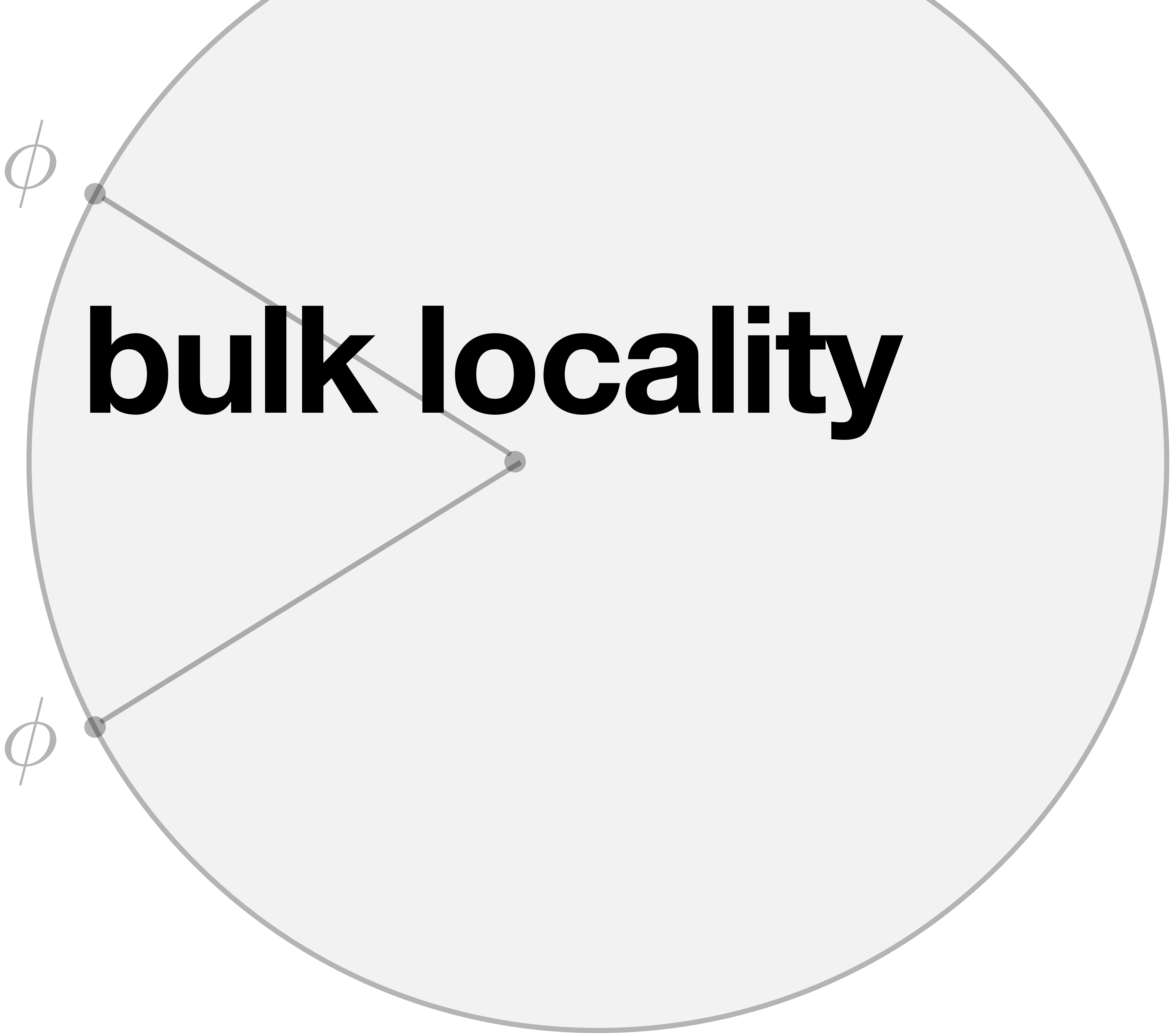
Bootstrapping bulk locality



Nat Levine

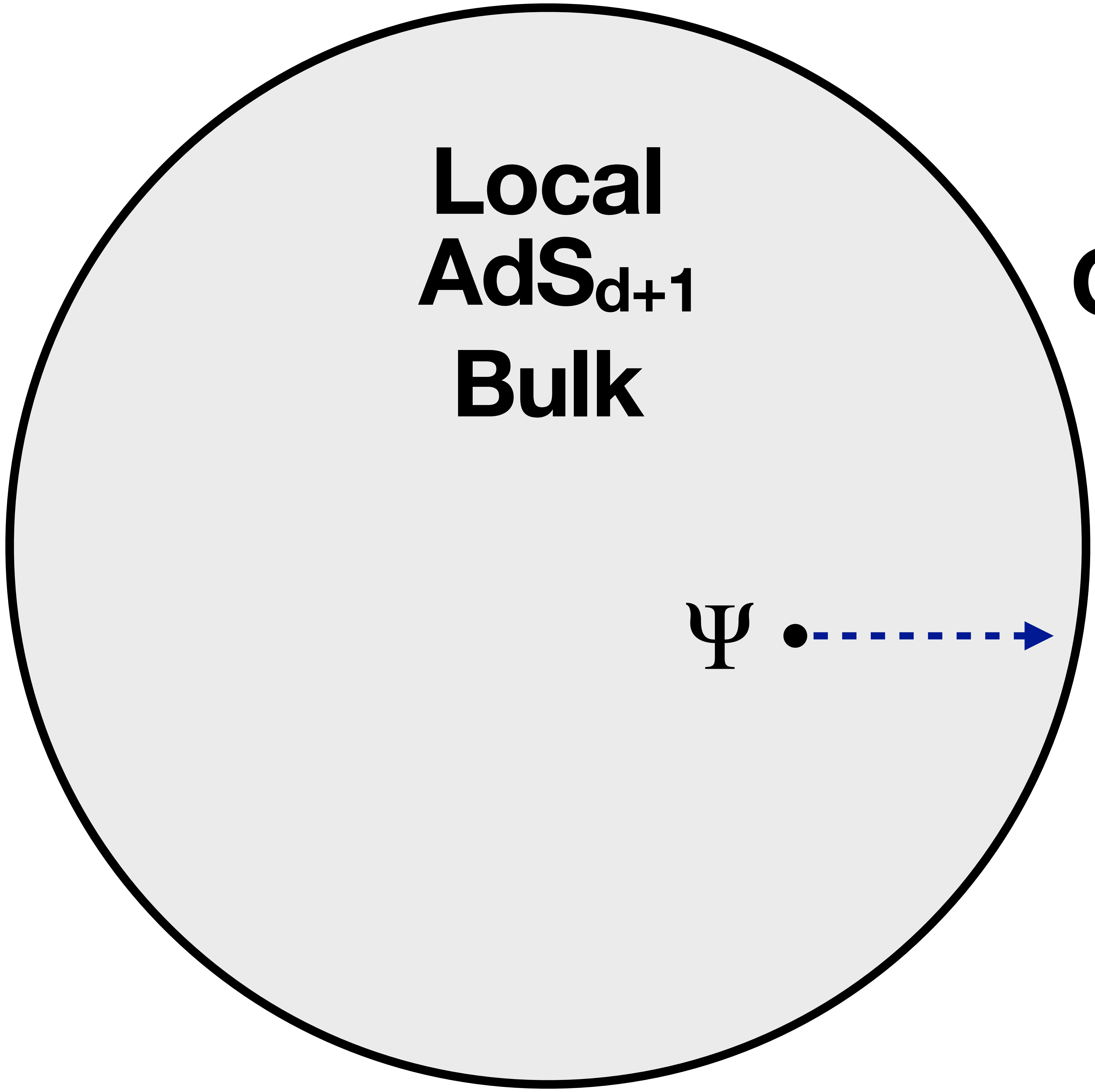
École Normale Supérieure

Strings 2024 Gong Show
CERN

[Part I: 2305.07078]
[Part 2: 24xx.xxxxx]

