Line Operators in Chern-Simons-Matter Theories and Bosonization in Three Dimensions

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short [2204.05262]

perturbative [22xx.xxxxx]

bootstrap [22xx.xxxxx]

Motivation

Chern-Simons theory posses a well established **non-perturbative** level-rank duality

Ones coupled to matter in the fundamental representation, this duality is expected to extend to a duality between **bosons** and **fermions**

extensive evidence, especially at large N

- correlation function of local operators
- spectrum of monopole/baryon operators
- thermal free-energies
- S-matrices
- relating non-susy dualities to well-established susy ones

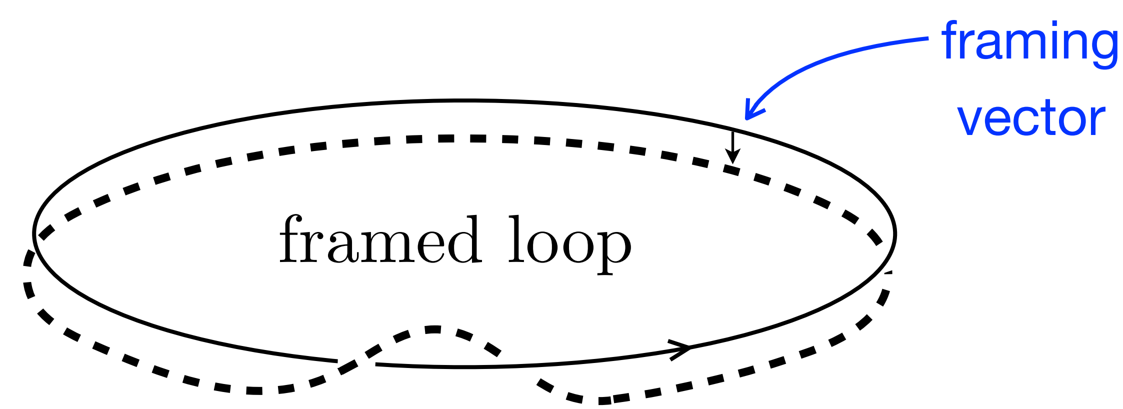
[Witten, Minwalla, Prakash, Trivedi, Wadia, Yin, Aharony, Gur-Ari, Yacoby, Maldacena, Zhiboedov, Giombi, Gaiotto, Kapustin, Hsin, Seiberg, Naculich, Schnitzer, Mlawer, Naculich, Riggs, Schnitzer, Camperi, Levstein, Zemba, Bedhotiya, Prakash, Gurucharan, Kirilin, Prakash, Skvortsov, Radivcevic, Jain, Yokoyama, Sharma, Takimi, Mandlik, Inbasekar, Mazumdar, Giveon, Kutasov, Benini, Closset, Cremonesi, …]

Motivation

The first evidence comes from the study of Chern-Simons itself [Witten 89]

self linking number

$$\left\langle W^{\mathfrak{f}}_{\text{unknot}} \right\rangle = e^{i\pi\lambda\mathfrak{f}} \times k\frac{\sin(\pi\lambda)}{\pi}$$

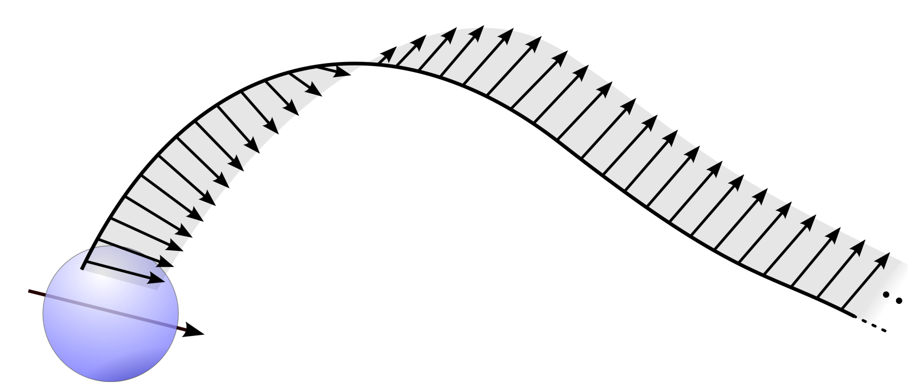


$SU(N)$ or $U(N)$ gauge group at level k

Only consider the $N \to \infty$ limit with $\lambda \equiv \frac{N}{k} \in [-1, 1]$ fixed

