A (non-)worldsheet description of strings backgrounds

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(**2311.04959** and **W.I.P** with Kim,
unpublished with Agmon, Collier and Yin
and **1811.00032** with Collier and Yin)

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- It will be great if there is a 'stringy' version of supergravity.
- **String field theory (SFT)** comes close at least conceptually (but with some limitations which we will discuss soon). Today, we discuss how it can be useful even **in practice** for some interesting cases.

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  - Bosonic SFT (9206084, 9705241 Zwiebach)
  - NSR II and heterotic (1508.05387 Sen, 1703.06410 de Lacroix, Erbin, Kashyap, Sen, Verma)
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- Once we plug $\mathcal{T}_0$ into SFT machinery, it produces a **path integral** Z.
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- **Computability**: If one is interested in computing observables up to a specific order in g_s and ϕ, terms in $S[\phi]$ beyond some finite order are not relevant. $S[\phi]$ provides the **Feynman rules** we can use to *systematically compute physical quantities order by order.*

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- Several interesting things: rigorous string perturbation theory around $\mathcal{T}_0$, D-instanton perturbation theory, mass renormalization, discovering new 2d CFT boudary states, tachyons, open-closed duality, ...
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- This talk: study **string backgrounds**!

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- $S_*[\varphi]$ provides a **spacetime** description of strings in the background ϕ_*. It still computes 'stringy' physics.

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- For superstrings, NS states should be in -1 picture while R states are in $-\frac{3}{2}$ and $-\frac{1}{2}$ pictures. GSO projections are also imposed. **RR fields** are also part of the string fields.

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- We expect the variation of the kinetic term of $S[\phi]$ to produce the free EOM $Q_B|\psi\rangle = 0$. This leads to $S_{kin}[\phi] = \frac{1}{2g_s^2}\langle\psi|c_0^- Q_B|\psi\rangle$ $\left(c_0^- = \frac{1}{2}(c_0 - \bar{c}_0)\right)$. In Siegel gauge, the corresponding propagator is given by $\sim \frac{b_0^+}{L_0^+}$, ($b_0^+ = b_0 + \bar{b}_0$, $L_0^+ = L_0 + \bar{L}_0$).

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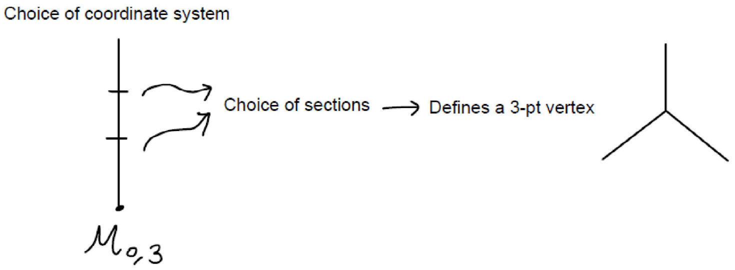
- It was shown (Hata, Zwiebach 93, Sen 14-15) that different choices of local charts are related via field redefinitions and thus does not affect the on-shell quantities.
- For superstrings, PCO locations also enter the definition of the vertices.

3-point vertex



- Example: For the insertion at $z = 0$, take the local chart w_0 with $|w_0| \leq 1$ to be $z = rw_0$ for some positive r. Similarly, $z = 1 - rw_1$ and $z = (rw_\infty)^{-1}$. Different r's correspond to different cubic vertices.

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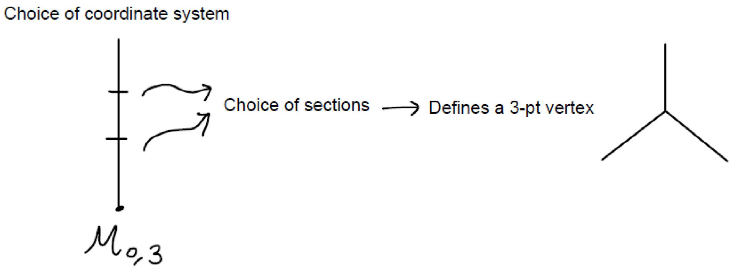

