Nonsupersymmetric branes in heterotic string theories

Kazuya Yonekura (Tohoku University.)

- [2303.17623] and [To appear]
  with J. Kaidi, K. Ohmori, Y. Tachikawa

- [2403.14933]

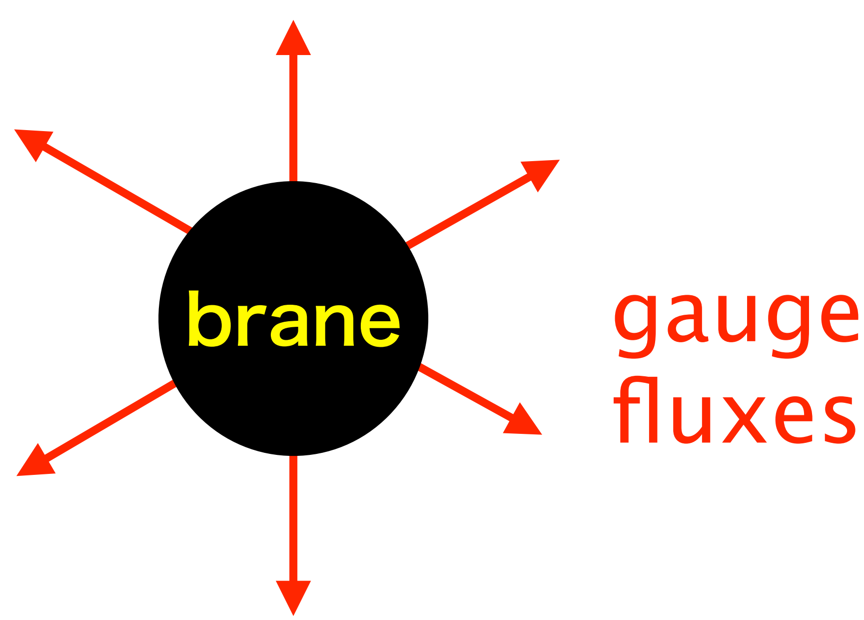
- [To appear]
  with M. Fukuda, S. Kobayashi, K. Watanabe

Introduction

In quantum gravity, it is believed that there exist dynamical objects for all possible charges.

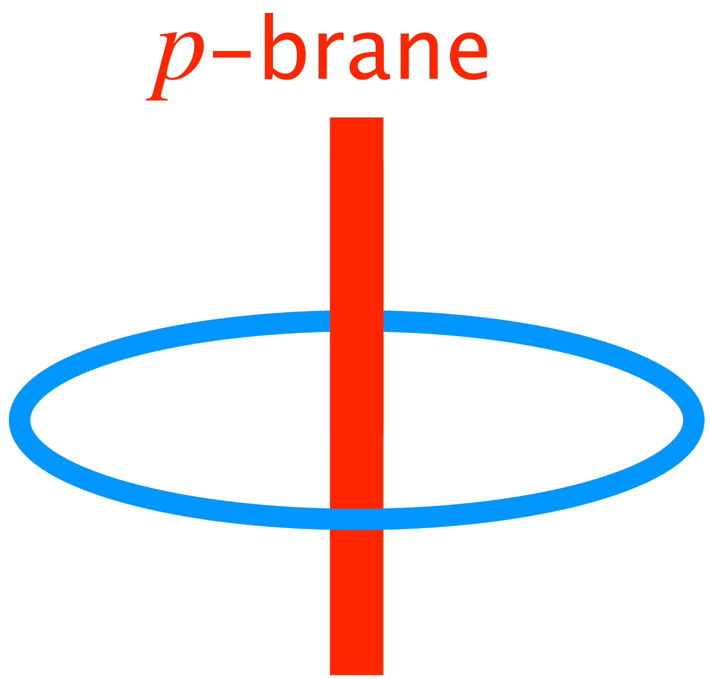
Black branes can often (but not always) realize such objects.

