Lessons from JT gravity

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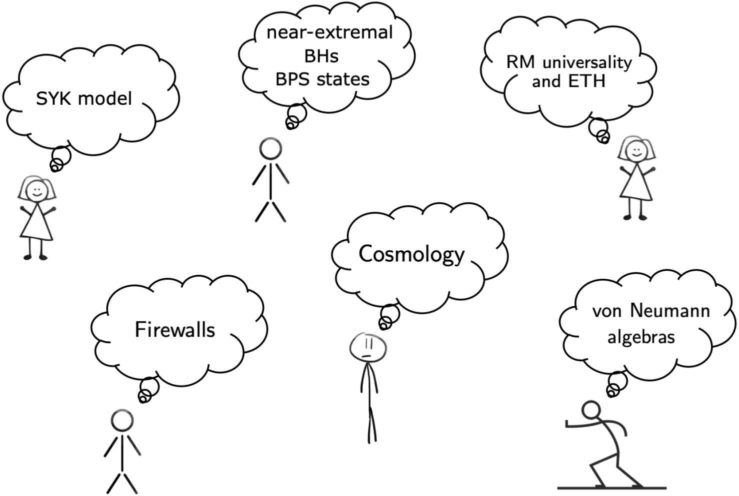
JT gravity is a simple model of quantum gravity that teaches us some lessons about the gravitational path integral.



Part 1: What is JT gravity?

2d Dilaton Gravity

In 2d, the Einstein-Hilbert action is topological and does not suppress fluctuations. To solve this problem introduce a dilaton Φ:

$$I = \underbrace{-\frac{S_0}{4\pi}\int_{\mathcal{M}}\sqrt{g}R}_{\text{topological}} \underbrace{-\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}(\Phi R + U(\Phi))}_{\text{dynamical}} + (\text{bdy terms})$$

- S_0: Parameter suppressing topology change.
- $U(\Phi)$: Dilaton potential determining the classical geometry.
- For black hole solutions $\Phi|_{\text{horizon}}$ determines the entropy while $U(\Phi)|_{\text{horizon}}$ determines the temperature.

Jackiw-Teitelboim (JT) Gravity

JT gravity corresponds to a linear dilaton potential $U(\Phi) = -\Lambda\Phi$ with action

$$I_{JT} = -\frac{S_0}{4\pi}\int_{\mathcal{M}} \sqrt{g}R - \frac{1}{2}\int_{\mathcal{M}} \sqrt{g}\Phi(R-\Lambda) + (\text{bdy terms})$$

- Solutions are spacetimes with constant curvature $R = \Lambda$. We have AdS ($\Lambda < 0$) and dS ($\Lambda > 0$) versions JT gravity[1].
- We will focus mostly on AdS and set $\Lambda = -2$.
- One can also include matter fields χ. In the simplest case they do not couple to the dilaton, e.g.

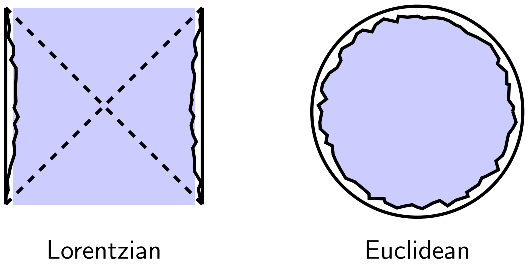
$$I_{\text{matter}}[g,\chi] = \frac{1}{2}\int \sqrt{g}\{(\partial\chi)^2 + m^2\chi^2\}.$$

For flat solutions pick $U(\Phi) = U_0$.

JT Gravity - Boundary Conditions

The integral over Φ imposes $R = -2$, so the only (gravitational) dynamics involves the location of the regularized boundary

$$\mathrm{d}s^2|_{\partial\mathcal{M}} = \frac{\mathrm{d}t^2}{\varepsilon^2}, \qquad \Phi|_{\partial\mathcal{M}} = \frac{\Phi_r}{\varepsilon}, \quad \varepsilon \to 0$$



Boundary curve is determined by $f(t) \in \mathrm{Diff}(S^1)/\mathrm{SL}(2,\mathbb{R})$. Also arises by considering large diffeomorphisms.

The action for the boundary curve (with matter sources turned off) is the Schwarzian theory

$$-I[f] = \underbrace{S_0}_{\text{from topological term}} + \underbrace{\Phi_r \int \mathrm{d}t \left\{ \tan \frac{\pi f(t)}{\beta}, t \right\}}_{\text{from dynamical terms}}$$

S_0 gives a finite contribution to entropy.

Φ_r is a dimensionful parameter, responsible for breaking the conformal symmetry (time reparameterizations).

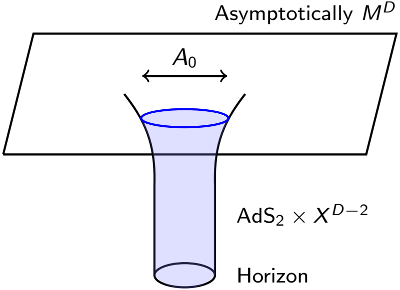
If Φ_r is zero, the symmetry is unbroken but fluctuations in the boundary shape are unsuppressed.

[Almheiri/Polchinski 14, Jensen 16, Maldacena/Stanford/Yang 16, Mertens/Englesoy/Verlinde 16]

Part 2: Where does it arise?