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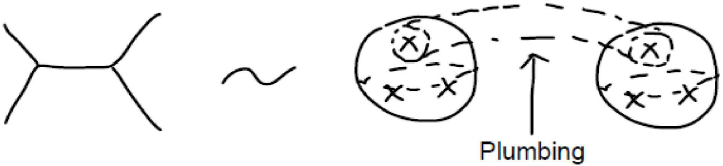
- $\frac{1}{3!}\{\{\psi^{\otimes 3}\}\} = \frac{1}{3!}\langle\psi^{\otimes 3}\rangle_r = \frac{1}{3!}\sum_{i,j,k}\mathcal{U}_{ijk}(r)\phi_i\phi_j\phi_k \subset g_s^2 S[\phi]$.

4-point diagram with a propagator



- Can use propagator and two cubic vertices to draw 4pt Feynman diagram with a propagator. For the Riemann surfaces, we have a plumbing construction of the four-punctured sphere via $w_0 \tilde{w}_0 = q := e^{-s-i\theta}$ with $0 \leq s,\ 0 \leq \theta < 2\pi$.

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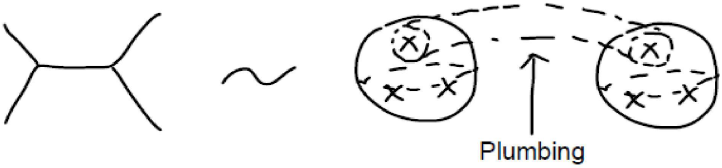

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- This produces two real parameter family of four-punctured sphere, equipped with local charts around four punctures induced from the cubic vertex.

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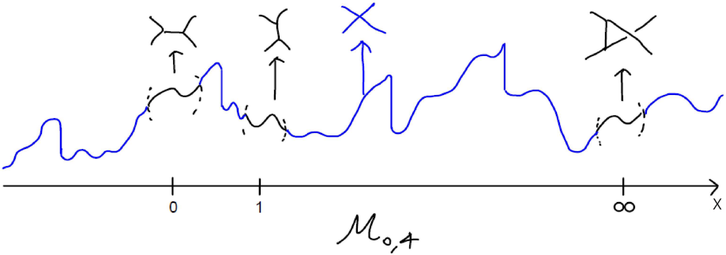

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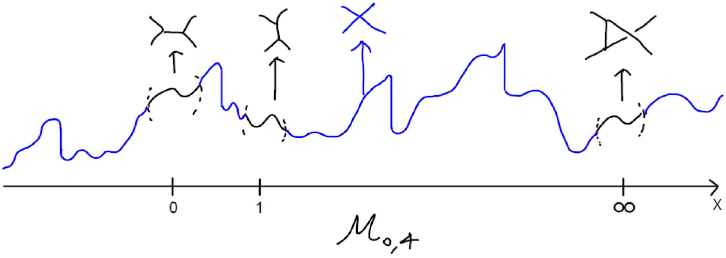

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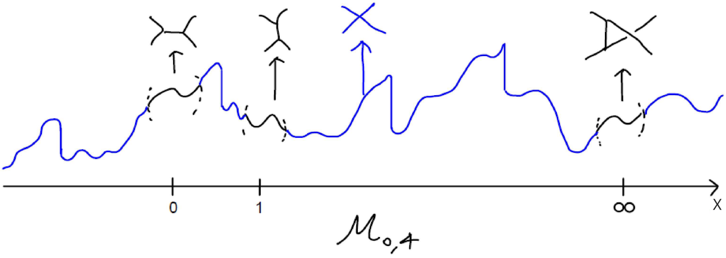

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- 4pt vertex is defined by a choice of local charts around each punctures over the blue region of $\mathcal{M}_{0,4}$.
- Any local chart choices are fine, as long as there are no 'boundaries.' This guarantees that *null-states decouple* from the on-shell amplitudes and thus is intimately related to the *gauge invariance*.