$\Longrightarrow$ Expect that the same dependance on the framing vector would lead to fractional statistics, ranging between boson (fermion) at $\lambda = 0$ to fermion (boson) at $|\lambda| = 1$

(Will be shortly proven)



level-rank $(k, \lambda) \quad \leftrightarrow \quad (-k, \lambda - \text{sign}(k))$

Plan

Consider CFTs in 3d

= fixed points of CS theory + fundamental bosons/fermions

$$S_E^{\rm bos} = S_{CS} + \int d^3x \, (D_\mu \phi)^\dagger \cdot D^\mu \phi + \frac{\lambda_6}{N^2}(\phi \cdot \phi^\dagger)^3$$ "quasi-boson"

$$S_E^{\rm fer} = S_{CS} + \int d^3x \, \bar{\psi} \cdot \gamma^\mu D_\mu \psi$$ "quasi-fermion"

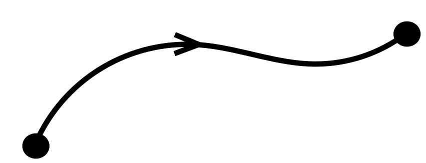
$$S_{CS} = \frac{ik}{4\pi} \int d^3x \, \epsilon^{\mu\nu\rho} \, {\rm tr}(A_\mu \partial_\nu A_\rho - \tfrac{2i}{3} A_\mu A_\nu A_\rho)$$

or their double trace deformation with respect to $\left(J^{(0)}\right)^2$

scalar current - $J^{(0)}_{\rm bos} = \phi^\dagger \cdot \phi, \quad J^{(0)}_{\rm fer} = \bar{\psi} \cdot \psi$

Study the most fundamental operators in CS-matter theory

"mesonic line operator"



= line operators along an arbitrary smooth path between a fundamental and an anti-fundamental boundary operators

Plan

Consider CFTs in 3d

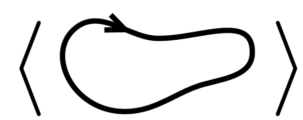
= fixed points of CS theory + fundamental bosons/fermions

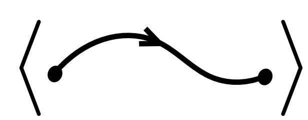
$$S_E^{\text{bos}} = S_{CS} + \int d^3x\,(D_\mu\phi)^\dagger \cdot D^\mu\phi + \frac{\lambda_6}{N^2}(\phi\cdot\phi^\dagger)^3$$ “quasi-boson”

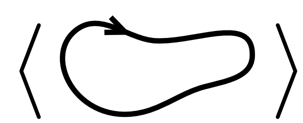
$$S_E^{\text{fer}} = S_{CS} + \int d^3x\,\bar\psi\cdot\gamma^\mu D_\mu\psi$$ “quasi-fermion”

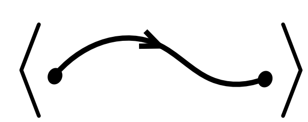
$$S_{CS} = \frac{ik}{4\pi}\int d^3x\,\epsilon^{\mu\nu\rho}\,\mathrm{tr}(A_\mu\partial_\nu A_\rho - \tfrac{2i}{3}A_\mu A_\nu A_\rho)$$