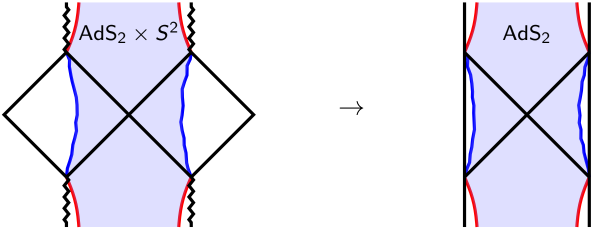
Near-Extremal Black Holes

Near-extremal black holes universally have an $\mathrm{AdS}_2 \times X^{D-2}$ throat with an emergent isometry that includes the 1d conformal group.



The asymptotic space M^D could be flat or AdS_D. Sharper statements can be made in the context of AdS/CFT.

It is useful to study the dynamics separately in the throat, and in the far-away region, and glue. Old idea implemented in multiple examples, recently [Nayak/Shukla/Soni/Trivedi/Vishal 18, Moitra/Trivedi/Vishal 18, Castro/Larsen/Papadimitriou 18, Sachdev 19]. E.g. for Reissner-Nordstrom:



Asymptotic observable determines boundary condition at the throat.

For example, entropy arises mostly from near-horizon physics. As another example, Hawking radiation spectrum is determined by AdS_2 boundary two-point function.

Higher-dimensional gravity in the throat is equivalent to JT gravity coupled to matter. Some comments:

JT-gravity sector:

- S_0: extremal Bekenstein-Hawking entropy.
- $g_{\mu\nu}$: metric along temporal and radial directions.
- Φ: Deviation of Area(X) from extremal value.
- $|\Phi| \ll S_0 \Rightarrow$ non-linear dilaton-potential terms suppressed by powers of $1/S_0$.

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Matter sector:

- Arises from all other KK modes of higher-dimensional metric and matter. $R_{\mathrm{AdS}} \sim R_X$ implies large number of light matter fields.
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Boundary Condition:

- Φ_r determined from gluing to far-away region [Almheiri/Kang 16].

Examples

1. Near-extremal BTZ in AdS_3. Near-horizon geometry: $AdS_2 \times S^1$.

$$S_0 = 2\pi\sqrt{\frac{cJ}{6}}, \qquad \Phi_r = \frac{c}{24}$$

2. Near-extremal Kerr-Newman in 4d flatspace. Near-horizon geometry: $AdS_2 \times S^2$.

$$S_0 = \pi\sqrt{Q^4 + 4J^2}, \qquad \Phi_r = \frac{G_N E_0}{\pi} S_0 \underset{J\to 0}{=} \sqrt{G_N}\, Q^3$$

3. Near-BPS black hole in $AdS_5 \times S^5$, dual to excitations of 1/16-BPS states in $\mathcal{N} = 4$ SYM. Near horizon geometry: $AdS_2 \times S^3 \times S^5$.

$$S_0 = N^2 \underbrace{s_0(Q/N^2, J/N^2)}_{O(1)}, \qquad \Phi_r = N^2 \underbrace{\phi_r(Q/N^2, J/N^2)}_{O(1)}$$

Relevant to Chang talk

SYK Model: QM model of large number of strongly interacting fermions with approximate 1d conformal symmetry (time reparameterizations) at low energies. At non-zero temperatures time reparametrizations get an action proportional to the Schwarzian [Sachdev 10, Kitaev 15, Maldacena/Stanford 16 . . .]

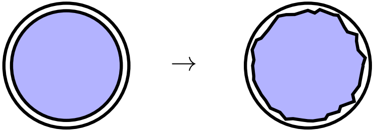
Non-critical String: 2d gravity coupled to matter CFT with central charge c. Described by JT gravity in the limit $c \to -\infty$.
[Saad/Shenker/Stanford 19, Seiberg/Stanford, Mertens/GJT 20 . . .]
Interesting developments described by Collier.

Part 3: What is it good for?

JT gravity is solvable in some regimes, and we will review implications of its solution.

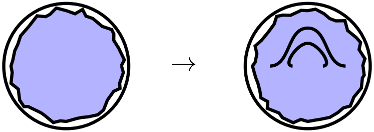
To begin, it is important to identify clearly the coupling constants of the theory.

1. Φ_r/β suppresses perturbative metric fluctuations (Schwarzian mode)



$\delta I[f] \sim \Phi_r/\beta$

2. S_0 suppresses non-perturbative fluctuations in topology (spacetime wormholes)



$\sim e^{-2S_0}$

In higher dimensions both roles are played by G_N. JT gravity with matter is exactly solvable for any Φ_r/β in the large S_0 limit.

Several approaches to solve JT gravity in the limit $S_0 \to \infty$ and Φ_r/β finite:

1. Liouville QM [Altland/Bagrets/Kamenev 16]
2. Localization [Stanford/Witten 17]
3. Limit of Liouville CFT [Mertens/GJT/Verlinde 17]
4. Particle in a magnetic field [Kitaev/Suh 18, Yang 18, Suh 19]
5. Double-scaled SYK [CGHPSSSST 16, Berkooz/Isachenkov/Narovlansky/Torrents 18]
6. Wheeler-de Witt Quantization [Harlow/Jafferis 18 and Lin]
7. JT gravity as a BF theory [Blommaert/Mertens/Verschelde 18, Iliesiu/Pufu/Verlinde/Wang 19]
8. Near-extremal limit of 2d CFT [Ghosh/Maxfield/GJT 19]
9. Limit of non-critical string [Mertens/GJT 20]

Different approaches emphasize different aspects of the physics, but are all in agreement! Results reviewed in [Mertens/GJT 22].

We continue with some applications that exploit the presence of these two separate coupling constants.

Application 1: Extremal Limit and BPS State Counting

Classical approximation to the gravitational path integral implies the near-extremal BH thermodynamic behavior

$$S \approx S_0 + 4\pi^2 \Phi_r T, \qquad E \approx E_0 + 2\pi^2 \Phi_r T^2.$$

This raises two puzzles:

1. Ground state at fixed charge is highly degenerate. Implausible without symmetry principle (3rd law of thermodynamics).

