Infrared Aspects of QFT and Quantum Gravity: Scattering and Coherence

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K.Prabhu, G.S., & R.M. Wald, Phys. Rev. D 106, 066005 (2022) [arXiv:2203.14334]

G.S., D. Danielson, & R.M. Wald (in prep.)

K. Prabhu & G.S. (in prep.)

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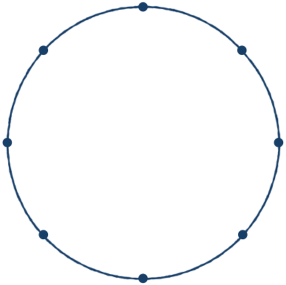
July 28, 2023

Gravitational and Electromagnetic Memory Effects



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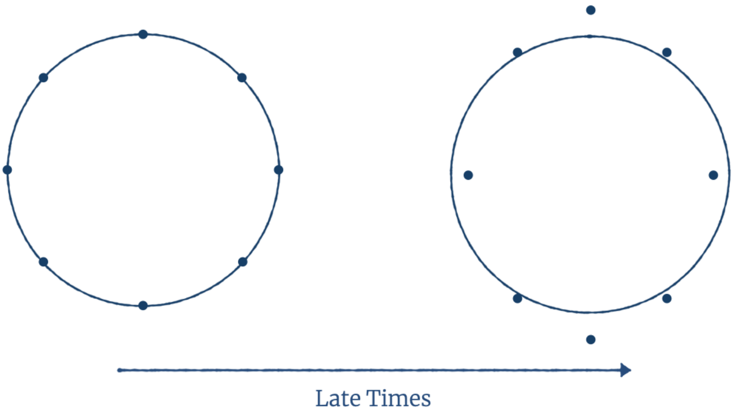

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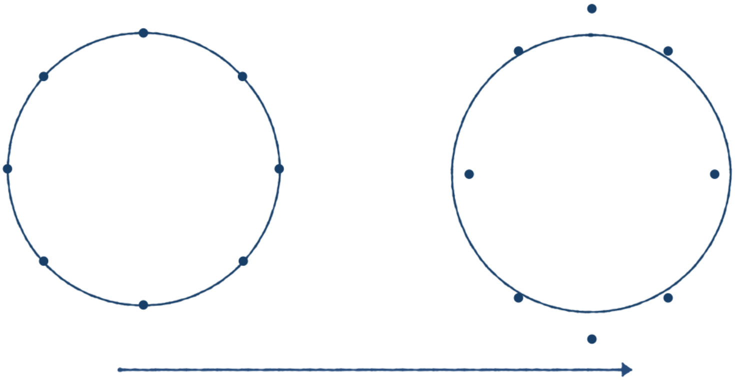
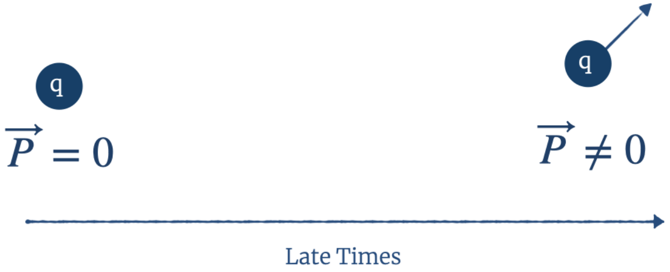


relative displacement

Gravitational Memory

$$\Delta\xi_a = \frac{\Delta_{ab}\xi^b_{(0)}}{r}$$

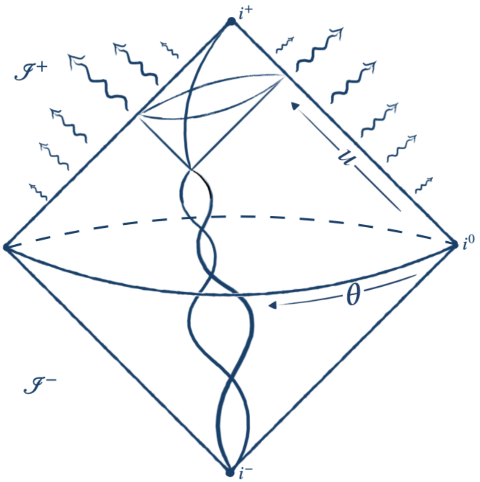
Gravitational and Electromagnetic Memory Effects



