Algebraic ER=EPR

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Based on NE, Folkestad '22
and work in progress with Hong Liu

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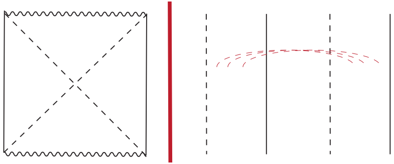
Spacetime Emergence from Entanglement

- General expectation: entanglement builds spacetime van Raamsdonk, Verlinde-Verlinde, Jensen-Sonner, Maldacena, Maldacena-Susskind, etc...

- Standard example:



- Similarly expressed as 'ER=EPR': $\mathcal{O}(G_N^{-1})$ entanglement builds spacetime; particularly relevant for the evaporating black hole van Raamsdonk, Maldacena Susskind, Verlinde-Verlinde.

Entanglement Builds Spacetime

"Classic ER=EPR": Entanglement Builds Spacetime

If there's enough entanglement in some bipartite state $|\psi_{R_1 R_2}\rangle$, and ρ_{R_1}, ρ_{R_2} each have a semiclassical gravitational bulk description, then the bulk dual to $|\psi_{R_1 R_2}\rangle$ is connected.

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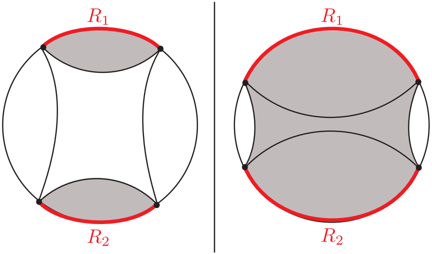
There's a similar expectation for a state which isn't pure $\psi_{R_1R_2}$:

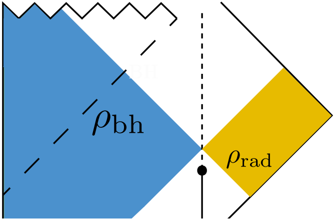


In this talk

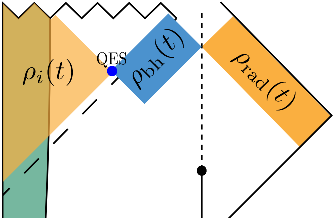
:

- Review of a counterexample to "entanglement builds spacetime' connectivity' NE, Folkestad '22
- An algebraic proposal for what builds spacetime connectivity work in progress w/ Liu

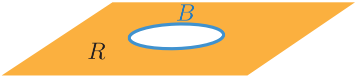
Evaporating AdS Black Holes Penington, AEMM



The entanglement wedge of the lower-dim'l CFT — B — has a complete time slice.

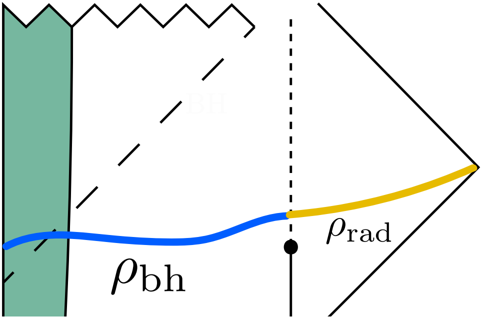


The entanglement wedge of B does not have a complete time slice.



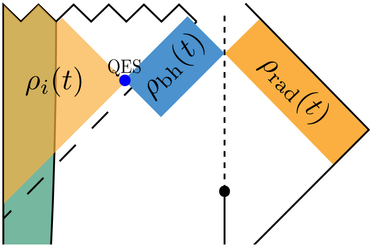
Universal Features: Pre- Page time

1. QES for ρ_{B} is empty (the classical extremal surface)
2. Pre-Page time, the entanglement wedge of ρ_{B} contains a Cauchy slice of the entire AdS bulk. The bulk state ρ_{bh} on this slice is not pure.



