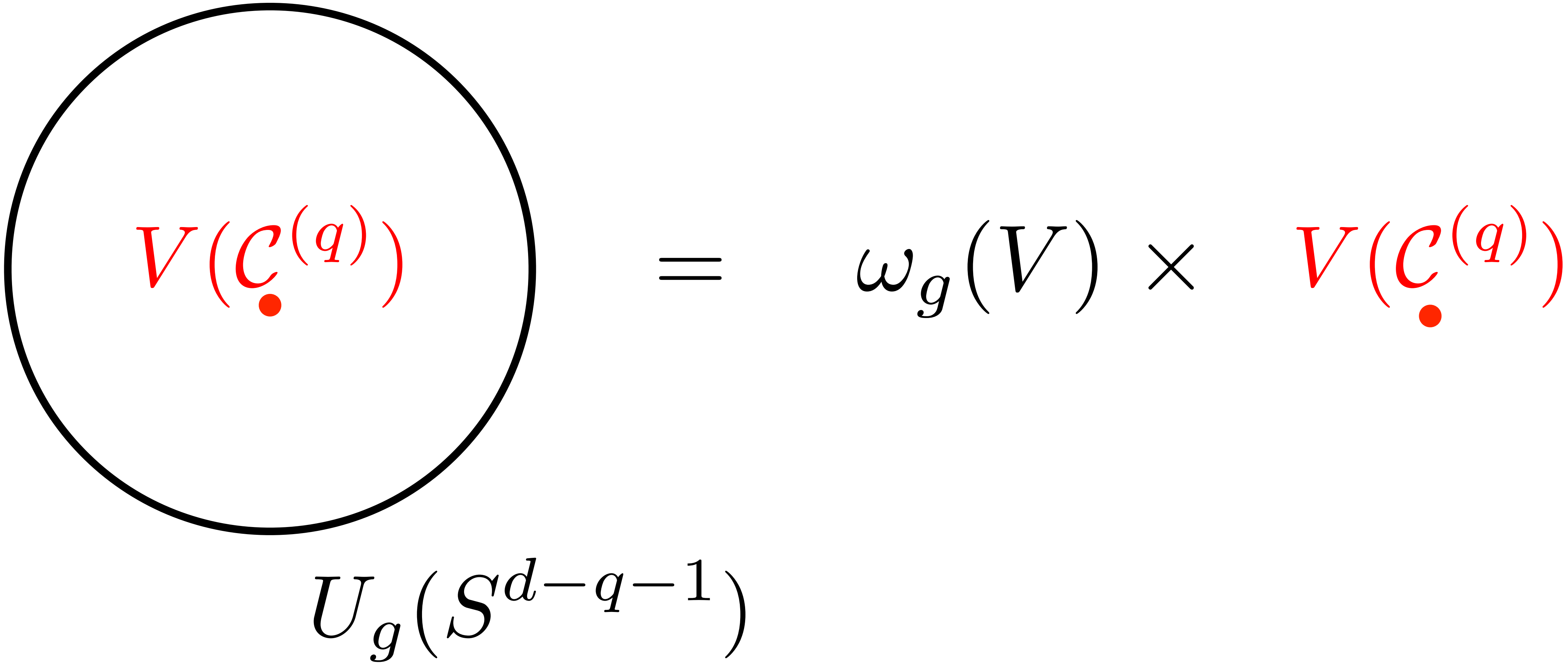
Ward Identities

- Pictorially,



Ward Identities

- Pictorially,



$$V(\mathcal{C}^{(q)}) = \omega_g(V) \times V(\mathcal{C}^{(q)})$$

$U_g(S^{d-q-1})$

Example: U(1) Gauge Theory

$$\mathcal{L} = -\frac{1}{2} F \wedge *F$$

$$G = U(1)_e^{(1)}$$

Noether current:

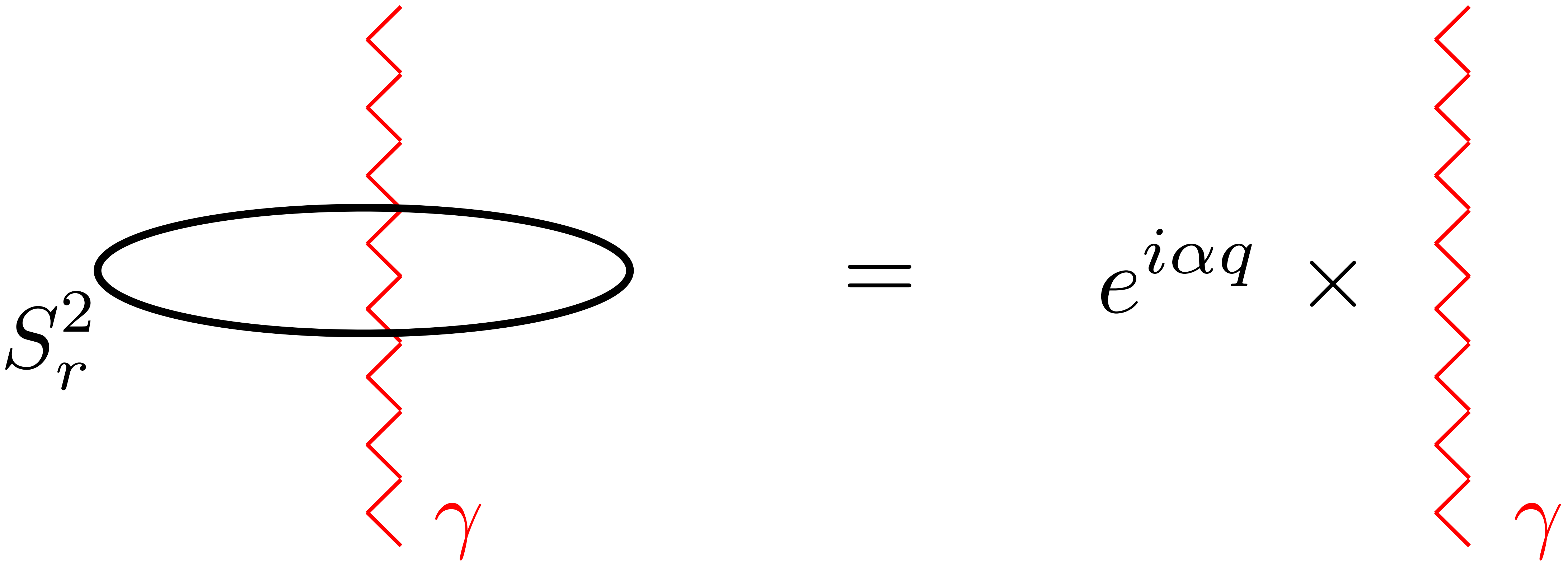
$$J_e^{(2)} = \frac{2}{g}(*F)$$

Charged operators:

$$V_e(\gamma, q) = e^{igq \oint_\gamma A}$$

Example: U(1) Gauge Theory

Ward identity:



$$e^{\frac{2i\alpha}{g}\int_{S_r^2} *F} e^{igq\oint_\gamma A} = e^{i\alpha q} e^{igq\oint_\gamma A}$$