Electromagnetic Memory

$$\Delta P_a = \frac{\Delta_a}{r}$$

Classical Scattering, Radiation and Memory



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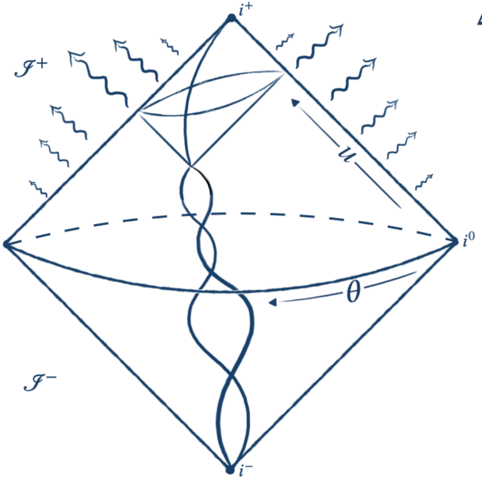

$$\Delta_a(\theta) = -\int_{-\infty}^{\infty} du\ E_a(u,\theta) \qquad E_a(u,\theta) = \partial_u A_a$$

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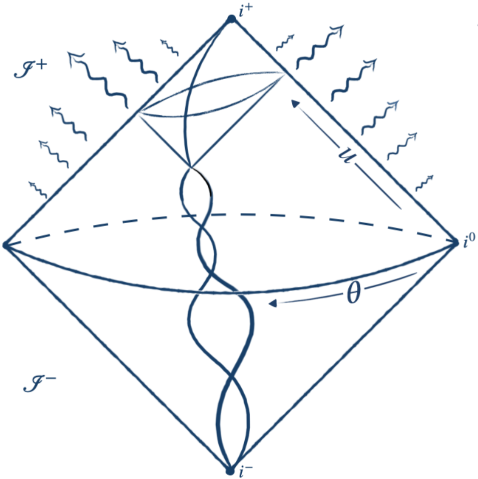

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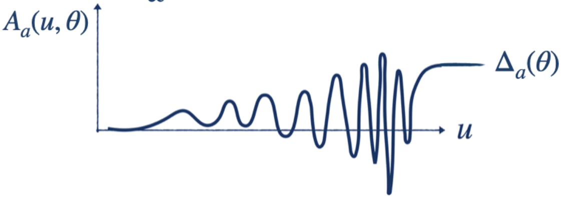
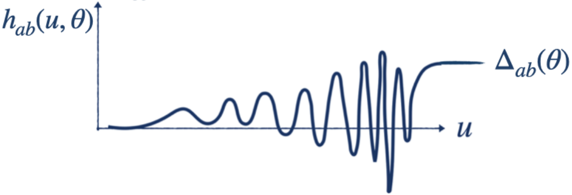
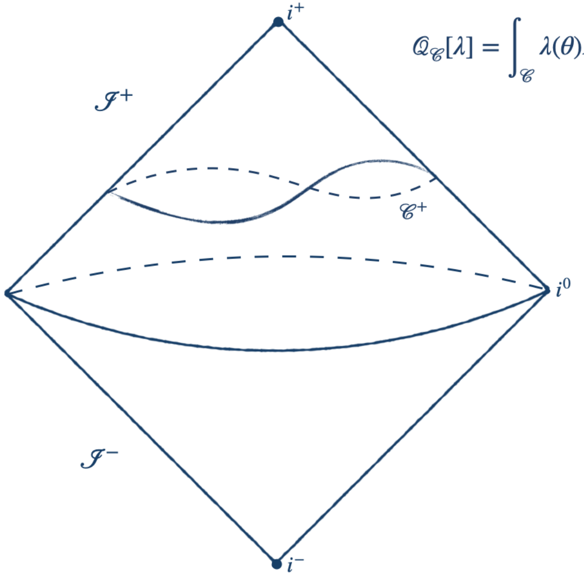


$$\Delta_{ab}(\theta) = \int_{-\infty}^{\infty} du\ N_{ab}(u,\theta) \qquad N_{ab}(u,\theta) = \partial_u h_{ab}$$



Memory and Charges



$$\mathcal{Q}_{\mathscr{C}}[\lambda] = \int_{\mathscr{C}} \lambda(\theta) F^{(2)}_{ur}(u,\theta)\; d\Omega$$