2. Breakdown of the statistical description of black hole microstates when

$$\frac{\delta T}{T} = \left(\frac{\partial T}{\partial E}\right)_Q \gtrsim O(1) \quad \text{when} \quad T \lesssim T_{\text{breakdown}} = 1/\Phi_r.$$

[Preskill/Schwarz/Shapere/Trivedi/Wilckez 91, Maldacena/Michelson/Strominger 98, Page 00]

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Why would deviations from the semiclassical approximation be important at low temperatures? Curvature can be made as small as we wish.

[Preskill/Schwarz/Shapere/Trivedi/Wilckez 91, Maldacena/Michelson/Strominger 98, Page 00]

Both puzzles are resolved by large perturbative quantum gravity effects at low temperatures. A soft mode in the metric arises that needs to be quantized.

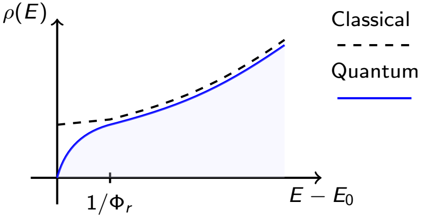
The JT gravity description of the throat makes their origin transparent. $\beta/\Phi_r = T_{\text{break.}}/T$ acts as a coupling constant for perturbative metric fluctuations. At large S_0 the theory is solvable allowing us to quantize it and correct the classical analysis.

Lets summarize the consequences...

[Ghosh/Maxfield/GJT 19, Iliesiu/GJT 20, Heydeman/Iliesiu/GJT/Zhao 20 ... Iliesiu/Murthy/GJT 22 ...]

Near-Extremal Limit

$$\rho(E) = e^{S_0} \sinh\left(\sqrt{8\pi^2 \Phi_r (E - E_0)}\right)$$



- Extremal black holes do not exist, even in absence of any decay channel!
- Expected gaps of order e^{-S_0} from non-perturbative physics.
- Holds in supergravity when the extremal limit does not preserve any supersymmetry.
- Correlation functions are affected by large quantum corrections when either $t \gtrsim \Phi_r$ or $\beta \gtrsim \Phi_r$. Conformal invariance lost in this regime, late time power-law decay in time.

These conclusions apply for near-extremal Kerr black holes [Kapec/Sheta/Strominger/Toldo 23, Rakic/Rangamani/GJT 23], although superradiance is likely to make the effect unobservable [Maldacena/Murthy, unpublished].

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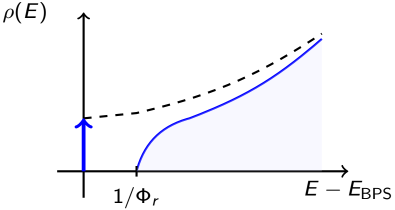
Conceptually similar quantum effects are also relevant in the near-Nariai limit of black holes in $D \geq 4$ de Sitter space [Maldacena/GJT/Yang 19].

Near-BPS Limit

$$\rho(E) = e^{S_0}\,\delta(E - E_{\text{BPS}}) + \dots$$



- Extremal black holes survive if they preserve two supercharges or more, with large quantum effects.
- Gap of order $1/\Phi_r$ visible in perturbation theory. Decoupling limit works.
- Correlation functions are affected by large quantum corrections when either $t \gtrsim \Phi_r$ or $\beta \gtrsim \Phi_r$. Correlators become constant at late times.
- The effect of matter is to shift S_0, a calculation pioneered by Ashoke Sen.

Bosonic symmetries do not lead to substantial modifications.
Supersymmetry leads to a richer set of possibilities:

Supercharges	(BPS) Ground States	Gap	Index	R-symmetry
$\mathcal{N}=0$	No	No	–	–
$\mathcal{N}=1$	No	No	0	–
$\mathcal{N}=2_{(\widehat{q},\delta)}$	Yes[1]	Yes[2]	0^3	U(1)
$\mathcal{N}=2_{(1,0)}$	Yes	Yes	=degeneracy	U(1)
$\mathcal{N}=4$	Yes	Yes	=degeneracy	SU(2)

Multiple $\mathcal{N}=2$ theories depending on $\widehat{q}$ (R-charge of the supercharge) and $\delta \mod \mathbb{Z}$ (background R-charge).

Exact correlation functions are known for $\mathcal{N} \leq 2$.

$\mathcal{N}=2$ case relevant for AdS_4 and AdS_5. $\mathcal{N}=4$ relevant for flat space and AdS_3.

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Q: Can geometric microstate constructions reproduce this behavior?

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These developments were used, in combination with earlier progress on localization in supergravity, to compute the gravitational path integral determining the index for 1/8-BPS black holes in $\mathcal{N} = 8$ supergravity, reproducing the precise **integer** value from string theory constructions.

[Banerjee/Banerjee/Gupta/Mandal/Sen 09, Dabholkar/Gomes/Murthy 10 to 14 . . . Iliesiu/Murthy/GJT 22 and questions raised by Sen 23]

Application 2: Pure JT Gravity as a Matrix Integral

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Near-extremal black holes: besides allowing for topology change, finite S_0 also has the effect of adding interactions in the matter and dilaton sector, that are not currently solvable with available techniques.

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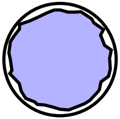
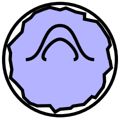
Toy model: consider pure JT gravity. Finite S_0 has the only effect of incorporating topology change via spacetime wormholes [Saad/Shenker/Stanford 19].

