Quantum Information and Spacetime

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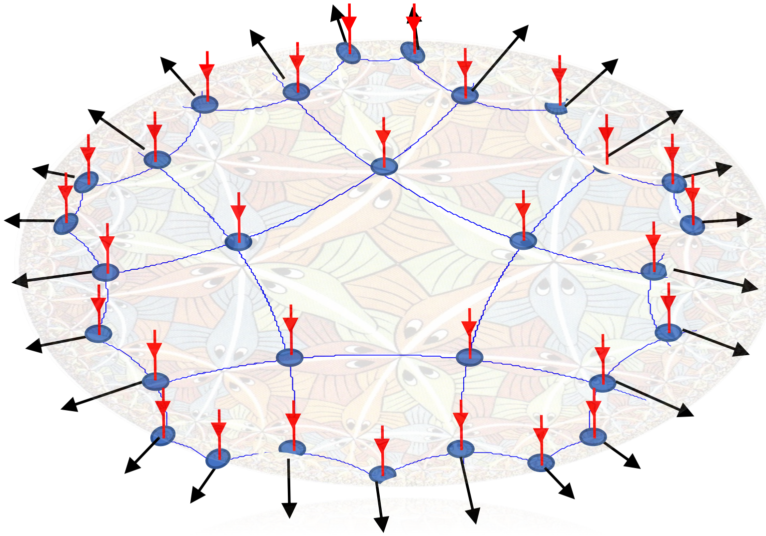

Strings, Vienna, 2022

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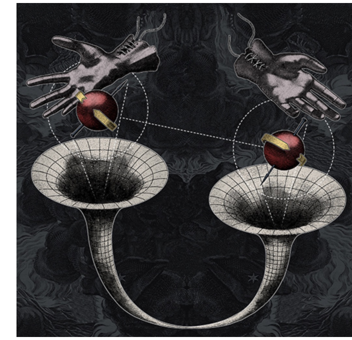
Quantum Information: Language and Toolbox

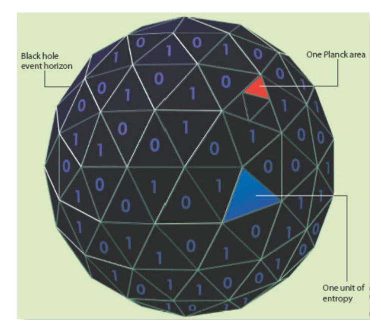


Quantum information is different: No cloning, uncertainty principle, Bell violations, entanglement, decoherence, …

QIT offers language + tools to study and exploit these phenomena. E.g.

Uncertainty principle ➔ quantum cryptography
Bell violations ➔ device-independent control
Entanglement ➔ many-body physics

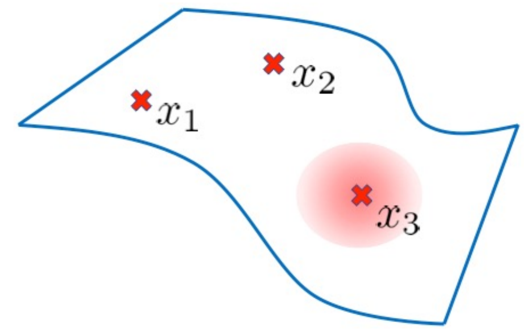
In recent years, exciting research at interface of quantum information with QFT and gravity.



Quantum information & field theory

Do quantum information tools apply to quantum field theory?

> Challenge: Notions such as *subsystems*, *entropy*, *approximation*, *circuits* more subtle!

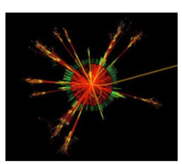
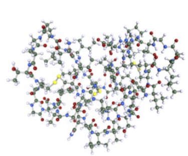
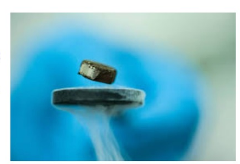


→ talk by Witten

Why bother?

1. New insights: Bekenstein bound from relative entropy, renormalization as QEC, c-theorem from subadditivity, ...

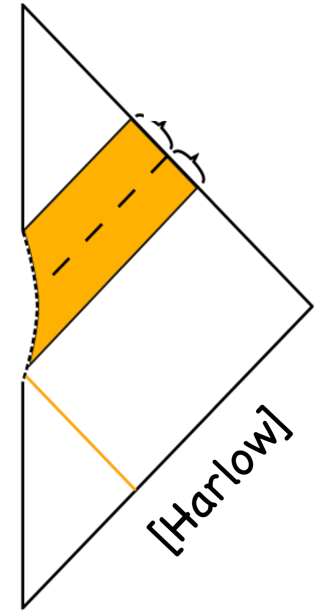
2. Quantum computers will be useful for simulating q. physics...

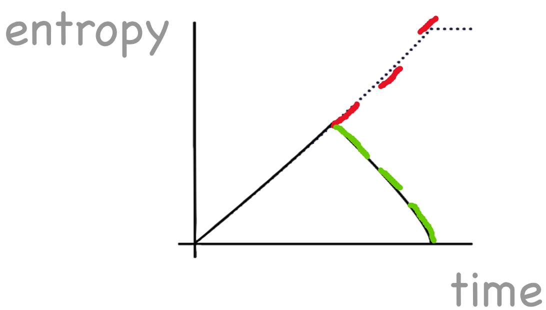

Feynman, Deutsch, ...

Can we simulate QFTs, or even theories of quantum gravity?

Motivation: Black hole information paradox(es)

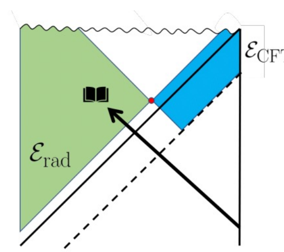


Suppose black hole is created from infalling matter and we watch it evaporate. What happens to Hawking radiation?



Paradox: Semiclassical calculation suggests entropy of radiation keeps increasing. But in *quantum* gravity, pure once fully evaporated!

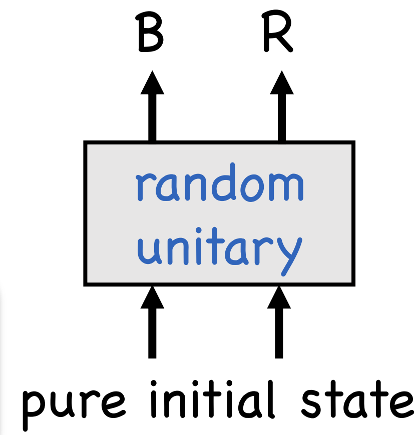
Similarly: If we throw diary into black hole, (when) can we decode it from Hawking radiation?



Many more puzzles: cloning, firewalls, ...

Recent breakthroughs shed new light on these problems in the holographic setting, drawing on quantum information ideas!