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- $\frac{1}{4!}\{\{\psi^{\otimes 4}\}\} = \frac{1}{4!}\int d\mathcal{M}_{0,4}^{vert}\langle\psi^{\otimes 4}\rangle_{vert} = \frac{1}{4!}\sum_{i,j,k,l}\mathcal{U}_{ijkl}\phi_i\phi_j\phi_k\phi_l \subset g_s^2 S[\phi].$

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- Can repeat the exercise of plumbing, filling in the missing pieces by vertices, etc. to higher points/genus. No-boundary condition to all orders is called geometric master equation whose solutions are known (e.g. MC 19). It implies that $S_Q[\phi]$ thus obtained solves the quantum BV master equation (a technical name for $S_Q[\phi]$ being consistent in the sense of gauge invariance).

1PI action and EOM

- Obtain $S_Q[\phi] = \frac{1}{g_s^2}\left(\frac{1}{2}\langle\psi|c_0^- Q_B|\psi\rangle + \sum_{g,n}\frac{1}{n!}\{\{\psi^{\otimes n}\}\}_{\mathcal{V}_{g,n}}\right)$.

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- For classical (sphere) solutions, we have
  $\mathcal{O}(\mu^1): \; Q_B U_1 = 0$
  $\mathcal{O}(\mu^2): \; Q_B U_2 + \frac{1}{2}[U_1^{\otimes 2}] = 0$ where $[U_1^{\otimes 2}]$ is variation of the sphere 3pt vertex involving two U_1's. It roughly corresponds to the OPE of two U_1's, but evaluated in some special local charts.

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- If $P_0[U_1^{\otimes 2}]$ is NOT Q_B-exact, it means that the EOM cannot be solved. This corresponds to the possible existence of *massless tadpoles*.

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- One can solve for the spectrum order by order in μ.

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- The solution can be obtained straightforwardly

$$w_2 = N_{NS} c\bar{c} \left(G_{2\mu\bar{\mu}} e^{-\phi} \psi^\mu e^{-\bar{\phi}} \bar{\psi}^{\bar{\mu}} - 3 G_{2\nu}^{\nu} (\eta \bar{\partial} \bar{\xi} e^{-2\bar{\phi}} - \partial \xi e^{-2\phi} \bar{\eta}) \right),$$

$$G_{2\mu\bar{\mu}}(X) = \delta_\mu^a \delta_{\bar{\mu}}^{\bar{a}} X_a X_{\bar{a}} - \delta_\mu^i \delta_{\bar{\mu}}^{\bar{i}} X_i X_{\bar{i}}, \quad N_{NS} = -\frac{1}{4\pi}, \; N_R = \frac{1}{8\pi}.$$

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- $G_{2\mu\bar{\mu}}(X)$ is just the $\mathcal{O}(R^{-2})$ part of the $AdS_5 \times S^5$ metric expanded in R^{-1}.

Example: $AdS_5 \times S^5$

- We can also solve for the spectrum using the linearized EOM e.g. axion field $P_0\varphi = f_{\alpha\beta} c\bar{c} e^{-\phi/2} S^\alpha e^{-\bar{\phi}/2} \bar{S}^\beta + \mathcal{O}(\mu^3)$, where $f = f_0 + \mu f_1 + \mu^2 f_2 + ... = \gamma^\mu f_\mu^{(1)}$ is the 1-form field strength.

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- Solution is given by:
  $f_{0\mu} = \partial_\mu c_0$ with $\partial_\mu \partial^\mu c_0 = 0$, $f_1 = 0$, $f_{2\mu} = \partial_\mu c_2 + k_\mu(X)$, where $k_\mu(X)$ is determined by c_0, and c_2 satisfies

$$\partial_\mu \partial^\mu c_2 + \left(X^a X^b \partial_a \partial_b + 5X^a \partial_a - X^i X^j \partial_i \partial_j - 5X^i \partial_i\right) c_0 = 0.$$

  This equation is nothing but $\nabla^2_{AdS_5 \times S^5} c = 0$ expanded to order R^{-2}.