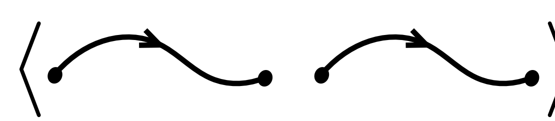
<u>At large N</u>

The matter contributes at $\mathcal{O}(1/N)$ to 

$\mathcal{O}(1)$ to 

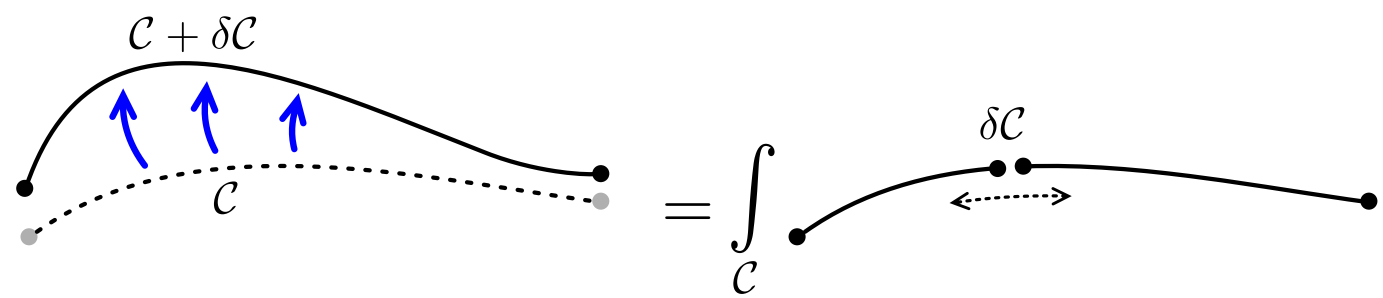
The double trace deformation contributes at $\mathcal{O}(1/N^2)$ to 

$\mathcal{O}(1/N)$ to 

$\mathcal{O}(1)$ to  connected

Results

- Classify the conformal line operators.
- Classify the operators on the lines and the ones the lines can and on at finite λ.
- The line operators satisfy a first order “evolution equation”



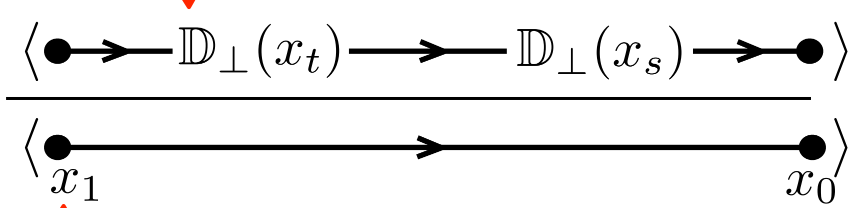
- Line bootstrap — [spectrum of boundary operators] + [evolution equation] uniquely determine the expectation values of the mesonic line operators, as well as the 1/N correction to closed loops expectation values.

- The conformal line operators of the bosonic and fermionic theories satisfy the same evolution equation and that their spectra of boundary operators are related to each other through the level-rank duality map $(k,\lambda) \leftrightarrow (-k, \lambda - \mathrm{sign}(k)) \;\Rightarrow\; \lambda_f = \lambda_b - \mathrm{sign}(k_b)$

Results

Demonstrate this by bootstrapping and computing explicitly the two-point function of the displacement operator

displacement operator



$$= \frac{\Lambda(\Delta)}{x_{st}^4} \left(\frac{x_{10} x_{st}}{x_{1s} x_{t0}} \right)^{2\Delta}$$

boundary operators of minimal dimension Δ and opposite transverse spin

$\Lambda(\Delta)$ is also the two-point function of the displacement operator on a circular loop.

$$\Lambda(\Delta) = -\frac{(2\Delta-1)(2\Delta-2)(2\Delta-3)\sin(2\pi\Delta)}{2\pi}$$

The dependance of Δ on λ depends on the line operator and whether we use λ_b or $\lambda_f = \lambda_b - \text{sign}(k_b)$.

Plan

Bosonic theory $\longrightarrow$ Fermionic theory $\longrightarrow$ Bootstrap

Without loss of generality, we assume $\mathrm{sign}(k_b) > 0 \quad \Rightarrow \quad \begin{array}{l} \lambda_b \in [0,1] \\ \lambda_f \in [-1,0] \end{array}$

(parity $k \leftrightarrow -k\,, \quad \lambda \leftrightarrow -\lambda$.)