Simplest toy model of evaporating black hole



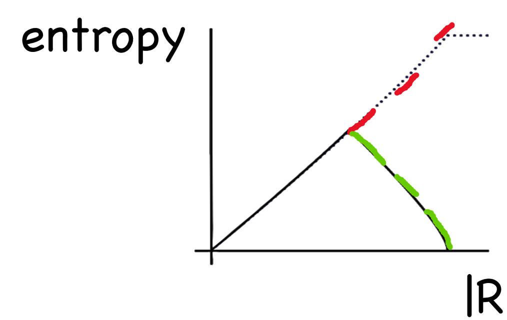
Model:

black hole = random unitary

time = relative size of radiation subsystem R

Page's theorem: For typical states,

$$\boxed{S(R) = \min(|B|,|R|) - O(1)}$$



Discussion:

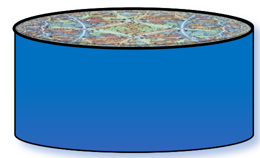
1. Use randomness to abstract away complicated technical detail.
2. Of course, want to derive results in q. gravity. Yet, toy models may help identify relevant principles + tools: Early radiation entangled with black hole, while late radiation entangled with early radiation.

Similarly, Hayden-Preskill: Black holes after Page time are "information mirrors". Relies on "decoupling principle" to diagnose information recovery.

The general plan

In holography, gravity emerges from complex QM system

boundary: d-dim
QM system (a CFT)



time

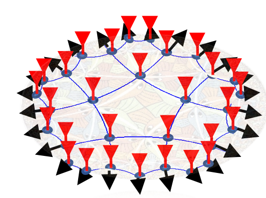
Controlled setup to study quantum gravity; including black holes, wormholes, ...

bulk: (d+1)-dim gravity (in AdS)

Understand this emergence using QIT

Build toy models that reproduce and explain

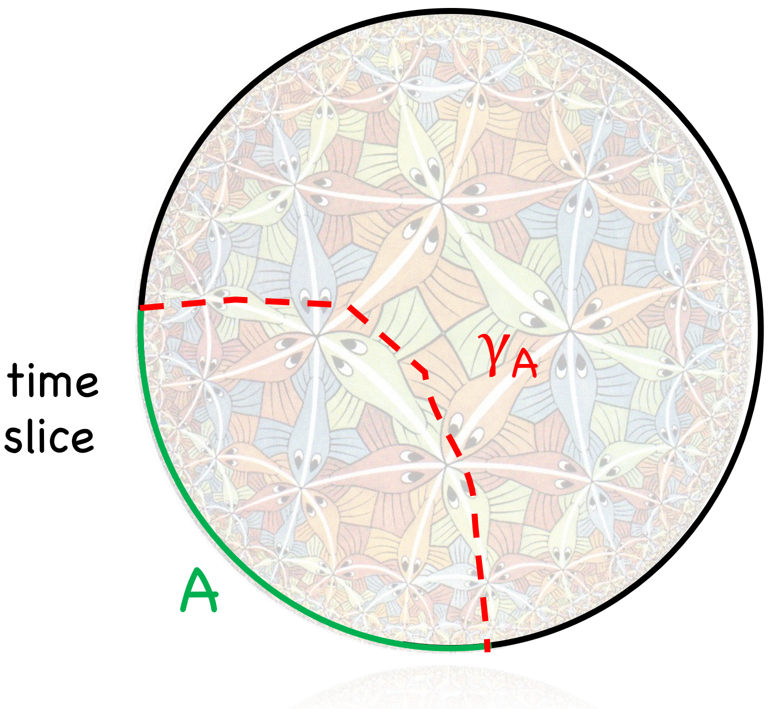
but also inspiration for other applications in QI, cond-mat, ...



Geometry vs Entanglement

Starting point: Entropy in holography

Ryu-Takayanagi's remarkable formula: Boundary (von Neumann) entropies are computed by areas of bulk minimal surfaces.

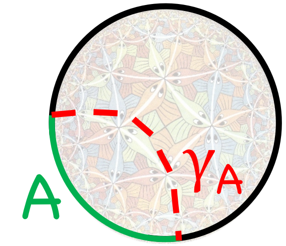


$$S(A) = \frac{|\gamma_A|}{4G} + \ldots$$

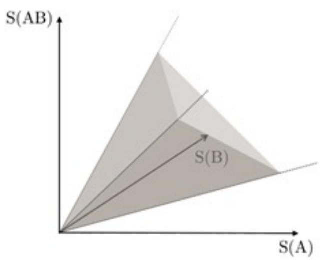
Ryu-Takayanagi (RT) law

What does it mean?

What do we know?



Entropy in holography is geometrized, implying constraints on either.



Infinitely many unusual entropy laws, can be organized systematically and interpreted.

Headrick et al
Bao-...-Ooguri-W,
Cui-...-W

However, can also go the other way and exploit known entropy laws to derive gravitational constraints. E.g., using relative entropy:

$\boxed{S(\rho\|\sigma) \geq 0}$

1st order: linearized Einstein equations
2nd order: positive energy inequalities

Faulkner et al, Lin et al, Lashkari et al

Already on the level of entropy, q. information offers new perspectives. More to be said... but why does the RT formula even make sense?

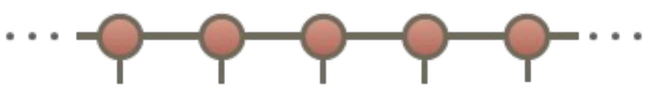
Tensor networks

Tensor network: define many-body state by contracting "local" tensors

$$|\Psi\rangle = \sum_{i_1,\ldots,i_n} \Psi_{i_1,\ldots,i_n} |i_1,\ldots,i_n\rangle$$

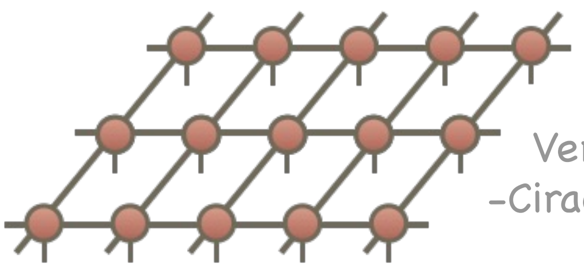
e.g.

MPS



White, Fannes-Nachtergaele-Werner, Östlund-Rommer

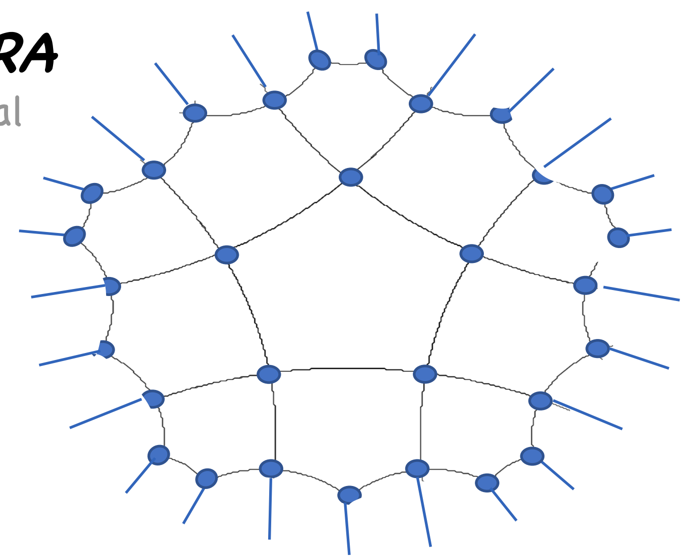
PEPS



Verstraete -Cirac

MERA

Vidal



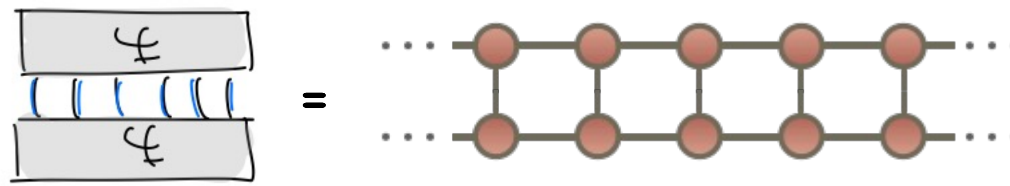
Numerical tool for many-body physics.

