Comparing the decoherence effects due to black holes versus ordinary matter

based on 2405.02227 with J. Maldacena

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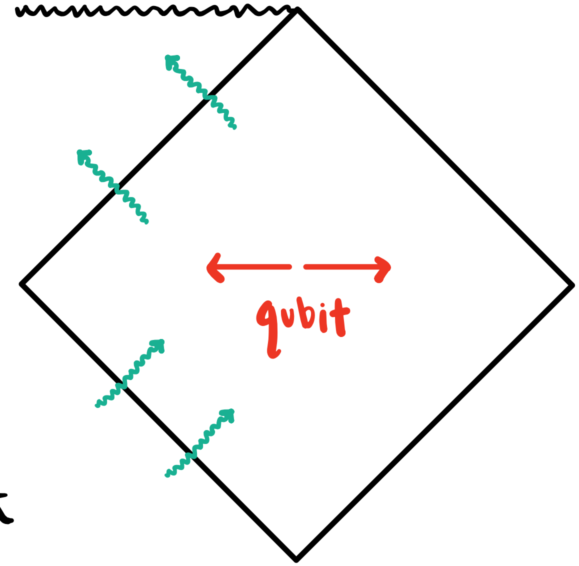
A thought experiment was recently discussed which involves the decoherence of a quantum system due to a black hole.

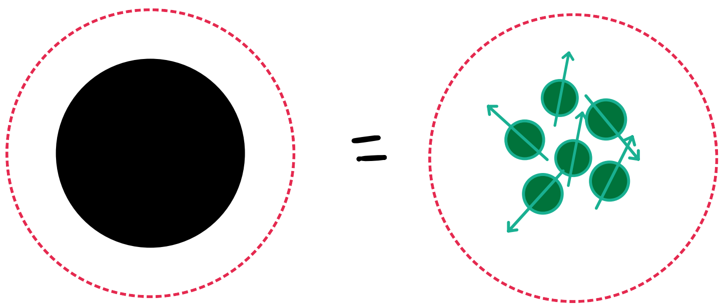
Wald, Satishandran, Danielson 2205.06279, 2301.00026

Gralla, Wei 2311.11461



It has been suggested that this effect may be unique to black holes and/or be of fundamental importance for their quantum description.

On the other hand, we expect that a black hole, when viewed from the outside, is described by an ordinary quantum system evolving unitarily.

