Black hole interiors

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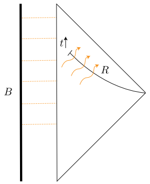
- String theory and especially AdS/CFT have given us a strong reason to accept (1) and (2).
- This leaves us with two options:

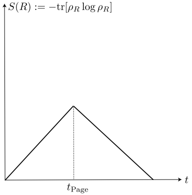
A. Give up on (3). The interior of the black hole is very different from semiclassical expectations.

B. Find some way to reconcile (3) with (1) & (2).

A version of the information problem

- Say we have a holographic CFT in some high energy state, coupled to a reservoir R, and we time evolve,





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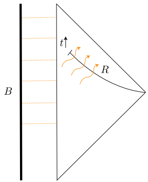
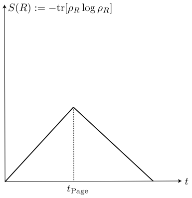
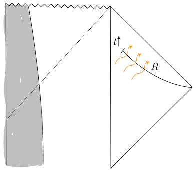
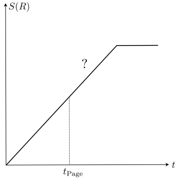




- Many of these states have bulk descriptions that look like evaporating black holes,





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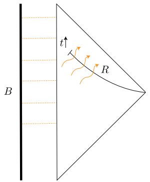

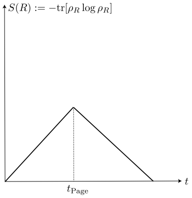

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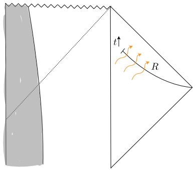

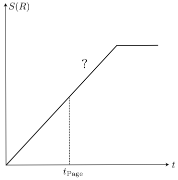

- This is an illustration of the information problem: the bulk description has an interior but naively not unitary.

Quantum extremal surfaces

- Amazingly, this apparent mismatch is resolved by the *quantum extremal surface formula*, here taking the form

  [QES: Ryu-Takayanagi '06, Hubeny-Rangamani-Takayanagi '07, Faulkner-Lewkowycz-Maldacena '13, Engelhardt-Wall '14]

  [This application: Penington '19, Almheiri-Engelhardt-Marolf-Maxfield '19]

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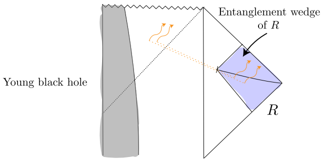

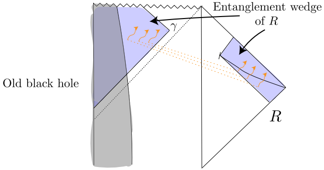

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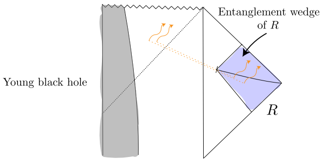
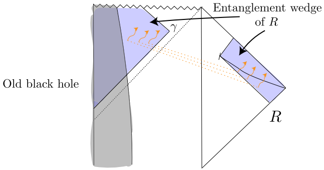
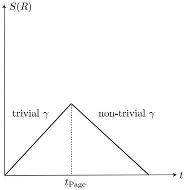




- The Page curve is correctly computed!



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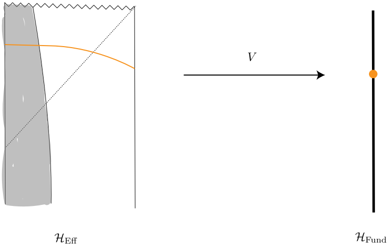
  [Ryu-Takayanagi '06, Hubeny-Rangamani-Takayanagi '07, Engelhardt-Wall '14]
- Then it was "derived" from the gravitational path integral in increasing generality [Lewkowycz-Maldacena '13, Faulkner-Lewkowycz-Maldacena '13, Dong-Lewkowycz '17, Penington-Shenker-Stanford-Yang '19, Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini '19]

QES and interiors from quantum codes

- There's also a directly Hilbert space way to understand both the QES formula and directly how to reconcile nice interiors with unitarity: "quantum codes." [Harlow '16, CA-Penington '21, CA-Engelhardt-Harlow-Penington-Vardhan '22]



