A holographic dual for Krylov complexity

or

Measuring the wormhole with Krylov complexity

Ruth Shir

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work with:
Eliezer Rabinovici, Adrián Sánchez-Garrido and Julian Sonner

Based on:

- E. Rabinovici, A. Sánchez-Garrido, RS and J. Sonner, "A bulk manifestation of Krylov complexity," arXiv:2305.04355[hep-th]
- E. Rabinovici, A. Sánchez-Garrido, RS and J. Sonner, "Krylov complexity from integrability to chaos," JHEP 07 (2022), 151
- E. Rabinovici, A. Sánchez-Garrido, RS and J. Sonner, "Krylov localization and suppression of complexity," JHEP 03 (2022), 211
- E. Rabinovici, A. Sánchez-Garrido, RS and J. Sonner, "Operator complexity: a journey to the edge of Krylov space," JHEP 06 (2021), 062
- J. L. F. Barbón, E. Rabinovici , RS and R. Sinha, "On The Evolution Of Operator Complexity Beyond Scrambling," JHEP 10 (2019), 264

Plan

- Why complexity? Which notion of complexity?
- Krylov complexity
- A holographic dual for Krylov complexity
- Open questions and future directions

A puzzle in the holographic dictionary

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What could be a corresponding observable on the boundary?

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Question

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Answer

Quantum complexity??? [Susskind 2014 –]

General time-dependent profile of complexity

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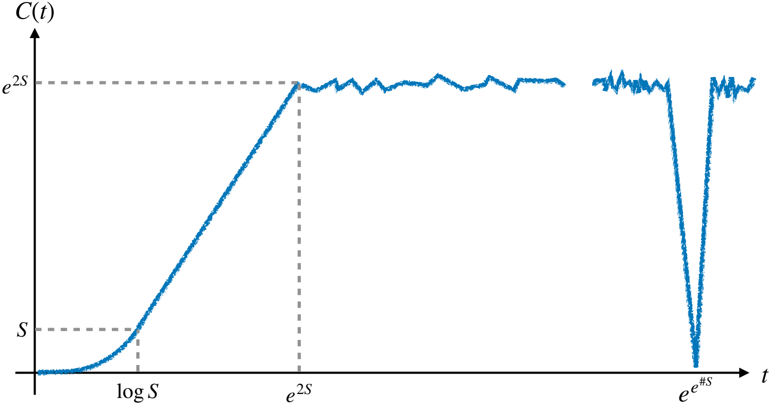

Different notions of quantum complexity

- **Circuit complexity:** The minimal number of local gates g_i needed to construct an operator U starting from an initial operator U_0, up to a tolerance parameter ϵ
- **Geometric complexity:** Choose a penalty metric on space of unitaries; define complexity as shortest path to reach U from U_0
- **Krylov complexity:** Defined using the system's Hamiltonian and initial state/operator

Can we find a precise bulk-boundary correspondence?

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[Rabinovici Sánchez-Garrido RS Sonner 2020]

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- No degeneracies in the spectrum of the Liouvillian $\omega_{ab} = E_a - E_b$ except for the D-fold degeneracy of zero frequencies $\omega_{aa} = E_a - E_a = 0$

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where K is the Krylov space dimension

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- Define **position** operator over Krylov basis

$$\hat{n} = \sum_{n=0}^{K-1} n\,|\mathcal{O}_n)(\mathcal{O}_n| \tag{8}$$

Krylov complexity:
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[Parker Cao Avdoshkin Scaffidi Altman 2018]

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- and as a measure of operator complexity at *all time scales* for *finite systems* in [Barbón Rabinovici RS Sinha 2019]

Dynamics of Krylov complexity

[Parker et al 2018] [Barbón Rabinovici RS Sinha 2019] [Rabinovici Sánchez-Garrido RS Sonner 2020]

n	b_n	wavefunction	K-complexity	time scales

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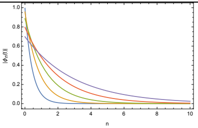
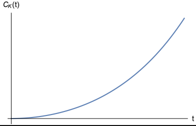
$n \ll S$	$b_n \sim n$			$t \lesssim \log S$