Conceptual tool: offers "dual" descriptions of complex phenomena → q. phases, topological order... geometry = entanglement pattern!

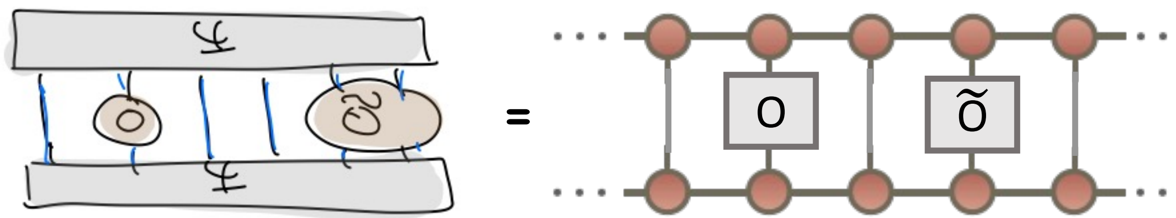
Computing with tensor networks

Very similar to path integral reasoning:

$\langle\psi|\psi\rangle =$



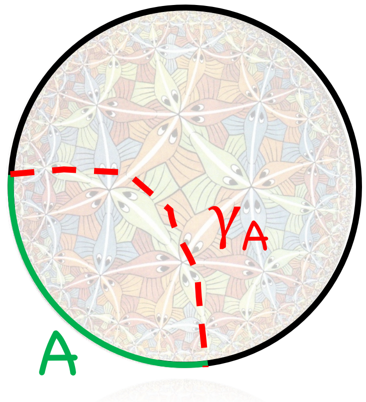
$\langle\psi|O_A \tilde{O}_B|\psi\rangle =$



Similarly for reduced density matrices etc.

Making sense of Ryu-Takayanagi

time slice

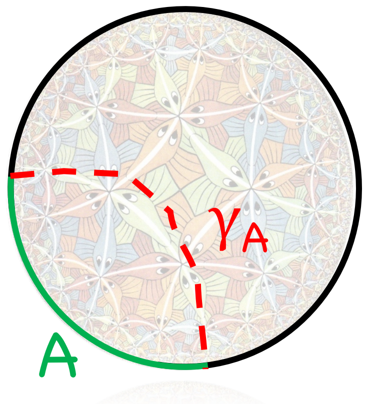


$$S(A) = \frac{|\gamma_A|}{4G} + \dots$$

Ryu-Takayanagi law

Making sense of Ryu-Takayanagi

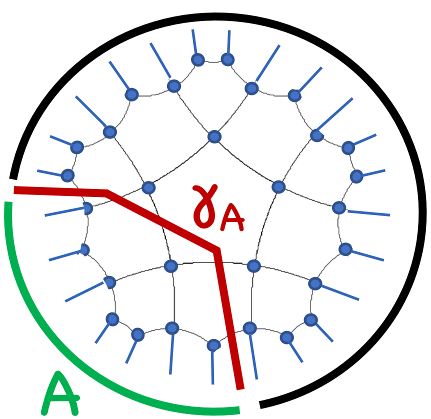
time slice



$$S(A) = \frac{|\gamma_A|}{4G} + \ldots$$

Ryu-Takayanagi law

Mysterious? Perhaps not! Tensor networks satisfy similar bound:



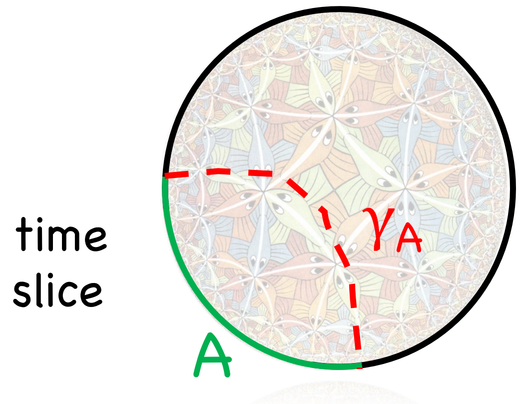
$$S(A) \leq N\,|\gamma_A|$$

N qubits per bond

Tantalizing: Picture shows "MERA" tensor network.
Used for critical states, looks like time slice of AdS...

Swingle

Making sense of Ryu-Takayanagi



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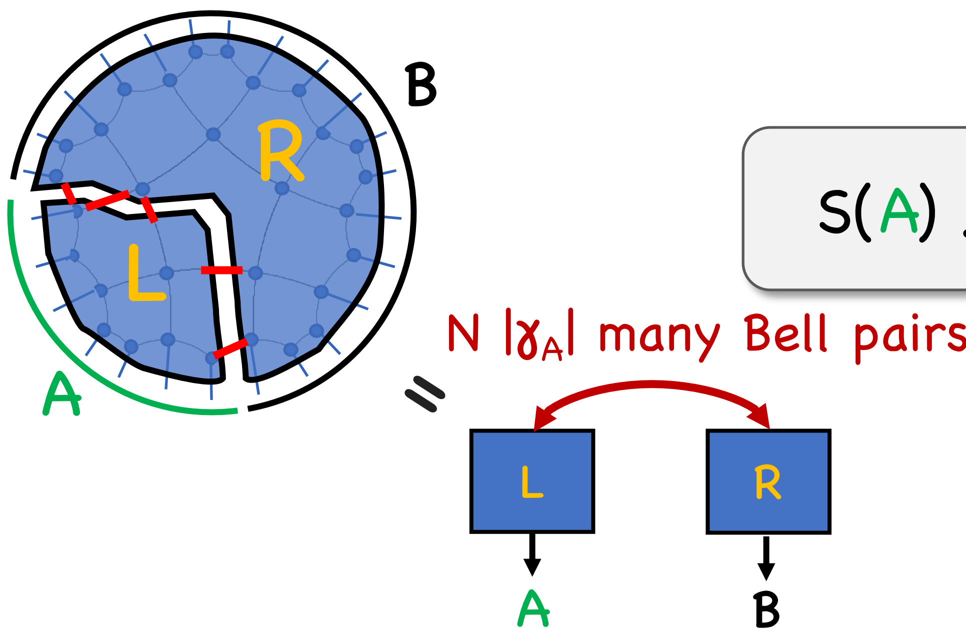
Reason:



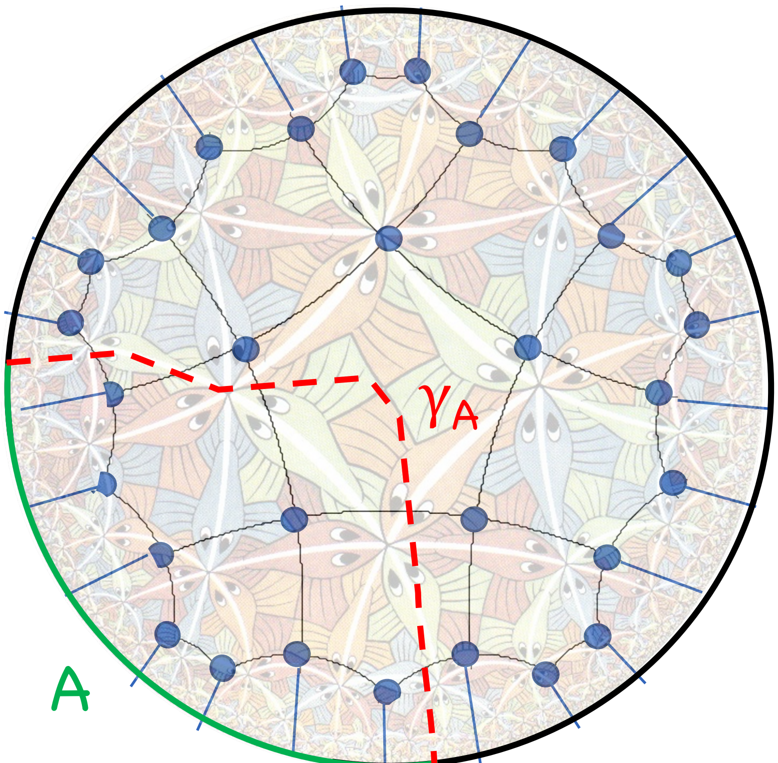
$$S(A) \leq N\,|\gamma_A|$$

N qubits per bond

Bound saturated if L, R are isometries!

Holography from tensor networks

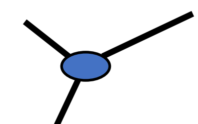
This suggest using TNS to define "exactly solvable" toy models:



Harlow et al, Hayden-...-W

Approach: Define boundary state via tensor network in bulk

simple bulk tensors, e.g. random tensors



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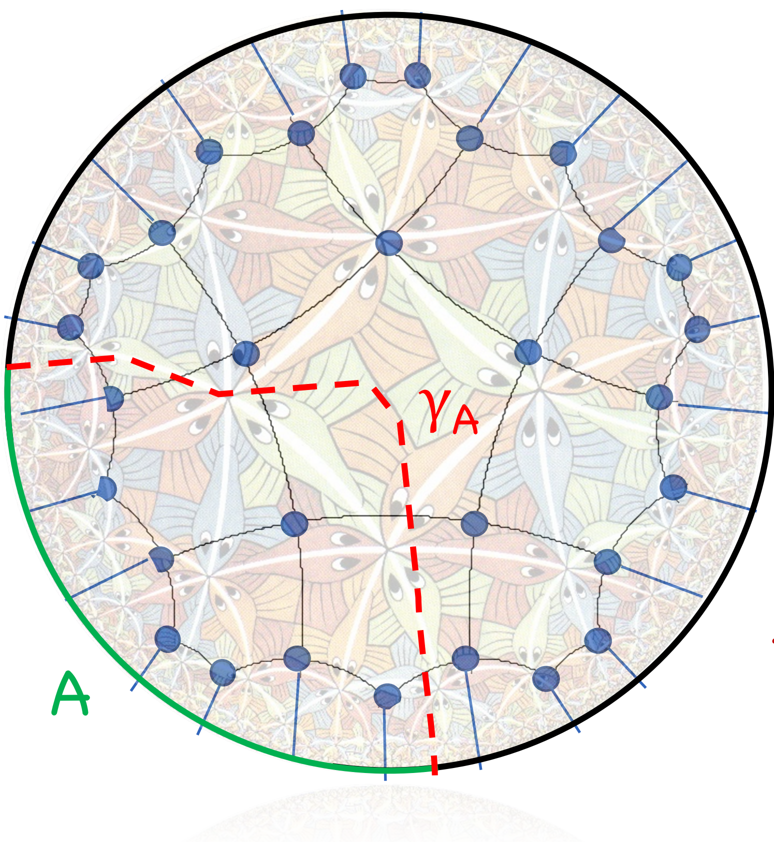

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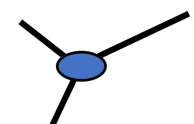


N qubits per bond

Ryu-Takayanagi law emerges for large N!

$$S(A) \simeq N\,|\gamma_A|$$

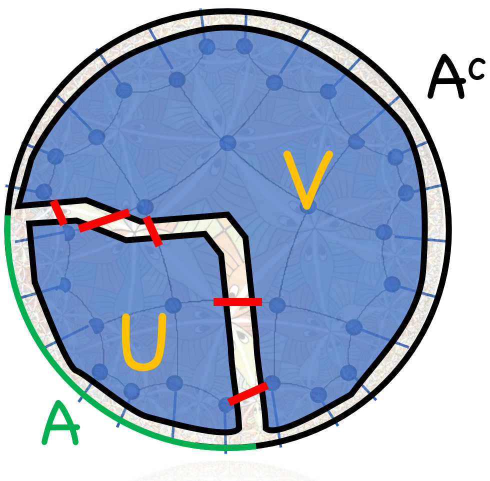
✓

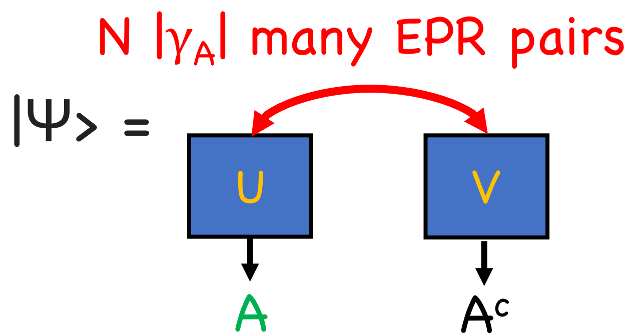
Mostly works in any geometry. By now, many variations known.

Three interpretations

Harlow et al,
Hayden-...-W

1. Random tensors ≈ isometric whenever possible

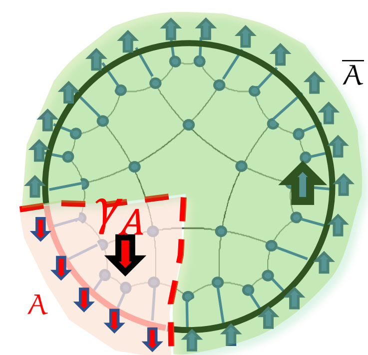




2. Entanglement distillation protocol

3. Replica trick + disorder average
➔ classical ferromagnetic S_n spin model

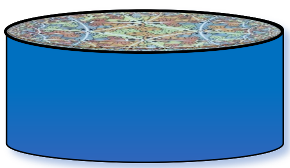
large N
= low T



E.g., Renyi-2 entropy => Ising model. Very versatile!