I will focus on generalizations of magnetic charges which are characterized by topology of gauge fields.



Introduction

In general, charges are measured by the behavior of gauge fields at infinity.



$$\mathbb{R}^{9,1} \sim \mathbb{R}^{p,1} \times \mathbb{R}_{\text{radial}} \times S^{8-p}$$

S^{8-p} : sphere at infinity surrounding the brane

A generalization of magnetic charge is a topologically nontrivial gauge configuration on the sphere S^{8-p}.

It is conjectured that for any given topology at infinity, there exist branes or at least some configurations.

[McNamara,Vafa,2019]

Introduction

I will talk about the heterotic superstring theories with gauge group G :

$$G = \mathrm{Spin}(32)/\mathbb{Z}_2 \;\; \text{or} \;\; (E_8 \times E_8) \rtimes \mathbb{Z}_2$$

The purpose:

To study branes characterized by topologically nontrivial gauge field configurations on the sphere S^{8-p} surrounding the brane.

It turns out that they have very nontrivial properties that are not seen before.

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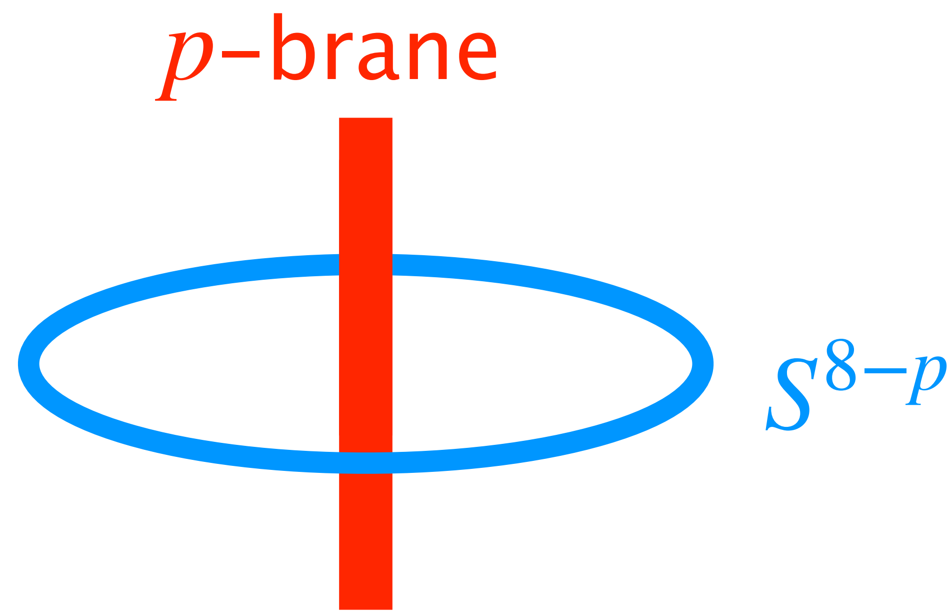
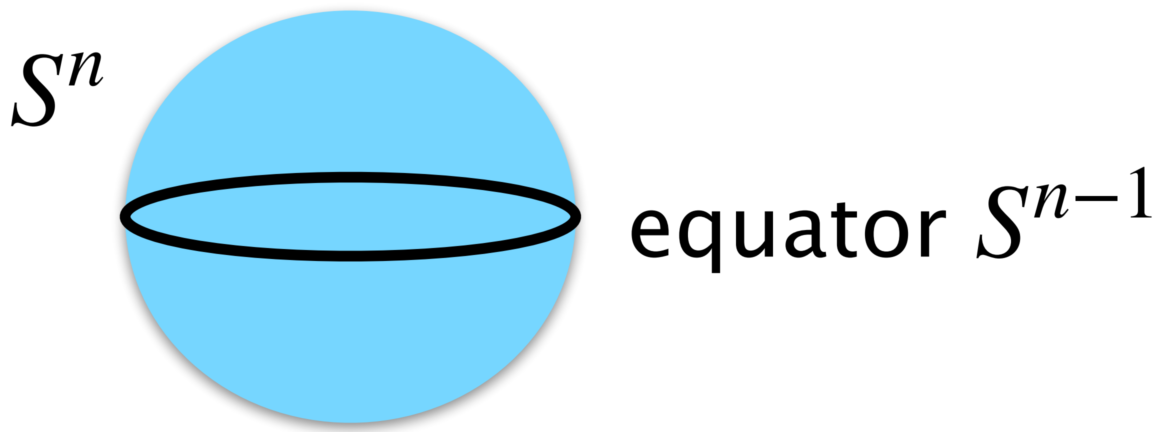
Gauge field topology