with Miguel Paulos

Setup

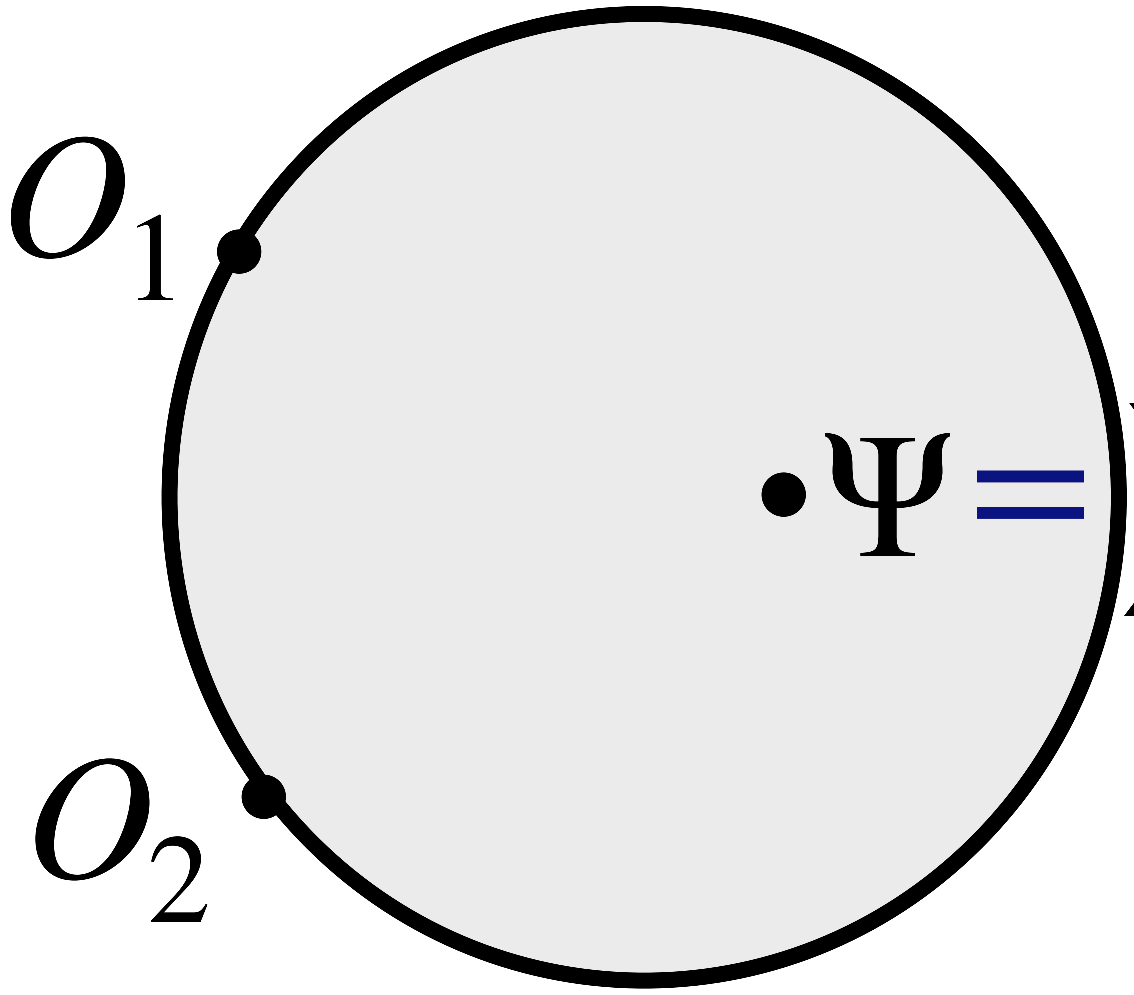


C(F)T$_d$

$$= \text{“}\sum_{\Delta} \mu_{\Delta} O_{\Delta}(x)\text{”}$$

cf. [Bena]
[Hamilton Kabat Lyfchitz Lowe]

