AdS/CFT @ loop order

Kostas Skenderis

STAG RESEARCH CENTRE
University of Southampton

Strings 2022
Vienna, Austria
21 July 2022

Outline

Outline

AdS/CFT @ tree-level

- The rules for AdS/CFT computations at tree-level have been understood since the early days of the AdS/CFT correspondence [GKP,W (1998)]:
  - One needs to compute the on-shell value of the action as a function of the fields parametrising the boundary conditions of bulk fields.
  - The on-shell action diverges due to the infinite volume of spacetime, but remarkably these IR divergences are local.
  - This allows to set up holographic renormalization [Henningson, KS (1998)] [de Haro, Solodukhin, KS (2000)]
  - This leads to well-defined rules to obtain renormalised CFT correlators.

Bulk=local QFT

- A trademark property of local QFT is that its UV divergences are local.
- UV/IR connection: UV properties of the boundary CFT are linked with IR properties of the bulk.
- Similarly, Ward identities and anomalies can be established using IR properties of the bulk only and agree exactly with what is expected from a local QFT.

- This provides structural evidence for AdS/CFT, independent of the details associated with any particular example:

  the bulk theory has the analytic structure of a d-dimensional local QFT.

AdS/CFT @ loop-level

- The purpose of this work is generalise these results to general loop-order.
- Many interesting results have been obtained in recent years regarding loop-corrections in AdS/CFT. [Penedones, Fitzpatrick, Kaplan, Aharony, Alday, Bissi, Aprile, Drummond, Heslop, Giombi, Sleight, Taronna, Yuan, Bertan, Sachs, Skvortsov, Ghosh, Ponomarev, Meltzer, Perlmutter, Sivaramakrishnan, Fichet, Carmi, ...]
- Our aim to present a systematic renormalisation procedure that deals with both UV and IR issues in the bulk.
- We will actually see that it is crucial that we deal with both issues.

- This talk is based on work in collaboration with
  Máx Bañados, Ernesto Bianchi, Iván Muñoz
  Bulk renormalisation and the AdS/CFT correspondence, 220x.xxxxx, and on-going

Bulk=String Theory

- Infrared divergences in QFT must cancel on their own.

  We do not add counterterms to cancel infrared divergences in QFT.

- By the UV/IR connection, the bulk should be finite in the UV, not renormalizable.

- It should be a string theory.

Bulk loops

- At low energies, the bulk is given by gravity coupled to matter, which should now be viewed as an effective field theory.
- Loop diagrams have bulk UV divergences and need to be renormalized.
- Loop diagrams also have bulk IR divergences: these need to be treated with holographic renormalisation.

  They will play an important role in what follows.

Outline

AdS/CFT

$$Z_{\text{AdS}}[\varphi_0; p_0^i] = \left\langle \exp\left(- \int_{\partial AdS} \varphi_0 \mathcal{O} \right) \right\rangle_{C_J}$$

- φ_0 denotes collectively boundary conditions/sources
- p_0^i denotes collectively all parameters that appear in the bulk action (masses and coupling constant).
- C_J denotes collectively all CFT data (CFT spectrum and OPE coefficients).

Renormalization

- Introduce UV regulator τ and IR regulator ε
- Introduce Z factors:

$$\varphi_0 = Z_\varphi \varphi, \qquad p_0^i = Z_{p^i} p^i$$

- A bulk theory is renormalised if one can find Z_ϕ, Z_{p^i} and boundary counterterm action $S_{\rm ct}$ s.t.

$$Z_{\rm AdS}^{\rm ren}[\varphi; p^i] = \lim_{\varepsilon \to 0} \lim_{\tau \to 0} Z_{\rm AdS}^{\rm reg}[\varphi_0, p_0^i; \varepsilon, \tau]$$

is finite.

- Renormalized correlators are then obtained by functionally differentiating w.r.t. φ.

- Renormalized correlators have scheme dependence:
- We need renormalisation conditions.

Regularisation

- Infrared regulator ε
  - Using coordinates where the AdS metric is:

$$ds^2 = \frac{dz^2 + d\vec{x}^2}{z^2},$$

  - Radial integrals are cut-off $r \geq \varepsilon$.
- UV regulator τ
  - On the CFT side, IR effects do break any of the conformal symmetries.
  - ➠ There must exist an AdS invariant UV regulator.