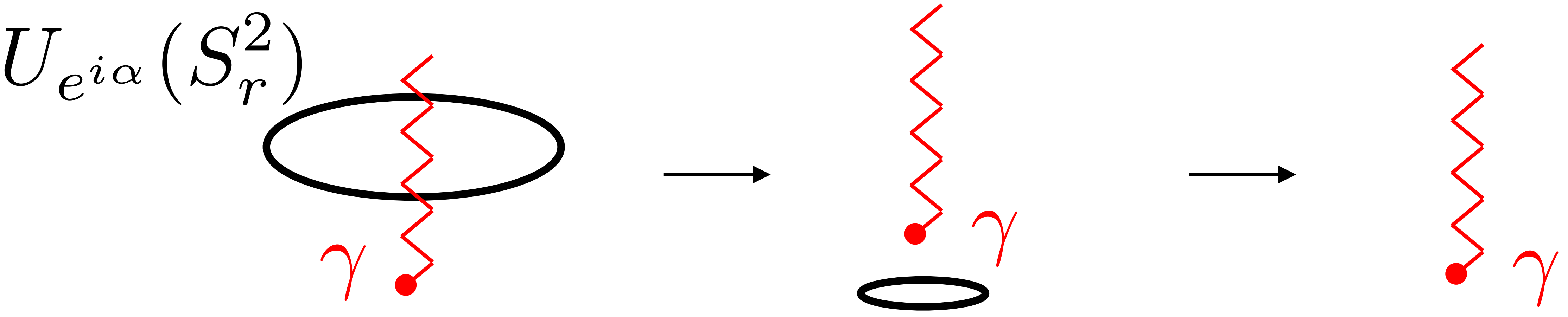
Symmetry Breaking

$$\mathcal{L} = -\frac{1}{2} F \wedge *F + \frac{1}{2} \bar{\psi} \not{D} \psi - m \bar{\psi} \psi$$