Warning — no detailed derivations

Line Operators in the Bosonic Theory

A Wilson line has a non-zero beta function on the line for the adjoint bi-scalar $\phi\phi^\dagger$

At the fix points we find the line operators

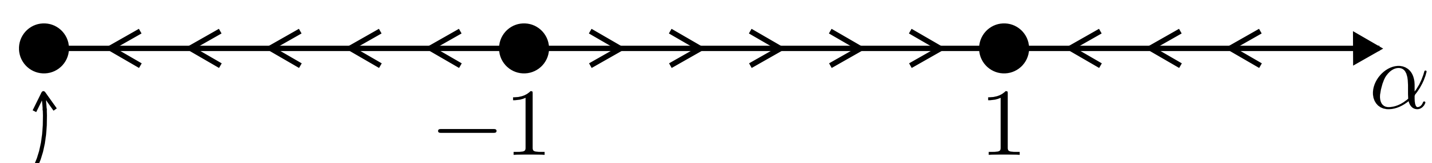
$\alpha = \pm 1$

$$\mathcal{W}^\alpha[\mathcal{C}, n] \equiv \left[\mathcal{P} e^{\,i\int_{\mathcal{C}}\left(A\cdot dx + i\alpha \frac{2\pi\lambda}{N}\phi\phi^\dagger |dx|\right)}\right]_n$$

path

framing vector

RG flow -



(almost) trivial line

There are also other conformal line operators — operators with d.o.f. on the line and non-unitary ones, that we will not describe in this talk.

Mesonic Line Operators

anti-fundamental boundary operator

fundamental boundary operator

Mesonic line operator $M = \mathcal{O}_L \xrightarrow{\mathcal{W}} \mathcal{O}_R$

Adjoint operators on the line $\mathcal{O}_{\text{inner}} = \mathcal{O}_R \times \mathcal{O}_L$

$\mathcal{O}_{R/L}$ are uniquely classified by
- Δ - conformal dimension
- $\mathfrak{s}$ - transverse spin

At tree level
line in the x^3- direction

$$\mathcal{O}^{(n,s)}_{R,\,\text{tree}} = \frac{1}{\sqrt{N}} \times \begin{cases} \partial^n_{x^3_R}\,\partial^s_{x^+_R}\phi & s \geq 1 \\ \partial^n_{x^3_R}\,\partial^{-s}_{x^-_R}\phi & s \leq 0 \end{cases}$$

tree level spin

$$\Delta^{(n,s)}_{\text{tree}} = 1/2 + n + |s|$$

$ds^2 = 2dx^+dx^- + (dx^3)^2$

An infinite straight line preserve an $SL(2,\mathbb{R}) \times U(1)$ conformal symmetry

The operators with $n = 0$ are $SL(2,\mathbb{R})$ primaries

Mesonic $\alpha = 1$ Line Operators

tree level spin

Spectrum -

$$\mathfrak{s}_L = s_L + \lambda/2$$
$$\mathfrak{s}_R = s_R - \lambda/2$$

$$\Delta^{(n,s)} = 1/2 + n + |\mathfrak{s}|$$

exact transverse spin

Operators with the same anomalous dimension are related by path derivatives (keeping framing perpendicular and fixed)

i.e. $$\mathcal{O}_L^{(0,s+1)} = \delta_+ \mathcal{O}_L^{(0,s)}\,, \qquad \mathcal{O}^{(n+1,s)} = \delta_3 \mathcal{O}_L^{(n,s)}$$

At the bottom of these four towers we have the operators

$$\{\mathcal{O}_L^{(0,0)}, \mathcal{O}_L^{(0,-1)}\} \quad \text{and} \quad \{\mathcal{O}_R^{(0,0)}, \mathcal{O}_R^{(0,1)}\}$$

opposite anomalous dimension

Mesonic $\alpha = 1$ Line Operators

tree level spin

Spectrum -

$$\mathfrak{s}_L = s_L + \lambda/2$$
$$\mathfrak{s}_R = s_R - \lambda/2$$

exact transverse spin

$$\Delta^{(n,s)} = 1/2 + n + |\mathfrak{s}|$$

This spectrum was derived by an explicate all loop computation

The relation between the dimension and spin can be shown to follow from supersymmetry