2d topologies can be classified by the number of boundaries n and genus g

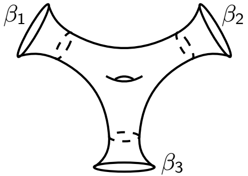
$Z_{\text{grav}}(\beta) = e^{S_0}$  $+ \; e^{-S_0}$  $+ \dots$

The first goal is to evaluate the gravitational path integral on such spaces.

Integrating out the dilaton leads to an integral over moduli spaces of hyperbolic surfaces. These have in general two contributions:

- Boundary Moduli: Schwarzian mode at each boundary.
- Bulk Moduli: Measure arises from one-loop determinants, efficiently evaluated using torsion [Stanford/Witten 19]. For orientable surfaces one-loop effects are trivial leading to Weil-Petersson measure (but not for non-orientable surfaces).

SSS found that the gravitational path integral over spacetimes with n boundaries and g wormholes (for example for $g = 1$ and $n = 3$)



is given by

$$Z_{g,n} = \prod_{i=1}^{n} \Big(\underbrace{\int_0^\infty b_i \mathrm{d} b_i}_{\text{WP measure}} \underbrace{Z_{\text{bdy grav.}}(\beta_i, b_i)}_{\text{bdy graviton (trumpet)}} \Big) \underbrace{V_{g,n}(b_1, \ldots, b_n)}_{\text{bulk moduli integral}}$$

The bulk integral includes the Weil-Petersson volumes $V_{g,n}$ with geodesic boundaries [Witten, Kontsevich, Mizakhani].

Weil-Petersson volumes satisfy the same topological recursion as the one appearing in the large N limit of matrix integrals [Eynard/Orantin 07].

SSS showed that this implies that the gravitational path integral (to all orders in the topological expansion) is

$$Z_{\text{grav.}}(\beta_1, \ldots, \beta_n) = \overbrace{\int_{L\times L} \mathrm{d}H\, P(H)}^{\text{average over Hamiltonians}} \quad \underbrace{\mathrm{Tr}\, e^{-\beta_1 H} \ldots \mathrm{Tr}\, e^{-\beta_n H}}_{\text{Product of partition functions on each bdy}}$$

Therefore, pure JT gravity is dual to a matrix integral! The probability distribution $P(H) = \exp(-L\,\mathrm{Tr}\,V(H))$ is determined by the disk density of states.

Douglas will discuss some generalizations of this result.

Doubly-nonperturbative Effects

Previous discussion was carried out in perturbation theory in e^{S_0} and the matrix integral provides a specific non-perturbative definition.

Non-perturbative contributions in the topological expansion are of order $e^{\# e^{S_0}}$. Some examples:

- Corrections to forbidden region (below threshold): One-eigenvalue instanton suppressed by $e^{-e^{S_0}}$ [Shenker,...].
- Corrections to spectrum: rapidly oscillating $e^{\mathrm{i}\# e^{S_0}}$, gas of FZZT branes [SSS 19].
- Another approach: Solve exact string equation numerically at finite e^{S_0} [Johnson].

Some comments

Quantum chaotic systems have a spectrum conjectured to share statistical features with a random matrix [Wigner ... Bohigas/Giannoni/Schmit ...].

The "pure JT gravity" toy model illustrates this mechanism in gravity; spacetime wormholes are responsible for quantum chaos in the black hole spectrum.
⚠ Doesn't mean every aspect of this model has to be taken literally!

Example: The double-cone wormhole [Saad/Shenker/Stanford 18, Chen/Ivo/Maldacena 23] is a universal contribution for any black hole in any dimensions displaying level repulsion, without implying the dual is a random matrix. Would be great to produce more results of this type.

Application 3: Quantum Chaos in BPS States

Thanks to the gap in the spectrum of near-BPS black holes we can isolate the BPS ground states. The decoupling limit works, but it is strongly coupled.

Most work on BPS microstates in string theory consists on matching S_0 between gravity and QM [Strominger/Vafa ...].

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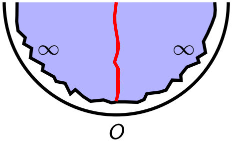
An answer was provided by [Lin/Maldacena/Rozenberg/Shan 22], showing one can use the late-time limit of correlation functions to test the randomness of BPS states wavefunction [Property of random matrix ensemble with extended susy].

Define a reference basis adapted to simple operators O. Project these simple operators to the BPS subspace

$$O \to \widehat{O} = P\,O\,P, \qquad P = e^{-\infty H}.$$

This projection can be implemented in the gravitational path integral by infinite Euclidean time evolution (comes with large boundary graviton effects!).



The idea is that the spectrum of the projected operators $\widehat{O}$ is chaotic. This is captured by spacetime wormhole contributions to correlators of projected operators $\mathrm{Tr}\big[\widehat{O}_1 \ldots \widehat{O}_n\big]$. [Lin/Maldacena/Rozenberg/Shan 22]

More comments by Lin later

Part 4: technical generalizations

Changing the dilaton potential

The dilaton gravity action

$$I = -\chi S_0 - \frac{1}{2}\int \sqrt{g}(\Phi R + U(\Phi)) + (\text{bdy term})$$

is specified by a number S_0 and a function $U(\Phi)$.