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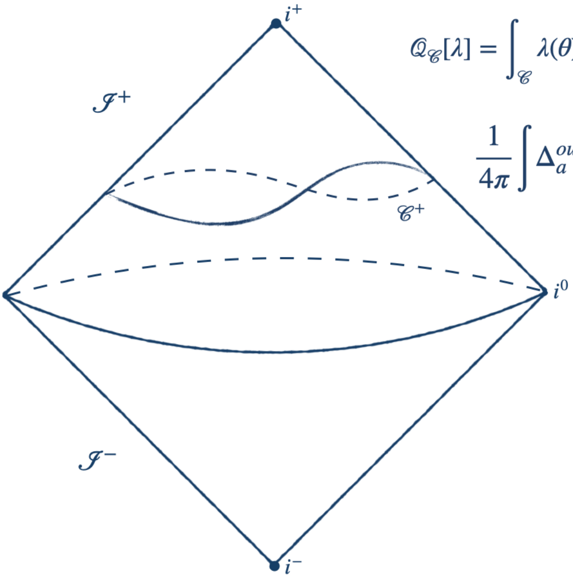

$$\mathcal{Q}_{\mathscr{C}}[\lambda] = \int_{\mathscr{C}} \lambda(\theta) F^{(2)}_{ur}(u,\theta)\ d\Omega$$

massless charge-current flux

$$\frac{1}{4\pi}\int \Delta^{out}_a \mathscr{D}^a \lambda\ d\Omega = \mathcal{Q}_{i^+}[\lambda] - \mathcal{Q}_{i^0}[\lambda] + \int_{\mathscr{I}^+} J_{out}\lambda$$

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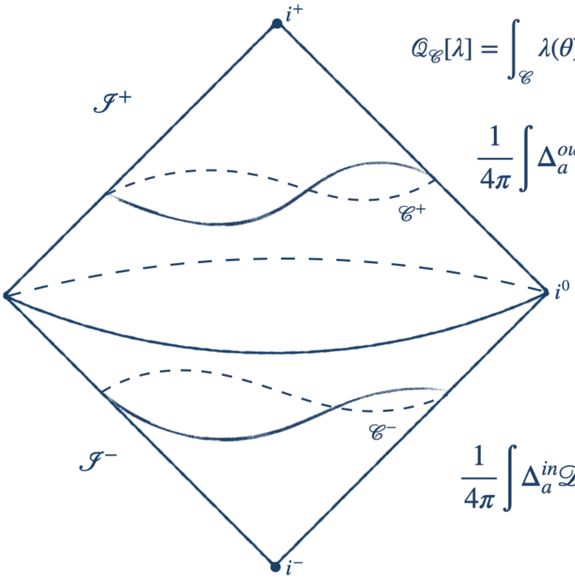

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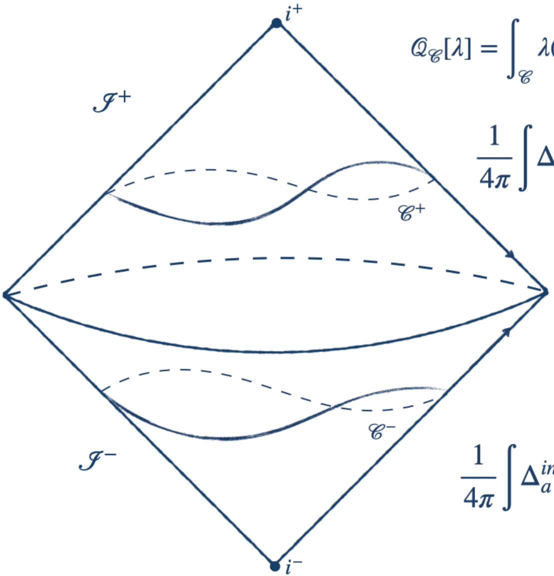

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where $\tilde{\lambda}(\theta) = \lambda(-\theta)$

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[Campiglia et al. 2017], [Prabhu, 2018] ...

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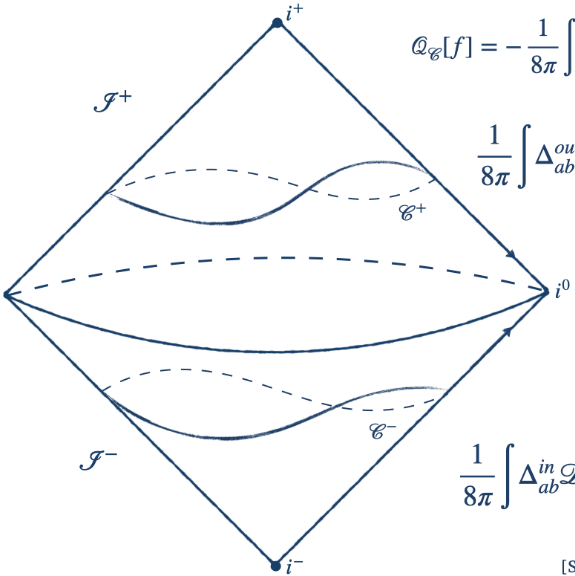

