Higgs-Confinement Transitions in QCD from Symmetry Protected Topological Phases

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Phases of Gauge Theory

- **E&M:** Coulomb phase of $U(1)_{\mathrm{e.m.}}$ gauge theory
- **QCD:** confining phase of $SU(3)_c$ gauge theory
- **Electroweak theory:** Higgs phase of

$$SU(2)_W \times U(1)_Y \to U(1)_{\mathrm{e.m.}}$$

- **Condensed matter physics:** conventional BCS superconductor is a Higgs phase with $U(1)_{\mathrm{e.m.}} \to \mathbb{Z}_2$

Here "phase" is used loosely…

Order Parameters and Symmetries

Sharp notion of phases and transitions typically requires order parameters and global symmetries **[Landau]**:

- **Order parameters:**
  large electric/magnetic loops
  [Wilson; Polyakov; Susskind; ’t Hooft]

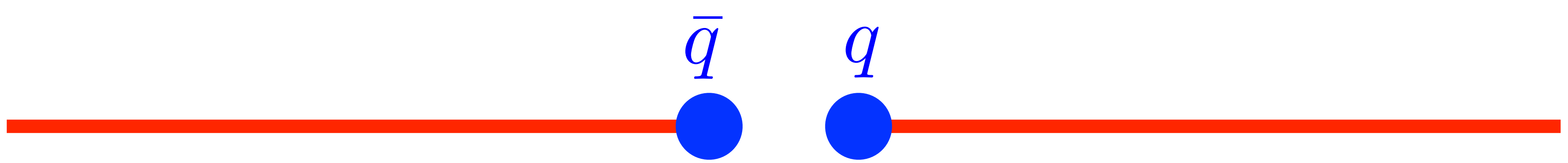
- **Symmetries:** generalized, one-form global symmetries **[Gaiotto, Kapustin, Seiberg, Willett]**, e.g. center symmetry.

- **Example:** pure $SU(3)_c$ gauge theory has a $\mathbb{Z}_3$ one-form center symmetry, unbroken in vacuum, broken at high T .

Fundamental Matter

One-form symmetries: always explicitly broken by fundamental matter. (They require non-generic matter.)

Example: QCD with fundamental quarks has no one-form center symmetry. Wilson lines can end; confining strings can break.



No sharp notion of confinement in QCD!

Finite T lattice simulations show smooth crossover.

Higgs-Confinement Continuity

Fundamental scalar fields can lead to complete Higgsing with a gapped vacuum, much like in the confining regime.

There are no symmetries that distinguish Higgsing and confinement: **they can be continuously connected, without a phase transition**.

- Rigorous lattice results **[Fradkin, Shenker; Banks, Rabinovici]**
- True in examples, especially with SUSY **[Intriligator, Seiberg; …]**
- **Continuity dogma is standard lore** **[Dimopoulos, Raby, Susskind]**

Plan for This Talk

- Examples of 3+1d gauge theories where the Higgs-confinement continuity dogma fails (there are many more, the mechanism works in any d).

- Unexpected Higgs-Confinement phase transitions.

- The reason is that Higgs and confining regimes are different symmetry protected topological (SPT) phases.

- Applications to QCD at $T = 0$ and finite baryon density (relevant for neutron star interiors)

What is an SPT?

- Condensed matter origins: IQHE, TIs, TSCs, …
- Anomaly inflow theories for background fields associated with global symmetries [Callan, Harvey]. Relativistic SPTs are mathematically well understood [Kitaev; Kapustin; Freed, Hopkins].
- Example: 2+1d gapped system with $U(1)$ symmetry and a quantized background Chern-Simons term:

$$\frac{ik}{4\pi}\int AdA \qquad k \in \mathbb{Z}$$

- Contributes a phase to the Euclidean partition function. A jump in k signals a phase transition (first order, unless the gap closes, e.g. free massive Dirac fermion).