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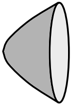
The disk path integral translates these to a density of states:



$$^{\beta} = e^{S_0}\int \mathrm{d}E\,\rho_0(E)e^{-\beta E}$$

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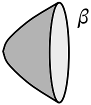

$$= e^{S_0}\int \mathrm{d}E\,\rho_0(E)e^{-\beta E}$$

Can go backwards, obtain $U(\Phi)$ from $\rho_0(E)$ and then use this data to compute higher orders (topologies)

$$\rho(E) = \underbrace{e^{S_0}\rho_0(E)}_{\text{input}} + \underbrace{e^{-S_0}\rho_1(E)}_{\text{genus 1}} + \underbrace{e^{-3S_0}\rho_2(E)}_{\text{genus 2}} + \dots$$

This is similar in structure to e.g. Hermitian one-matrix integrals

$$\int_{L \times L} \mathrm{d}H \, e^{-L \mathrm{Tr} V(H)}$$

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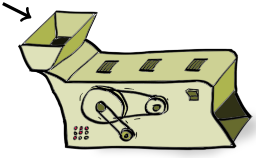
By solving the inverse problem, one can determine $V(H)$ from $\rho_0(E)$. Then in principle one can determine everything else. (In practice, the "loop equations" give a shortcut that doesn't require solving for $V(H)$).

$$\rho(E) = \underbrace{L\rho_0(E)}_{\text{input}} + \underbrace{L^{-1}\rho_1(E) + L^{-3}\rho_2(E) + \ldots}_{\text{determined by loop eqns}}$$

So, both dilaton gravity and matrix integrals are machines like this:

$\rho_0(E)$



$\rightarrow \{\rho_1(E), \rho_2(E), \ldots, \rho_1(E, E'), \ldots\}$

Are the machines identical? [Maxfield/Turiaci 20, Witten 20]

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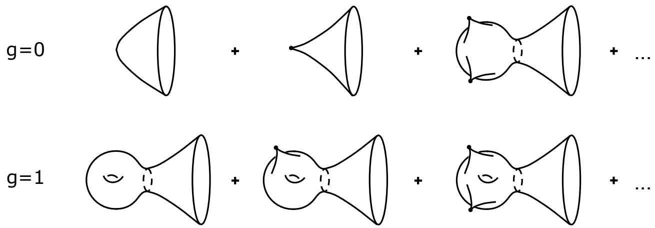
and expand in λ. This generates the stat mech of a gas of conical defects in otherwise hyperbolic space [Maxfield/Turiaci 20, Witten 20]:



If $\alpha < 1/2$, the defects are "sharp" enough to repel geodesics...

... which allowed [Maxfield/Turiaci 20] to use a theorem from [Eynard/Orantin 07] to show that the resulting theory remains exactly dual to a matrix integral. The proof works for

$$U(\Phi) = 2\Phi \to 2\Phi + \int_0^{1/2} \mathrm{d}\alpha \lambda(\alpha) e^{-2\pi(1-\alpha)\Phi}.$$

For the case of blunt defects ($1/2 < \alpha < 1$), the analysis is harder but some checks have been done that suggest the theory remains dual to a matrix integral [Turiaci/Usatyuk/Weng 20, Eberhardt/Turiaci 23].

It would be nice to prove directly that dilaton gravity satisfies the loop equations ("topological recursion"). It would also be nice to realize the older "$(2, p)$ minimal string" using dilaton gravity.

Adding symmetries

In addition to picking $\rho_0(E)$, one can also choose to include various symmetries in a matrix ensemble.

JT gravity can be dressed up so that it is dual to such ensembles:

$$\begin{aligned} \text{time-reversal} &\Longleftrightarrow \\ \text{global symmetry} &\Longleftrightarrow \\ \text{supersymmetry} &\Longleftrightarrow \\ \text{anomalies} &\Longleftrightarrow \end{aligned}$$

Some interesting points:

1.

2.

3.

4.

[Iliesiu, Johnson, Kapec, Mahajan, DS, Turiaci, Witten]

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Some interesting points:

1. Mirzakhani's recursion can be generalized to non-orientable and super-Riemann surfaces
2. the Altland-Zirnbauer 10-fold classification of RMT can (should?) be regarded as a classification of random Hermitian supercharges
3. supersymmetric (e.g. BPS) ground states are random vectors – they do not live in some special subspace that is immune to the chaos
4. the $\mathcal{N} = 4$ case is open :)

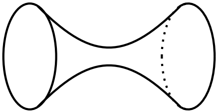
[Iliesiu, Johnson, Kapec, Mahajan, DS, Turiaci, Witten]

Part 5: Where is it going?

The big puzzle

JT is dual to a random ensemble of Hamiltonians, not a particular Hamiltonian. This is because spacetime wormholes lead to a nonzero variance for e.g. the thermal partition function

$$\langle Z(\beta)^2 \rangle - \langle Z(\beta) \rangle^2 \quad =$$



What happens to these wormholes if the bulk theory is dual to a particular boundary theory?

This **factorization problem** is particularly urgent in $d > 1$ where there is no obvious ensemble of boundary theories.

It has been discussed several times at Strings, so we will not focus on it.

Three dimensions

A first step towards higher dimensions could be pure 3d gravity.

Does the sum over topology make sense? Is it dual to an approximate ensemble of CFTs? [Cotler/Jensen 20, Chandra/Collier/Hartman/Maloney 22, Belin/de Boer/Jafferis/Nayak/Sonner 23]

Does the sum over topology diverge in a way that represents the difficulty of defining CFTs? E.g. 3d gravity tries to be modular invariant by including a sum over $SL(2, Z)$, but the sum diverges [Hartman].

Important technical progress in 3d gravity will hopefully help to answer these questions, [Collier/Eberhardt/Zhang 23].

Eberhardt and Sonner are going to comment on 3d gravity in the second part of this session, and Wong on Thursday.

Even higher dimensions

Could one try to make sense of the gravity path integral in higher dimensions, and could its wormholes tell us something about the statistical features of boundary theories?