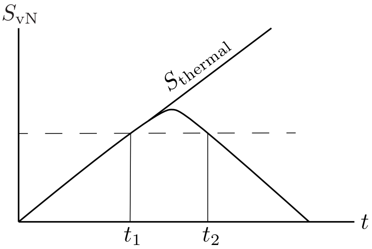
Universal Features: Post-Page time

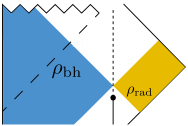
1. QES for $\rho_{\rm B}$ is far from the old classical RT surface.
2. Entanglement wedge of $\rho_{\rm B}$ no longer consists of a complete Cauchy slice of the entire bulk.
3. Entanglement wedge of radiation includes the now nontrivial island.

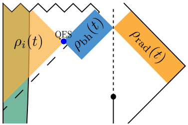


Pick two times so

$$S[\rho_B(t_1)] = S[\rho_B(t_2)]$$







B and R are not connected (in a gravitating spacetime).

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The Canonical Purification

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1. Take some density matrix in the diagonal basis

$$\rho = \sum p_i |\rho_i\rangle\langle\rho_i|$$

   and a Hilbert space $\mathcal{H}$.

2. Double the Hilbert space and define the pure state in the doubled Hilbert space:

$$|\sqrt{\rho}\rangle = \sum_i \sqrt{p_i}\, |\rho_i\rangle|\rho_i\rangle$$

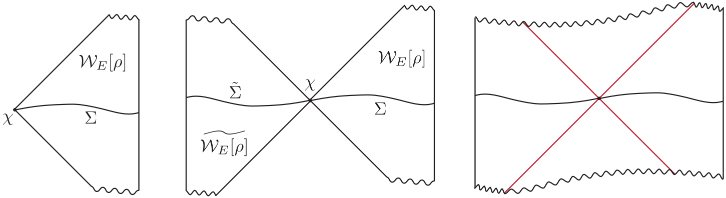
3. Can think of it as "flipping bras to kets".

4. Clearly tracing out new d.o.f. returns ρ.

The gravity dual of this construction is obtained by CPT conjugation around the minimal QES.

Holographic Dual of the Canonical Purification NE, Wall; Bousso, Chandrasekaran, Shahbazi-Moghaddam

Given a CFT in some mixed state ρ with a semiclassical dual entanglement wedge $\mathcal{W}_E[\rho]$, $|\sqrt{\rho}\rangle$ is given by a CPT conjugation of the spacetime around the QES χ.



TFD Canonical Purification

Gibbs state:

$$\rho = \frac{1}{Z} \sum e^{-\beta E_n} |n\rangle\langle n|$$

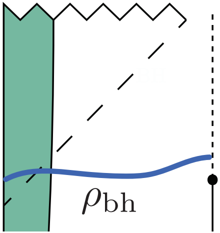
Can purify by doubling the Hilbert space; we get TFD:

$$|\text{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum e^{-\beta E_n/2} |n\rangle |n\rangle$$

Which gives us the complete (maximally extended) Schwarzschild-AdS black hole Maldacena '01, the CPT conjugation around the QES of a single side's entanglement wedge.

The Puzzle

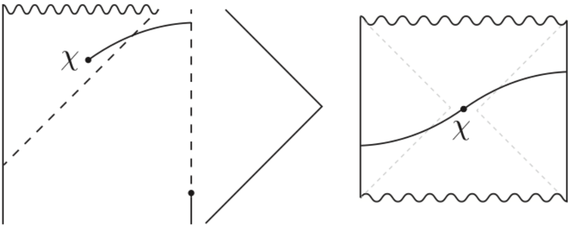
Now take B at t_1.



Its canonical purification is just two disconnected copies: CPT conjugation around the empty set just gives a second copy not geometrically connected to the first.

The Puzzle

Now take B at t_2.



Its canonical purification is a single connected geometry.

The Puzzle

Pre- vs Post-Page

Two semiclassical holographic spacetimes, with two boundaries and *the same von Neumann entropy*.