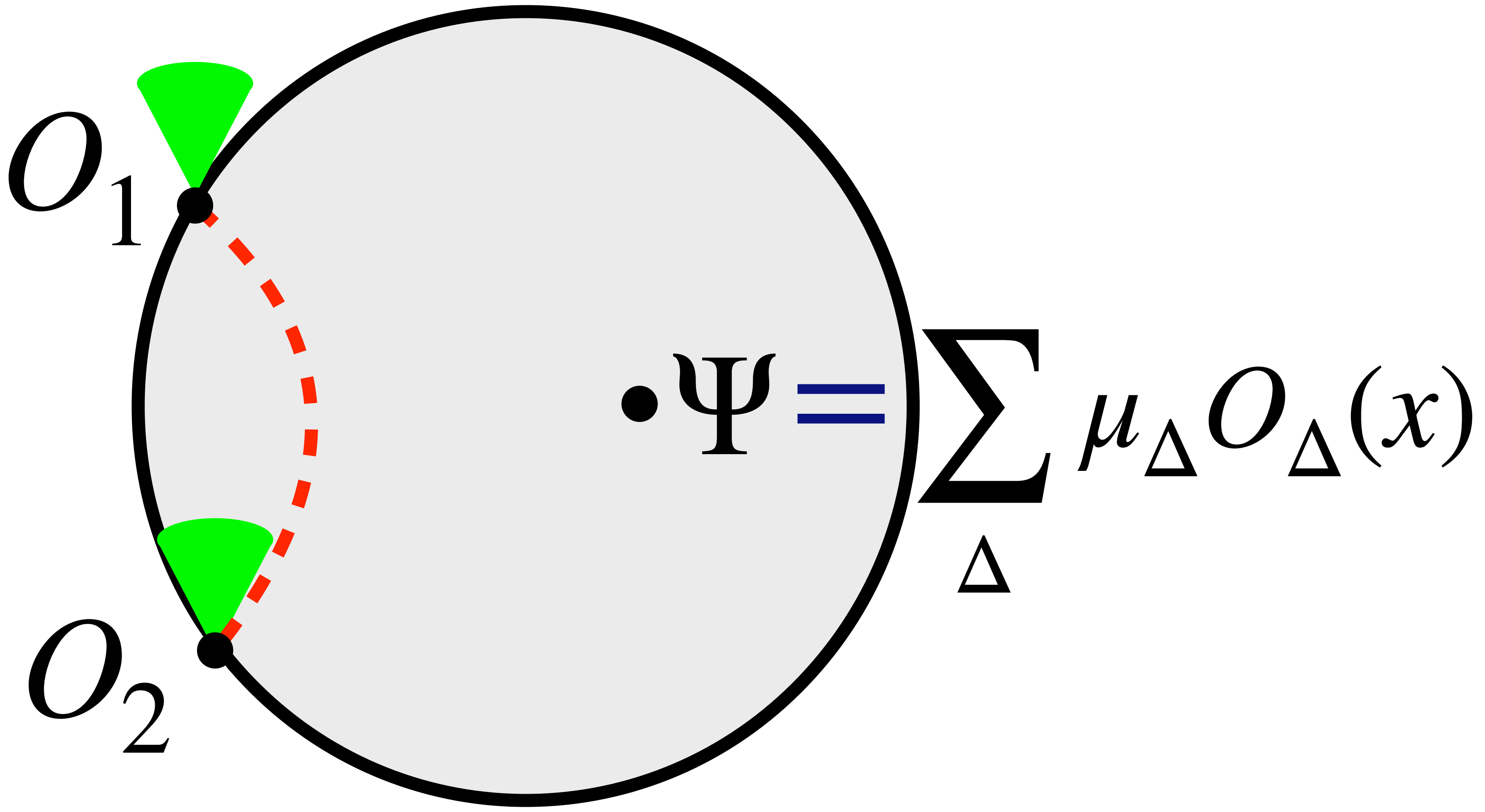
Setup



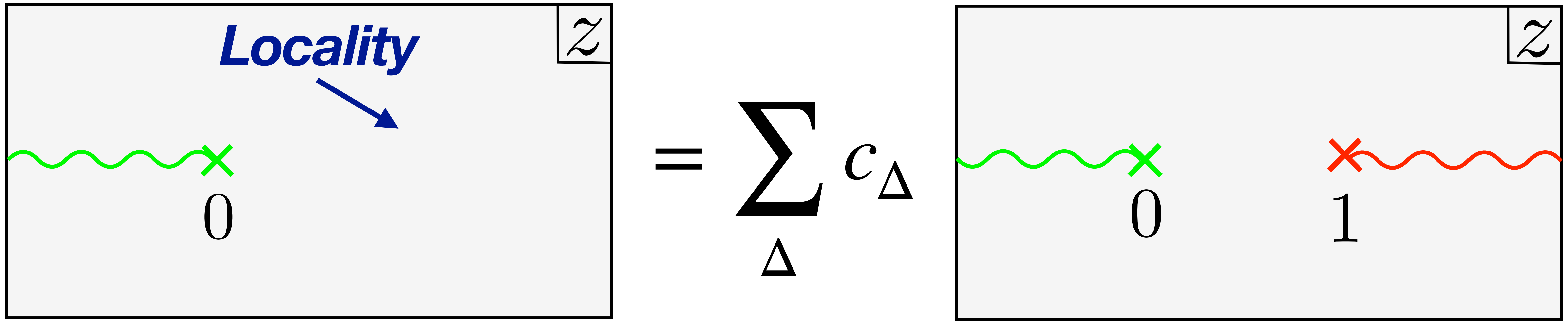
$$\Psi = \sum_{\Delta} \mu_{\Delta} O_{\Delta}(x)$$

$$\langle \Psi \,|\, O_1 O_2 \rangle = F(z) = \sum_{\Delta} c_{\Delta}\, g_{\Delta}(z) \qquad (c_{\Delta} = \mu_{\Delta}\, \lambda_{\Delta}^{12})$$