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So let's discuss 3, within the laboratory environment of 2d.

Adding matter to JT

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After including a matter field ϕ in the bulk, the AdS/CFT dictionary gives us another observable $\mathcal{O}$. So from a boundary perspective the system becomes a type of two matrix integral [Jafferis/Kolchmeyer/Mukhametzhanov/Sonner 22].

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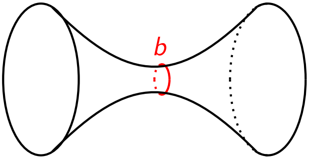
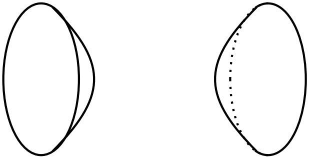
From the bulk perspective, this leads to a problem. On wormholes, dynamical matter fields lead to divergences in the "small wormhole" part of moduli space



$$Z_{\text{matter}} \sim e^{\frac{\#}{b}}.$$

Familiar from the tachyon divergence in worldsheet string theory. Ideas:

1. include ghosts in the bulk so that the total central charge is zero [Kitaev,. . .].
2. change the b contour or deform the bulk theory so that b is discrete [Jafferis/Kolchmeyer/Mukhametzhanov/Sonner 22]
3. use a fancier model where something removes the small b part of the moduli space. E.g. in [Maldacena/Qi 18] SYK analysis, there is a transition to



   Similar to [Horowitz/Polchinski 96].
4. A practical approach is to assume the divergence is cured *somehow* and focus on things that receive contributions from big wormholes (no small cycles)...

JT gravity as a matrix integral as a red herring

In the very special case of dilaton gravity without matter, you can do nice exact computations including small wormholes, and they exactly match random matrix theory.

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This could be a distraction from a more general correspondence

$$\begin{array}{c}\text{random matrix } \textbf{universality} \\ \text{and ETH and } \ldots\end{array} \quad \longleftrightarrow \quad \text{big semiclassical wormholes}$$

Some related issues have been discussed in the quantum chaos literature [Berry, Sieber/Richter, Muller/Heusler/Braun/Haake/Altland]. There the configurations can be analyzed semiclassicaly but they are not classical solutions.

Cosmology

Ideas for applying JT-style concepts to cosmology

1. islands [Dong/Qi/Shagnan/Yang, Krishnan, Hartman/Jiang/Shaghoulian]
2. $-$AdS wormholes [Maldacena/Turiaci/Yang, Cotler/Jensen/Maloney]
3. dS in AdS [Anninos/Hofman]
4. double-scaled SYK [Susskind/Rahman,Verlinde/Narovlansky]
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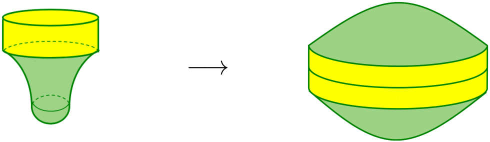
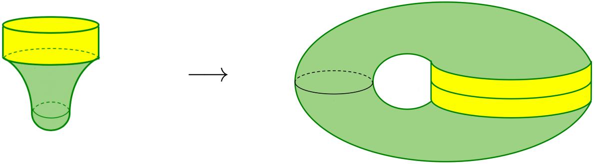
Is dS somehow related to a quantum system with finite entropy S_{dS}?

More on 5: idea is to look for wormhole effects in the wave function or density matrix of the universe.



or



Suppressed by $e^{-2S_{\mathrm{dS}}}$, but enhanced from matter contributions if the universe is sufficiently large.

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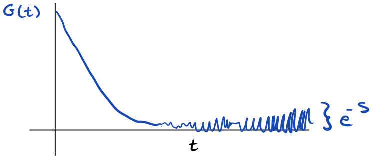
For AdS-JT black holes, [Saad 19] showed that wormholes lead to two-point functions that do not decay forever in time, in keeping with expectations of ETH in a finite-entropy system [Maldacena 01].



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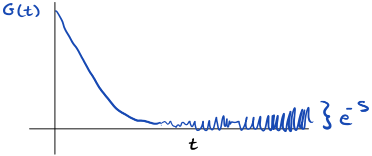

In de Sitter, do two-point functions decay forever? Perhaps wormhole effects lead to non-decay [Mirbabayi 23]. Or perhaps they do decay.

Wormholes and infalling observers

(Back to AdS...)

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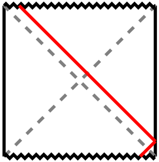
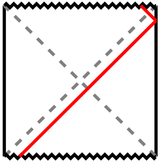
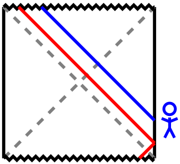
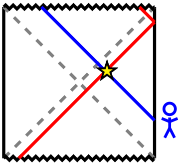
Consider two states of a large two-sided AdS black hole

$\phi(\text{early})|\text{TFD}\rangle =$ 

$\phi(\text{late})|\text{TFD}\rangle =$ 

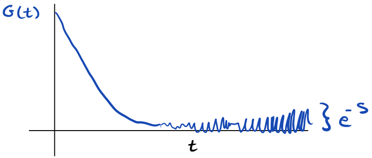
These seem very different from the perspective of an infaller:





Puzzle – these states are not orthogonal, even if the early and late times become very large [Saad 19] (in fact, they span the same space!):

$$\langle \mathrm{TFD}|\phi(t)\phi(0)|\mathrm{TFD}\rangle =$$



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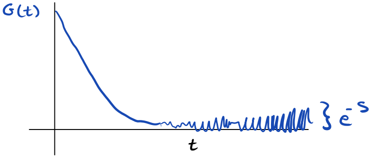

If you prepare $\phi(\text{early})|\mathrm{TFD}\rangle$ is the horizon safe, or could you hit something? **Is the noise dangerous?** [Polchinski 17 unpublished]

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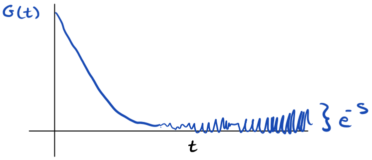

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Practical idea: study infallers on the wormhole geometry that Saad uses to reproduce the (averaged) noise in $G(t)$.