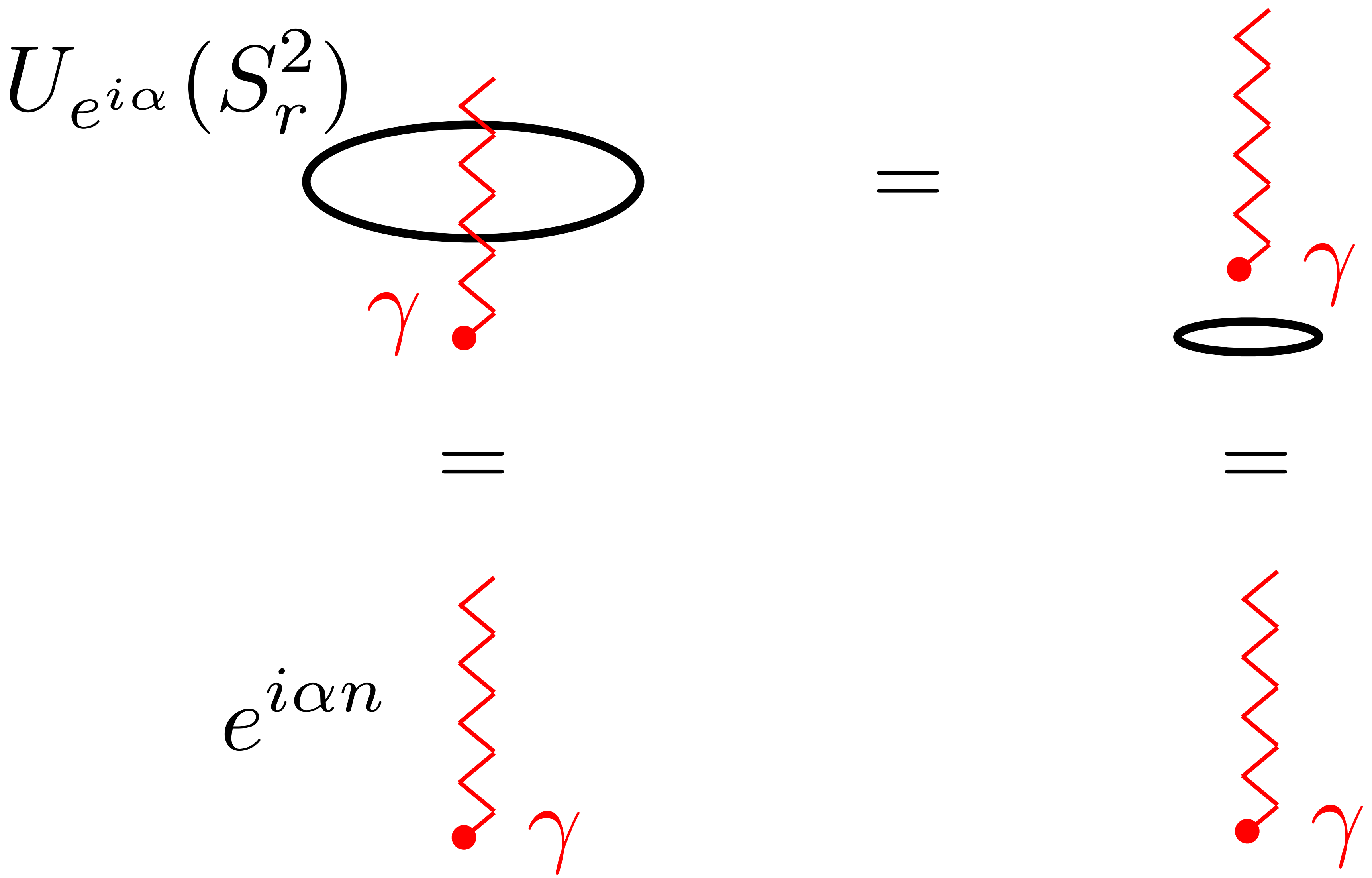
$$D_\mu = \partial_\mu + ignA_\mu$$

$$J_e^{(2)} = \frac{2}{g}(*F) \qquad d(*F) = -gn\bar{\psi}\gamma^\mu\psi$$

Charge n Wilson line can end:



Symmetry Breaking



$$\Rightarrow \; e^{i\alpha n} = 1 \; \Rightarrow \; \alpha = \frac{2\pi k}{n}$$

$$\Rightarrow \quad U(1)^{(1)}_e \to \mathbb{Z}^{(1)}_n$$

Non-Invertible Symmetries and the Completeness Hypothesis

Completeness Hypothesis

In G gauge theory coupled to quantum gravity, there must exist states in every representation of G.

It is often noted that this is related to the absence of global symmetries.

Polchinski ’04

Example: U(1) Gauge Theory

- Pure U(1) gauge theory $\Rightarrow$ U(1) electric 1-form global symmetry
- States of charge n $\Rightarrow \mathbb{Z}_n$ electric 1-form global symmetry
- States of every charge $\Rightarrow$ no electric 1-form global symmetry

Completeness ⇔ No electric 1-form global symmetry

Counterexample: S_3 Gauge Theory

- Pure S_3 gauge theory ⇒ no 1-form global symmetry

Harlow, Ooguri ’18

- But…there do exist more general topological codimension-2 operators

Invertible symmetry: $U_g \times U_{g'} = U_{gg'}$

Non-invertible symmetry: $T_a \times T_b = \sum_c T_c$

- Completeness of S_3 gauge theory ⇔ no non-invertible 1-form symmetry

TR, Shao, ‘20

General Statement

- Complete spectrum $\Leftrightarrow$ no 1-form invertible electric global symmetry for G compact, connected Lie group or finite abelian group
- Complete spectrum $\Leftrightarrow$ no 1-form (possibly non-invertible) electric global symmetry for G compact Lie group
- So, the completeness hypothesis is equivalent to the absence of (possibly non-invertible) 1-form electric global symmetries

TR, Shao, ’20
Heidenreich, McNamara, Montero, Reece, TR, Valenzuela, ‘21

Approximate Global Symmetries and the Weak Gravity Conjecture

Weak Gravity Conjecture (WGC)

In any d-dimensional U(1) gauge theory coupled to quantum gravity, there must exist a “superextremal” particle of charge *q*, mass *m*, with

$$\frac{q}{m} \geq \left.\frac{Q}{M}\right|_{\text{ext}} \gtrsim \frac{1}{M_{\mathrm{Pl};d}^{\frac{d-2}{2}}}$$

$$\left(q = \underset{\text{coupling constant}}{g}\,\overset{\text{charge quantum}}{n}\right)$$

Arkani-Hamed, Motl, Nicolis, Vafa, ’06

Symmetry Breaking and the WGC

$$\mathcal{L} = -\frac{1}{2} F \wedge *F + \frac{1}{2} \bar{\psi} \not{D} \psi - m \bar{\psi} \psi$$