$$\mathcal{Q}_{\mathscr{C}}[f] = -\frac{1}{8\pi}\int_{\mathscr{C}} f(\theta) C^{(3)}_{urur}(u,\theta)\, d\Omega \quad \text{when} \quad N_{AB} = 0$$

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[Strominger et al. 2013], [Troessaert, 2018], [Prabhu, 2018] ...

Fock Quantization

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$$||h||^2 = 16\pi \int_0^\infty \int_{\mathbb{S}^2} d\omega d\Omega \; \omega |\tilde{h}_{ab}(\omega, \theta)|^2$$

where $\tilde{h}_{ab}(\omega, \theta)$ is Fourier transform of $h_{ab}(u, \theta)$.

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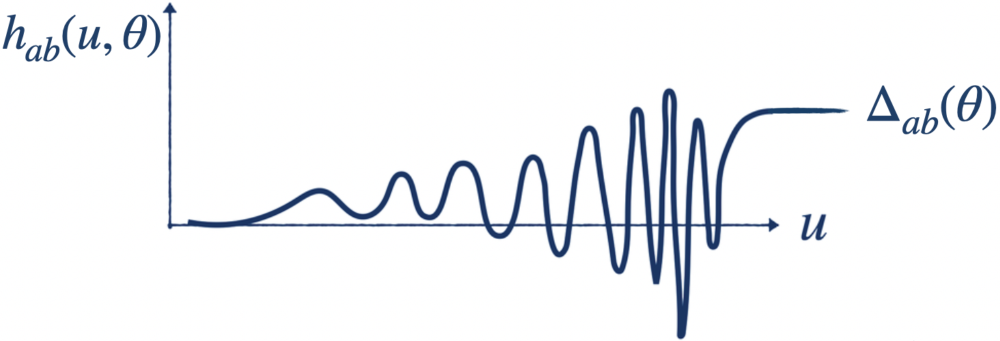
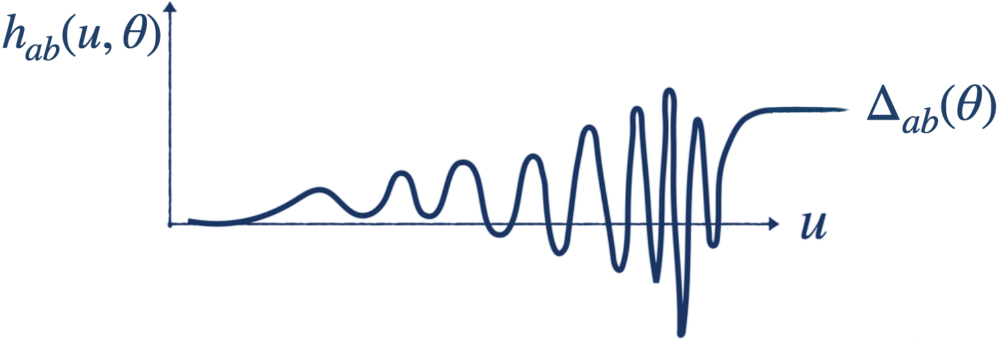


$$\Delta_{ab} \neq 0 \implies \Delta h_{ab} \neq 0 \implies \tilde{h}_{ab}(\omega, \theta) \sim \frac{1}{\omega}$$



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- There are an *uncountably infinite* number of "in/out" Fock spaces labeled by all possibles memories $\Delta^{\text{in/out}}$. Memory is not conserved. To go beyond "inclusive cross sections" and have a well-defined S-matrix one needs to include states with memory.

Massive QED - Faddeev-Kulish Hilbert Space

$$\mathcal{Q}_{i^0}(\lambda) = \mathcal{Q}_{i^-}(\lambda) - \frac{1}{4\pi}\int_{\mathbb{S}^2} \Delta_a^{\text{in}} \mathscr{D}^a \lambda$$

- Key Idea: The charge at spatial infinity is conserved. Therefore "in" Hilbert space of eigenstates of the charge $\boldsymbol{\mathcal{Q}}_{i^0}(\lambda)$ with eigenvalue $\mathcal{Q}_{i^0}(\lambda)$ will will map to an "out" Hilbert space of eigenstates with eigenvalue $\mathcal{Q}_{i^0}(\tilde{\lambda})$ [Faddeev & Kulish, 70']...