Setup

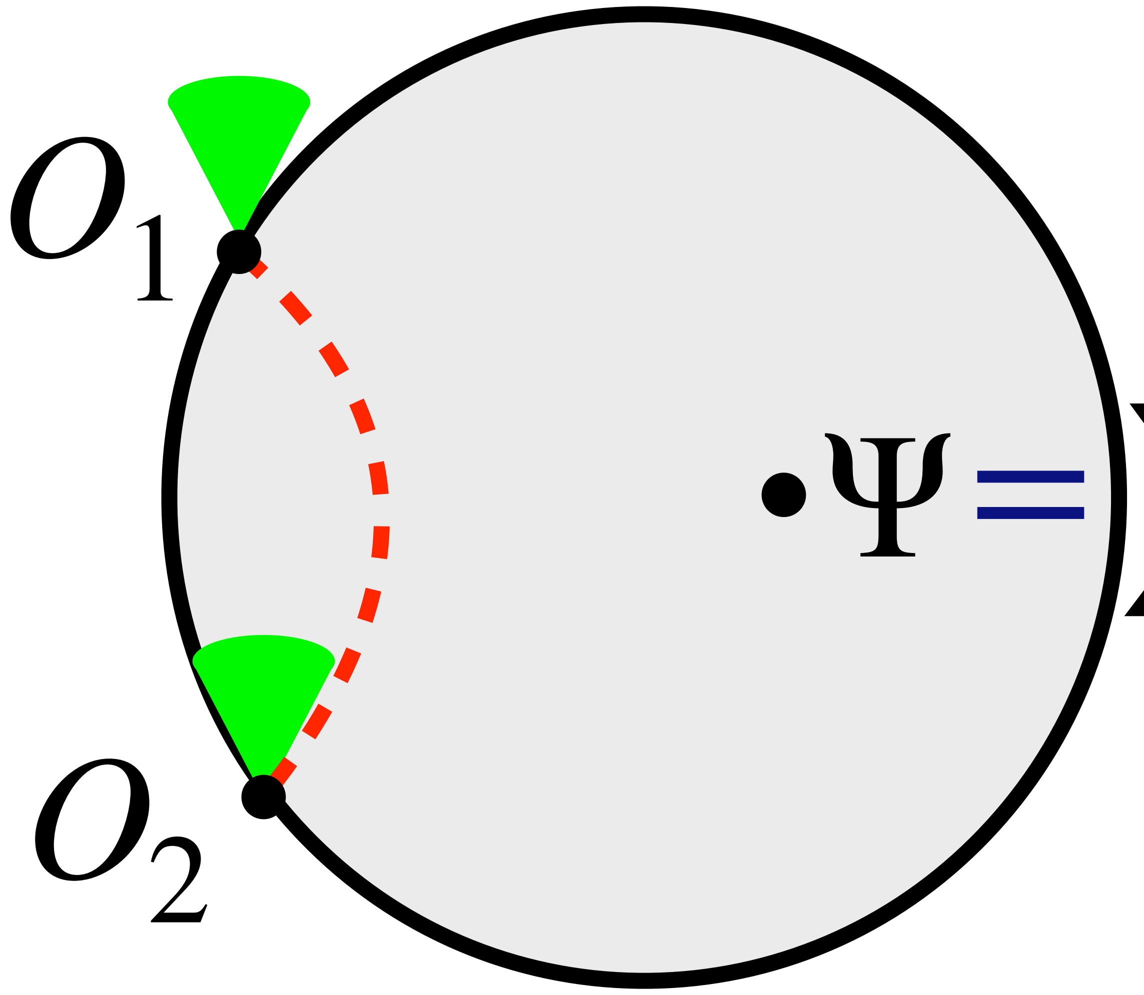


$$\langle \Psi | O_1 O_2 \rangle = F(z) = \sum_\Delta c_\Delta \, g_\Delta(z) \qquad (c_\Delta = \mu_\Delta \, \lambda_\Delta^{12})$$



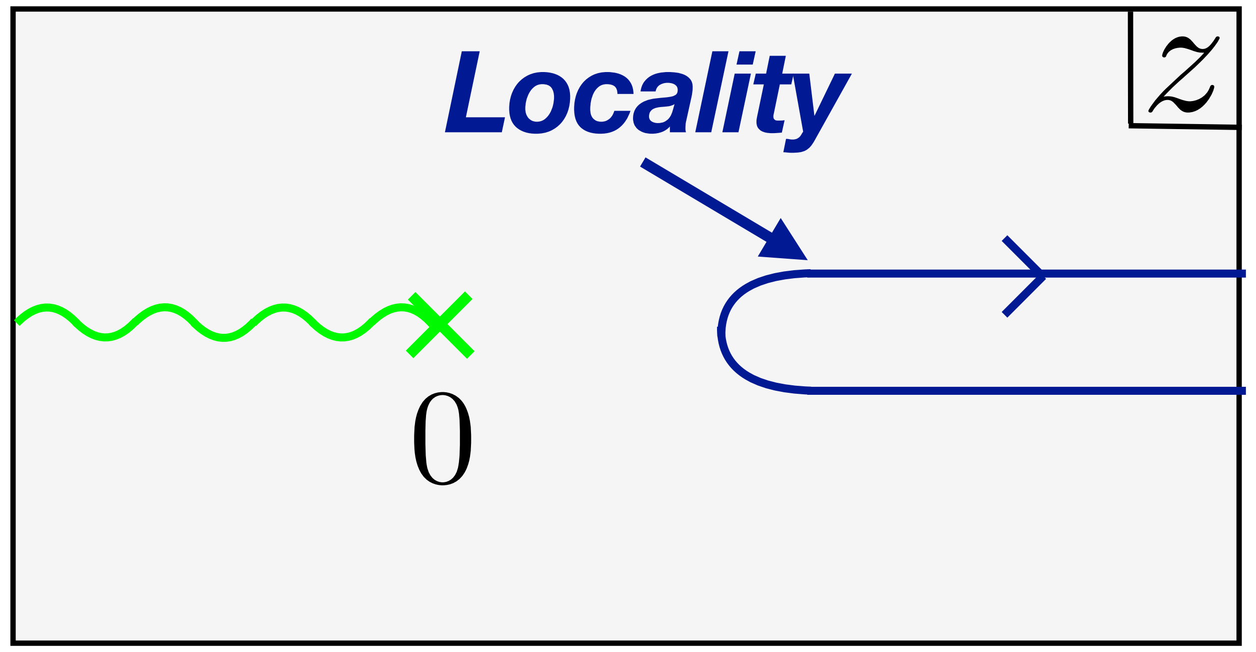
[Kabat Lifschytz]