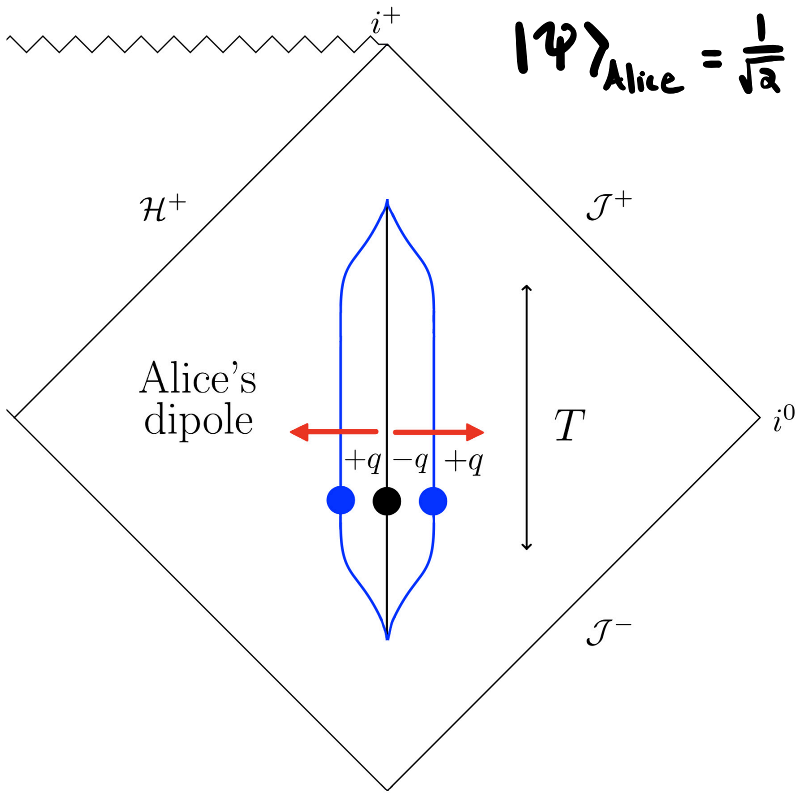


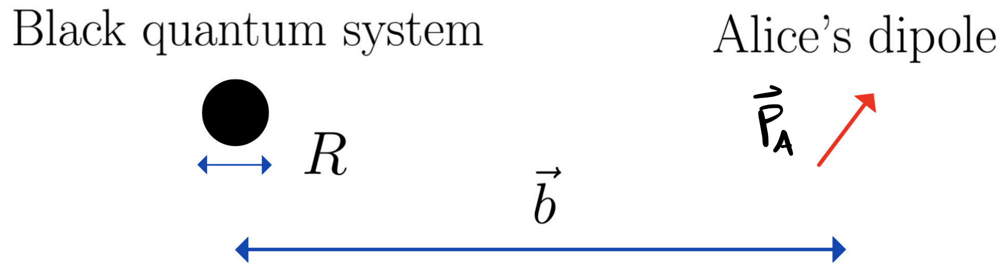
Today: we replace the black hole by an ordinary quantum system at finite temperature and obtain the same qualitative effect.

The idea is to analyze the problem in terms of an effective theory that applies equally well for the black hole case as for an ordinary matter system.

Review of setup for decoherence thought experiment

$$|\psi\rangle_{Alice} = \frac{1}{\sqrt{2}}\left(\left|-\tfrac{d}{2}\right\rangle + \left|+\tfrac{d}{2}\right\rangle\right)$$



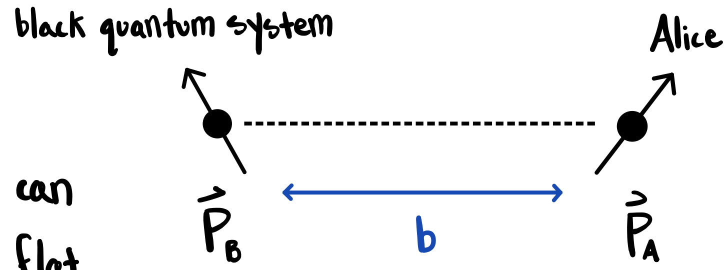


$$T \gg b \gg R$$

The black hole will destroy the coherence of the superposition at a constant rate, i.e. the off-diagonal elements of Alice's density matrix decay as $e^{-\Gamma t}$ for some constant Γ. Wald, Satishandran, Danielson 2205.06279, 2301.00026

There is an analogous gravitational version of the effect (which involves the superposition of a massive particle)

Effective theory picture



The fields sourced by $\vec{P}_A$ have wavelength $\lambda \sim T \gg R$, so we can approximate the black quantum system by a point particle in flat space. Interactions are captured by multipole operators living on the worldline.

$$S_{\text{int}} = -\frac{e^2}{4\pi b^3} \int dt \left(\vec{P}_A \cdot \vec{P}_B - 3 \left(\vec{P}_A \cdot \hat{b} \right) \left(\vec{P}_B \cdot \hat{b} \right) \right) \sigma_3$$

$R\omega \ll 1$

$T \gg b$

Classical vector, indicating the size of Alice's dipole

Operator acting on the Hilbert space of the black quantum system

Pauli matrix acting on Alice's system (parameterizing the two configurations)

When we compute the time evolution of Alice's density matrix under this interaction, we find a constant decoherence rate of the form

$$\Gamma \propto \left(\frac{e^2}{b^3} \right)^2 P_A^2 \int dt \, \langle P_B(t) P_B(0) \rangle$$

In this framework, the decoherence is arising from thermal or quantum fluctuations of the electric dipole operator describing the black quantum system.