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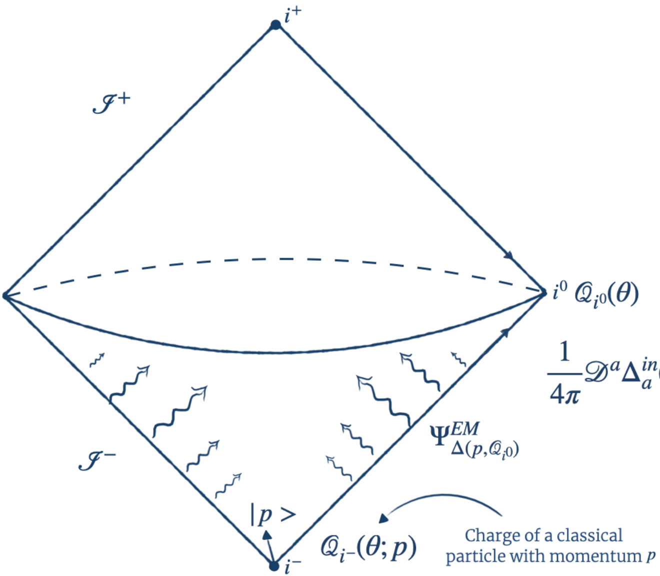

$$\frac{1}{4\pi}\mathscr{D}^a\Delta_a^{in}(\theta;p,\mathcal{Q}_{i^0}) = \mathcal{Q}_{i^-}(\theta;p) - \mathcal{Q}_{i^0}(\theta)$$

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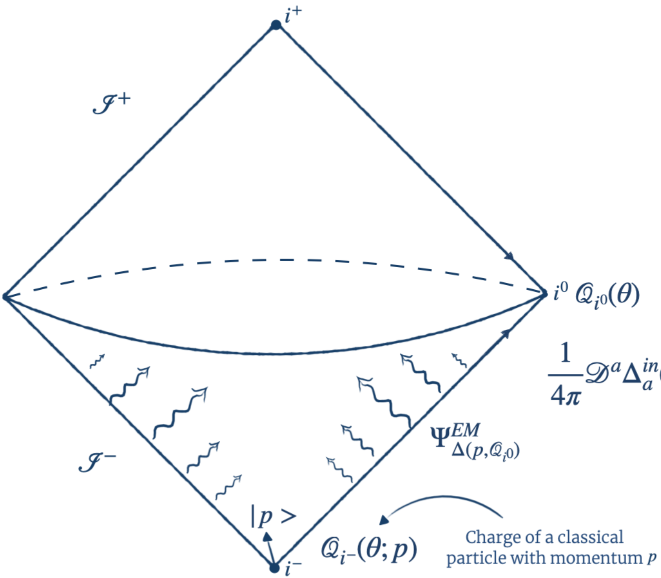

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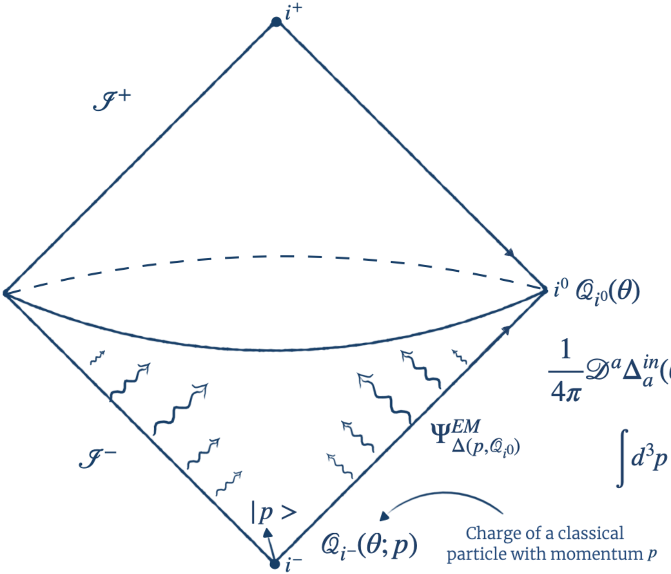

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- This construction fails in *all* other theories including quantum gravity.