What do we know?

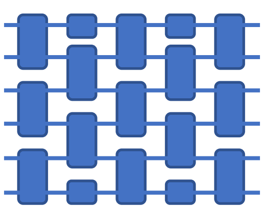
Random tensor networks (RTN) offer versatile toy model where geometry emerges from entanglement. Reproduce Ryu-Takayanagi (+ much more). Easy to analyze using replica trick.



Significance to holography? Match fixed area states (incl. nonperturbative corrections). Point to interesting new effects such as replica symmetry breaking. ✓

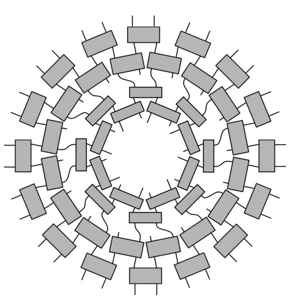
Dong-Harlow-Marolf, Penington et al, …, Dong-Qi-W, …, Akers et al, Cheng-…-W

Surprisingly, there are also applications beyond gravity:



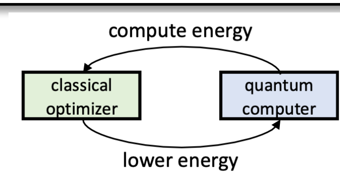
Similar techniques apply to random quantum circuits.

relevant to "quantum supremacy" proposals, condensed matter theory, …



Inspired research on q. circuits for critical systems.

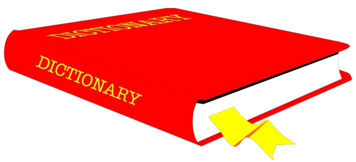
promising for near-term quantum computers



Kim-Swingle, …, Witteveen-W

Dualities as Quantum Codes

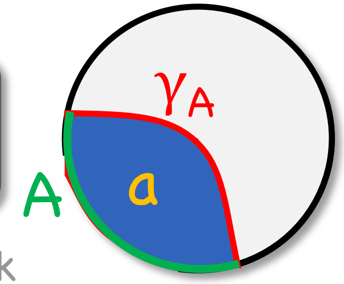
Holographic dictionary



Every local bulk operator should be dual to *some* boundary operator…

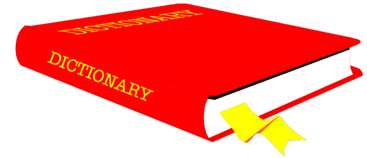
Subregion duality:

Any bulk operator in "entanglement wedge" a can be written as boundary operator in A!



Dong-Harlow-Wall, cf. Hamilton et al, Banks et al, Heemskerk et al, Cotler-…-W, …, Harlow TASI, talk by Liu

Holographic dictionary



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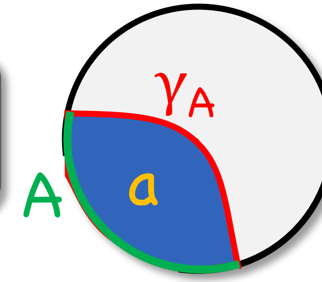
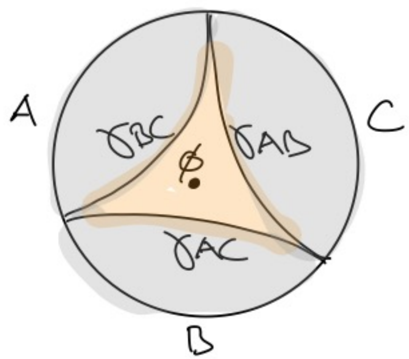


Leads to a puzzle:



$$\phi = O_{AB} = O_{AC} = O_{BC}$$

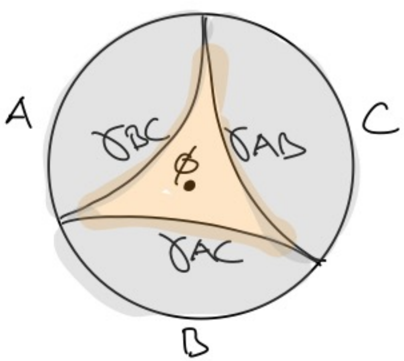
no common support ↯

$AB \cap AC \cap BC = \emptyset$

Resolution: Only few states correspond to any particular bulk. "ϕ=O" holds (makes sense!) only on small "code subspaces" of CFT Hilbert space.

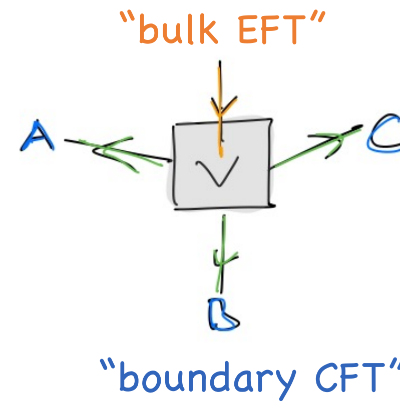
Almheiri-Dong-Harlow, Verlinde$^{\otimes 2}$, ...

Terminology? Redundancy in puzzle is precisely feature of quantum error correcting codes...



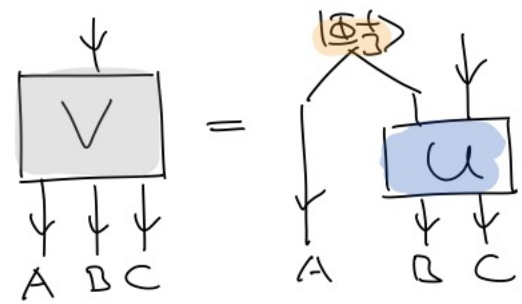
Three-qutrit code

$$V|\color{orange}{i}\rangle = \frac{1}{\sqrt{3}} \sum_{j=0}^{2} |j, j + \color{orange}{i}, j - \color{orange}{i}\rangle$$



Isometry that encodes 3-dim "bulk" into 3^3=27-dim "boundary".

Key observation:



likewise if interchange roles of A, B, C

Subregion duality: can decode from BC alone!

NB: learn nothing from A alone

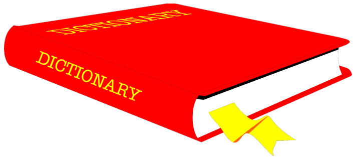
"Erasure code" that can correct loss of any qutrit!