Wormholes may give a large probability of hitting the particle [DS/Yang 22, Iliesiu/Levine/Lin/Maxfield/Mezei 24, Blommaert/Chen/Nomura 24]. But the principles for doing this calculation are unclear...

... for example, [Penington/Witten 23] define a nice procedure to decompose wormhole path integrals into Hilbert space inner products, and this slicing suggests that you do *not* hit the particle.

- what is JT gravity?
- some applications of its solvability (near-extremal and extremal BHs, JT as a matrix integral, chaos in BPS states, . . .)
- generalizations that preserve duality with matrix integral (changing the potential, adding symmetries)
- areas for future work (factorization, higher dimensions, cosmology, infalling observer,. . .)

Now we move to part two of the session, with further comments from:

Maxfield, Lin, Maldacena, Eberhardt, Sonner

Henry Maxfield

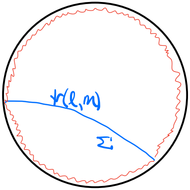
Non-perturbative effects on the gravitational Hilbert space

spatial metric
matter field config.

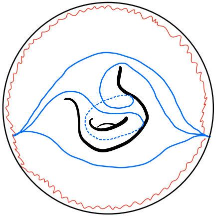
Bulk Hilbert space:

- Wavefunction of local data on a Cauchy surface Σ?
- Needs **gauge fixing:** e.g., $\mathrm{Tr}\,K = 0$ (geodesic).
- Good gauge **perturbatively:** either → geometry close to AdS
  → diffs close to identity
- Get local, non-degenerate inner product

[Witten '22]



$$\langle \ell', m' | \ell, m \rangle = \delta(\ell - \ell')\,\delta_{mm'}$$



- **Non-perturbatively** can be many Σ satisfying gauge condition!
- Discrete **residual diffeomorphisms**
- Results in **degenerate (& non-local) inner product**
- **Null states** with gravitational gauge symmetry explanation

Explicit realisation in JT gravity (with any matter)

[Iliesiu,Levine,Lin,Maxfield,Mezei]

Perturbative: $S_0 \to \infty$

$$\langle \ell' | \ell \rangle = \delta(\ell - \ell') = \int dE\, \rho_0(E)\, \phi_E(\ell)\, \phi_E(\ell')$$

$$\Downarrow \; \Sigma$$

$$\boxed{\langle \ell' | \ell \rangle = e^{-S_0} \sum_i \phi_{E_i}(\ell)\, \phi_{E_i}(\ell')}$$

Non-perturbative $\{E_i\}$ spectrum of dual QM

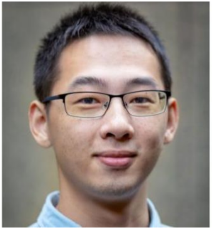
Alternative approach:
complete gauge-fixing.
Global min length (Euclidean)
[Penington,Witten]

Henry Lin

Chaos in BPS black holes

Henry Lin, Stanford University

based on 2207.00407, 2207.00408 [HL, Maldacena, Rozenberg, Shan]
+ work in progress with Yiming Chen & Steve Shenker



see also Strings '22 talk by Maldacena

Examples of BPS black holes governed by $\mathcal{N} \geq 2$ Schwarzian:

1. ground states of $\mathcal{N} = 2$ SYK (regular or double scaled) [Fu, Gaiotto, Maldacena, Sachdev; Berkooz, Brukner, Narovlansky, Raz; Boruch, HL, Yan]
2. 3-charge black hole, $\frac{1}{8}$-BPS sector of D1-D5 CFT
3. $\frac{1}{16}$-BPS "fortuitous" states in $\mathcal{N} = 4$ SYM [Chang & YH Lin; Budzik, Murali, Viera; Chang & Yin, ..., Boruch, Heydeman, Iliesiu, Turiaci]

super Schwarzian: more than just the microstate count

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Consider BPS black holes in asymptotically AdS_{d+1}.

$V_i = \mathrm{CFT}_d$ primary dual to a BPS black hole microstate.

$$C_{\mathrm{i}O\mathrm{j}} = \langle V_i | \, O_{\mathrm{simple}} \, | V_j \rangle$$

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Statistics of these OPE coefficients, e.g.,

$$\sum_{\mathrm{i,j}} C_{\mathrm{i}O\mathrm{j}} C_{\mathrm{j}O\mathrm{i}} \sim \mathrm{Tr}\Big(e^{-\infty H} O_{\mathrm{simple}} e^{-\infty H} O_{\mathrm{simple}} \Big)$$

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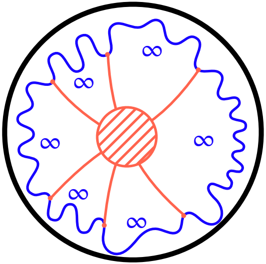
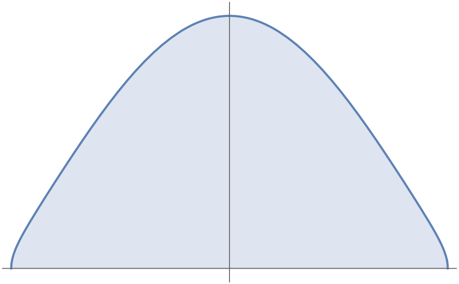
Compute RHS using the gravity path integral:



Upshot: C_{iOj} should behave like a random matrix



BPS chaos $\approx$ eigenvalue repulsion

With Yiming Chen and Steve Shenker, studying some **non-examples** that lack a horizon:

1. $\frac{1}{2}$-BPS states in $\mathcal{N} = 4$ SYM/Lin-Lunin-Maldacena geometries
2. $\frac{1}{4}$-BPS states in $\mathcal{N} = 4$ SYM at 1-loop
3. 2-charge fuzzballs, or $\frac{1}{4}$-BPS states in the D1-D5 CFT.