S^{8-p} : angular sphere surrounding the brane



Topologies of gauge fields on S^n ($n = 8 - p$) for a gauge group G are classified by the homotopy group $\pi_{n-1}(G)$.



Transition function:

$g : S^{n-1} \to G$

$[g] \in \pi_{n-1}(G)$

Homotopy groups

Four nontrivial homotopy groups of
$G = \mathrm{Spin}(32)/\mathbb{Z}_2$ or $(E_8 \times E_8) \rtimes \mathbb{Z}_2$

- $\pi_0((E_8 \times E_8) \rtimes \mathbb{Z}_2) = \mathbb{Z}_2$ → 7–brane
- $\pi_1(\mathrm{Spin}(32)/\mathbb{Z}_2) = \mathbb{Z}_2$ → 6–brane
- $\pi_3((E_8 \times E_8) \rtimes \mathbb{Z}_2) = \mathbb{Z} \times \mathbb{Z}$ → 4–brane
- $\pi_7(\mathrm{Spin}(32)/\mathbb{Z}_2) = \mathbb{Z}$ → 0–brane

In this talk, I will mainly focus on the 6–brane.
Other branes have similar properties.

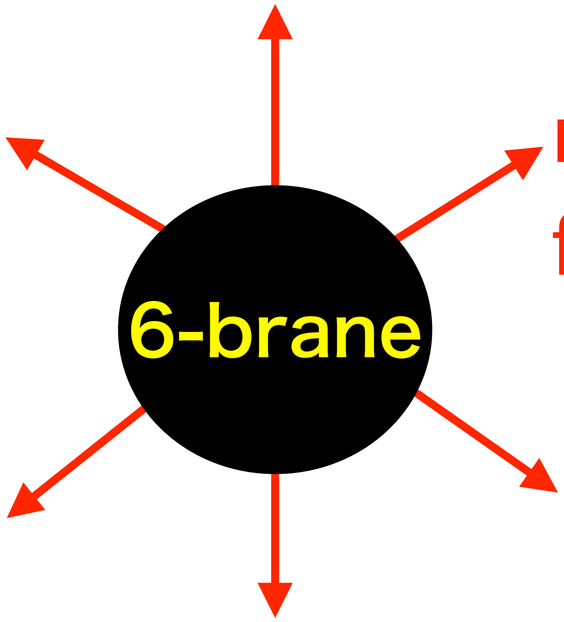
Early work on 0 and 4–branes:
[Polchinski,2005], [Bergshoeff,Gibbons,Townsend,2006]

6-brane magnetic flux

$\pi_1(\mathrm{Spin}(32)/\mathbb{Z}_2) = \mathbb{Z}_2$
Possible magnetic field configurations on $S^{8-p} = S^2$:

$$F_{\mu\nu} = \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & q_i \\ -q_i & 0 \end{pmatrix} \cdot \frac{\epsilon_{\mu\nu}}{2}$$

($\epsilon_{\mu\nu}$: volume form on S^2)



- If all $q_i \in \mathbb{Z}$, then this $F_{\mu\nu}$ corresponds to $0 \in \mathbb{Z}_2$
- If all $q_i \in \frac{1}{2} + \mathbb{Z}$, then $F_{\mu\nu}$ corresponds to $1 \in \mathbb{Z}_2$

The stable configuration is $q_i = \frac{1}{2}$.