Mesonic $\alpha = 1$ Line Operators

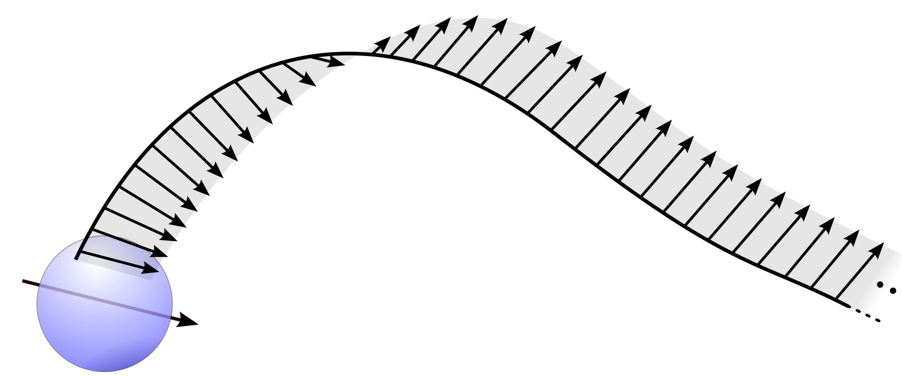
tree level spin

Spectrum - $$\mathfrak{s}_L = s_L + \lambda/2$$
$$\mathfrak{s}_R = s_R - \lambda/2$$

exact transverse spin

$$\Delta^{(n,s)} = 1/2 + n + |\mathfrak{s}|$$

This confirms the expectation that the dependance on the framing vector leads to fractional statistics, ranging between boson at $\lambda = 0$ to fermion at $\lambda = 1$



The operator **on the line** with minimal dimension is the bi-scalar adjoint

$\mathcal{O}_R^{(0,0)} \times \mathcal{O}_L^{(0,0)}$ with $\Delta_{\min} = \Delta_L^{(0,0)} + \Delta_R^{(0,0)} = 1 + \lambda > 1$

$\Rightarrow$ The $\alpha = 1$ line is a stable fix-point

The evolution equation ($\alpha = 1$)

Expectation values take the form

transverse spin

conformal invariant functional of the path

$$\left\langle \mathcal{O}_L^{(0,0)} \mathcal{W} \mathcal{O}_R^{(0,0)} \right\rangle = \frac{\left(n_L^+ n_R^-\right)^{\lambda/2}}{|x_L - x_R|^{1+\lambda}} \times F^{(0,0)}[x(\cdot)]$$

conformal dimension

Under a smooth variation of the path $x(\cdot) \mapsto x(\cdot) + v(\cdot)$

$$\delta \mathcal{W} = \int ds\, |\dot{x}(s)|\, v^\mu(s)\, \mathcal{P}\left[\mathbb{D}_\mu(s) \mathcal{W}\right]$$

$\Delta_R^{(0,0)} = 1/2 + \lambda/2$ $\quad$ $\Delta_L^{(0,-1)} = 3/2 - \lambda/2$