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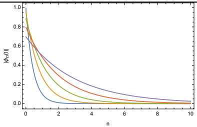
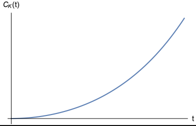
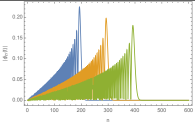
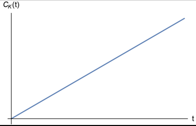
$n \ll S$	$b_n \sim n$			$t \lesssim \log S$
$n > S$	$b_n \sim \Lambda S$			$t > \log S$

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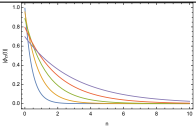
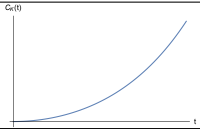
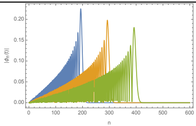
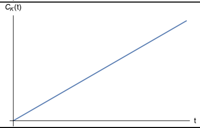
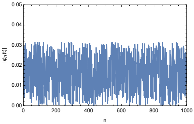
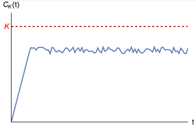
$n \ll S$	$b_n \sim n$			$t \lesssim \log S$
$n > S$	$b_n \sim \Lambda S$			$t > \log S$
$n \sim e^{2S}$	“descent”			$t \gtrsim e^{2S}$

Numerical results: SYK_4 setup

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- SYK_4 is a maximally chaotic many-body system. Consider complex SYK_4 with L fermions

$$H_{SYK} = \sum_{i,j,k,l}^{L} J_{ij,kl} c_i^\dagger c_j^\dagger c_k c_l \tag{10}$$

where $\{c_i, c_j^\dagger\} = \delta_{ij}$ and $\{c_i, c_j\} = 0 = \{c_i^\dagger, c_j^\dagger\}$

- The coupling constants are taken from a Gaussian distribution with

$$\overline{J_{ij,kl}} = 0 \quad \overline{|J_{ij,kl}|^2} = \frac{3! J^2}{L^3} \tag{11}$$

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- Operator taken to be the hopping operator

$$\mathcal{O} = c_{L-1}^\dagger c_L + c_L^\dagger c_{L-1} \tag{12}$$

shown in [Sonner Vielma 2017] to satisfy ETH

Non-perturbative “Descent”

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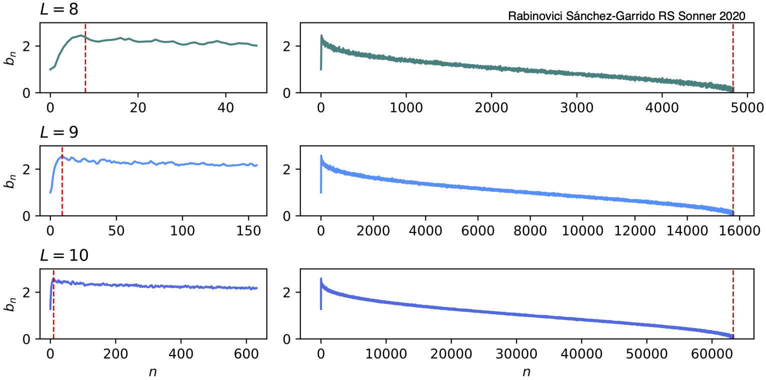

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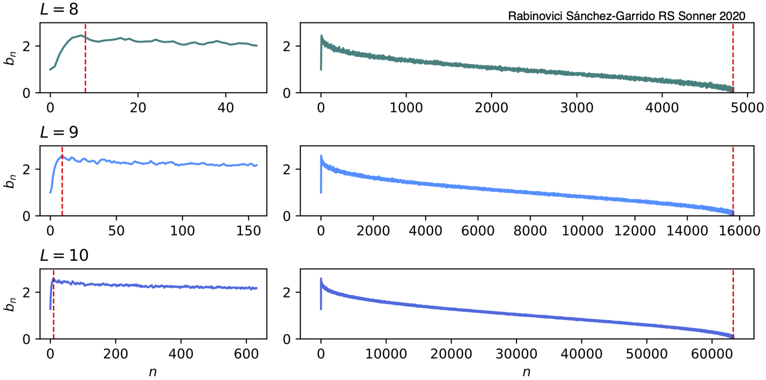