Black 6-brane solution

Supergravity solutions for the 6-brane have been basically obtained long time ago.

[Horowitz-Strominger,1991]
based on early work

Similar solutions for the 4 and 0-brane may be possible.

[Fukuda-Kobayashi-Watanabe-KY, to appear]

The solutions are qualitatively similar to those of NS5-branes.

I will only present the extremal limit.

Extremal 6-brane solution

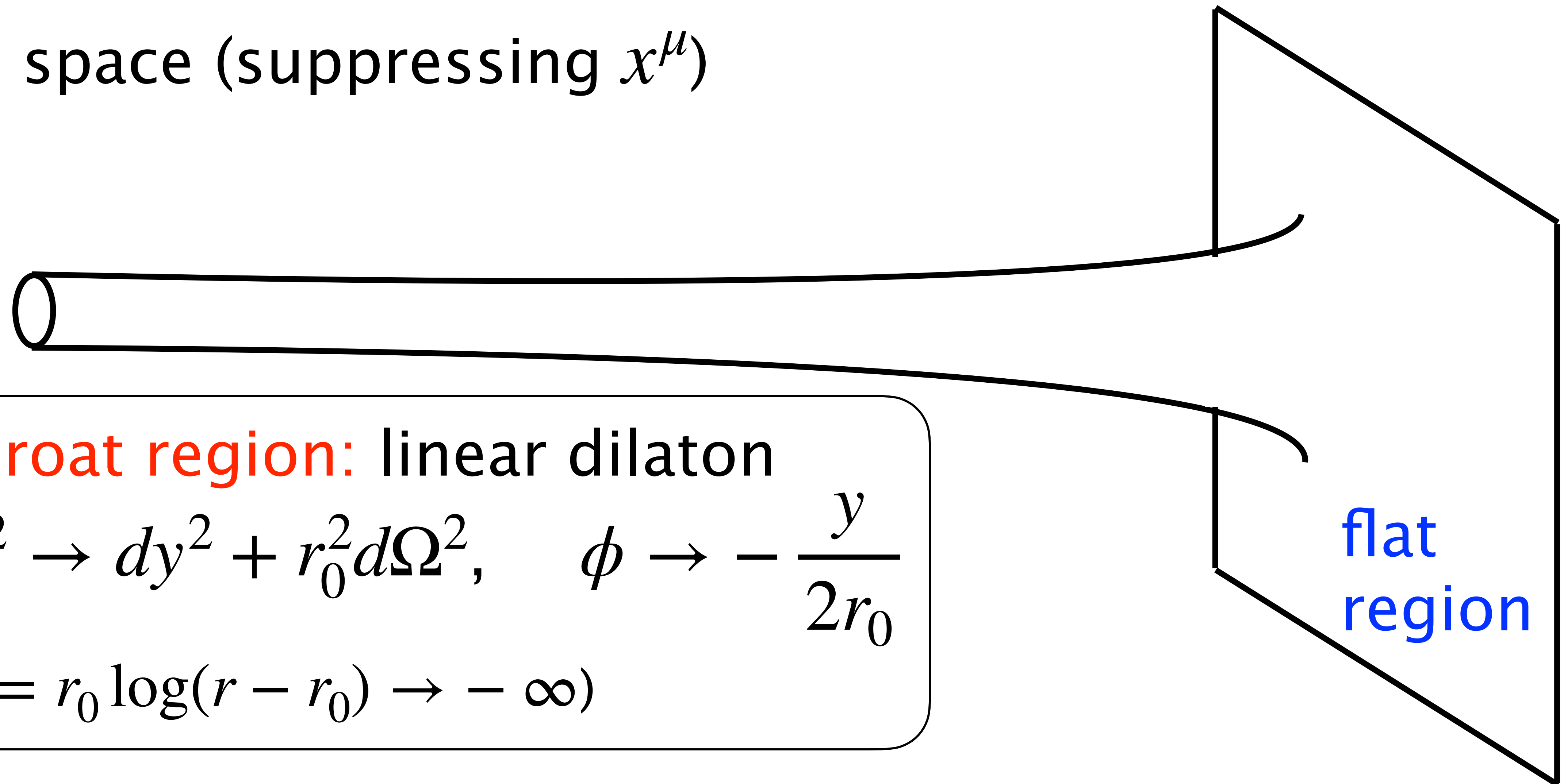
metric : $$ds^2 = dx^\mu dx_\mu + \frac{dr^2}{(1 - r_0/r)^2} + r^2 d\Omega_2^2$$

