Burns holography

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Work with K. Costello & N. M. Paquette

- Celestial holography: quantum gravity in asymptotically flat spacetimes is dual to a CFT on the celestial sphere

[Strominger '17]
[Pasterski, Pate, Raclariu '21]

- This talk: build an explicit example on *Burns space*

Costello, Paquette, AS
[PRL 061602]
[2306.00940]

[Costello, Paquette '22]

- Main takeaway: in our example

Celestial holography	=	Twisted holography	+	Twistor strings

- Twistor space of M = projective 2-spinor bundle

$$S^2 \to Z \to M$$

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[Atiyah, Hitchin, Singer '78]

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$$Z \simeq \mathbb{SL}_2(\mathbb{C}) + \text{boundaries}$$

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- Recycle holography for B-model on deformed conifold

$$\mathbb{SL}_2(\mathbb{C}) \simeq \mathrm{AdS}_3 \times S^3$$

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- KK reduce along S^2 fibers of $Z \to M$

- Burns metric is a Euclidean signature, asymptotically flat, Kähler metric that is scalar-flat (but not Ricci-flat)

Kähler potential

$$K = |u|^2 + N \log |u|^2 \qquad u_i \in \mathbb{C}^2 - 0$$

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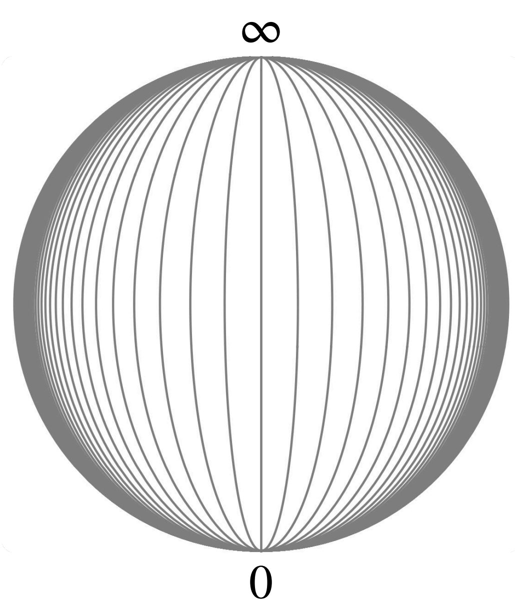
Foliation of AdS_3 *by the* S^2 *fibers of* $Z \to M$



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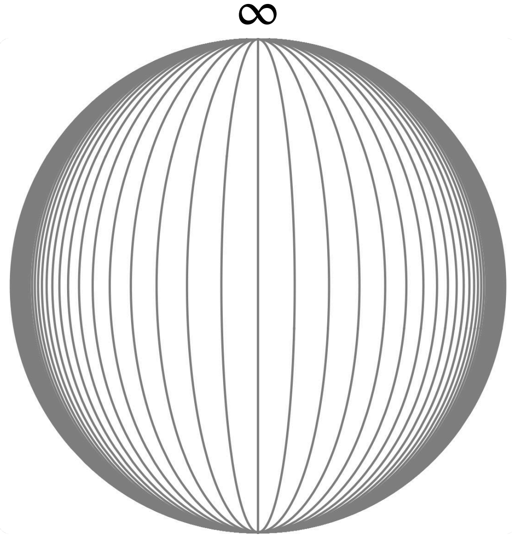

$\times \; S^3$ —*Twistor fibration*→ $\mathrm{Blowup}_0(\mathbb{C}^2)$

Type I B-model —*KK reduction*→ Mabuchi gravity + 4d WZW model

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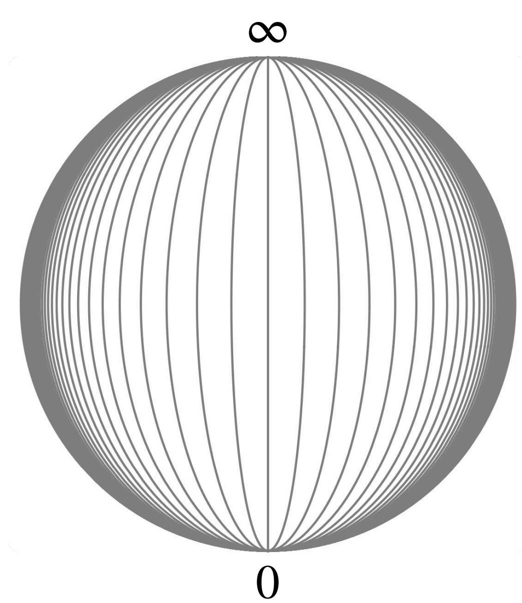