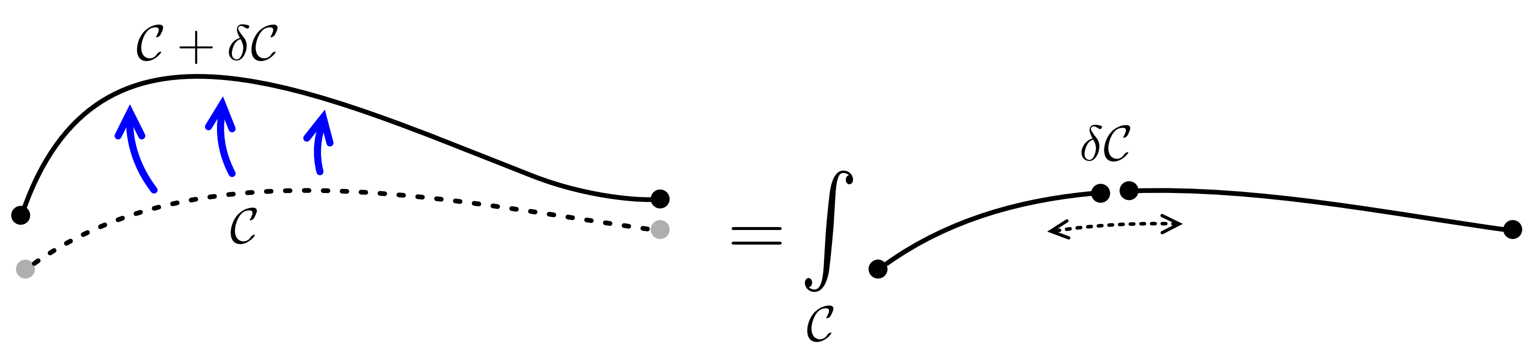
Displacement operator

$$\mathbb{D}_- = -4\pi\lambda\, \mathcal{O}_R^{(0,0)}\, \mathcal{O}_L^{(0,-1)} \qquad \mathbb{D}_+ = -4\pi\lambda\, \mathcal{O}_R^{(0,1)}\, \mathcal{O}_L^{(0,0)}$$

$\mathfrak{s}_R = -\lambda/2$ $\quad$ $\mathfrak{s}_L = -1 + \lambda/2$ $\quad$ normalization dependant

$$\Rightarrow \qquad \Delta(\mathbb{D}_\pm) = 2\,, \quad \mathfrak{s}(\mathbb{D}_\pm) = \pm 1$$

First order chiral Schwinger-Dyson equation



Boundary equation ($\alpha = 1$)

The boundary operators also satisfy an operator equation.

It relates $SL(2,\mathbb{R})$ primaries from the same tower as

part of a smooth deformation of the line

$$\delta_- \mathcal{O}_L^{(0,s+1)} = \bar{\beta}\, \mathcal{O}_L^{(2,s)} \,, \qquad s \geq 0$$

$$\delta_+ \mathcal{O}_L^{(0,-s-1)} = \beta\, \mathcal{O}_L^{(2,-s)} \,, \qquad s \geq 1$$

unique operator with the same quantum numbers

Bootstrap or explicate computation $\quad \Rightarrow \quad \beta = \bar{\beta} = -\dfrac{1}{2}$

Mesonic $\alpha = -1$ Line Operators

Spectrum - the same with only the anomalous dimension of $\widetilde{\mathcal{O}}_R^{(n,0)}$ and $\widetilde{\mathcal{O}}_L^{(n,0)}$ are flipped

$\Rightarrow$ four towers with the bottom operators

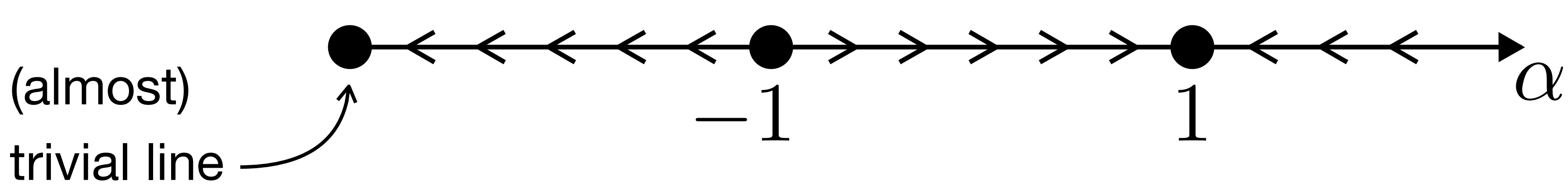
$$\Delta_L^{(0,0)} = \Delta_R^{(0,0)} = \frac{1-\lambda}{2}$$

$$\{\widetilde{\mathcal{O}}_L^{(0,0)}, \widetilde{\mathcal{O}}_L^{(0,1)}\} \quad \text{and} \quad \{\widetilde{\mathcal{O}}_R^{(0,0)}, \widetilde{\mathcal{O}}_R^{(0,-1)}\}$$

$$\Delta_L^{(0,1)} = \Delta_R^{(0,-1)} = \frac{3+\lambda}{2}$$

The operator on the line with minimal dimension is the bi-scalar adjoint

$\widetilde{\mathcal{O}}_R^{(0,0)} \times \widetilde{\mathcal{O}}_L^{(0,0)}$ with $\widetilde{\Delta}_{\text{min}} = \widetilde{\Delta}_L^{(0,0)} + \widetilde{\Delta}_R^{(0,0)} = 1 - \lambda < 1$



Plan

Bosonic theory $\longrightarrow$ Fermionic theory $\longrightarrow$ Bootstrap

Mesonic Line Operators in the fermonic theory

A Wilson line is a good conformal operator $W[x(\cdot)] = \mathcal{P}e^{i\int A_\mu dx^\mu}$

We have repeated the same steps as in the bosonic theory.
We find a perfect match with the $\alpha = 1$ mesonic line operators for both, the **spectrum** and the form of the **evolution equation**.