dilaton : $$e^{-2\phi} = g_s^{-2}(1 - r_0/r)$$

- $x^\mu = (t, \vec{x})$: 7-dimensions parallel to the 6-brane
- r : radial direction
- Ω_2 : angular S^2 surrounding the brane
- r_0 : constant with $r_0^2 = \frac{1}{8}\alpha' \sum_{i=1}^{16} q_i^2$

Extremal 6-brane solution

(r, Ω) space (suppressing x^μ)



Throat region: linear dilaton

$$ds^2 \to dy^2 + r_0^2 d\Omega^2, \qquad \phi \to -\frac{y}{2r_0}$$

$(y := r_0 \log(r - r_0) \to -\infty)$

The throat may be a holographic dual of the worldvolume theory. (Little string theory in the case of NS5–branes).

[Aharony,Berkooz,Kutasov,Seiberg,1997]

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Worldsheet for flat space

The radius of S^{8-p} in the throat region is stringy: $r_0^2 \sim \alpha'$.
A worldsheet analysis to all orders of α' is more appropriate.

In flat 10 dimensions, the string worldsheet theory is

$$\mathbb{R}^{1,9} \times G_1$$

- $G = \mathrm{Spin}(32)/\mathbb{Z}_2$ or $(E_8 \times E_8) \rtimes \mathbb{Z}_2$
- G_1 : current algebra theory at level 1

Worldsheet for throat region

The worldsheet theory for the throat region:

[Kaidi,Ohmori,Tachikawa,KY, 2023]

$$\mathbb{R}^{1,p} \times \mathbb{R}_{\text{linear dilaton}} \times H_k$$

- $\mathbb{R}^{1,p}$: $\mathcal{N} = (1,0)$ sigma model with flat target space
- $\mathbb{R}_{\text{linear dilaton}}$: $\mathcal{N} = (1,0)$ linear dilaton CFT
- H_k : current algebra theory with group H and level k

For the 6-brane ($p = 6$),

$$H = SU(16)/\mathbb{Z}_4, \quad k = 1$$

Very brief sketch of derivation

Dualities of Current algebra : $G_1 \sim H_k \times \mathrm{SO}(8-p)_1$

$$\mathbb{R}^{1,9} \times G_1 \sim [\mathbb{R}^{1,p} \times \underline{\mathbb{R}_{\text{radial}}} \times \underline{S^{8-p}}] \times [H_k \times \underline{\mathrm{SO}(8-p)_1}]$$

$[\mathcal{N} = (1,1)\ S^{8-p}$ multiplets$]$

$\mathbb{R}_{\text{linear dilaton}}$

gapped in the IR (i.e. disappears)

We get $\mathbb{R}^{1,p} \times \mathbb{R}_{\text{linear dilaton}} \times H_k$. (I omit the details.)

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Worldvolume theory

The worldvolume theories on the p-branes are very likely to be strongly coupled, "non-Lagrangian", nonsupersymmetric theories in higher dimensions.

"Non-Lagrangian" nature is suggested by considerations of massless fermions and anomalies.