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The Puzzle

Pre- vs Post-Page

Two semiclassical holographic spacetimes, with two boundaries and *the same von Neumann entropy*. One is connected and the other is not. **Entanglement does not always build spacetime.** What is the relevant difference between the two states that permits connectivity in one and not the other?

Things to try that fail to distinguish...

- Complexity
- The difference $S_{max} - S_{vN}$
- Reflected entropy
- Various other entanglement measures...

The Relevant Difference: Topology of the QES

Before the Page time: Σ is inextendible; after the Page time, it is extendible.

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Of course, the topology of the QES is notoriously difficult to diagnose from the dual theory.

So...

The Relevant Difference: Topology of the QES

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Of course, the topology of the QES is notoriously difficult to diagnose from the dual theory.

So...

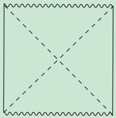
Question

What is the best diagnostic of a connected spacetime dual to some bipartite state $|\psi_{R_1 R_2}\rangle$ (where R_1 R_2 are complete boundaries)?

Some Intuition

If the spacetime is connected...

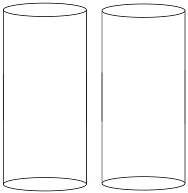
1. The entangled wedges of R_1 and R_2 share a common, nontrivial edge.



2. The bulk Fock space of low energy perturbations should not contain states that factorize into a product state on W_{R_1} and W_{R_2}. (Equivalently, the GNS Hilbert space constructed from $|\psi_{R_1R_2}\rangle$ should fail to factorize around into R_1, R_2.)
3. The algebra of bulk operators (not including cross product improvements) in the large-N limit should be type III.

More Intuition

Whereas a "disconnected" spacetime should fail all of these criteria, and in AdS we expect that the algebra of bulk operators should be type I.



Statements about algebras of bulk operators can be translated into boundary language using subregion duality Harlow; Dong, Harlow, Wall; Liu Leutheusser;...: the boundary subalgebra of operators acting on the GNS Hilbert space is type III in the connected case.

A Proposal: Part 1

Algebraic ER=EPR, Part I

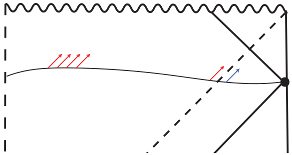
- There is a classical wormhole connecting R_1 to R_2 if $\mathcal{A}_{R_1}$, $\mathcal{A}_{R_2}$ are type III.
- R_1 to R_2 are disconnected if $\mathcal{A}_{R_1}$, $\mathcal{A}_{R_2}$ are type I.

where $\mathcal{A}_R$ is the relevant boundary subalgebra of operators built. We assume here the spacetime is semiclassical: all fluctuations are suppressed in positive powers of G_N.

What about type II?

The Pre-Page Operator Algebra

Let's decouple the bath and evolve with the decoupled Hamiltonian.



$$S[\rho_B(t_1)] \sim \mathcal{O}(G_N^{-1})$$

This diverges in the $G_N \to 0$ limit. It should not be type I (no pure states).

But we have a bulk volume form and a very clear geometry whose fluctuations go as $G_N^{a>0}$. So it seems that we can define a trace, and if so it should not be type III.

So perhaps it is type II.

A Potential Type II

Recall our connectivity criteria:

If the spacetime is connected...

1. The entanglement wedges of R_1 and R_2 share a common, nontrivial edge.
2. The GNS Hilbert space does not factorize.
3. The algebra of bulk operators in the large-N limit should be type III.

The Pre-Page black hole satisfies (2) but not (1) or (3).

So is this somewhat connected? Not quite connected but not quite disconnected?