Ryu-Takayanagi:

S(A) = log(3)

S(BC) = log(3) + S(ρ)

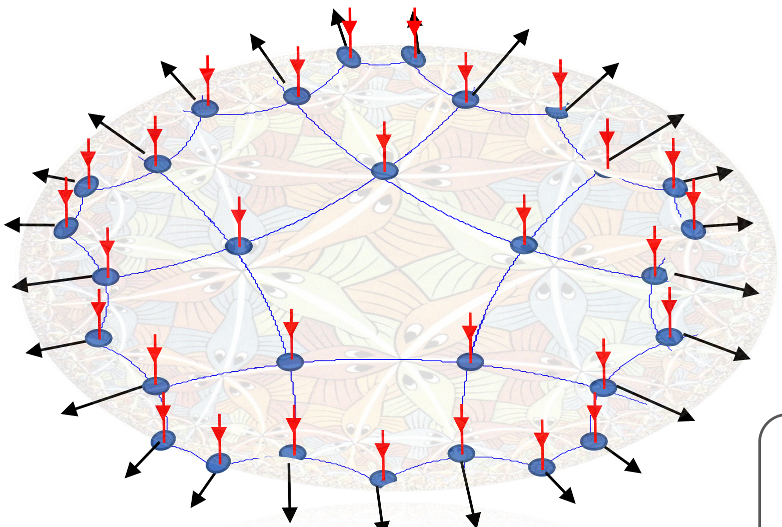
Holographic codes



How can we combine both toy models of AdS/CFT "dictionary"?

Approach: Define bulk-boundary mapping via tensor network

= glue together many small codes

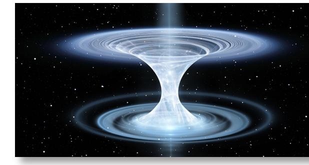


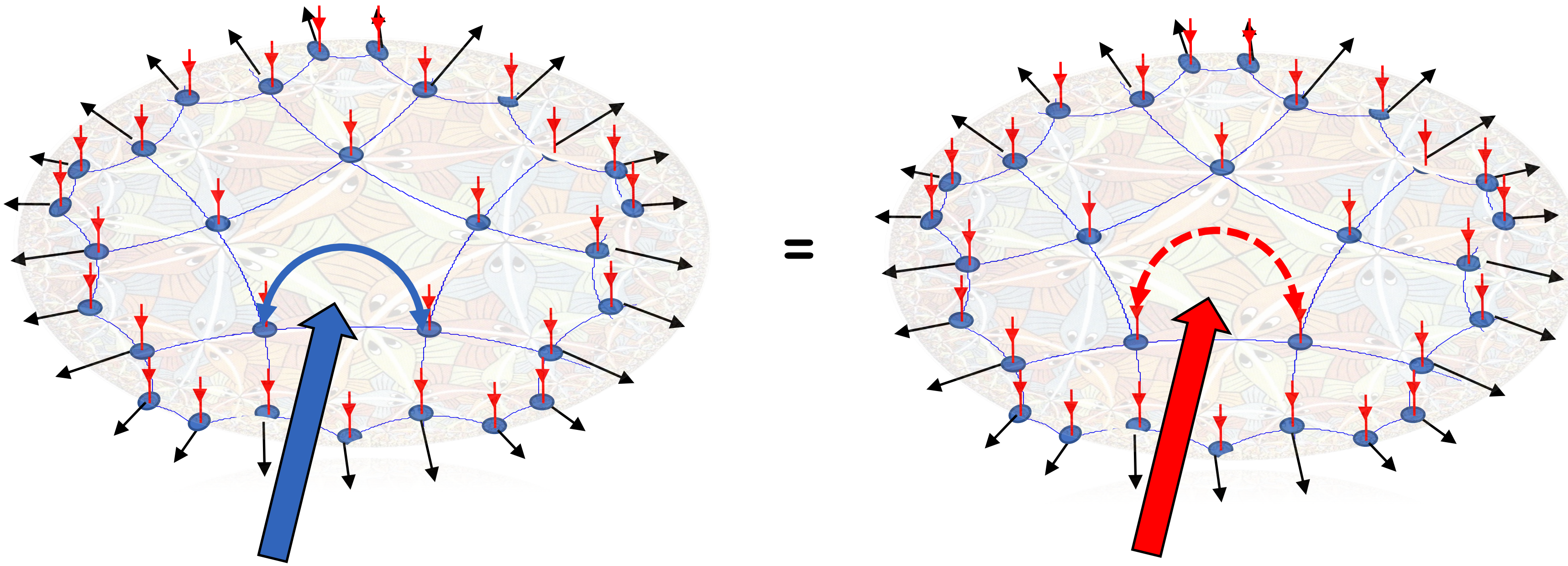
red legs: bulk degrees
black legs: boundary degrees

"logical" bulk states are encoded in "physical" boundary Hilbert space

Toy model of how bulk quantum fields get encoded in CFT.

Entanglement vs Geometry



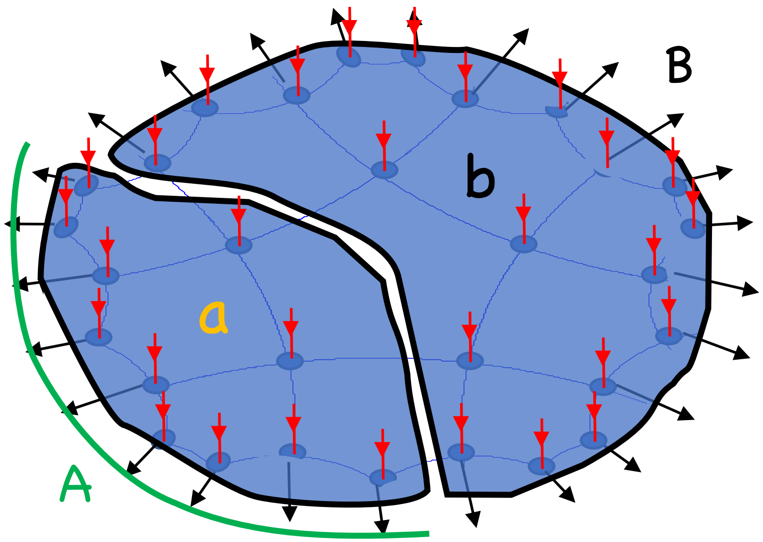


add link to geometry input entangled state

Natural ambiguity between geometry and entanglement, realizing Maldacena-Susskind's vision of "ER = EPR".

Locality & error correction

If bulk legs have small dimension, obtain error correcting code that satisfies "subregion duality":



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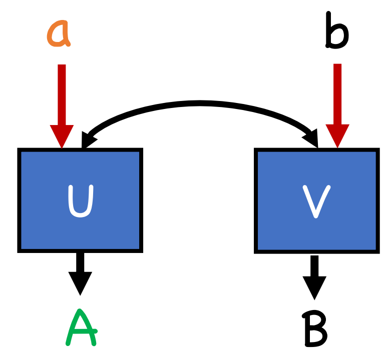


...for isometries U, V.

Bulk degrees in "entanglement wedge" a encoded in boundary subsystem A.

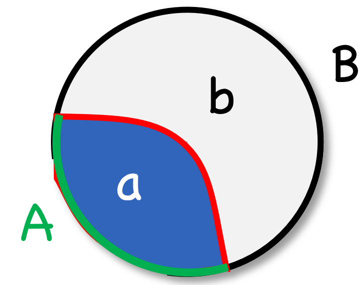
In particular, bulk corrections to entropy: $S(A) \approx N\,|\gamma_A| + S(a)$ ✓

Dong-Harlow-Wall, Faulkner et al, Engelhardt-Wall, ...