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Plausibility argument for chaos in the $\frac{1}{16}$-BPS case based on fortuity ideas/continuation in N (the rank of the gauge group).

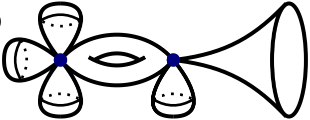
Question for the fuzzball community: does $\widehat{O}_{\text{simple}}$ have a chaotic[1] spectrum for 3-charge fuzzballs? Could known 3-charge fuzzballs be less chaotic than a typical black hole?

[1] strong chaos can be distinguished from weak chaos by the scale $\delta\lambda_{\text{thouless}}$ in which RMT statistics sets in.

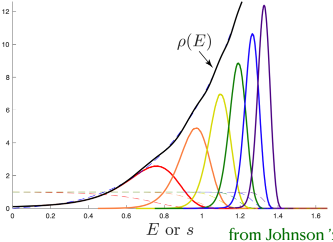
Lorenz Eberhardt

(Doubly) non-perturbative effects

- ZZ-instantons in the worldsheet description of the $(2,p)$ minimal string or the Virasoro minimal string [SSS '19, Collier, LE, Mühlmann, Rodriguez '23]



- Introduces boundaries to the worldsheet
- Bulk computation necessitates string field theory [Eniceicu, Mahajan, Murdia, **Sen** '20 - …] [Majahan's talk]
- These effects can be of order $e^{ie^{S_0}}$
- They probe the discreteness of the matrix model spectrum [SSS '19, Johnson '22, …]
- Their presence can be predicted from resurgence on the asymptotic genus expansion [SSS '19, …]



from Johnson '22

Questions and prospects

- Can we compute these contributions directly using the gravitational path integral?

  [Post, van der Heijden, Verlinde '22]

- Can we probe the plateau of the spectral form factor?

  cf. tau-scaling limit [Blommaert, Kruthoff, Yao '22, Saad, Stanford, Yang, Yao '22]

- Higher-d gravity reduces to JT-gravity in a near extremal limit. Can we uplift such (doubly) non-perturbative effects e.g. to 3d gravity?

Julian Sonner

Ensembles for random (2D)CFT

[Belin, de Boer, Jafferis, Nayak, JS]

Lessons from JT: chaotic Hilbert space of BH states = matrix model

Chaotic CFTs should correspond to a form of random ensemble à la RMT. [Belin, de Boer; Cotler, Jensen; Altland, JS; Chandra, Collier, Hartman, Maloney,…]
This will be a joint statistical model of the data for 2D CFT:

$$\{h_i, \bar{h}_i, C_{ijk}\}$$

[Belin, de Boer, Jafferis, Nayak, JS]

random matrix ($\sim L_{ij}$) random tensor ($\sim C_{ijk}$)

Want:

- CFT constraints satisfied on average: $\overline{\text{crossing}}$ ~ 0
- Fluctuations of constraints small $\boldsymbol{\sigma}^{(2)}$(crossing) ~ 0
- Note: this is not an ensemble of exact CFTs

Matrix/tensor model $\longleftrightarrow$ 3D gravity

[Belin, de Boer, Jafferis, Nayak, JS]

We construct a joint probability distribution of matrix/tensor d.o.f.

$$\int [dL]\,[dC]\,(\,\cdot\,)\,e^{-\mathrm{tr}V(L,C)}$$

random matrix ($\sim L_{ij}$)

random tensor ($\sim C_{ijk}$)

CFT observable

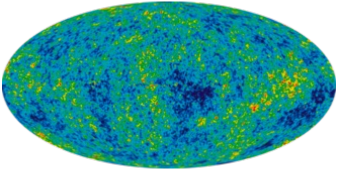
Resulting diagrammatic expansion suggests discrete triangulated 3-geometries.

- Tensor sector: non-Gaussianities & crossing; “Virasoro-Regge theory”
- Matrix/modular sector: Cardy density on average, chaotic spectral statistics
- On-shell agreement with VirTQFT, off-shell extension? [see talk by Wong]

Juan Maldacena

The gravitational path integral and cosmology

- We have a semiclassical approach to the early universe via the theory of inflation.
- It successfully predicts the small primordial curvature fluctuations that we observe in the CMB and the large scale structure.

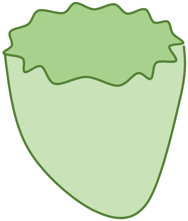


- There is a minor extension of this formalism that leads to a problem…

A Problem with the path integral in cosmology

- The Hartle-Hawking proposal for the wavefunction of the universe.
- It gives the wrong value for the overall <u>curvature</u> of the universe. (Even when applied purely in the context of inflation.)
- It gives the right values for the small scale <u>curvature</u> fluctuations.

- What exactly are we doing wrong ?
  - Quantum corrections becoming large ?
  - Other topologies ?
  - Taking the full string landscape into account?
  - Another principle to select the state of the universe?



It is an important problem!