Features a non-perturbative e^{-2S} slope

[Rabinovici Sánchez-Garrido RS Sonner 2020]

K-complexity for SYK_4 with 10 complex fermions

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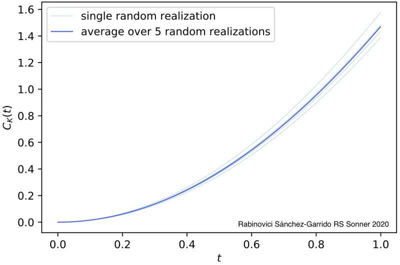

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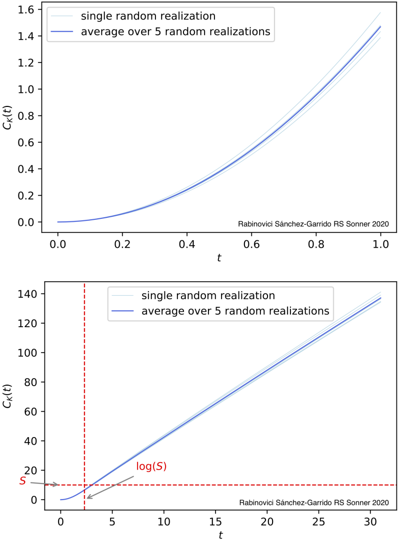

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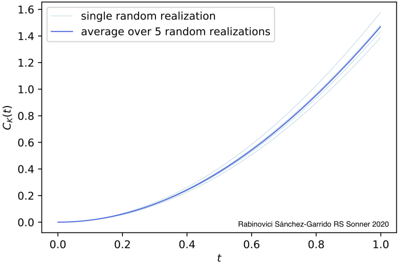

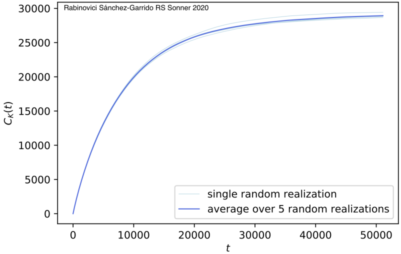

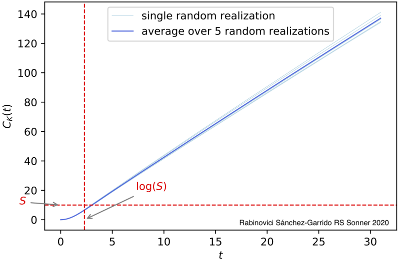

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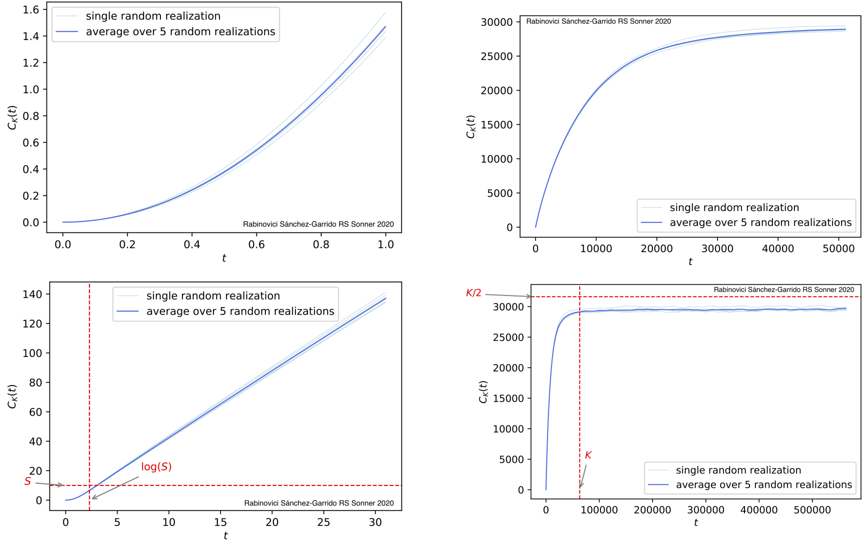