Vacuum Gravity - Failure of Faddeev-Kulish Hilbert Space

$$\mathcal{Q}_{i^0}^{\mathrm{GR}}(f) = -\frac{1}{8\pi}\int_{\mathbb{S}^2} \Delta_{ab}^{\mathrm{in}}\mathscr{D}^a\mathscr{D}^b f(\theta) + \int_{\mathscr{I}^-} f(\theta)N^2$$

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- Intuition: Memory and Energy flux are not independent! In gravity, the gravitational radiation "sources" (i.e. via energy flux) its own memory. Matching the memory to the energy flux introduces more radiation! This introduces more energy flux and so on...

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There does not appear to be *any* "preferred" Hilbert space for scattering in QG ("Non-Faddeev-Kulish" representations also fail)

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- Given any state $|\Psi\rangle$ in a Hilbert space $\mathscr{H}$ one can express that state as a list of correlation functions of operators in an Algebra $\mathscr{A}$. For example,

$$\langle\phi(x)\rangle_\Psi, \langle\phi(x_1)\phi(x_2)\rangle_\Psi, \dots, \langle\phi(x_1)\dots\phi(x_n)\rangle_\Psi, \dots$$

  Conversely, given a list of correlation functions on $\mathscr{A}$ (satisfying commutation relations, positivity, ...) one can construct (by GNS) a Hilbert space where this list of correlation functions is packaged as a vector $|\Psi\rangle \in \mathscr{H}$. Thus viewing a state as a list of correlation functions on $\mathscr{A}$ or as a vector in a Hilbert space are essentially equivalent. [Witten, 2022],[Hollands & Wald, 2014]

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- However, by considering states as lists of correlation functions one is now freed from choosing in advance a particular Hilbert space!

Algebraic Scattering Theory

- Given a set of correlation functions Ψ_{in} on the "in" algebra $\mathscr{A}_{\text{in}}$ (i.e. thus specifying the "in" state) then what is the expected value of any "out" observable in $\mathscr{A}_{\text{out}}$ (which would then specify the "out" state)?

$$\langle a_{\text{out}} \rangle_{\Psi_{\text{in}}} \text{ for any } a_{\text{out}} \in \mathscr{A}_{\text{out}}.$$

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- To compute $\langle a_{\text{out}} \rangle_{\Psi_{\text{in}}}$ we can use the Heisenberg equations of motion to define an (invertible) map between the "in" and "out" Algebras [Källén, '49]

$$S : \mathscr{A}_{\text{out}} \to \mathscr{A}_{\text{in}} \implies \langle a_{\text{out}} \rangle_{\Psi_{\text{in}}} = \langle S[a_{\text{out}}] \rangle_{\Psi_{\text{in}}}$$

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- The (perturbative) formulation of algebraic scattering theory for a massive scalar field coupled to a massless scalar field can be straightforwardly constructed and one can compute the correlation functions of any "out" observables (fields, memory, charges, ...) to any order in perturbation theory [G.S., K. Prabhu, in prep.].

Bad Things Happen to "Good" Scattering Data

- In any gauge theory, the charges $\mathcal{Q}_{i^0}(\lambda)$ have serious implications for coherence. This ultimately comes from the charges "superselect". Any local gauge invariant observable $\mathcal{O}$ commutes with all of the charges

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- A more familiar case of superselection is the total electric charge $\mathcal{Q}(1)$. Given states Ψ_{q_1} and Ψ_{q_2} with total charge q_1 and q_2, a standard argument shows that the matrix element $\langle \Psi_{q_1} | \boldsymbol{O} | \Psi_{q_2} \rangle$ for any local gauge invariant observable $\boldsymbol{O}$ must vanish

$$\langle \Psi_{q_1} | [\mathcal{Q}(1), \boldsymbol{O}] | \Psi_{q_2} \rangle = (q_1 - q_2) \langle \Psi_{q_1} | \boldsymbol{O} | \Psi_{q_2} \rangle = 0$$

  Therefore, if $q_1 \neq q_2$ then $\langle \Psi_{q_1} | \boldsymbol{O} | \Psi_{q_2} \rangle = 0$. In other words, for any local gauge invariant observable $\boldsymbol{O}$, a superposition of Ψ_{q_1} and Ψ_{q_2} is an *incoherent* superposition — these states cannot interfere.

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- An incoming electron Ψ_f has definite total electric charge but is a superposition of incoming momenta and therefore superposition of large gauge charges.