$\times \quad S^3$ $\xrightarrow{\textit{Twistor fibration}}$ $\mathrm{Blowup}_0(\mathbb{C}^2)$

Conformal symmetry broken to the $\mathbb{C}^*$ preserving $0, \infty$

$\Longrightarrow$ *the dual theory has defects at* $0, \infty$

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$\xrightarrow{\textit{KK reduction}}$

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+
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The theory on Burns space

- $\mathcal{K} = K + \rho$
- $\mathrm{g} : \text{Burns space} \to \mathrm{SO}(8)$

[Mabuchi '86] [Donaldson '85] [Nair '91]
[Losev, Moore, Nekrasov, Shatashvili '95]
[Ooguri, Vafa '91] [Yang '77]

$$S_{4\mathrm{d}} = \int_M \mathrm{Ric}(\mathcal{K}) \wedge \partial\mathcal{K} \wedge \bar{\partial}\mathcal{K} + \int_M \partial\bar{\partial}\mathcal{K} \wedge \mathrm{tr}\left(\mathrm{g}^{-1}\partial\mathrm{g} \wedge \mathrm{g}^{-1}\bar{\partial}\mathrm{g}\right) - \frac{1}{3}\int_{M\times[0,1]} \partial\bar{\partial}\mathcal{K} \wedge \mathrm{tr}\left(\tilde{\mathrm{g}}^{-1}\mathrm{d}\tilde{\mathrm{g}}\right)^3$$

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- Eoms:

$$R(\mathcal{K}) = 0$$
$$F^-(A) = 0\,, \quad A = -\bar{\partial}\mathrm{g}\,\mathrm{g}^{-1}$$

- The Wess-Zumino term imposes quantization of N.

The celestial chiral algebra

- Theory of N D1s + 8 D5s + O5 in twistor space of $\mathbb{C}^2$ [Costello '21]
- $\mathrm{Sp}(N)$ gauge theory with $\mathrm{SO}(8)$ flavor symmetry [Costello, Li '19]

D1-D1 $\quad X_{iab}(z) \in \mathbb{C}^2 \otimes \wedge^2 \mathbb{C}^{2N} \qquad X_{iab}(z)\, X_{jcd}(w) \sim \dfrac{1}{z-w}\, \varepsilon_{ij}\varepsilon_{a[c|}\varepsilon_{b|d]}$

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Defect boundary conditions

$$X \sim 1/z \text{ at } z = 0, \infty$$

$$I \sim 1/\sqrt{z} \text{ at } z = 0, \infty$$

"Ramond puncture"

Most general poles consistent with regularity of bulk-brane couplings (Koszul duality)

[Costello, Paquette '20, '22]
[Paquette, Williams '21]

Testing the duality: an example

- Gauge invariant operators:

Soft currents

[Strominger '21]

[Guevara, Himwich, Pate Strominger '21]

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- Planar correlator without defects reproduces 2-pt amplitude of Hawking, Page & Pope '80.

$$\left\langle J_{pq}(\omega_1, z_1, \bar{z}_1)\, J_{rs}(\omega_2, z_2, \bar{z}_2)\right\rangle = -\frac{N}{z_{12}^2}\, J_0\left(\sqrt{4N\omega_1\omega_2 z_1 z_2 \frac{\bar{z}_{12}}{z_{12}}}\right) \mathrm{tr}(\mathfrak{t}_{pq}\mathfrak{t}_{rs})$$

Further tests & future directions

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- Can we discover dualities for self-dual Einstein gravity?

  [Skinner '13]
  [Bittleston, Heuveline, Skinner '23]