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- IIB supergravity background:

$$ds^2 = R^2(ds^2_{AdS_3} + ds^2_{S^3}) + ds^2_{T^4}$$
$$H_3 = 2qR^2(w_{AdS_3} + w_{S^3}), \quad F_3 = 2\sqrt{1-q^2}R^2(w_{AdS_3} + w_{S^3}),$$

where $q = 1 - \frac{\mu^2}{2} + \mathcal{O}(\mu^3)$, $qR^2 = \alpha' k$, $k \in \mathbb{N}$.

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- $q = 1$: pure NSNS background with exact worldsheet CFT $SL(2,\mathbb{R})_{k+2} \oplus SU(2)_{k-2} \oplus U(1)^4 \oplus 10$ free fermions (Maldacena, Ooguri 00-01). We take this background as $\mathcal{T}_0$.

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- We study the mixed flux solution. Turn on F_3 profile at the leading order: $U_1 \sim c\bar{c}e^{-\phi/2}S_+^{\alpha\alpha'}\Theta_+ e^{-\bar{\phi}/2}\tilde{S}_+^{\beta\beta'}\tilde{\Theta}_+ \left(V^{sl}_{j=-\frac{1}{2}}\right)_{\alpha\beta}\left(V^{su}_{j'=\frac{1}{2}}\right)_{\alpha'\beta'}$.

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Spectrum of pulsating strings

- Study spectrum of pulsating strings in mixed flux. At the pure NSNS point, the corresponding vertex operator is

$$\varphi_0 = c\tilde{c}e^{-\phi-\tilde{\phi}}V_{j_0,j',n}, \;\; V_{j_0,j',n} \sim \psi^-\tilde{\psi}^-(J_{-1})^n(\tilde{J}_{-1})^n V^{sl}_{j_0,j_0,j_0} V^{su}_{j',j',j'} V_{T^4}.$$

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- At order μ, due to $P_0[U_1\varphi_0] = 0$, we take the same solution as the zeroth order solution.

Spectrum of pulsating strings (non-BPS)

- To order μ^2, we take the solution of the form

$$\varphi_2 = c\tilde{c}e^{-\phi-\tilde{\phi}}V_{j_0+\mu^2 j_2, j', n} + (\text{ghosts, descendants}).$$

Plugging this into the linearized EOM, we obtain

$$\frac{2j_2(2j_0-1)}{k} = \mathcal{A}(\varphi_0, \varphi_0, U_1, U_1),$$

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- Sample results for $\delta h = -\frac{\alpha' \delta m^2}{4} = \frac{\mu^2 j_2(2j_0-1)}{k}$ with $n = 1, h_{T^4} = 0$:

k \ j'	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
7	4.51353	7.7253			
8	2.61214	3.18173	5.03926	38.0435	
9	1.97318	2.21068	2.76008	4.25035	15.9923

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- Due to RR fluxes, stringy observables in GKP backgrounds have been considered very tough to study e.g. stringy corrections to Kahler potential.
- Maybe SFT can help? The relevant open-closed-unoriented super-SFT has recently been constructed (Moosavian, Sen, Verma 19).

Example: Flux compactification

- Consider IIB flux compactification:

$$ds^2 = G_{AB}dX^A dX^B = e^{2A(y)}dx_\mu dx^\mu + e^{-2A(y)}g_{ij}(y)dy^i dy^j,$$
$$\tilde{F}_5 = (1 + *_{10})d\alpha(y)dx^0 dx^1 dx^2 dx^3.$$

$\mu = 0, ..., 3$ are non-compact flat directions while $i = 1, ..., 6$ are along the compact CY 3-fold $\mathcal{X}$. Consider the orientifold $\mathcal{X}/\mathcal{I}$ with $\mathcal{I}^2 = 1$.