Fermion SPTs in 3+1d

- Free 2-component Weyl fermion ψ_α with time-reversal T

$$\mathcal{L} = -i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi - \frac{m}{2}(\psi\psi + \bar{\psi}\bar{\psi}) \qquad m \in \mathbb{R}$$

- For either sign of m, the theory is gapped and trivial. Related by $U(1)_{\mathrm{axial}}$, which has mixed gravity anomaly. This leads to the following SPTs in the two phases:

$$\frac{i\theta_g}{384\pi^2}\int_{M_4} \mathrm{tr}(R \wedge R) \qquad \theta_g = 0, \pi$$

- T-symmetry quantizes θ_g, SPT jump at $m = 0$ enforces a phase transition. Without T: can avoid the transition by dialing through complex masses $m \in \mathbb{C}$.

QCD, Positivity, and (no) SPTs

QCD with quark mass $m_q > 0$ can be regulated in such a way that its Euclidean path-integral measure is positive (i.e. all theta-angles vanish) [Vafa, Witten; Weingarten].

Partition function on any four-manifold M_4 with arbitrary vector-like background gauge fields A is positive:

$$Z[M_4, A] > 0 \qquad \Longrightarrow \qquad \text{trivial SPT}$$

Our SPT examples will violate positivity via Yukawas or a chemical potential (another option: $m_q < 0$ [Tachikawa, Yonekura]).

Example 1: SU(3) Higgs-Yukawa-QCD

Start with three-flavor QCD: $SU(3)_c$ gauge theory with

3 Dirac = 6 Weyl quarks: $\Psi_i^a = (\psi_i^a, \bar{\chi}_i^a) \qquad a, i = 1, 2, 3$

quark mass $m_q > 0$, flavor symmetry $U(1)_B \times SU(3)_f$

Add: anti-fundamental Higgs field h_a^i + potential + Yukawas:

$$\mathcal{L}_{\text{Yukawa}} = y \varepsilon_{abc} \varepsilon^{ijk} \bar{h}_i^a \left(\psi_j^b \psi_k^c + \bar{\chi}_j^b \bar{\chi}_k^c \right) \qquad y > 0$$

Preserves T, P. Higgs exchange pairs quarks in color/flavor anti-symmetric channel (like gluon exchange in QCD).

Phases of SU(3) HYQCD

(C) $M_h^2 \to +\infty$: integrate out Higgs. Leaves QCD with $m_q > 0$. Gapped, trivial SPT.

(H) $M_h^2 \to -\infty$: Engineer a color-flavor locking Higgs vev:

$$h_a^i = v\delta_a^i \qquad v \in \mathbb{C}$$

$SU(3)_c$ fully Higgsed, $SU(3)_f$ unbroken by identifying $a = i$.

Additionally: $U(1)_B$ spontaneously broken: $\langle \det(h_a^i) \rangle = v^3$

A single massless NGB, otherwise gapped.

The Higgs Phase as a Gapless SPT

Massless $U(1)_B$ NGB can in principle render the $\theta_g = \pi$ SPTs meaningless, but this require a mixed anomaly between $U(1)_B$ and gravity, which is absent. Thus we can still meaningfully ask whether $\theta_g = 0, \pi$ in the Higgs phase.