Massless fermions after compactification on S^{8-p} are obtained by computing either

- fermion zero modes on S^{8-p} in supergravity, or
- fermion spectrum using $\mathbb{R}^{1,p} \times \mathbb{R}_{\text{linear dilaton}} \times H_k$

Anomalous fermions

It turns out that fermions have anomalies under H and require Green–Schwarz mechanism.

The case of 6–brane :

- $S^{8-p} = S^2$ with magnetic flux $F_{\mu\nu} = \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & 1/2 \\ -1/2 & 0 \end{pmatrix} \cdot \frac{\epsilon_{\mu\nu}}{2}$
- The commutant of $F_{\mu\nu}$ in $G \sim SO(32)$: $H \sim SU(16)$

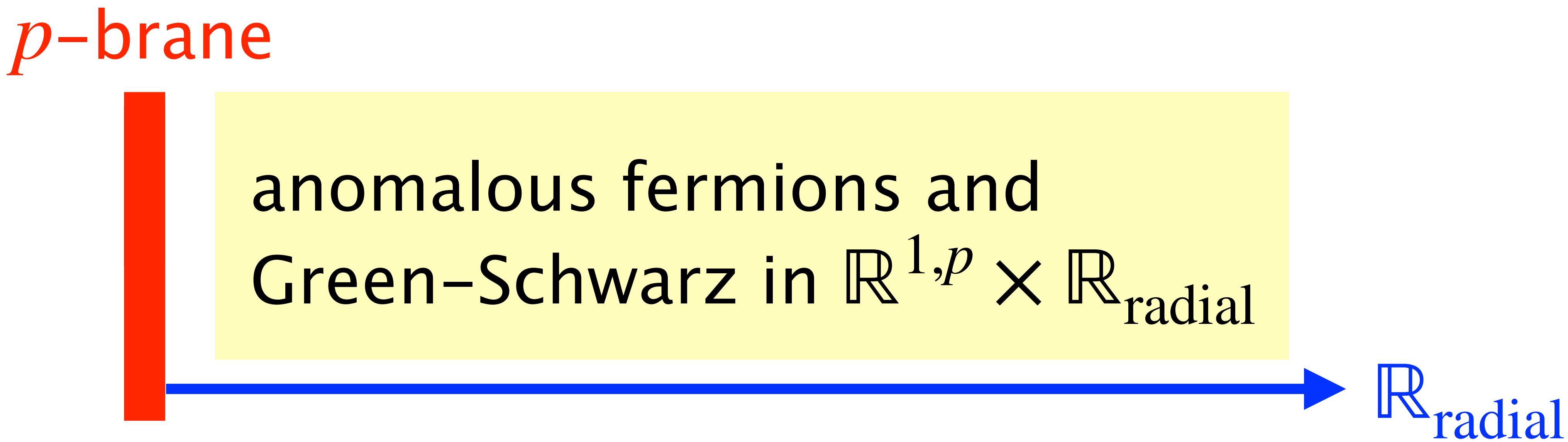
Green–Schwarz in 10 dim. $B_2 \wedge \mathrm{tr}\,(F_{SO(32)})^4$

↓ S^2 compactification with $F_{\mu\nu}$

Green–Schwarz in 8 dim. $B_2 \wedge \mathrm{tr}\,(F_{SU(16)})^3$

Brane as a boundary condition

After dimensional reduction on S^{8-p} :