The Fluctuation-Dissipation Theorem

To determine $G_W(\omega) \equiv \int dt\, e^{i\omega t} \langle P_B(t) P_B(0) \rangle$ for various systems of interest, we compute the response function $\chi(t) \equiv i\theta(t) \langle [P_B(t), P_B(0)] \rangle$ which is related to $G_W(\omega)$ by the fluctuation-dissipation theorem:

$$G_W(\omega) = 2(n_B(\omega)+1)\,\mathrm{Im}\,\chi(\omega), \quad n_b(\omega) \equiv \frac{1}{e^{\beta\omega}-1} \overset{\beta\omega \ll 1}{\approx} \frac{1}{\beta\omega}$$

For an ordinary matter system, $\chi(\omega)$ depends on transport coefficients such as the conductivity and viscosity.

For black holes, the low-frequency expansion of $\chi(\omega)$ is given by the (static and dynamical) "Love numbers",

$$\chi(\omega) = A + i\omega B + \mathcal{O}(\omega^2)$$

$A = 0$ for black holes in 4d, but $B \neq 0$.

From a computation of the Love numbers one can read off $G_W(\omega)$ and reproduce the black hole decoherence rates of Wald, Satishandran, Danielson

Comparison to ordinary matter

The response functions set the decoherence rate, so comparing χ_{BH} and χ_{matter} is the same as comparing the decoherence effects.

We compare objects of the same size and at the same temperature.

Response function of spherical conductor with resistivity ρ:

$$\mathrm{Im}\,\chi_{\mathrm{cond}}(\omega) \sim \frac{1}{e^4}\omega\rho R^3$$

BH response function:

$$\mathrm{Im}\,\chi^{e}_{\mathrm{BH}}(\omega) \sim \frac{1}{e^2}\omega r_s^4$$

To be comparable to a black hole, $\rho/R \sim e^2$.

A small ball of some impure metal will do (ex. Al alloy)

Comparison to ordinary matter - gravitational case

We can make a similar comparison for the gravitational effect.

(Ex. Compare to a self-gravitating fluid or an elastic solid with some viscosity)

An ordinary object of the same mass as the BH typically has a larger decoherence effect, while the opposite is true for an object of the same size (when all comparisons are made at the same temperature).

Summary

- The decoherence effect is consistent with the hypothesis that, from the outside, black holes are described by ordinary quantum systems.
- It is qualitatively present for ordinary matter at finite temperature.
- The decoherence arises from thermal fluctuations of the multipole moments of the black hole/matter system.
- For the electromagnetic effect, the decoherence can be of equal magnitude for black holes and ordinary objects.
- For the gravitational effect, ordinary matter produces a weaker effect, if we compare objects of the same size and temperature.

The connection to absorption

$G_W(\omega)$ also governs the absorption of low-frequency fields.

Let us discuss scalar fields for simplicity, where

$$S_{int} = \int dt \, \underbrace{\mathcal{O}(t)}_{\text{operator describing black quantum system}} \, \underbrace{\phi(t)}_{\text{bulk scalar field}}$$

Amplitude for transition from $|i\rangle$ to $|f\rangle$ of black quantum system:

$$\mathcal{M}_{i\to f} = \frac{1}{\sqrt{2\omega}} \int_0^T dt e^{-i\omega t} \langle f|\mathcal{O}(t)|i\rangle$$

$\Gamma_{i\to f} = \sum_f |\mathcal{M}_{i\to f}|^2$, so

$$\Gamma_{i\to f} = \frac{T}{2\omega} \int_0^T dt e^{i\omega t} \langle \mathcal{O}(t)\mathcal{O}(0)\rangle$$

For σ_{abs}, divide $\Gamma_{i\to f}$ by T and incoming particle flux.

$$\sigma_{\text{abs}}(\omega) = \frac{1}{2\omega} \int dt e^{i\omega t} \langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \frac{1}{2\omega} G_W(\omega) \qquad \omega R \ll 1$$

σ_{abs} is related to the classical absorption cross section σ_{abs}^{clas} by

$$\sigma_{\text{abs}}(\omega) = (n_b + 1)\sigma_{\text{abs}}^{\text{clas}}(\omega) \implies \sigma_{abs}^{clas}(\omega) \sim \frac{\beta}{2} G_W(\omega) \quad \text{for } \beta\omega \ll 1$$

Comparison to ordinary matter - more details...