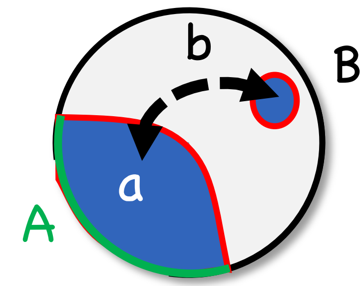
Quantum minimal surfaces and islands

So far assumed fixed minimal surface in code subspace (no backreaction).

In general, have state-dependent "quantum minimal surface"
minimizing "generalized entropy": Engelhardt-Wall



$$\boxed{S(A) \simeq \min \{ N\,|\gamma_A| + S(a) \}} \quad \checkmark$$



E.g., if add highly entangled state between distant bulk sites, obtain "island" disconnected from boundary.

Quantum minimal surfaces and islands

So far assumed fixed minimal surface in code subspace (no backreaction).

In general, have state-dependent "quantum minimal surface" minimizing "generalized entropy":

Engelhardt-Wall

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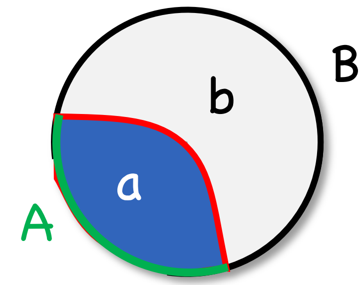

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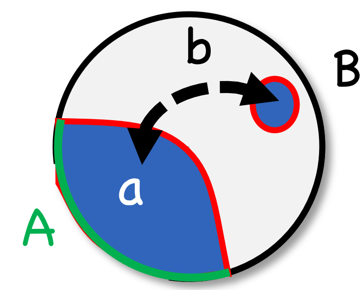
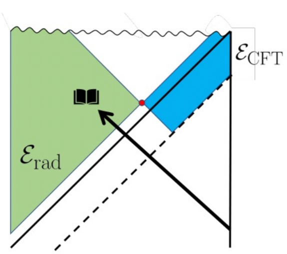


More subtle if entanglement not maximal.

Akers-Penington, …,
Cheng-…-W-Witteveen



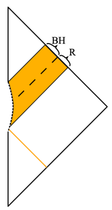
These ideas feature crucially in recent breakthroughs on black hole information paradox that give bulk picture of evaporation. Approach: Collect radiation in reservoir and analyze associated quantum extremal surfaces.

Dynamic spacetime → q. "extremal" surface.
Hubeny-Rangamani-Takayanagi, Engelhardt-Wall

Penington
Almheiri et al

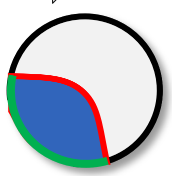
What do we know?

Quantum error correction appears to be the correct language to reason about quantum information in holography.



Key ingredient in recent celebrated progress on black hole information paradox.

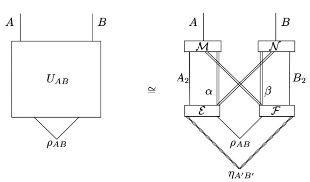
Penington, Almheiri et al



Holographic codes based on RTN reproduce key features, from ER=EPR to emergence of "islands".

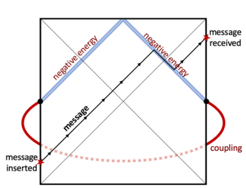
Harlow et al
Hayden-...-W

Again, there are surprising applications beyond gravity:



Suggests nonlocal q. computation needs little entanglement.

very surprising if true, implies strong attacks on position-based crypto, ...

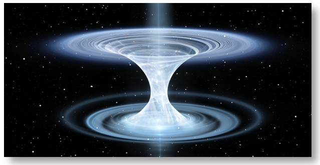


Inspired new quantum protocols from bulk physics.

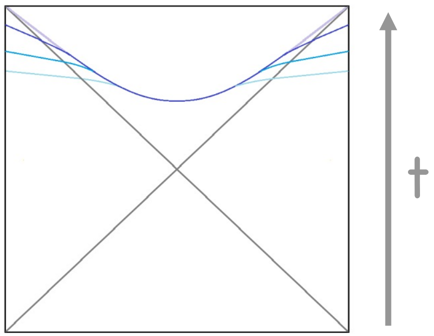
e.g., many-body teleportation inspired by particle moving through wormhole

Bonus: Dynamics, Complexity, and Quantum Protocols (pick one ☺)

Complexity = ?



So far, focus on static phenomena. In the last part of this talk will discuss some dynamical aspects. E.g., recall wormhole growth paradox:



In two-sided eternal AdS black hole, volume of wormhole grows for exp. long times, while natural quantities in dual CFT equilibrate rapidly.

Susskind's proposal: Wormhole volume = circuit complexity of bdry state?

Intuition: Dynamics so chaotic that no "shortcuts" in

$$e^{-iHt}\,|TFD\rangle = U\cdots U\,|TFD\rangle = U^t\,|TFD\rangle$$

→ many nontrivial investigations, checks, refinements... talks Shira & Vijay

such unitaries exist → Haferkamp et al

Complexity vs Pseudorandomness

Bouland-Fefferman-Vazirani

Some computational complexity-theoretic uneasiness:

$\boxed{\text{Volumes easy to estimate} \Leftrightarrow \text{complexity difficult to estimate}}$!?

e.g. pseudorandom ensembles: no efficient algorithm can distinguish them from Haar random (hence from each other), but complexity can vary

In fact, can construct ensembles of states resembling of Shenker-Stanford-like "shock states", for which complexity=volume thought to apply:

$$\boxed{U\ P_k\ U\ P_{k-1}\ U\ \ldots\ U\ P_1\ U\ |TFD\rangle}$$

like a quantum block cipher

black box unitary

Pauli "shocks"

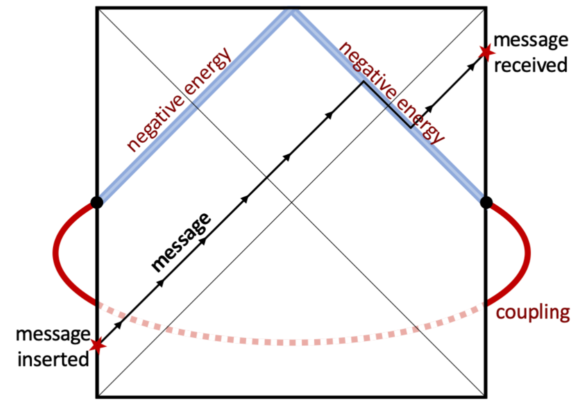
Toy model suggests intriguing possibility:

Could AdS/CFT dictionary be exponentially hard to compute?