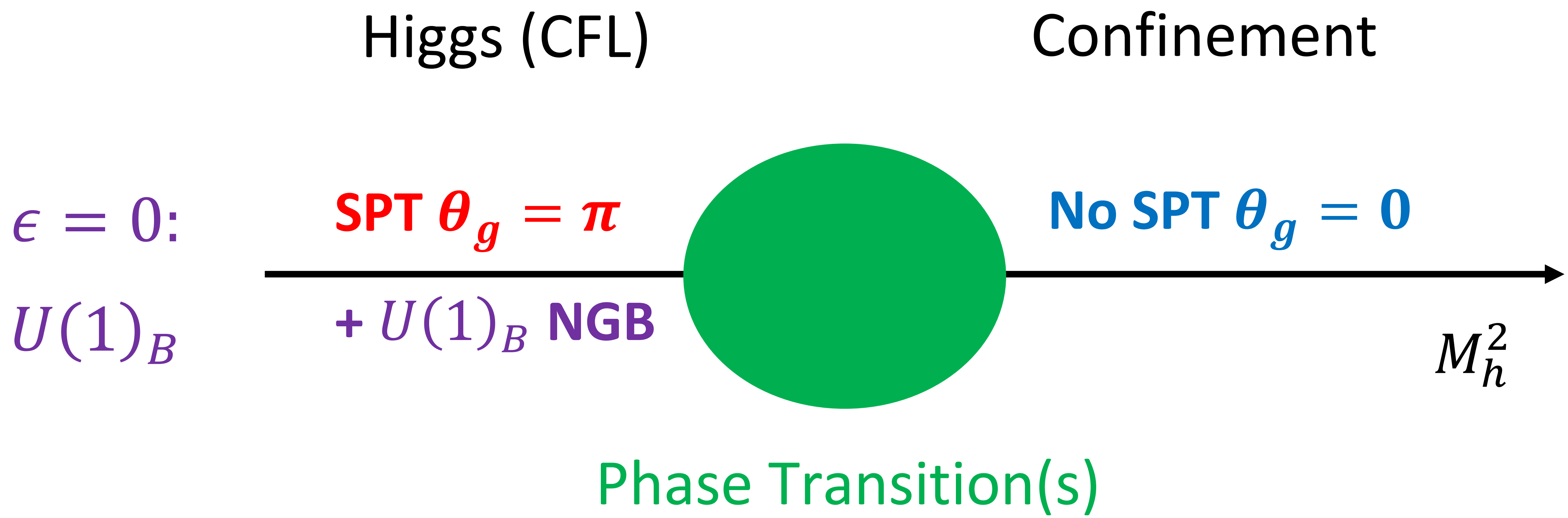
Fermions transform under unbroken $SU(3)_f$:

$$\psi_i^a \to \psi_i^j = \mathbf{1} \oplus \mathbf{8} \qquad \chi_a^i \to \chi_j^i = \mathbf{1} \oplus \mathbf{8}$$

Masses: $M_{\mathbf{1}} = m_q \pm 4yv \qquad M_{\mathbf{8}} = m_q \pm 2yv$

At large vev's, 9 fermions flip sign: $\theta_g = \pi$ (and also $\theta_f = \pi$)

Phase Diagram



Two arguments for a phase transition — are they related?

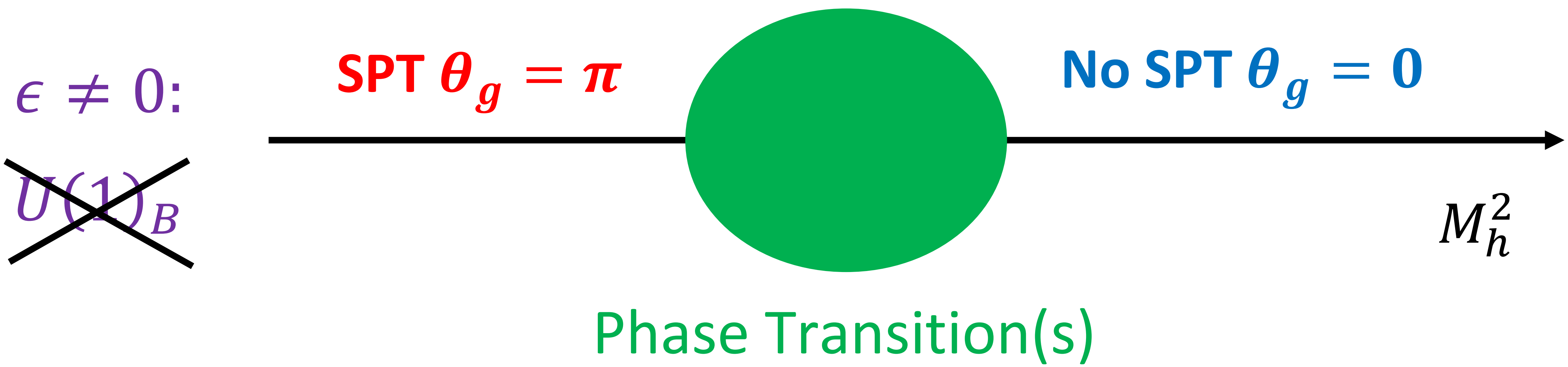
- Realization of $U(1)_B$ (Landau)
- SPT jump $\Delta\theta_g = \pi$ (non-Landau)