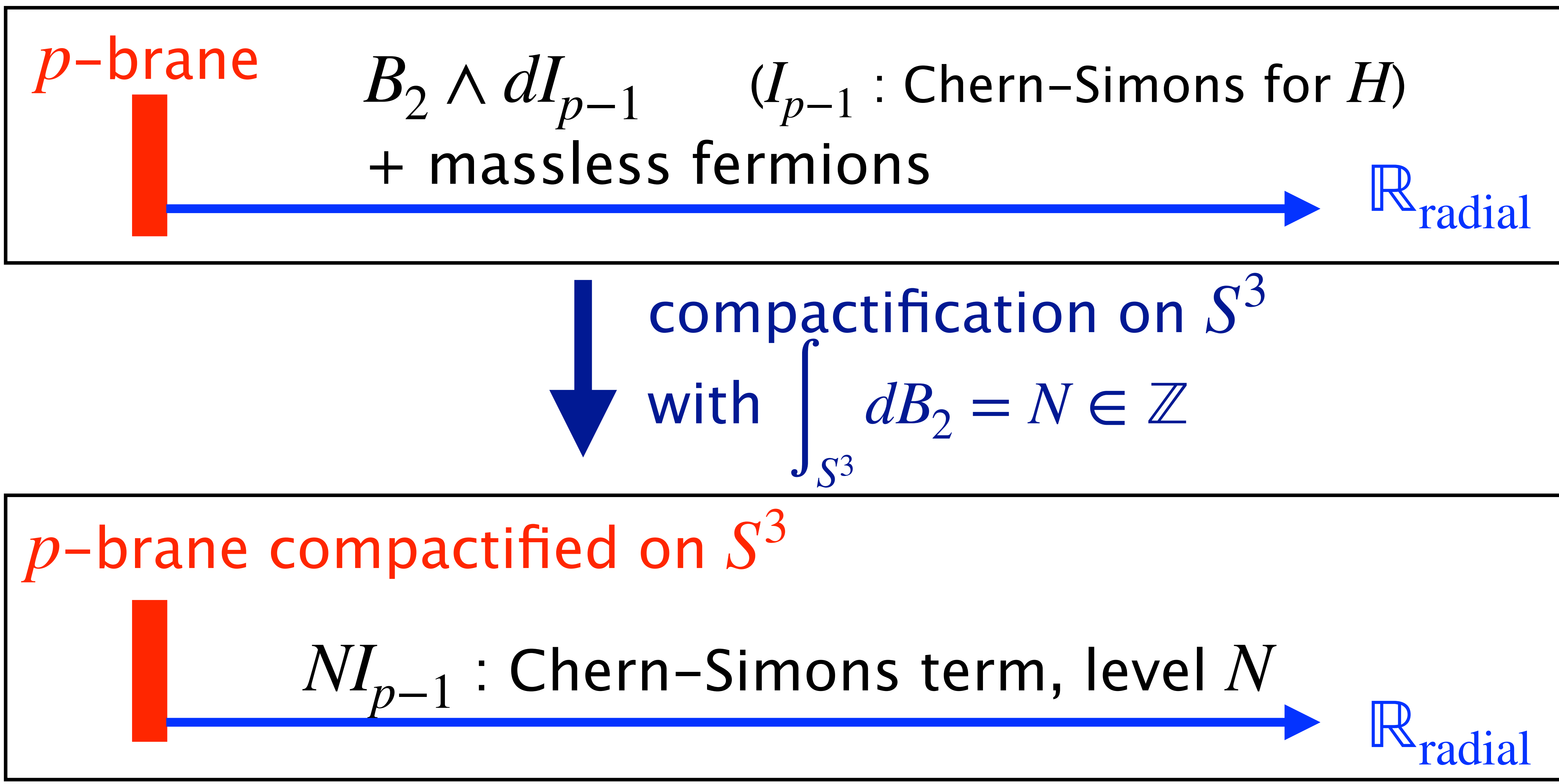


It is very hard to imagine an explicit boundary condition for chiral fermions at the p–brane.

The existence of anomalies suggests that the boundary condition should mix the fermions and the B–field.
It is very likely to be non–Lagrangian.