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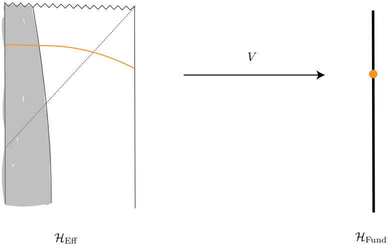

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  But only more recently have we understood how to extend that story to black hole interiors,

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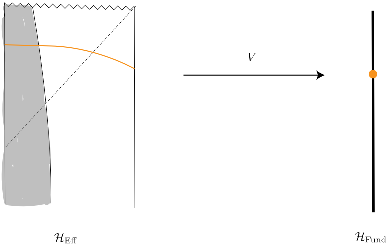

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- The new subtlety was that $|\mathcal{H}_{\mathrm{Eff}}| > |\mathcal{H}_{\mathrm{Fund}}|$, "non-isometric code". Inner products are not preserved – information loss??

Quantum codes: toy model

- To illustrate the QES formula and a non-isometric code, consider

$$V\left|n\right\rangle_r = \frac{1}{\sqrt{|B|}}\sum_{b=1}^{|B|} e^{i\theta(n,b)}\left|b\right\rangle_B$$

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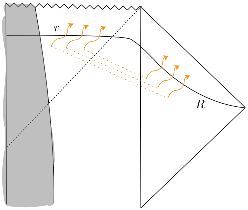
- This models the nice property that inner products are approximately preserved

$$\langle n'|V^\dagger V|n\rangle = \begin{cases} 1 & n' = n \\ O\left(1/\sqrt{|B|}\right) & n' \neq n \end{cases}$$

even for $|r| \gg |B|$.

QES from quantum codes: toy model

- The analog of the "Hawking state" is



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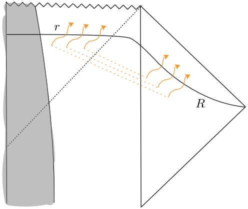

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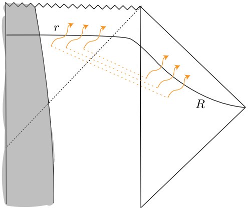

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- One can then compute e.g.

$$\text{tr}(\rho^2) = \frac{1}{|B|^2} \sum_{\substack{n,n' \\ b,b'}} D_n D_{n'} e^{i\left(\theta(n,b) - \theta(n',b) + \theta(n',b') - \theta(n,b')\right)} \approx \sum_n D_n^2 + \frac{1}{|B|}$$

$$S_2(\rho_R) := -\frac{1}{2} \log \text{tr}(\rho_R^2) \approx \min\left(S_2(\psi_{\text{Hawk,R}}), \log |B|\right)$$

Operator reconstruction

- Either way we think about the QES formula, it has dramatic implications for thinking about operators in quantum gravity, because of "entanglement wedge reconstruction." [Czech-Karczmarek-Noguiera-van Raamsdonk '12, Jafferis-Lewkowycz-Maldecena-Suh '15, Dong-Harlow-Wall '16, Harlow '16, CA-Penington '21]

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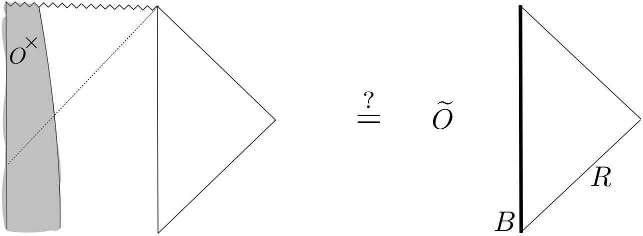

What operator $\widetilde{O}$ on BR satisfies

$$\widetilde{O}V\left|\psi\right\rangle \simeq VO\left|\psi\right\rangle$$

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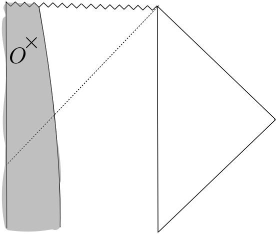
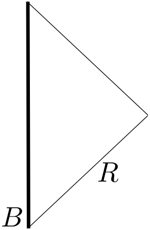


$\stackrel{?}{=}$ $\widetilde{O}$



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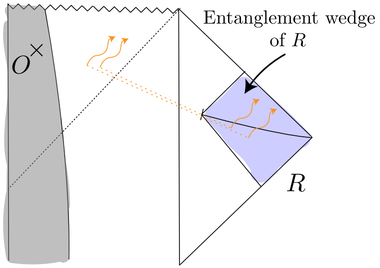
- Theorem: $\widetilde{O}$ can have support on only R (or B, or BR) iff O acts inside the "entanglement wedge" of R (or B, or BR), before and after it acts.

