Quantum Error Correction For Gravitational Algebras @ large N

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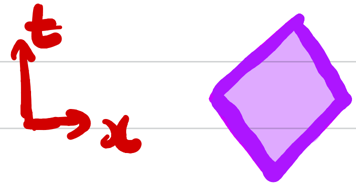
+ 2 Papers to appear

Gravitational Algebras:

In Algebraic Approach to QFT:



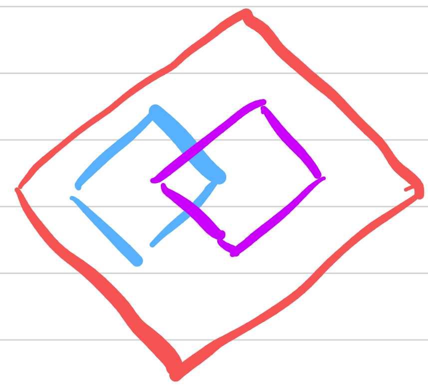
$\longrightarrow \quad \mathcal{A}(\diamond) \subset \mathcal{B}(\mathcal{H})$

Roughly $\sim$ $f($smeared local operators in $\diamond)$

+ Products & Sums

Axioms: Local nature of the QFT

(e.g Haag Duality)



$\Rightarrow$ von Neumann Algebras, type III_1 factors

$\subset$ Bounded$(\mathcal{H}_{QFT})$

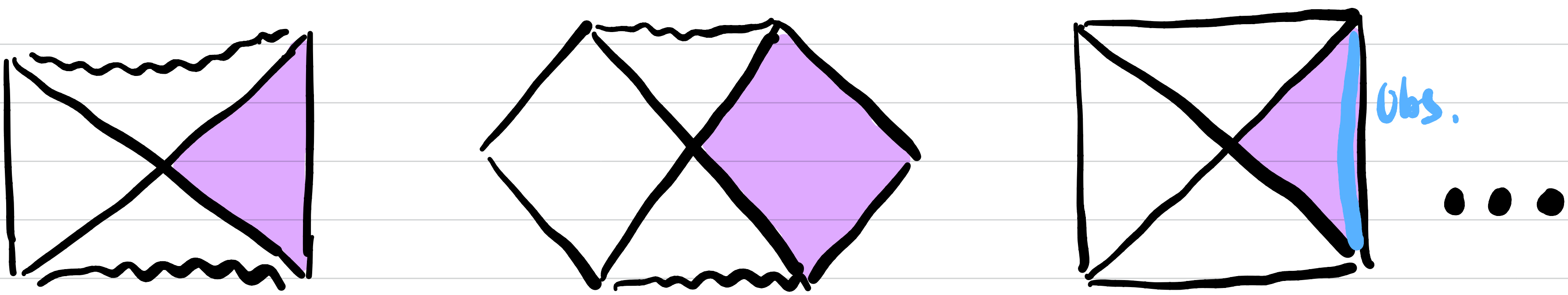
$A \cap A' =$ trivial

Natural to wonder what happens in Quantum Grav.

Subtle: localized regions, diff. invariant, background independent.

Semiclassical limit: Progress; Liu, Leutheusser; CPLW, ...

Roughly: $G_N \to 0$, QFT + diffeo. constraints



AdS BH — Sch. BH — dS

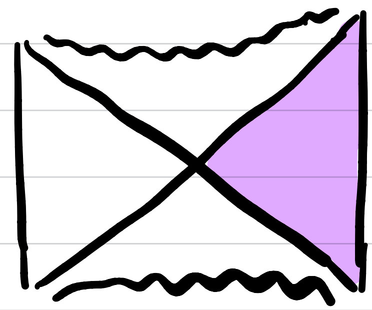
type III_1 / type II_∞ ← type II_1

Different ∞ entanglement structure allowed.