AdS invariant point-splitting

- Let x, y two points and $\xi(x,y)$ the chordal distance. Coincident points have $\xi(x,x) = 1$.
- Consider now the geodesic,

$$x_0(\tau) = \frac{x_0}{\cosh\tau}, \qquad \vec{x}(\tau) = \vec{x} + x_0 \tanh\tau \; \hat{n} \,,$$

where $\hat{n}^2 = 1$ and $\hat{n} \cdot (\vec{x} - \vec{y}) = 0$

- A short computation shows that

$$\xi(x(\tau), y) = \frac{\xi(x,y)}{\cosh\tau}$$

and thus $\xi(x(\tau), y)$ is invariant under AdS isometries and $\xi(x(\tau), y) \neq 1$ if $\tau \neq 0$. The use of rescaled ξ as regulator has already appeared in [Bertan, Sachs (2018)].

- Regulated bulk-to-bulk propagator:

$$G(x,y) = G(\xi(x,y)) \to G(x(\tau), y) = G\left(\xi(x,y)/\cosh\tau\right)$$

⇒ Using this propagator suffices to regulate UV infinities.

Conformal invariance and bulk loops

- Conformal invariance fixes 2- and 3-point functions up to constants and higher point functions up to functions of cross-ratios.
- Since we can regulate the bulk UV divergences while respecting AdS isometries, we should be able to establish the same results holographically to all order in bulk loops.
- In a CFT conformal correlators do need renormalization, and the conformal dimensions of operators may also renormalize.

  This renormalisation should be due to IR divergences in the bulk.

Outline

- Φ^4 theory in a fixed AdS background:

$$S[\Phi] = \int d^{d+1}x\sqrt{g}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{m_0^2}{2}\Phi^2 + \frac{\lambda_0}{4!}\Phi^4\right]$$

- In this case: $p_0^i = \{m_0^2, \lambda_0\}$
- We write $\Phi = \phi + h$, where ϕ satisfies the classical field equation with Dirichlet BC $\varphi_0(x)$,

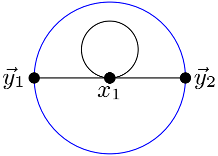
$$\phi = z^{d-\Delta}\varphi_0 + \ldots$$

and we integrate out the quantum fluctuations h perturbatively.
- Regulated theory:

$$Z_{\text{AdS}}^{\text{reg}}\left[\varphi_0; m_0^2, \lambda_0; \varepsilon, \tau\right] = e^{-S_{\text{sub}}[\phi;\varepsilon,\tau]}(\text{path integral over } h)$$

⇒ Tree-level diagrams are computed from $S_{\text{sub}} = S_{\text{reg}}^{\text{on-shell}} + S_{\text{ct}}$.

One-loop correction to 2-point function

$T_1 =$ 

$$= G\left(\frac{1}{\cosh\tau}\right)\int [dx_1] K(x_1,\vec{y}_1) K(x_1,\vec{y}_2)$$

K: bulk-to-boundary propagator, and the regulated bulk-to-bulk propagator at coincident point is

$$G\left(\frac{1}{\cosh\tau}\right) \sim \frac{1}{\tau^{d-1}} + \dots$$

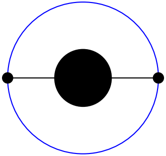
- Change variables in the integral that amounts to conformal transformations in the boundary and
- use the transformation properties K under conformal transformations

$$T_1 = \frac{1}{(\vec{y}_1 - \vec{y}_2)^{2\Delta}}\left(\frac{1}{\tau^{d-1}} + \dots\right)\alpha_0$$

where α_0 is a constant.

2-point function to all orders

- This discussion generalises to all orders:

$T = \vec{y}_1$  $\vec{y}_2 = \int [dx_1][dx_2] K(x_1, \vec{y}_1) F(x_1, x_2) K(x_2, \vec{y}_2)$

- As long as $F(x_1, x_2)$ is constructed out of bulk-to-bulk propagators (with all internal vertices integrated), the results is the same,

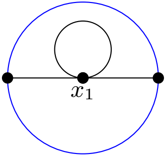
$$T = \frac{1}{(\vec{y}_1 - \vec{y}_2)^{2\Delta}} \alpha_\tau$$

- α_τ is a constant that depend on the regulator τ may be set to one by renormalising the source φ_0. This suggests there are no anomalous dimensions

BUT ...