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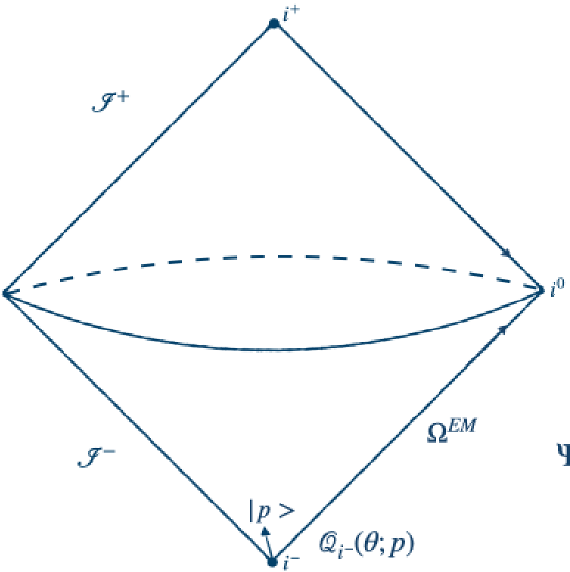

$$\mathcal{Q}_{i^0}(\theta) = \mathcal{Q}_{i^-}(\theta;p)$$

$$\Psi_f = \int d^3p\, f(p)\,|p> \otimes \Omega^{EM}$$

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The expected value $\langle \boldsymbol{O}(x) \rangle_{\Psi_f}$ is spacetime translation invariant for any gauge invariant observable $\boldsymbol{O}$ [D. Danielson, **G.S.** & R. M. Wald, in prep.]

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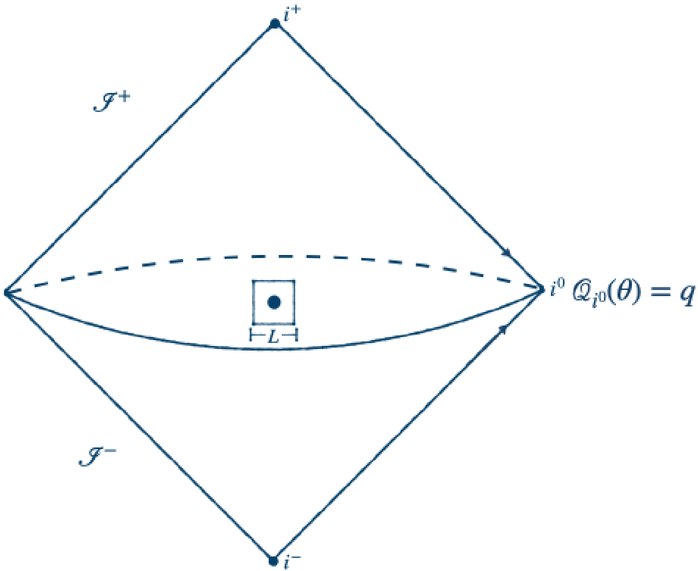

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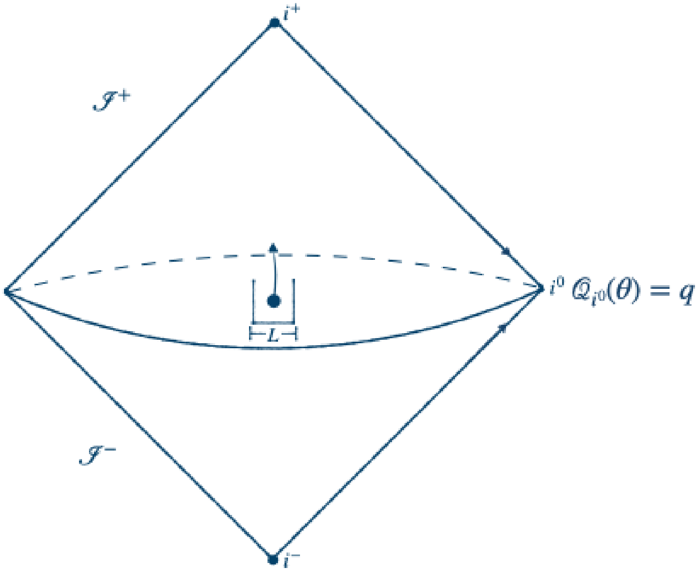