Operator reconstruction: upshot

Young black hole

$\widetilde{O}_R$	no
$\widetilde{O}_B$	yes



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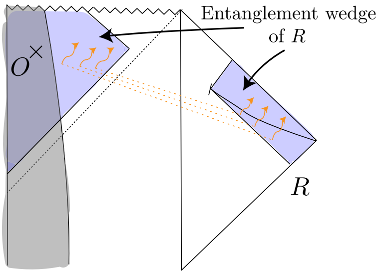

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- This leads to an interesting idea to resolve an issue with the fact that $|\mathcal{H}_{\rm Eff}| > |\mathcal{H}_{\rm Fund}|$: it is impossible that for all states

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  Maybe we should only demand sub-exponentially complex operators make sense in semiclassical gravity.
- This leads to a self-consistent picture: semiclassical gravity is valid for *simple* operators, but *not all* operators. The locality structure of the semiclassical description can fail if you do exponentially complex operations.

Complexity versus information paradox

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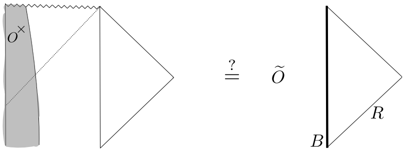
- We can summarize one lesson: there is *no* tension between

  1. A finite black hole entropy
  2. A unitary black hole S-matrix

  3^*. A black hole interior described to a good approximation by gravitational EFT *for low-complexity operators*

State dependence

- Consider again: for O in the interior, what is $\widetilde{O}$ like?



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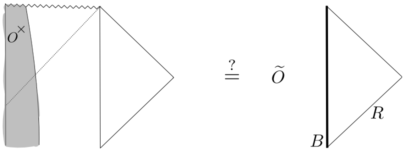

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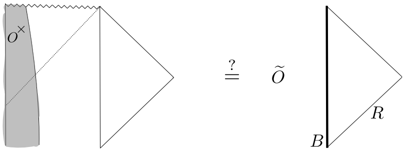

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Old BH: $\widetilde{O}$ must act on R

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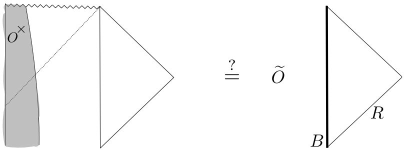

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- No! We find a contradiction if we even demand $\widetilde{O}$ works on states different by a simple U_R: [c.f. CA-Engelhardt-Harlow-Penington-Vardhan '22 theorem 5.1]

$$\int_{\text{simple}} \langle \psi_2 | U_R^\dagger V^\dagger \widetilde{O} V U_R | \psi_1 \rangle \propto \langle \psi_2 | V^\dagger \operatorname{tr}_R[\widetilde{O}] V | \psi_1 \rangle$$

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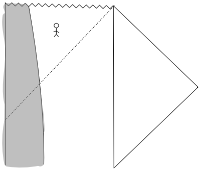
- For example, say Alice makes a measurement of the interior.



Her projection operators do not admit a representation on $\mathcal{H}_{\mathrm{Fund}}$ that works for all states.

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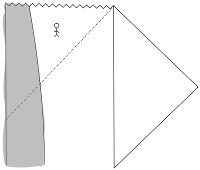
- For example, say Alice makes a measurement of the interior.



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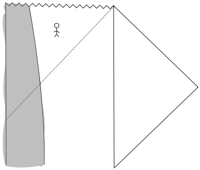

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- However, any measurement on more than $O\left(\log \left|\mathcal{H}_{\mathrm{Fund}}\right|\right)$ modes cannot fit into such a $\mathcal{H}_{\mathrm{code}}$. Unclear how to describe the statistics of these measurements.

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- What's the right answer? How might we settle this? Will this depend on our measurement theory for interior observers?