Setup

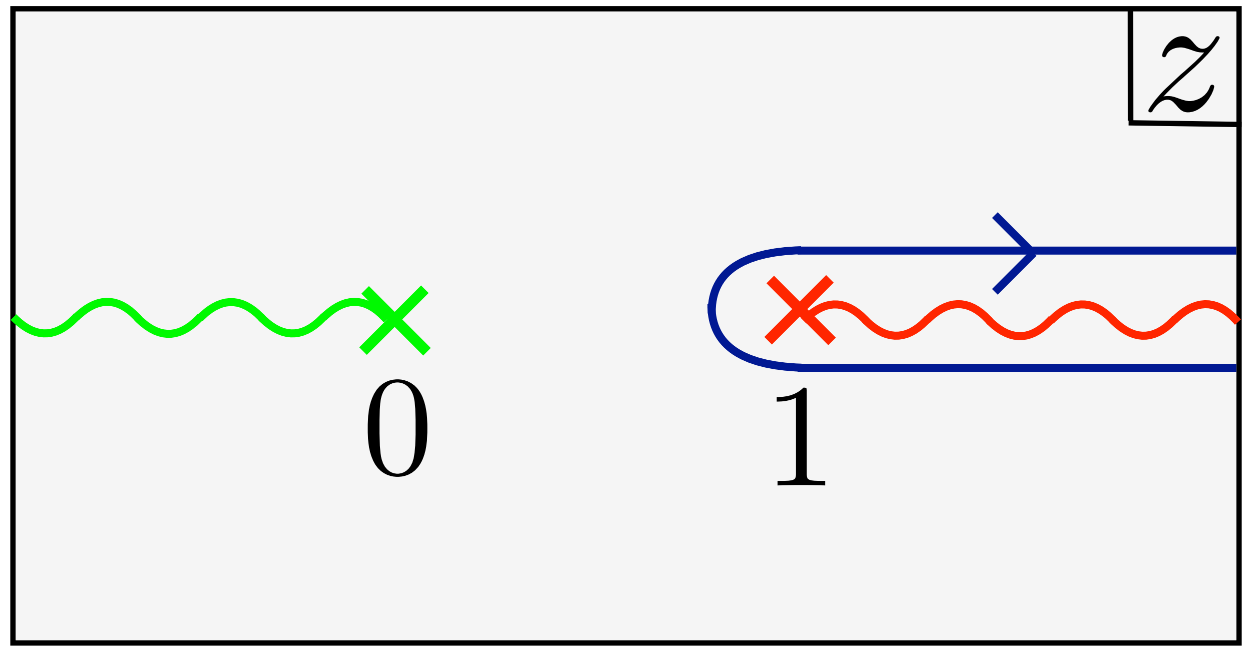


$$\bullet\Psi = \sum_{\Delta} \mu_\Delta O_\Delta(x)$$

$$\langle \Psi | O_1 O_2 \rangle = F(z) = \sum_{\Delta} c_\Delta\, g_\Delta(z) \qquad (c_\Delta = \mu_\Delta\, \lambda_\Delta^{12})$$



$$= \sum_{\Delta} c_\Delta$$



[Kabat Lifschytz]