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- H_3 and F_3 fluxes only have i, j components, and D3-branes and O3-planes are spacetime-filling along x^μ (no D7/O7 for simplicity).
- Plug the ansatz into IIB supergravity EOM:

$$\text{ISD}: \quad *_6 G_3 = iG_3, \qquad G_3 = F_3 - \tau H_3,$$
$$\text{Bianchi}: \quad dH_3 = dF_3 = 0, \quad d\tilde{F}_5 = H_3 \wedge F_3 + 2\mu_3\kappa_{10}^2\rho_{D3}^{loc}dVol_{\mathcal{X}/\mathcal{I}},$$
$$\text{where} \quad \rho_{D3}^{loc} = \sum_{y_{D3}} \delta^{(6)}(y - y_{D3}) - \frac{1}{4}\sum_{y_{O3}} \delta^{(6)}(y - y_{O3}),$$
$$\text{and} \quad e^{4A(y)} = \alpha(y) \quad \Rightarrow \quad \nabla^2\left(\alpha^{-1}\right) = \frac{1}{3\text{Im}\tau}|G_3|^2 + \mu_3\kappa_{10}^2\rho_{D3}^{loc}.$$

Example: Flux compactification

- For $\mathcal{O}(1)$ quantized fluxes, ISD condition requires $g_{ij}(y)$ *to be correlated with* g_s. This may be achieved by having special choices of 3-form fluxes and large complex structure moduli ($\sim g_s^{-1}$) (Demirtas, Kim, McAllister, Moritz 19), where perturbative 4d $\mathcal{N} = 1$ SUSY is preserved simultaneously.

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- An explicit example of toroidal orientifold $T^6/\mathcal{I}$ (Cicoli, Licheri, Mahanta, Maharana 22):

$$
\begin{aligned}
&T^6: \; Z^i \sim Z^i + 1 \sim Z^i + u_i, \quad Z^i = y^{2i-1} + u_i y^{2i}, \quad i = 1, 2, 3, \\
&\mathcal{I}: \; (Z^1, Z^2, Z^3) \to -(Z^1, Z^2, Z^3) \quad \Rightarrow \; 64 \text{ O3}, \\
&\frac{1}{(2\pi)^2\alpha'} F_3 = 4dy^2dy^3dy^5 - 2dy^1dy^4dy^5 - 2dy^1dy^3dy^6, \\
&\frac{1}{(2\pi)^2\alpha'} H_3 = 4dy^1dy^4dy^6 - 2dy^2dy^3dy^6 - 2dy^2dy^4dy^5.
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Example: Flux compactification

- At string tree-level, 4d $\mathcal{N} = 1$ superpotential is given by $W_{tree} = \int_{\mathcal{X}/\mathcal{I}} G_3 \wedge \Omega$ (Gukov, Vafa, Witten 99). Non-renormalization theorem states that this is perturbatively exact in α' and g_s: $W_{pert} = W_{tree}$ (Burgess, Escoda, Quevedo 05).

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- For the specific case of interest,

$$W_{pert} = -2u_2u_3 + u_1u_3 + u_1u_2 - \tau(2u_1 - u_2 - u_3),$$

$$\Rightarrow \; u_1 = u_2 = u_3 = \tau \text{ solves F-term equations.}$$

In particular, $W_{pert} = 0 \;\Rightarrow\; \mathcal{N} = 1$ SUSY perturbatively preserved.

Integrated Bianchi identity $\Rightarrow$ number of spacetime-filling D3 $= 4$.

Example: Flux compactification

- At string tree-level, 4d $\mathcal{N}=1$ superpotential is given by $W_{tree} = \int_{\mathcal{X}/\mathcal{I}} G_3 \wedge \Omega$ (Gukov, Vafa, Witten 99). Non-renormalization theorem states that this is perturbatively exact in α' and g_s: $W_{pert} = W_{tree}$ (Burgess, Escoda, Quevedo 05).
- For the specific case of interest,

$$W_{pert} = -2u_2u_3 + u_1u_3 + u_1u_2 - \tau(2u_1 - u_2 - u_3),$$
$$\Rightarrow \; u_1 = u_2 = u_3 = \tau \text{ solves F-term equations.}$$

In particular, $W_{pert} = 0 \;\Rightarrow\; \mathcal{N}=1$ SUSY perturbatively preserved.