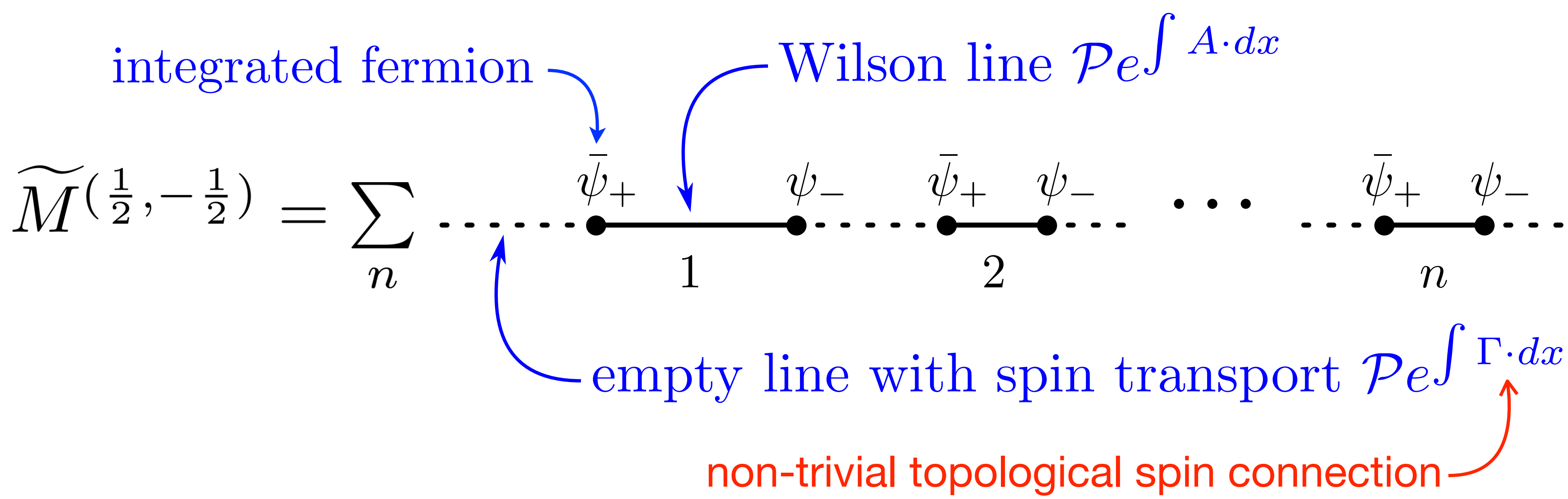
Fermionic	Tree	Bosonic	Tree	$\mathfrak{s}$	Δ
$\mathcal{O}_R^{(0,-\frac{1}{2})}$	ψ_-	$\mathcal{O}_R^{(0,0)}$	ϕ	$-\frac{\lambda_b}{2}$	$\frac{1+\lambda_b}{2}$
$\mathcal{O}_R^{(0,\frac{1}{2})}$	ψ_+	$\mathcal{O}_R^{(0,1)}$	$\partial_+\phi$	$\frac{2-\lambda_b}{2}$	$\frac{3-\lambda_b}{2}$
$\mathcal{O}_L^{(0,\frac{1}{2})}$	$\bar{\psi}_+$	$\mathcal{O}_L^{(0,0)}$	$\phi^\dagger$	$\frac{\lambda_b}{2}$	$\frac{1+\lambda_b}{2}$
$\mathcal{O}_L^{(0,-\frac{1}{2})}$	$\bar{\psi}_-$	$\mathcal{O}_L^{(0,-1)}$	$\partial_-\phi^\dagger$	$\frac{\lambda_b-2}{2}$	$\frac{3-\lambda_b}{2}$