functionals → **sum rules**

$$\theta_f[\,-\,] = \oint dz\, f(z)\,(\,-\,)$$

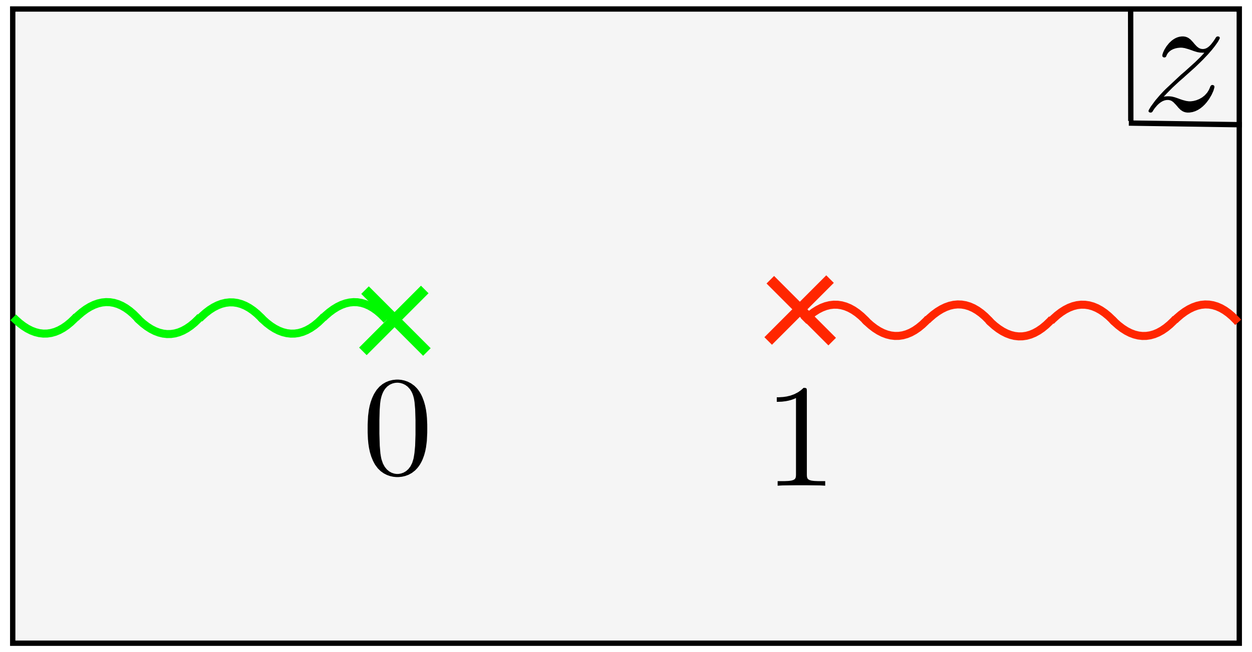
$$\theta_f[F] = \sum_{\Delta} c_\Delta\, \theta_f[g_\Delta] = 0$$

Why?

- Additional constraints on top of crossing

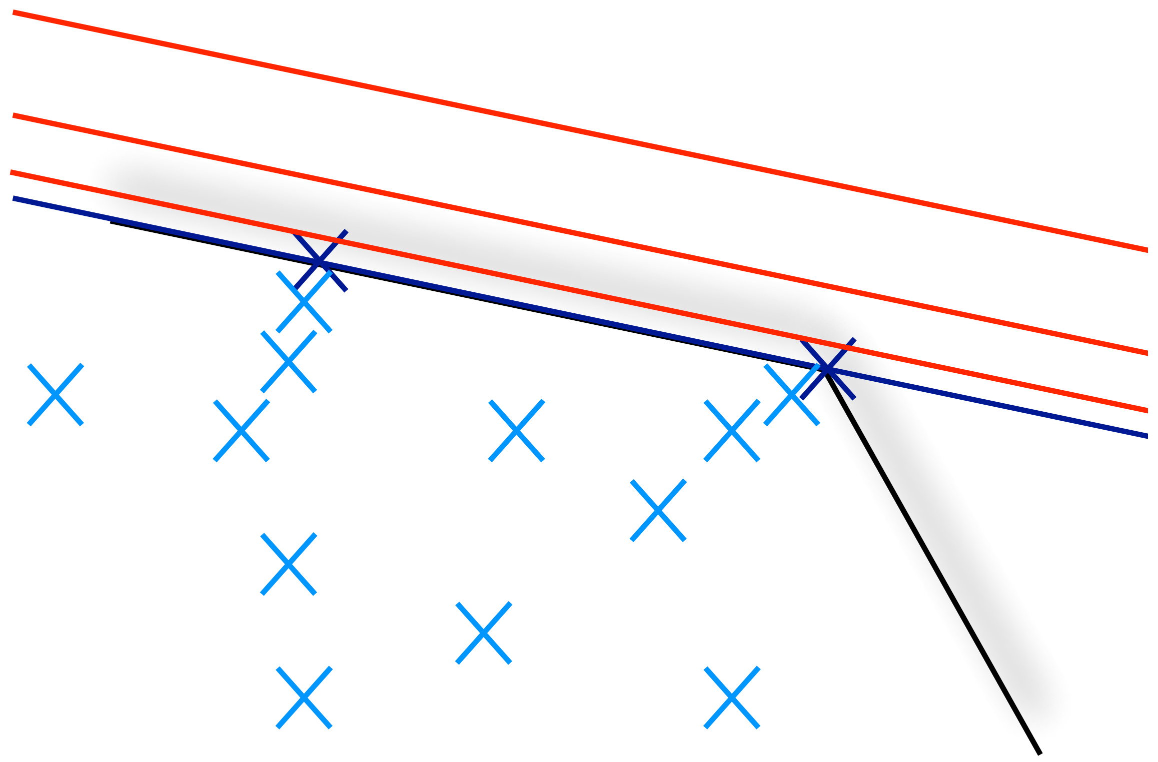
$$\theta_f[F] = \sum_{\Delta} c_\Delta\, \theta_f[g_\Delta] = 0$$