Integrated Bianchi identity $\Rightarrow$ number of spacetime-filling D3 $= 4$.

- Complex structure moduli u_i are inversely proportional to the string coupling g_s. Therefore, *string perturbation theory should treat u_i^{-1} on the same footing as g_s.*

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- How do we perform the string perturbation theory / SFT for this case? We start with open-closed-unoriented IIB on $\mathbb{R}^{1,3} \times T^6/\mathcal{I}$ with 4 D3 and 64 O3 as $\mathcal{T}_0$.

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- Worldsheet CFT correlators on $T^6/\mathcal{I}$ explicitly depend on $u_i \sim g_s^{-1}$. In order to systematically count $(u_i)^{-1} \sim g_s$, we introduce vielbein: $\lambda^{i'} = e_i^{i'} \psi^i$ such that $\lambda^{i'}(z)\lambda^{j'}(0) \sim \delta^{i'j'}/z$. Then, $e_{2i}^{2i'} \sim g_s^{-1/2}$, $e_{2i'-1}^{2i'-1} \sim g_s^{1/2}$.

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- For example, consider B-field vertex operator ($H_3 = dB$): $\frac{1}{4\pi} c\bar{c}B_{ij}e^{-\phi}\psi^i e^{-\bar{\phi}}\bar{\psi}^j$. Acting with a holomorphic PCO, we get $\sim c\bar{c}H_{ijk}\psi^i\psi^j e^{-\bar{\phi}}\bar{\psi}^k + \ldots \sim g_s^{1/2}$ for the explicit flux choices we had before (H_{ijk} is of $\mathcal{O}(g_s^0)$).

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- We can take these H_3 and F_3 as the leading order ($g_s^{1/2}$) perturbative solution of SFT, and subsequently build higher order solutions $\Rightarrow$ <u>SFT solution in $\mu = g_s^{1/2}$ expansion.</u>

Example: Flux compactification

- $\mathcal{O}(\mu)$ solution:
$\mu U_1 = \frac{1}{4\pi} c\bar{c} B_{ij} e^{-\phi}\psi^i e^{-\bar{\phi}}\bar{\psi}^j + \frac{ig_s\sqrt{\alpha'}}{3!16\sqrt{2}\pi} c\bar{c} F_{ijk} e^{-\phi/2} S_\alpha (\Gamma^{ijk})^{\alpha\beta} e^{-\bar{\phi}/2}\bar{S}_\beta .$

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$$P_0 U_2 \sim c\bar{c}\Big(B_{ij}B^{ij}(\eta\bar{\partial}\bar{\xi} e^{-2\bar{\phi}} - \partial\xi\bar{\eta} e^{-2\phi}) - 2B_{ik}B^{kj} e^{-\phi}\psi^i e^{-\bar{\phi}}\bar{\psi}_j$$
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- If we started with some generic choice of $u_i \sim g_s^{-1}$, quantized H_3 and F_3, then the requirement that RHS of $Q_B P_0 U_2 = \ldots$ is Q_B-exact leads to the *integrated Bianchi identity and ISD conditions* that GKP had in IIB supergravity analysis.

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- Linearized EOM at ghost number one in (R,NS)/(NS,R) sector leads to the expected *Killing spinor equations*, where nontrivial spinor solutions exist only if G_3 is (2,1)-form so that W_{pert} vanishes.

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- SFT provides a systematic framework for studying interesting backgrounds such as *AdS* and flux compactifications. Observables such as *AdS* Virasoro-Shapiro amplitude (Alday, Hansen, Silva 22, Alday, Hansen 23) and g_s-corrections to Kahler potential in GKP should be computable in this framework.

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- In principle, SFT knows how to deal with **time-dependent backgrounds**. An example which was studied intensively in the past is the open string rolling tachyon (Sen 02). Can we address *closed string cosmology* (Rodriguez 23)?

THANK YOU