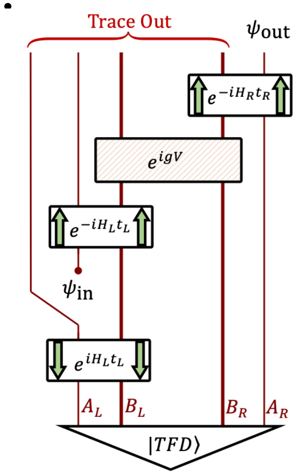
Holographic teleportation

Gao-Jafferis-Wall, Maldacena-Stanford-Yang, ...

[Quanta]



Holography allows for traversable wormholes connecting two bulk regions. We can throw in a qubit on the left, it exits on the right....



Boundary: Remarkable "holographic teleportation" protocol between two CFTs: "self-decoding" even though CFT time evolution highly scrambling!

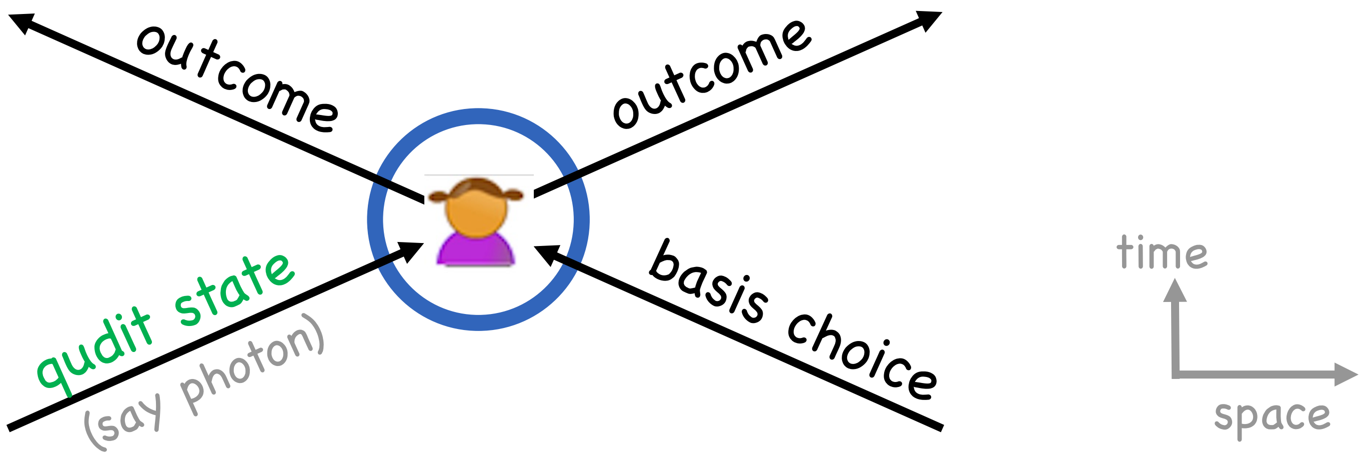
Recent work proposed concrete many-body protocols (using e.g. random unitaries) and general QI mechanism.

Yoshida-Yao, Brown-...-W, ...

scrambling vs size winding

Position-based quantum cryptography

Task: Verify party's **spacetime location.**   Idea:



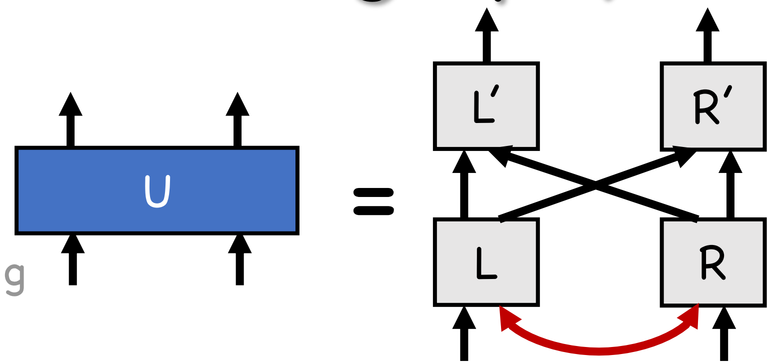
Party must perform computation with quantum input. Secure by no cloning?!

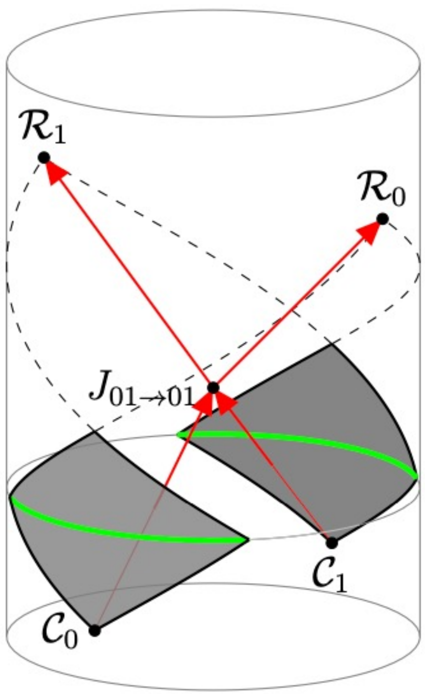
No! Colluding parties can attack <u>any</u> such scheme if they share exponential entanglement (essentially teleportation)!

Buhrman-Chandran
-..., Beigi-König

Nonlocal computation via holography

More generally, any two-party unitary circuit has nonlocal realization at exponential entanglement cost. Beigi-König





[May]

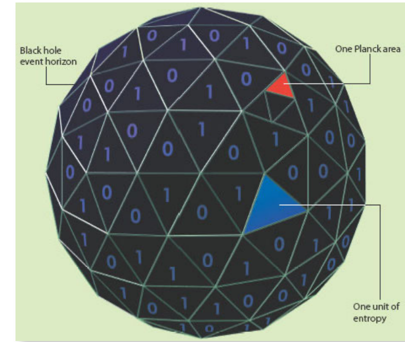
Holography suggests another way: run original unitary circuit in bulk, use dictionary to obtain equivalent nonlocal implementation on boundary.

Conjecture: $\boxed{\text{entanglement} \sim \text{complexity}}$ May

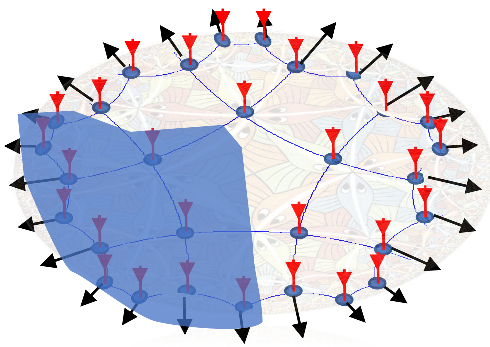
of nonlocal implementation — of "original" unitary

Really true? Concrete protocols? Crypto implications?

Summary



Holography offers challenges, puzzles, and paradoxes...



...pushing the boundary of quantum information, which can offer new tools, models, mechanisms.

Ongoing research to exploit connections both ways!

Motivation ranges from trying to understand emergence of spacetime from quantum mechanics to learning how dualities can help simulate complex quantum systems on (quantum) computers...

Thank you for your attention!