$$D_\mu = \partial_\mu + igA_\mu$$

$$J_e^{(2)} = \frac{2}{g}(*F) \qquad d(*F) = -g\bar{\psi}\gamma^\mu\psi$$

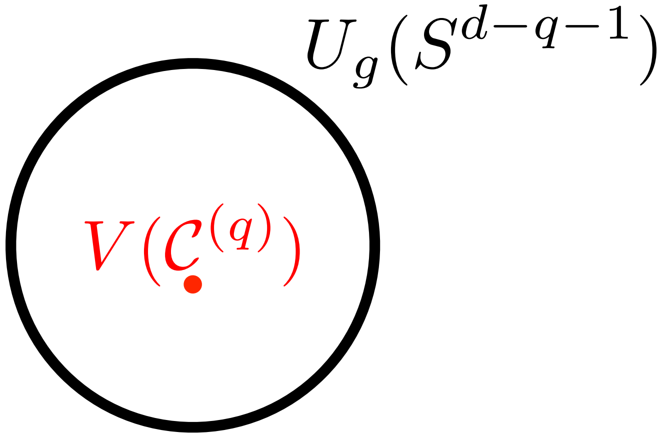
$$g \to 0, \quad m \to \infty \;\Rightarrow\; d(*F) = 0$$

$\Rightarrow$ 1-form symmetry restored

Symmetry Breaking and the WGC

- To avoid approximate 1-form symmetry at the Planck scale, cannot have g too small or $m/M_{\rm Pl}$ too large
- Can make this intuition more precise by studying the modification of the Ward identity due to charged particles, deriving the WGC bound

$$\left\langle \frac{q}{m} \right\rangle \gtrsim \frac{1}{M_{\rm Pl}}$$



$$\approx \omega_g(V) \times V(\mathcal{C}^{(q)})$$

i.e., the average particle is superextremal

- Points towards stronger versions of the WGC

Heidenreich, Reece, TR ’15,’16, Andriolo, Junghans, Noumi, Shiu '18

Córdova, Ohmori, TR, ‘22

Higher-Group Global Symmetries and Weak Gravity Conjecture Mixing

Higher-Group Symmetries

- Suppose we have a collection of p_k-form U(1) global symmetries, k = 1, …, n
- Each may be coupled to a background (p_k+1)-form gauge field $A^{(k)}_{p_k+1}$
- Gauge transformations may be "mixed up":

$$A^{(k)}_{p_k+1} \to A^{(k)}_{p_k+1} + d\Lambda^{(k)}_{p_k} + \sum_i \alpha_{p_k+1-p_i}(A^{(j)}_{p_j})\Lambda^{(i)}_{p_i} + \text{ Schwinger terms}$$

- This is the hallmark of a higher-group symmetry

Example: Axion Electrodynamics

$$S = \int_X \Big[-\frac{1}{2g^2} F \wedge \star F - \frac{1}{2} f_\theta^2 d\theta \wedge \star d\theta + \frac{K}{8\pi^2} \theta F \wedge F \Big]$$

Couple to background fields:

$$S = \int_X \Big[-\frac{1}{2g^2}(F_2 - a_2^{(e)}) \wedge \star(F_2 - a_2^{(e)}) - \frac{1}{2} f_\theta^2 d\theta \wedge \star d\theta \\ -\frac{1}{2\pi} \theta \wedge dc_3^{(m)} \Big] + \frac{K}{8\pi^2} \int_Y d\theta \wedge (F_2 - a_2^{(e)}) \wedge (F_2 - a_2^{(e)})$$

Background gauge transformations:

$$a_2^{(e)} \to a_2^{(e)} + d\lambda_1^{(e)}$$

$$c_3^{(m)} \to c_3^{(m)} + d\lambda_2^{(m)} - \frac{K}{2\pi} a_2^{(e)} \wedge \lambda_1^{(e)} - \frac{K}{4\pi} \lambda_1^{(e)} \wedge d\lambda_1^{(e)}$$

Example: Axion Electrodynamics

Gauge invariant field strengths:

$$f_3^{(e)} = da_2^{(e)}$$

$$g_4^{(m)} = dc_3^{(m)} + \frac{K}{4\pi} a_2^{(e)} \wedge a_2^{(e)}$$

Turning on background for $a_2^{(e)}$ leads to background for $g_4^{(m)}$. This implies a relation on the emergence scales of the electric 1-form symmetry for A_1 and the magnetic 2-form symmetry for θ:

$$\Lambda_A^e \lesssim \Lambda_\theta^m$$

Brennan, Córdova ‘20

Higher-Group Scale Relations

- Have from consistency of the higher-group structure:

$$\Lambda_A^e \lesssim \Lambda_\theta^m$$

- 1-form electric symmetry of A is broken by charged particles:

$$m \lesssim \Lambda_A^e$$

- 2-form magnetic symmetry of θ is broken at or below the charged string scale:

$$\Lambda_\theta^m \lesssim M_s \equiv \sqrt{2\pi T}$$

$$\Rightarrow m \lesssim M_s$$

Brennan, Córdova ’20

WGC Mixing in 4d

$$S = \int_X \Big[- \frac{1}{2g^2} F \wedge \star F - \frac{1}{2} f_\theta^2 d\theta \wedge \star d\theta + \frac{K}{8\pi^2} \theta F \wedge F \Big]$$

Axion WGC: $f_\theta S_{\mathrm{inst}} \lesssim M_{\mathrm{Pl}}$

WGC for strings: $T \lesssim f_\theta M_{\mathrm{Pl}}$

Further assume: $S_{\mathrm{inst}} \sim 8\pi^2 / g^2$ Fan, Fraser, Reece, Stout '21

$$\Rightarrow T \lesssim \frac{g^2}{8\pi^2} M_{\mathrm{Pl}}^2 \Rightarrow M_s \sim \sqrt{2\pi T} \lesssim g M_{\mathrm{Pl}}$$

Higher-group: $m \lesssim M_s \lesssim g M_{\mathrm{Pl}} \Rightarrow$ WGC for A_1!

Heidenreich, Reece, TR '21, Kaya, TR '22

Chern-Weil Global Symmetries and Dissolved Branes

Chern-Weil Global Symmetries

- Gauge theories have conserved currents of the form

$$J = F \wedge F \qquad\qquad J = H \wedge F$$

$$J = \mathrm{Tr}(F^k) \, := \mathrm{Tr}(\underbrace{F \wedge F ... \wedge F}_{k})$$

- Their conservation follows from Bianchi identities, e.g. $dF = 0\,, \quad dH = 0\,, \quad dF + [A, F] = 0$
- In quantum gravity, we expect these symmetries must be either gauged or broken

Heidenreich, McNamara, Montero, Reece, TR, Valenzuela, ‘20

CW Symmetries in Type IIA SUGRA

- Consider CW current $G_4 \wedge H_3$, where $G_4 = dC_3 + ...$ is the gauge-invariant 4-form field strength
- Naively gauged via CS coupling $C_3 \wedge G_4 \wedge H_3$
- But, broken by D4-branes, $d(G_4 \wedge H_3) = \delta_5^{\mathrm{D4}} \wedge H_3 \neq 0$
- Resolution: additional D4 worldvolume d.o.f.:

$$S_{\mathrm{CS}} = \int_{\mathrm{D4}} (C_5 - C_3 \wedge \mathcal{F}_{\mathrm{D}p} + C_1 \wedge \mathcal{F}_{\mathrm{D}p} \wedge \mathcal{F}_{\mathrm{D}p})$$

- Gauged 7-form current instead given by

$$G_4 \wedge H_3 + \mathcal{F}_{\mathrm{D}p} \wedge \delta_5^{\mathrm{D4}}$$

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Dissolved Branes

- More generally, consistent gauging and breaking of CW symmetries implies:
  - the existence of certain extended objects
  - the existence of worldvolume degrees of freedom
  - the ability for certain bulk degrees of freedom to dissolve/ end on others
- In theories with higher-form abelian gauge fields, such as Type II supergravity, these consistency conditions lead to a cascade of dissolved charges, reproducing much of the known structure of D-brane interactions in Type II string theory

Heidenreich, McNamara, Montero, Reece, TR, Valenzuela, ‘20

Conclusions

The Big Picture

- Our understanding of quantum field theory has been revolutionized in recent years by the discovery and development of generalized notions of global symmetries
- Meanwhile, recent years have seen renewed interest in identifying universal features of quantum gravity
- In this talk, I have argued that these two areas of research are closely related:
  - Much of our understanding of quantum gravity can be viewed as a consequence of the absence of global symmetries
  - Generalized global symmetries hold power not only for QFT, but also for quantum gravity
  - For more, see Miguel Montero’s discussion session on “Strings and the Real World”

Thank You!