Gravitational case:

BH response function: $\mathrm{Im}\,\chi^{g}_{\mathrm{BH}}(\omega) \propto \omega \frac{r_s^6}{G}$

a) Same mass and same temperature comparison:

$\nu \equiv$ kinematic viscosity
$R_s \equiv$ Schwarzschild radius of fluid

Response function of self-gravitating fluid: $\mathrm{Im}\,\chi(\omega) \sim \omega \frac{\nu R^6}{G R_s}$

Example: self-gravitating ball of water at room temp. $r_s \sim 0.6\,\mu m$, $R \sim 5 \times 10^5$ m

$$\frac{\mathrm{Im}\chi_{\mathrm{fluid}}}{\mathrm{Im}\chi^{g}_{\mathrm{BH}}} \sim \frac{\nu}{\nu_{\mathrm{BH}}}\left(\frac{R}{r_s}\right)^6 \sim 10^{64} \qquad \frac{R}{r_s} \sim 10^{12} \qquad \frac{\nu}{\nu_{\mathrm{BH}}} \sim 10^{-8}$$

A ball of water absorbs gravitons more easily than a black hole of the same mass.

b) Same size and same temperature comparison:

Response function of elastic solid with some viscosity ν: $\mathrm{Im}\chi(\omega)_{\mathrm{metal}} \sim \omega\nu \frac{R^5}{G}\left(\frac{R_s}{R}\right)\left(\frac{c}{c_s}\right)^4$

Example: lead ball at ~ 1.5 K. $r_s \sim 100\,\mu m$, $R_s \sim \ell_p$

$R_s \equiv$ Schwarzschild radius of the solid
$c_s \equiv$ speed of sound in solid

$$\frac{\mathrm{Im}\chi_{\mathrm{metal}}}{\mathrm{Im}\chi^{g}_{\mathrm{BH}}} \sim 10^{-19} \qquad \frac{\nu}{\nu_{\mathrm{BH}}} \sim 10^{-10} \qquad \frac{R_s}{R} \sim 10^{-30} \qquad \left(\frac{c}{c_s}\right)^4 \sim 10^{21}$$

The metal absorbs fewer gravitons than the black hole.

Zero temperature black holes

The near-horizon geometry of an extremal black hole develops an approximate SL(2) symmetry that fixes the form of the correlators.

Scalar effect: the decoherence is not linear in T but $\propto \ln T$

A massless field corresponds to an operator of dimension $\Delta = 1$ so that

$$\int_0^T dt \int_0^T dt' \langle \mathcal{O}(t)\mathcal{O}(t')\rangle \propto \int_0^T dt \int_0^T dt' \frac{1}{(t-t'-i\epsilon)^2}$$

$$\propto \ln T + \text{endpoints}$$

This answer was derived using different methods for a Kerr black hole in Gralla, Wei 2311.11461

For operators with dimension $\Delta > 1$,

$$\int_0^T dt \int_0^T dt' \langle \mathcal{O}(t)\mathcal{O}(t')\rangle \propto \int_0^T dt \int_0^T dt' \frac{1}{(t-t'-i\epsilon)^{2\Delta}}$$

$$\propto \left(\frac{1}{T}\right)^{2(\Delta-1)} + \text{end points}$$

The T-dependent term $\to 0$ in the $T \to \infty$ limit.

No T-dependent contribution to the decoherence at long times.