Emergence of Geometry

$G_N \to 0$ Finite: N or G_N

AdS/CFT / Holography:



$\to \sum_i e^{-\beta E_i} |E_i\rangle^* \otimes |E_i\rangle$

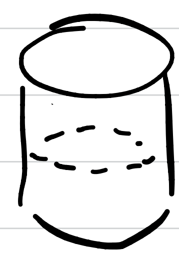
TFD $\in \mathcal{H}_L \otimes \mathcal{H}_R$ Right obs: $\mathbb{I}_L \otimes B(\mathcal{H}_R)$

type I_∞: $\exists$ Pure States

e.g. $|E_i\rangle\langle E_i|$

So type $I_\infty \xrightarrow[N\to\infty]{}$ type III_1 / II_∞

Large-N limit in AdS/CFT



$\mathcal{H}_N^R$

Sequence of theories $N \in \mathbb{N}$,

e.g.: $CFT^L_N \otimes CFT^R_N$ with $\mathcal{H}_N = \mathcal{H}^L_N \otimes \mathcal{H}^R_N$

on L, R CFTs

- $\phi^i_N = \mathrm{tr}_N(M_1 M_2 \ldots)$ Single trace fields
- $|\Psi_N\rangle$ sequence of states $\in \mathcal{H}_N$ (e.g. $|TFD^N_\beta\rangle$)
- Correlation functions: $\phi^i_N \to \phi^i_N - \langle \phi^i_N \rangle_{\Psi_N}$

$$\langle \phi^1_N \phi^2_N \phi^3_N \phi^4_N \rangle_{\Psi_N} \xrightarrow[N\to\infty]{} \langle \phi^1_N \phi^2_N \rangle_{\Psi_N} \langle \phi^3_N \phi^4_N \rangle_{\Psi_N}$$

+ perms.

- Large N fields behave like GFFs

$\phi^i \equiv \phi^i_\infty$; abstract ops.

- $\mathcal{H}_{bulk} = \mathcal{H}_{GNS} \sim \phi^1 \phi^2 \ldots \phi^k |1\rangle$ / null states.

$$\langle 1 | (\tilde\phi^1 \ldots \tilde\phi^q)(\phi^1 \ldots \phi^k) | 1 \rangle = \lim_N \langle \tilde\phi^1_N \ldots \tilde\phi^q_N \, \phi^1_N \ldots \phi^k_N \rangle_{\Psi_N}$$

Emergent von Neumann algebra: Liu, Leutheusser

$$\pi(\phi^1)\,\phi^2 \ldots \phi^k|1\rangle = \phi^1\phi^2 \ldots \phi^k|1\rangle$$

$C(R)$ = closure alg. gen. $\underbrace{\pi(\phi^i)}$

Right CFT

$\subset B(\mathcal{H}_{GNS})$

Supported on Right CFT

Type III_1: guaranteed by properties $\langle\phi^i\phi^j\rangle$

type II ~ include H_{CFT}

(E. Gesteau's talk)

Q1: How do we get back the type I_∞ algebra?

$\rightsquigarrow$ How do we see microstates?

Q2: type II algebras have entropies

$S_{II}(\rho) + C$ defined using max. mixed state

what is C? what sense is:

$S_N(\rho_N) \to S_{II}(\rho)$ true?

Q3: How does string theory fit in?

Today: construct a Quantum Error Correcting code that attempts to address Q1-3

claim: natural extension of above discussion

QEC: How does bulk theory fit into microscopics

(code) $\mathcal{H} = \mathcal{H}_{GNS}$ $\boxed{\mathcal{K}_N \text{ fixed } N}$ (physical)

Need.

$V_N : \mathcal{H} \to \mathcal{K}_N$ Sequence of maps.

Funny: $\dim \mathcal{H} = \infty$ $\dim \mathcal{K}_N \sim e^{2 S_N}$ finite

Construct as follows: $\gamma_N : \phi^i \to \phi^i_N$

γ_N gives the correct representative of

single trace field $= \underset{N \times N}{\mathrm{Tr}} M_1 \cdots M_k$

Def. of large-N limit. @ Fixed N

✓ GNS.