K-entropy

[Barbón Rabinovici RS Sinha 2019]

- K-entropy measures the amount of disorder in the wavefunction

$$S_K(t) = -\sum_{n=0}^{K-1} |\phi_n(t)|^2 \log |\phi_n(t)|^2 \tag{13}$$

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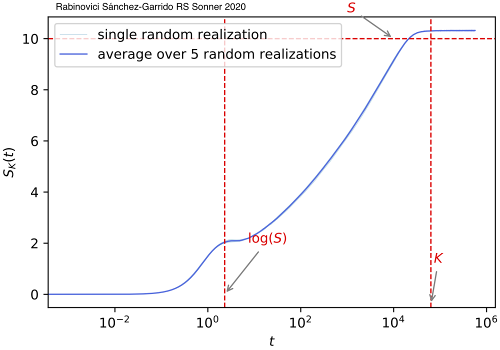

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Suppression of K-complexity in XXZ

- Anderson localization on the Krylov chain

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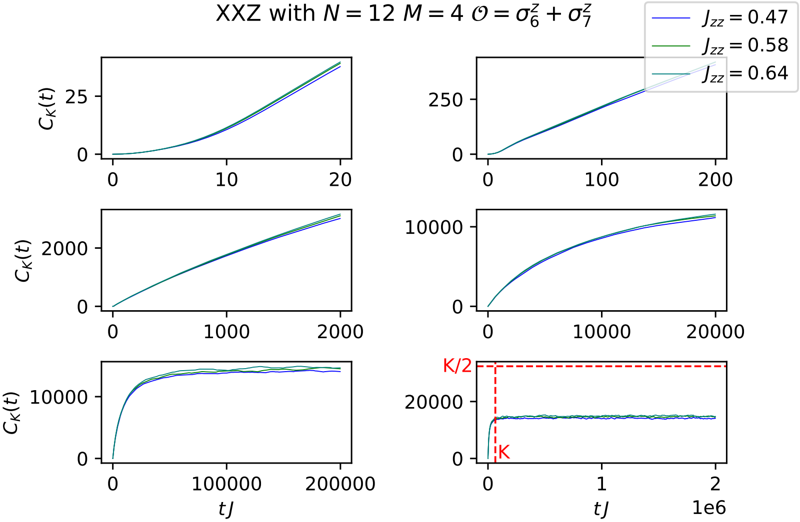

Krylov space dimension $K = 64771$

[Rabinovici Sánchez-Garrido RS Sonner 2021]

Krylov complexity for states

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- Studied in [Balasubramanian Caputa Magan Wu 2022]

A holographic dual for Krylov complexity

The boundary: DSSYK

[Berkooz Narayan Simon 2018][Berkooz Isachenkov Narovlansky Torrents 2018]

- Start with SYK with N fermions $\{\psi_i, \psi_j\} = 2\delta_{ij}$

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- Start with SYK with N fermions and p-body interactions

$$H_{SYK} = i^{p/2} \sum_{1 \leq i_1 \leq i_2 \leq \cdots \leq i_p \leq N} J_{i_1 i_2 \ldots i_p} \, \psi_{i_1} \psi_{i_2} \ldots \psi_{i_p} \tag{16}$$