Compactification on S^3

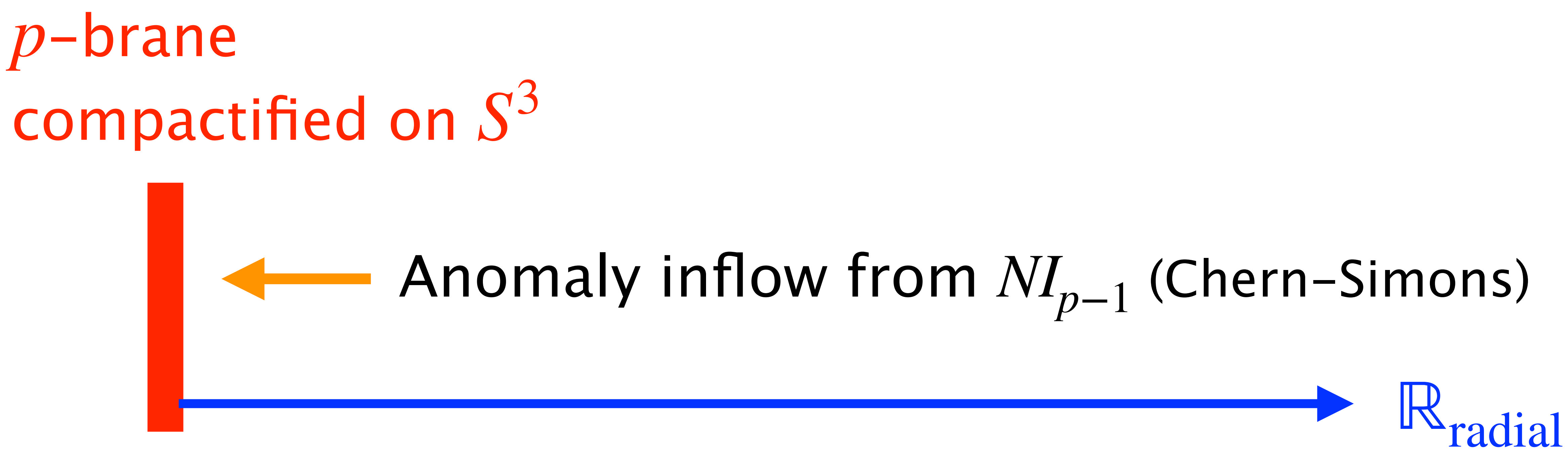
Some further nontrivial properties may be revealed by compactification on S^3.



Anomaly inflow

H is a global symmetry from the point of view of the brane.

The p–brane compactified on S^3 with the B–field flux N has a 't Hooft anomaly determined by anomaly inflow from NI_{p-1}.



Duality?

What is the explicit theory after compactification on S^3 ?

Conjecture: [KY,2024]

p–brane compactified on S^3 with $\int dB_2 = N$

↕ dual?

N NS5–branes compactified on S^{8-p}
with the gauge flux associated to $\pi_{7-p}(G)$

For the 6–brane, NS5–branes are compactified on S^2 with

$$F_{\mu\nu} = \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & 1/2 \\ -1/2 & 0 \end{pmatrix} \cdot \frac{\epsilon_{\mu\nu}}{2}$$

Duality?

The 6-brane on S^3 with the B-field flux N:

- 4-dim. $SU(N)$ gauge theory
- $H \sim SU(16)$ global symmetry
- chiral fermions
  - $2 \times$ symmetric representation of $SU(N)$
  - $2 \times$ antisymmetric representation of $SU(N)$
  - bifundamental of $SU(N) \times SU(16)$ ← match the $SU(16)$ anomaly from inflow

It is very hard to imagine a 7-dim. Lagrangian whose compactification on the odd-dim. sphere S^3 gives 4-dim. chiral fermions having (perturbative) 't Hooft anomalies.

7-dim. theory is very likely to be non-Lagrangian.

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Summary

- In heterotic superstring theories, there exist p-branes whose charges are characterized by the topology of gauge fields on the sphere S^{8-p} surrounding the brane.

- The exact worldsheet theory for the near horizon region of the p-brane is
$$\mathbb{R}^{1,p} \times \mathbb{R}_{\text{linear dilaton}} \times H_k$$
H_k : some current algebra theory

- The worldvolume theories on the branes may be non-Lagrangian and have mysterious properties related to anomalies and chiral fermions.