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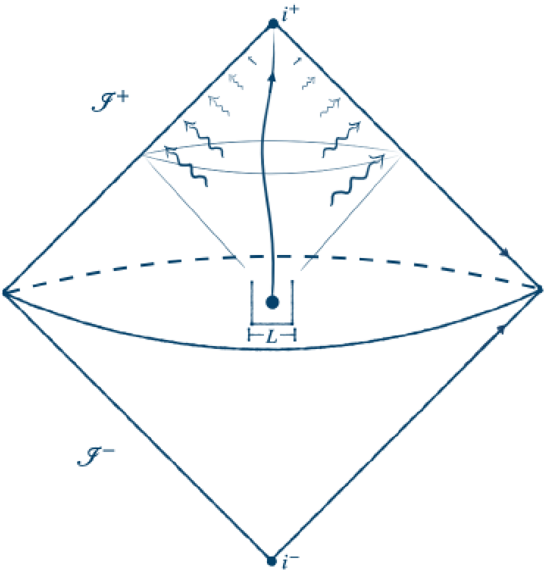

$$\Psi_{FK} = \int d^3p\, f(p)\,|p> \otimes \Psi^{EM}_{\Delta(q,p)}$$

$$\frac{1}{4\pi}\mathscr{D}^a \Delta_a^{out}(\theta; p, q) = \mathcal{Q}_{i^-}(\theta; p) - q$$

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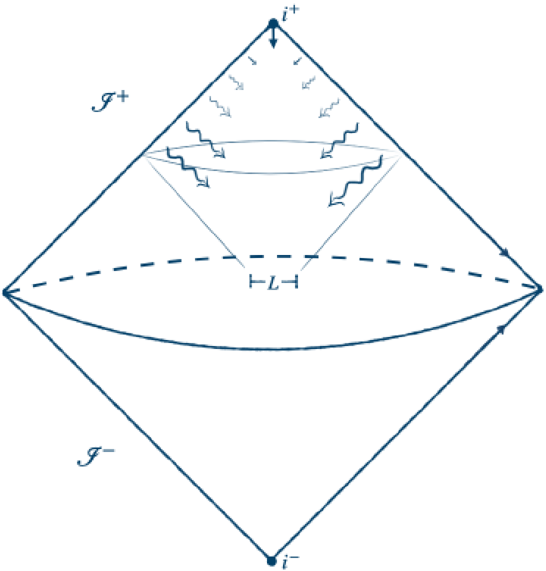

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- The (extremely fine tuned!) entanglement and absorption of “soft radiation” of the FK state is responsible for the localization of the electron in the bulk!

Summary

- IR divergences arise from sticking a state in a Hilbert space to which it doesn't belong.
- In massive QED the Faddeev-Kulish representation is a preferred representation but, as opposed to a "proof of principle" it is actually a "fluke"!
- Non-Kulish-Faddeev representations don't work
- A well-defined (IR-finite) scattering theory can be constructed by simply evolving "in" correlation functions to "out" correlation functions.
- Due to the infrared properties of the theory, the space of asymptotic states in QED (or Yang Mills) which correspond to physical "bulk" states are highly fine-tuned and there are many states that are "junk"!

Failure of FK: Massless QED and Linearized Gravity

$$\mathcal{Q}_{i^0}(\lambda) = \mathcal{J}(\lambda) - \frac{1}{4\pi}\int_{\mathbb{S}^2} \Delta_a^{\text{in}} \mathscr{D}^a \lambda$$

- In massless QED, the analogous construction is to pair eigenstates of the incoming charge-current flux with memory. However, the eigenvalue is now a δ-function on $\mathbb{S}^2$. The required "dressings" have "collinear divergences" and therefore have infinite energy! All "FK states" are unphysical except the vacuum

[Kinoshita,'62],[Lee & Nauenberg, '64]

$$\mathcal{Q}_{i^0}^{\text{GR}}(f) = -\frac{1}{8\pi}\int_{\mathbb{S}^2} \Delta_{ab}^{\text{in}} \mathscr{D}^a \mathscr{D}^b f(\theta) + \int_{\mathscr{I}^-} f(\theta) T_{vv}(v,\theta)$$

- In linearized quantum gravity one can again repeat the FK construction. [Akhoury & Choi, 2017] In this case there are no collinear divergences so the "dressings" are not singular. However, we cannot set $\mathcal{Q}_{i^0}^{\text{GR}} = 0$ since this would set the total four-momenum to zero! (*Can't hide mass behind the moon!*) All "FK states" have undefined angular momentum except the vacuum