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- Define the ratio parameter $\lambda \equiv \frac{2p^2}{N}$
- In the limit $N \to \infty$, $p \to \infty$ and λ fixed, the **ensemble averaged** effective Hamiltonian is

$$H = \frac{J}{\sqrt{\lambda(1-q)}} \begin{pmatrix} 0 & \sqrt{1-q} & & & \\ \sqrt{1-q} & 0 & \sqrt{1-q^2} & & \\ & \sqrt{1-q^2} & 0 & \sqrt{1-q^3} & \\ & & \sqrt{1-q^3} & 0 & \ddots \\ & & & \ddots & \ddots \end{pmatrix}$$

where $q \equiv e^{-\lambda}$

Krylov complexity for DSSYK

- In [Rabinovici Sánchez-Garrido RS Sonner 2023] we showed that this Hamiltonian is written in the Krylov basis for H with initial state

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$$C_K(t) = \langle \hat{n}(t) \rangle \tag{19}$$

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$$H = -\frac{2J}{\lambda} + 2\lambda J\left(\frac{k^2}{2} + 2e^{-\tilde{l}}\right) \tag{21}$$

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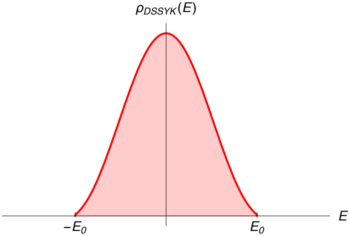

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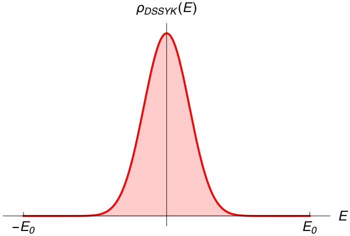

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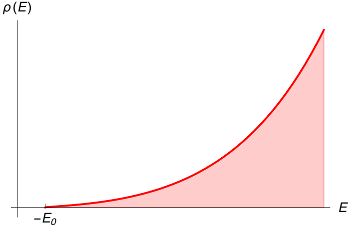
- In the triple-scaled limit we take $\lambda \to 0$ and zoom-in near the ground state of DSSYK



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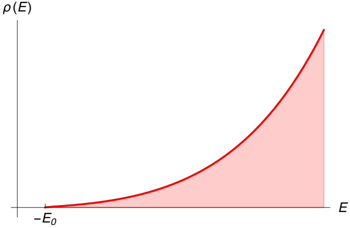

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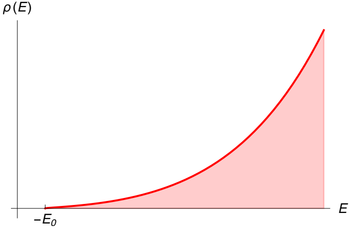

- The d.o.s. in the triple-scaled limit is $\rho(E) \propto \sinh(2\pi\sqrt{E})$
- The **Liouville** form of the triple-scaled Hamiltonian connects DSSYK with the Hamiltonian of JT gravity

K-complexity in the triple-scaled limit of SYK

[Rabinovici Sánchez-Garrido RS Sonner 2023]

- Recall that $l = \lambda n$ and hence $C_K(t) = \frac{\langle l(t) \rangle}{\lambda}$
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The bulk: JT gravity

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- The action of JT gravity

$$S_{JT} = \int_{\mathcal{M}} d^2x \sqrt{-g}\Big[\Phi_0 R + \Phi(R+2)\Big] + 2\int_{\partial\mathcal{M}} dx\sqrt{\gamma}\Big[\Phi_0 K + \Phi(K-1)\Big] \qquad (23)$$

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- Equations of motion

$$0 = R + 2 \Longrightarrow \mathrm{AdS}_2 \tag{25}$$

$$0 = (\nabla_\mu\nabla_\nu - g_{\mu\nu})\Phi \tag{26}$$

Wormhole (ERB) length

[Harlow Jafferis 2018]

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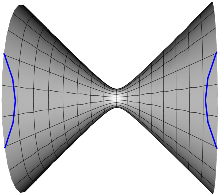