Disentangling the Transitions

Explicitly break $U(1)_B$ by $B = 2$ flavor singlet operator:

$$\Delta\mathcal{L} = \varepsilon \det(h_a^i) \qquad \varepsilon \in \mathbb{R}$$

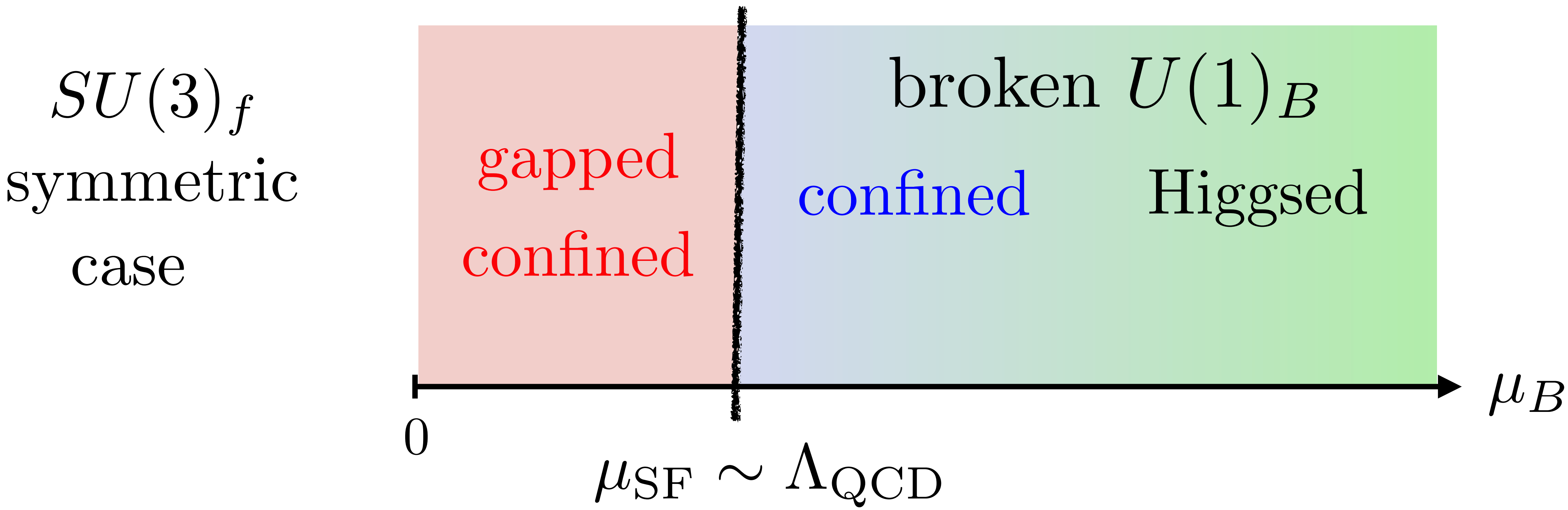
Preserves P , T and lifts NGB, but SPT jump remains:



The SPT forces a (generically) distinct, unexpected transition between the Higgs and the confining phases.

Example 2: Finite Density QCD

Ordinary QCD: 3 degenerate flavors, quark mass $m_q > 0$
$U(1)_B$ chemical potential μ_B (preserves P, T; sign problem)



[Schäfer, Wilczek] conjecture: Higgs-confinement (or quark-hadron) continuity in the $U(1)_B$ breaking superfluid phase.

The High-Density Higgs Phase

When $\mu_B \gg \Lambda_{\text{QCD}}$ QCD is weakly coupled: 1-gluon exchange in color/flavor anti-symmetric channel leads to a CFL vev for a composite Higgs field [Alford, Rajagopal, Wilczek; ...]