$\lambda_f = \lambda_b - 1$

Mesonic Line Operators in the fermonic theory

We have also found the fermionic dual of the $\alpha = -1$ operators

It has fundamental fermion condensed in the exponent



Fermionic	Tree	Bosonic	Tree	$\mathfrak{s}$	Δ
$\widetilde{\mathcal{O}}_R^{(0,-\frac{1}{2})}$	$\mathbf{1}$	$\widetilde{\mathcal{O}}_R^{(0,0)}$	ϕ	$-\frac{\lambda_b}{2}$	$\frac{1-\lambda_b}{2}$
$\widetilde{\mathcal{O}}_R^{(0,-\frac{3}{2})}$	$\partial_-\psi_-$	$\widetilde{\mathcal{O}}_R^{(0,-1)}$	$\partial_-\phi$	$-\frac{2+\lambda_b}{2}$	$\frac{3+\lambda_b}{2}$
$\widetilde{\mathcal{O}}_L^{(0,\frac{1}{2})}$	$\mathbf{1}$	$\widetilde{\mathcal{O}}_L^{(0,0)}$	$\phi^\dagger$	$\frac{\lambda_b}{2}$	$\frac{1-\lambda_b}{2}$
$\widetilde{\mathcal{O}}_L^{(0,\frac{3}{2})}$	$\partial_+\bar{\psi}_+$	$\widetilde{\mathcal{O}}_L^{(0,1)}$	$\partial_+\phi^\dagger$	$\frac{2+\lambda_b}{2}$	$\frac{3+\lambda_b}{2}$

$\lambda_f = \lambda_b - 1$

Bootstrap

Main point — [spectrum of boundary operators] + [evolution equation] uniquely determine the expectation values of the mesonic line operators

Strategy —

- Expand in the deformation away from a straight line. It is a sort of conformal perturbation theory on the line.
- At any order in the deformation, include all relevant and marginal bulk and boundary operators.
- Pick a regularization scheme and fix all coefficients by demanding
  - Conformal symmetry with the appropriate spectrum of boundary operators.
  - At linear order around any path we have the evolution equation.

We find that these conditions are sufficient to systematically fix all the coefficients.

Bootstrap

For example, at second order we have the two-point function of the displacement operator

$$\frac{\langle \bullet \!\rightarrow\! \mathbb{D}_\perp(x_t) \!\rightarrow\! \mathbb{D}_\perp(x_s) \!\rightarrow\! \bullet \rangle}{\langle \underset{x_1}{\bullet} \!\longrightarrow\! \underset{x_0}{\bullet} \rangle} = \frac{\Lambda(\Delta)}{x_{st}^4}\left(\frac{x_{10}x_{st}}{x_{1s}x_{t0}}\right)^{2\Delta}$$

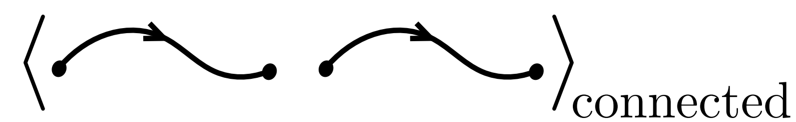
with $$\Lambda(\Delta) = \Lambda(2-\Delta) = -\frac{(2\Delta-1)(2\Delta-2)(2\Delta-3)\sin(2\pi\Delta)}{2\pi}$$

Here, Δ is the dimension of any of the four bottom operators.
For example, for the $\alpha = 1$ line operator it is $\Delta = (1+\lambda_b)/2$

or $\Delta = (3-\lambda_b)/2$

Future directions

- To complete the derivation of the duality at the planar level, one should also match the connected piece of the correlation functions between mesonic line operators, as well as the local single trace operators.



We expect that known [spectrum of single trace local operators]
+[spectrum of boundary operators]
+[evolution equation]

would be sufficient.

- Find an explicit solution for the expectation values.

- Derive the holographic dual — parity breaking versions of Vasiliev's higher-spin theory (is our original motivation)

Thank you