(a) Global coordinates
$ds^2 =$
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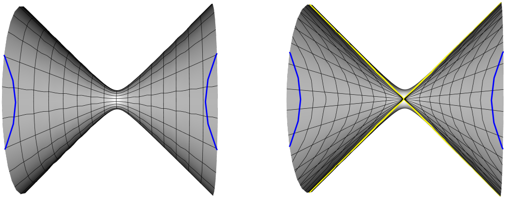

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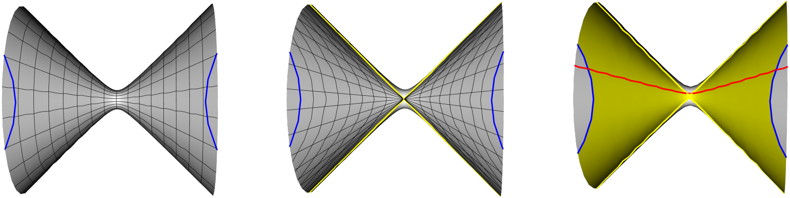

(a) Global coordinates $ds^2 = -(1+x^2)d\tau^2 + \frac{dx^2}{1+x^2}$ $\Phi(x,\tau) = \Phi_h\sqrt{1+x^2}\cos\tau$

(b) Schwarzschild coordinates $ds^2 = -(r^2 - r_s^2)dt^2 + \frac{dr^2}{r^2 - r_s^2}$ $\Phi(r,t) = \phi_b r$

(c) The **wormhole** length defined as the geodesic distance between two points on the boundaries

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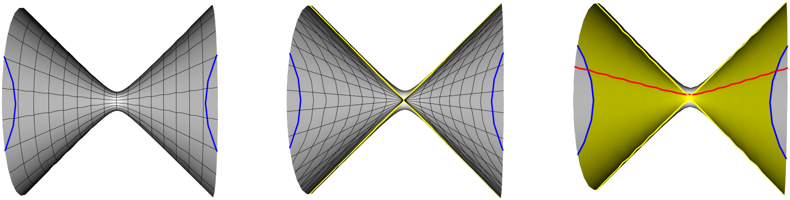

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(b) Schwarzschild coordinates $ds^2 = -(r^2 - r_s^2)dt^2 + \frac{dr^2}{r^2-r_s^2}$ $\Phi(r,t) = \phi_b r$

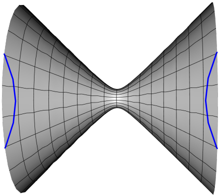
(c) The **wormhole** length defined as the geodesic distance between two points on the boundaries

- Wormhole length

$$\tilde{l} = 2\log\left(\frac{2\phi_b}{\epsilon}\right) + 2\log\left[\cosh\left(\frac{\Phi_h}{\phi_b}t_b\right)\right] - 2\log\Phi_h \qquad (27)$$

Wormhole (ERB) length

[Harlow Jafferis 2018]



(a) Global coordinates
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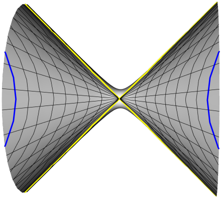
$\Phi(x,\tau) = \Phi_h\sqrt{1+x^2}\cos\tau$



(b) Schwarzschild coordinates
$ds^2 = -(r^2 - r_s^2)dt^2 + \frac{dr^2}{r^2-r_s^2}$
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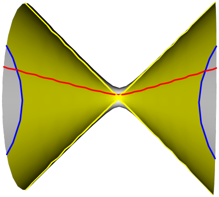
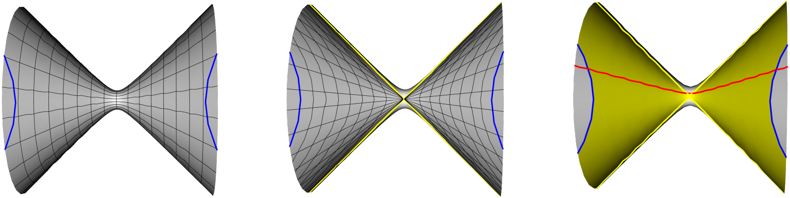


(c) The **wormhole** length defined as the geodesic distance between two points on the boundaries