IR divergences

$T_1 =$

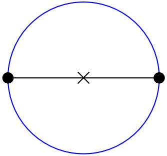


$$= G\left(\frac{1}{\cosh\tau}\right)\int_{z\geq\varepsilon}[dx_1]K(x_1,\vec{y}_1)K(x_1,\vec{y}_2)$$

- The integral is IR divergent,

$$\frac{\varepsilon^{-(2\Delta-d)}}{(2\Delta-d)}\delta(\vec{y}_1-\vec{y}_2)+\cdots-\frac{c_\Delta}{|\vec{y}_1-\vec{y}_2|^{2\Delta}}\left[2\ln\left(\frac{\varepsilon}{|\vec{y}_1-\vec{y}_2|}\right)+\psi(\Delta)-\psi(\nu)\right]$$

- This now requires a mass counterterm



$$\sim \delta m^2\Phi^2,$$

leading to an anomalous dimension γ and renormalisation of φ_0.

Higher-point functions

- Higher-point functions work the same way:
  - Ignoring the IR divergences one recovers the expected form of correlators due to conformal invariance.
  - IR divergences give rise to anomalous dimensions, confirming the values determined by the analysis of 2-point functions.
  - Bulk UV divergences are removed by renormalizing the mass and bulk coupling constant λ.
- We explicitly carried out the above up to 4-point functions to order λ^2 for the Φ^4 theory.
- For this theory and for these correlators up to $d = 6$ the bulk theory is renormalisable in the sense no new couplings are required for renormalisation.
- When $d > 6$ one needs additional derivative couplings of the schematic form $\partial^n \Phi^4$.

(Preliminary) results for AdS$_4$, $\Delta = 2$

- Coupling constant renormalization: $\lambda_0 = Z_\lambda \lambda$

$$Z_\lambda = (1 + \lambda S_1) - \frac{3\lambda}{16\pi^2} \log \tau \quad \Rightarrow \quad \beta_\lambda = \frac{3\lambda^2}{16\pi^2}$$

  Same beta function as in flat space, as it should.

- Mass renormalization: $m_0^2 = m^2 + \delta m^2$

$$m^2 = -2 - \lambda \left(\frac{1}{24\pi^2} - S_2 \right) - \lambda^2 (S_3 + \ldots)$$

$$m^2 = \Delta(\Delta - 3) \quad \Rightarrow \quad \Delta = 2 + \gamma$$

$$\delta m^2 = \frac{\lambda}{8\pi^2 \tau^2} \left(1 + \lambda \left(S_1 - \frac{1}{96\pi^2} \right) \right) + \frac{\lambda^2}{768\pi^4} \log \tau \left(\frac{18}{\tau^2} - 1 \right)$$

- Renormalization of source: $\varphi_0 = \varepsilon^{-\gamma} S_4 \varphi$
- S_1, S_2, S_3, S_4: functions related to scheme dependence.

Correlators to order λ^2

- Results consistent with conformal invariance.
- Scheme-dependence affects the values of Δ, λ and normalization of dual operator. We need additional UV information:
  - spectrum and interactions from string theory in AdS, or equivalently the same information from dual CFT.
- 4-point function:

$$\langle \mathcal{O}_R(\vec{y}_1)\mathcal{O}_R(\vec{y}_2)\mathcal{O}_R(\vec{y}_3)\mathcal{O}_R(\vec{y}_4)\rangle = -\lambda c_\Delta^4 \left(\left(1 + \frac{3\lambda}{16\pi^2}\right) D_{\Delta,\Delta,\Delta,\Delta} + \frac{\lambda}{32\pi^2} \left(\frac{d}{d\alpha} \left(D_{\Delta,\Delta,\Delta+\alpha,\Delta+\alpha} y_{34}^{2\alpha}\right)\Big|_{\alpha=0} + 2 \text{ permutations} \right) \right)$$

where $D_{\Delta,\Delta,\Delta,\Delta}$ is the tree-level contact diagram and $\Delta = 2 + \gamma$.

Conclusions/Outlook

- We set up bulk renormalisation in the AdS/CFT correspondence.
- Bulk renormalisation is completely consistent with what one would anticipate based on the AdS/CFT duality.
- This provides further structural support for the duality.

- There are a lot of things to be done ...
  - Include graviton exchanges in the bulk.
  - Work out to all-loops for general n-point function.
  - Connect with recent discussions based on the conformal bootstrap.
  - .....