A Speculative Proposal

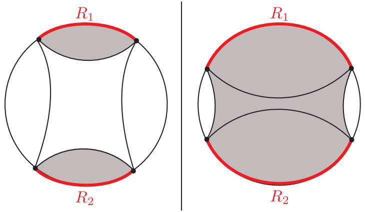
Working in the large-N limit and only with the bulk QFT operators:

Algebraic ER=EPR

- There is a classical wormhole connecting R_1 to R_2 if $\mathcal{A}_{R_1}$, $\mathcal{A}_{R_2}$ are type III.
- *(Speculation:) We define R_1 to R_2 if $\mathcal{A}_{R_1}$ to be connected via a quantum wormhole $\mathcal{A}_{R_2}$ are type II.*
- R_1 to R_2 are disconnected if $\mathcal{A}_{R_1}$, $\mathcal{A}_{R_2}$ are type I.

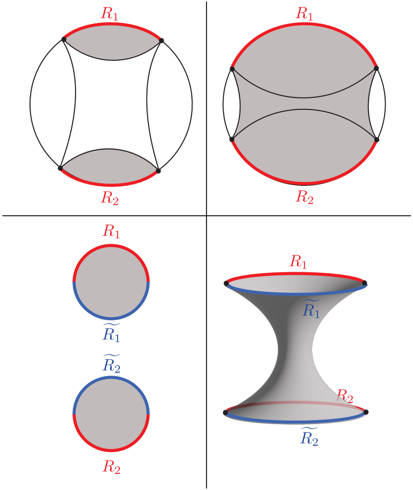
What about multipartite states?

In this case it seems the algebra is type III no matter what:

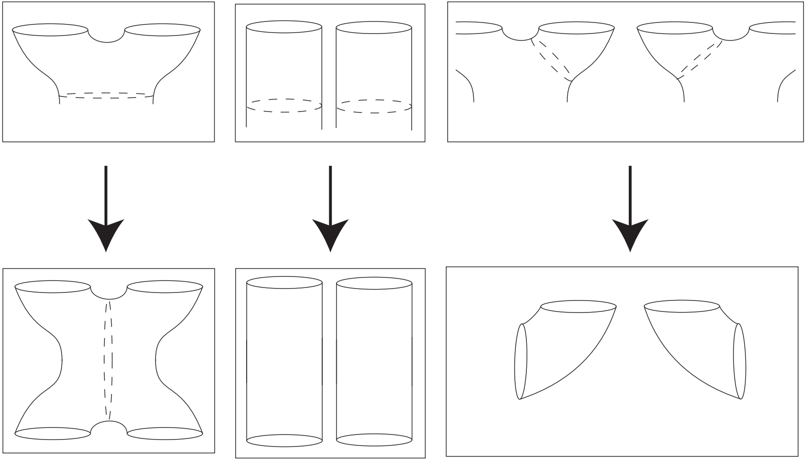


To give a connectivity criterion here, we purify the state using, once again, canonical purification:

$$\psi_{R_1R_2} \to |\psi_{R_1R_2\widetilde{R_1R_2}}\rangle$$



The algebra $\mathcal{A}_{R_1\widetilde{R}_1}$ can only be type III if R_1 and R_2 are classically connected.



We simply treat $|\psi_{R_1 R_2 \widetilde{R_1 R_2}}\rangle$ as a bipartite state and use the previous definitions.

Upshot

- The standard expectation that entanglement builds spacetime is flawed: it is possible to build semiclassical, holographic, well-behaved spacetimes with large von Neumann entropy and no wormhole.
- Standards probes typically related to spacetime emergence also fail to diagnose connectivity.
- But algebra type does seem to distinguish.
- This also works for diagnosing connectivity of subregions or multiple boundaries.

Some Further Comments

- We speculate that perhaps the right definition of quantum connectivity – the absence of a nontrivial QES despite a large amount of entropy – is a type II von Neumann algebra where we might have naively expected a type I.

- By working a little bit harder (separating the algebra into complex and simple subalgebras), we can also see that by this definition the island is “quantum connected” to the radiation.

Thank you!