Def: $\gamma_N(\phi^i \ldots)\,|\psi_N\rangle = V_N\,\pi(\phi^i \ldots)\,|1\rangle$

(technically V_N could be badly behaved, but possible to project $V_N \to V_N \Pi_N$ to fix)

Then: show $V_N^\dagger V_N \to \mathbb{I}_{\mathcal{H}}$ Pointwise!

$\forall$ matrix elements $\langle \xi_1 | \cdot | \xi_2 \rangle$ on $\mathcal{H}$

$$\langle \xi_1 | V_N^\dagger V_N | \xi_2 \rangle \to \langle \xi_1 | \xi_2 \rangle$$

Key: allows $V_N : \underset{\infty\text{-dim}}{\mathcal{H}} \to \underset{\text{finite dim}}{K_N}$ to work.

Also: in ∞ $\not\Rightarrow$ $\| V_N^\dagger V_N - \mathbb{I}_{\mathcal{H}} \| \to 0$

this would be uniform convergence.

Typical QEC $\quad V: \mathcal{H} \to K \qquad \left.\begin{array}{l} V^\dagger V = \mathbb{1} \\ VV^\dagger = e \end{array}\right\}$ Isometry

Approximate QEC $\quad \| V^\dagger V - \mathbb{1} \| < \varepsilon$

Here: "pointwise approximate QEC" $\longrightarrow$

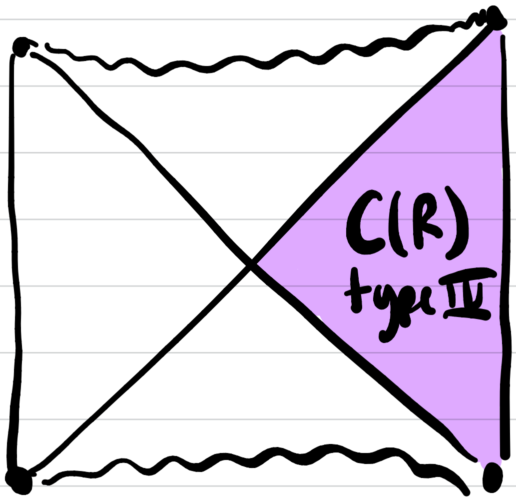
"Asymptotically Isometric Codes" TF, Li

But does it work like other QEC codes?

YES!

Operator Reconstruction: (Dong, Harlow, Wall; Harlow)

$M_N(L)$



$M_N(R)$

$\downarrow$

$\mathbb{I} \otimes B(K_N^R)$
type-I

Defined by:

$$\beta_N^R(c_R)\, V_N - V_N c_R \to 0$$

pointwise sense.

$$\forall \; c_R \in C(R)$$

$$\beta_N^R : C(R) \to M_N(R)$$

Proved 2 main Theorems; Pointwise Approximate

1. Information Disturbance Tradeoff (no-cloning)

$\rightarrow$ maintain bdry causality in bulk

$$\exists \; \beta_N^R \quad \leftrightarrow \quad [\, V_N^\dagger X_N^L V_N ,\; C(R) \,] \to 0$$