- **Renormalized** wormhole length

Wormhole (ERB) length

[Harlow Jafferis 2018]



(a) Global coordinates
$ds^2 = -(1+x^2)d\tau^2 + \frac{dx^2}{1+x^2}$
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$ds^2 = -(r^2 - r_s^2)dt^2 + \frac{dr^2}{r^2-r_s^2}$
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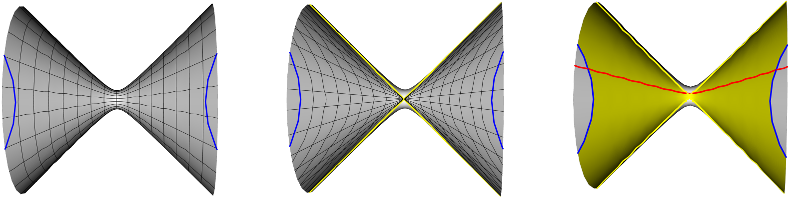
(c) The **wormhole** length defined as the geodesic distance between two points on the boundaries

- **Renormalized** wormhole length

$$\tilde{l} = 2\log\left[\cosh\left(\frac{\Phi_h}{\phi_b}t_b\right)\right] - 2\log\Phi_h \qquad (27)$$

Wormhole (ERB) length

[Harlow Jafferis 2018]



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- **Renormalized** wormhole length

$$\tilde{l} = 2\log\left[\cosh\left(\sqrt{\frac{E}{2\phi_b}}t_b\right)\right] - \log\left(\frac{\phi_b E}{2}\right) \qquad (27)$$

Krylov complexity = wormhole length

[Rabinovici Sánchez-Garrido RS Sonner 2023]

K-complexity in triple-scaled SYK

$$\lambda C_K(t) = 2\log\left[\cosh\left(\sqrt{\lambda J E}\, t\right)\right] - \log\left(\frac{E}{4\lambda J}\right) \qquad (28)$$

Krylov complexity = wormhole length

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Wormhole length in JT gravity

$$\tilde{l}(t) = 2\log\left[\cosh\left(\sqrt{\frac{E}{2\phi_b}} t\right)\right] - \log\left(\frac{\phi_b E}{2}\right) \qquad (29)$$

Summary, open questions and future directions

Result

Krylov basis in triple-scaled SYK = Wormhole length basis in JT gravity

Summary, open questions and future directions

Result

Krylov basis in triple-scaled SYK = Wormhole length basis in JT gravity

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Krylov basis in triple-scaled SYK = Wormhole length basis in JT gravity

Krylov complexity in triple-scaled SYK = Wormhole length in JT gravity

- Quantum corrections?

Summary, open questions and future directions

Result

Krylov basis in triple-scaled SYK = Wormhole length basis in JT gravity

Krylov complexity in triple-scaled SYK = Wormhole length in JT gravity

- Quantum corrections?
- Going away from the small λ limit

Summary, open questions and future directions

Result

Krylov basis in triple-scaled SYK = Wormhole length basis in JT gravity

Krylov complexity in triple-scaled SYK = Wormhole length in JT gravity

- Quantum corrections?
- Going away from the small λ limit
- Saturation of K-complexity?

Summary, open questions and future directions

Result

Krylov basis in triple-scaled SYK = Wormhole length basis in JT gravity

Krylov complexity in triple-scaled SYK = Wormhole length in JT gravity

- Quantum corrections?
- Going away from the small λ limit
- Saturation of K-complexity?
- The Krylov basis with operators is richer, what is the precise geometric interpretation of operator insertions in DSSYK

Summary, open questions and future directions

Result

Krylov basis in triple-scaled SYK = Wormhole length basis in JT gravity

Krylov complexity in triple-scaled SYK = Wormhole length in JT gravity

- Quantum corrections?
- Going away from the small λ limit
- Saturation of K-complexity?
- The Krylov basis with operators is richer, what is the precise geometric interpretation of operator insertions in DSSYK
- Higher dimensional gravity?