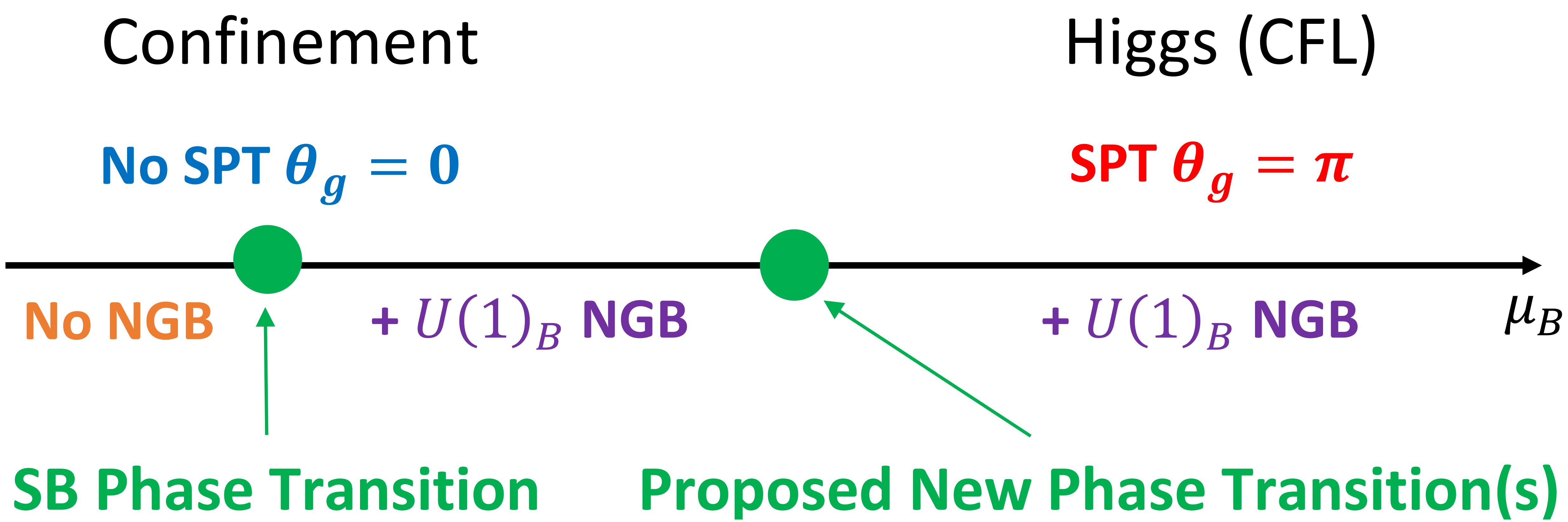
$$\varepsilon_{abc}\varepsilon^{ijk}\langle\psi_j^b\psi_k^c + \bar{\chi}_j^b\bar{\chi}_k^c\rangle \sim \mu_B\delta_a^i$$

Same quantum numbers as fundamental h_a^i in HYQCD, with same consequences: the two Higgs phases are qualitatively identical, e.g. the QCD gaps are [Son]:

$$\Delta_{\mathbf{1}} = 2\Delta_{\mathbf{8}} \sim \mu_B e^{-1/g(\mu_B)}$$

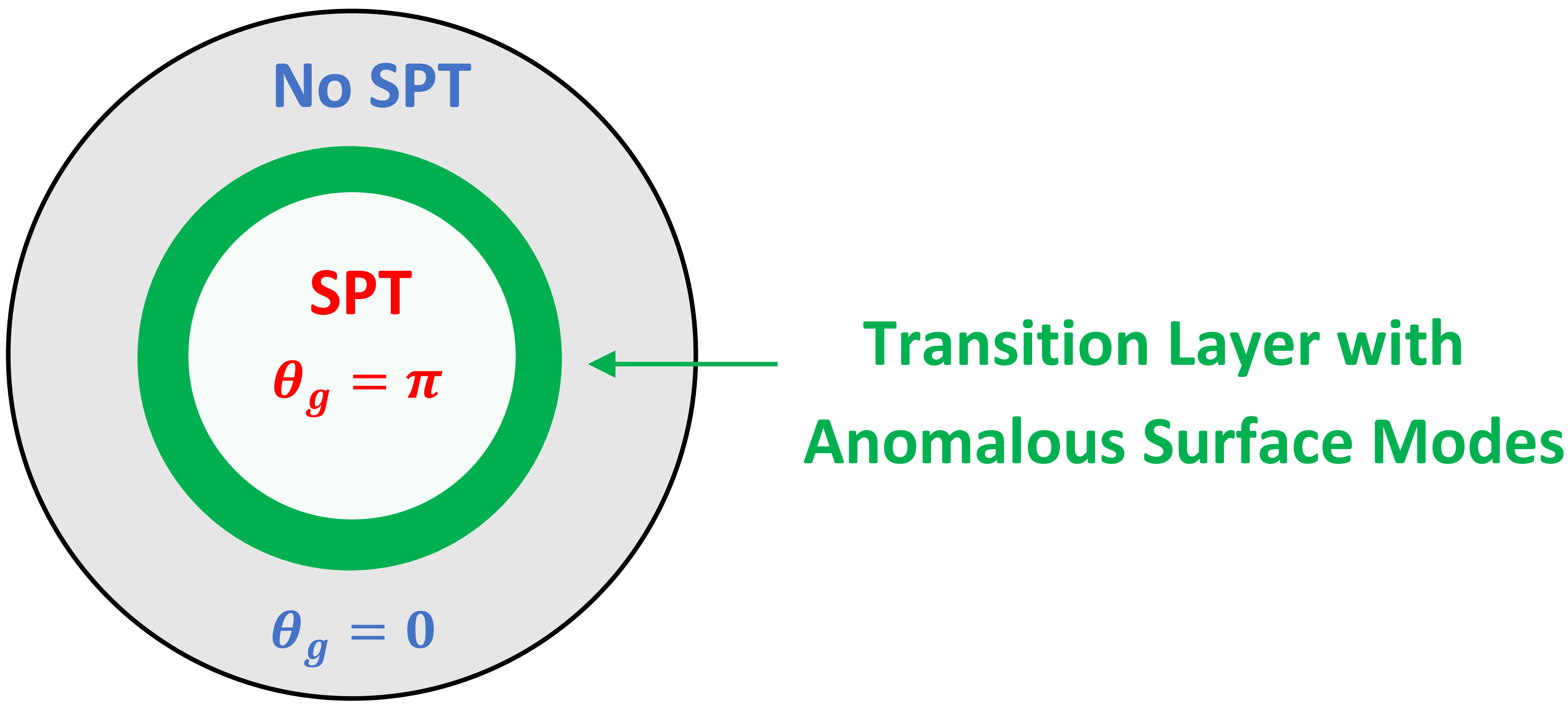
High-Density QCD is a Gapless SPT

The SPTs are also identical: $\theta_g = \pi$ (proof: relate the two weakly-coupled Higgs phases by a deformation that maintains the fermion gap). **Motivates new QCD phase transitions**, in tension with [Schäfer, Wilczek] conjecture.



$\theta_g = \pi$ and Neutron Stars

Neutron star cores may be dense enough to activate the Higgs phase with $\theta_g = \pi$ at their cores. The domain wall separating the phases must harbor gapless fermions or other anomalous edge modes, due to the SPT jump.



Conclusions and Outlook

- Higgs-confinement continuity can fail if the two regimes are in different SPT phases (general, many examples).

- High-density QCD is a gapless SPT with $\theta_g = \pi$

- Consequences for QCD phase diagram, and (optimistically) for neutron star interiors.

- QCD in the vacuum has no stable strings, but rotating neutron star interiors are baryon superfluids and harbor vortex strings. How are those affected by the SPT?

Thank You for Your Attention!