2. Reconstruction wedges: gen $\mathcal{E}(L) \subset \mathcal{E}(R)'$

Largest algebras $\mathcal{E}(L), \mathcal{E}(R) \subset B(\mathcal{H})$

Reconstructed from $M_N(L), M_N(R) \subset B(K_N)$

Then

① Maximal Reconstruction

$$\mathcal{E}(L) = \mathcal{E}(R)'$$

all that commute with $\mathcal{E}(R)$

Haag Duality Bulk

② $S_{rel}(\psi_N | \phi_N ; M_N(R)) \to S(\psi | \phi ; \mathcal{E}(R))$

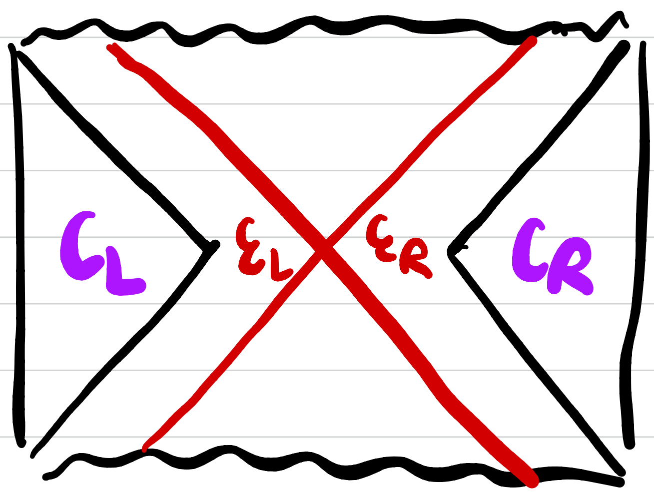
$\forall \psi, \phi \in \mathcal{H}$ & $\psi_N - V_N \psi \to 0$ $\quad \phi_N - V_N \phi \to 0$

③ $$\Delta^{is}_{M_N(R)} V_N - V_N \Delta^{is}_{\mathcal{E}(R)} \to 0$$

Δ^{is} : Tomita Takesaki Modular Op: $\rho^{is}_{N,L} \otimes \rho^{-is}_{N,R}$

Assumptions in 1, 2 trivially satisfied by BH case

More complicated settings:



C_L, C_R easy to reconstruct: use γ_N, $\equiv$ Causal wedges

Q: what conditions guarantees such maximal reconstructions?

$\mathcal{E}_L = \mathcal{E}_R'$ follows by HRT formula in many situations $\rightarrow$ Grav. Path Integral

WIP w/ Li

Entropy works as follows: if $\mathcal{E}(R) = \mathcal{E}(L)'$ type II_1 then:

$\exists$ $a_N \to \infty$ numbers s.t

$$\lim_{N\to\infty} \left(S_{vN}(\psi_N; M_N(R)) - a_N \right) = S_{\mathrm{II}}(\psi; C(R))$$

$\forall$ $\psi_N - V_N \psi \to 0$

(Modulo some extra assumptions on code & "smoothing" of S_{vN})

$\underline{\underline{a_N}}$: Give the unknown shift in the type-II entropy.

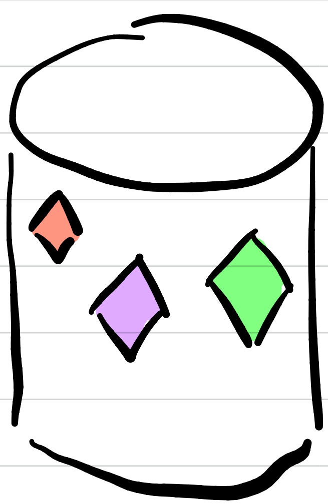
✓ one copy

For vacuum codes: i.e. $K_N = K_N^{CFT}$ $\psi_N = \Omega_N$

Where $\mathcal{H}$ has a representation $U(g)$ of $SO(d,2)$ CFT symm's.

s.t. $U_N(g)\, V_N - V_N\, U(g) \to 0$

Then:

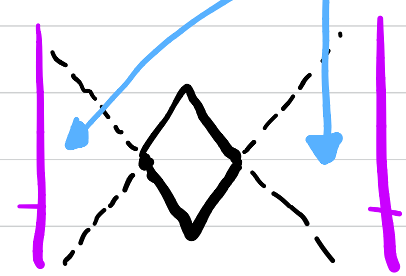


$M_N(\Diamond\text{'s})$ Bdy Algebras

$$\downarrow$$

$\mathcal{C}(\Diamond)$ or $\mathcal{E}(\Diamond)' = \mathcal{E}(\Diamond')$

Without P.I what can we say?



Vacuum Codes: WIP

1.  Double Cone then

$\mathcal{E}(\Diamond) = C(\Diamond)$ uses algebraic version Bisognano Wichmann.

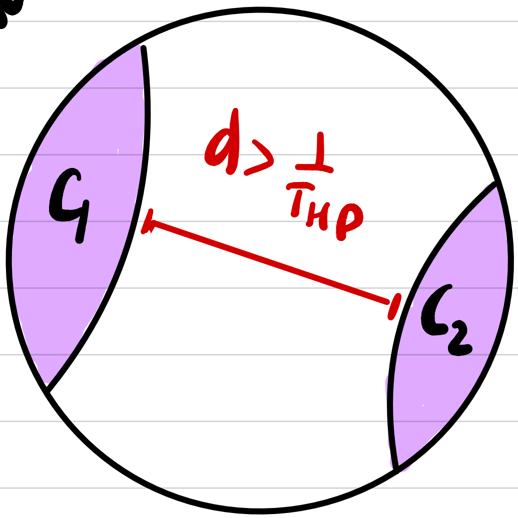
2. For two such regions $\Diamond_1 \cup \Diamond_2$

then, prove: $\mathcal{E}(\Diamond_1 \cup \Diamond_2) = C(\Diamond_1 \cup \Diamond_2)$

if diamonds are sufficiently separated $d > d_c$

with $\mathrm{Tr}_{\mathcal{K}_N}\, e^{-\beta H_N} \to \mathrm{Tr}_{\mathcal{H}}\, e^{-\beta H}$ $\forall$ $T < 1/d_c = T_{HP}$

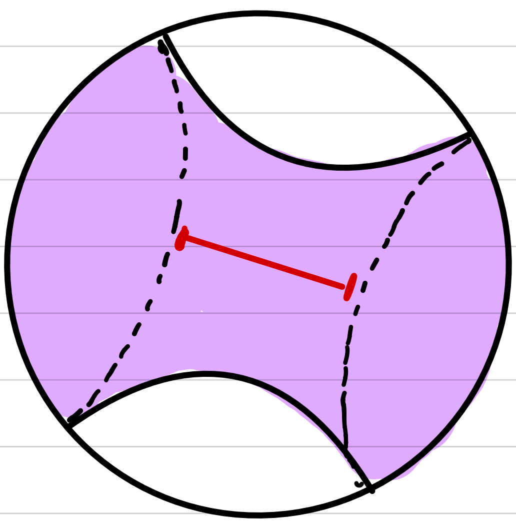
Slice AdS:



d: exactly the minimal geodesic distance b/w minimal surfaces C_1, C_2

Non-trivial dynamical statement

<u>Note</u>: $d_{HP} > d_{MI}$ so more work required to reproduce AdS/CFT predictions



Proof of this fact use same ideas relating split property to thermodynamics. Longo et al.

Hagedorn Divergence in Bulk Theory

$$\mathrm{Tr}_{\mathcal{H}}\, e^{-\beta H} = \infty$$

$$T > T_{Hag}$$

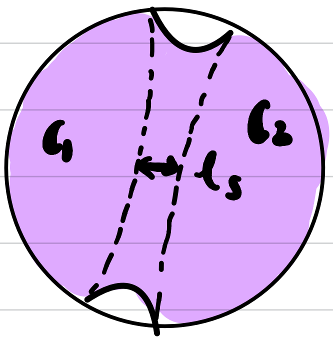
$\mathcal{N}=4$ SYM at finite λ

→ order of limits: $\lim_{T \to T_{Hag}} \lim_{N \to \infty}$

Plausibly Leads To Violations of the Split Property:

(Split: $\mathcal{C}_1 \vee \mathcal{C}_2 \cong \mathcal{C}_1 \otimes \mathcal{C}_2$)

For $d < 1/T_{Hag} \sim 1/\lambda^{1/4}$



Can be detected using the code.

Thank You!