- Toy model for 1d crossing



Want to understand **“extremal”** solutions

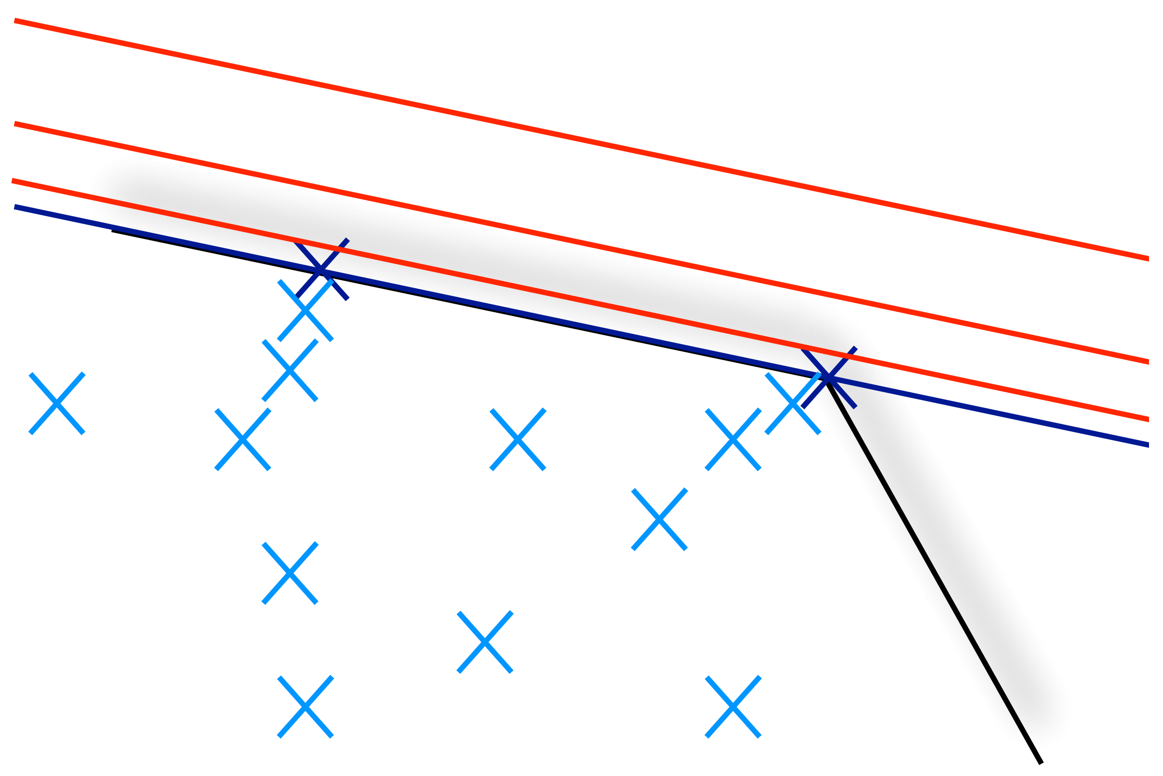
[El-Showk Paulos] [Mazac] [Mazac Paulos] [Paulos Zan]



Why?

Want to understand **"extremal"** solutions

[El-Showk Paulos] [Mazac] [Mazac Paulos] [Paulos Zan]



Recipe

basis of functionals $\theta_n[\,-\,] = \oint dz\, f_n(z)\,(\,-\,)$

1. **Complete**
2. **Dual** to a "sparse" spectrum: $\theta_n[g_{\Delta_m}] = \delta_{mn}$

→ **"extremal" solution**

$$F_\Delta^{\Delta_n} = g_\Delta - \sum_n \theta_n[g_\Delta]\, g_{\Delta_n}$$

Result

Explicit bases of functionals θ_n

dual to any $\Delta_n = 2\Delta_\phi + 2n + \gamma_n$

$\sim n^{-\epsilon}$ for large n

analytic for large n

Result

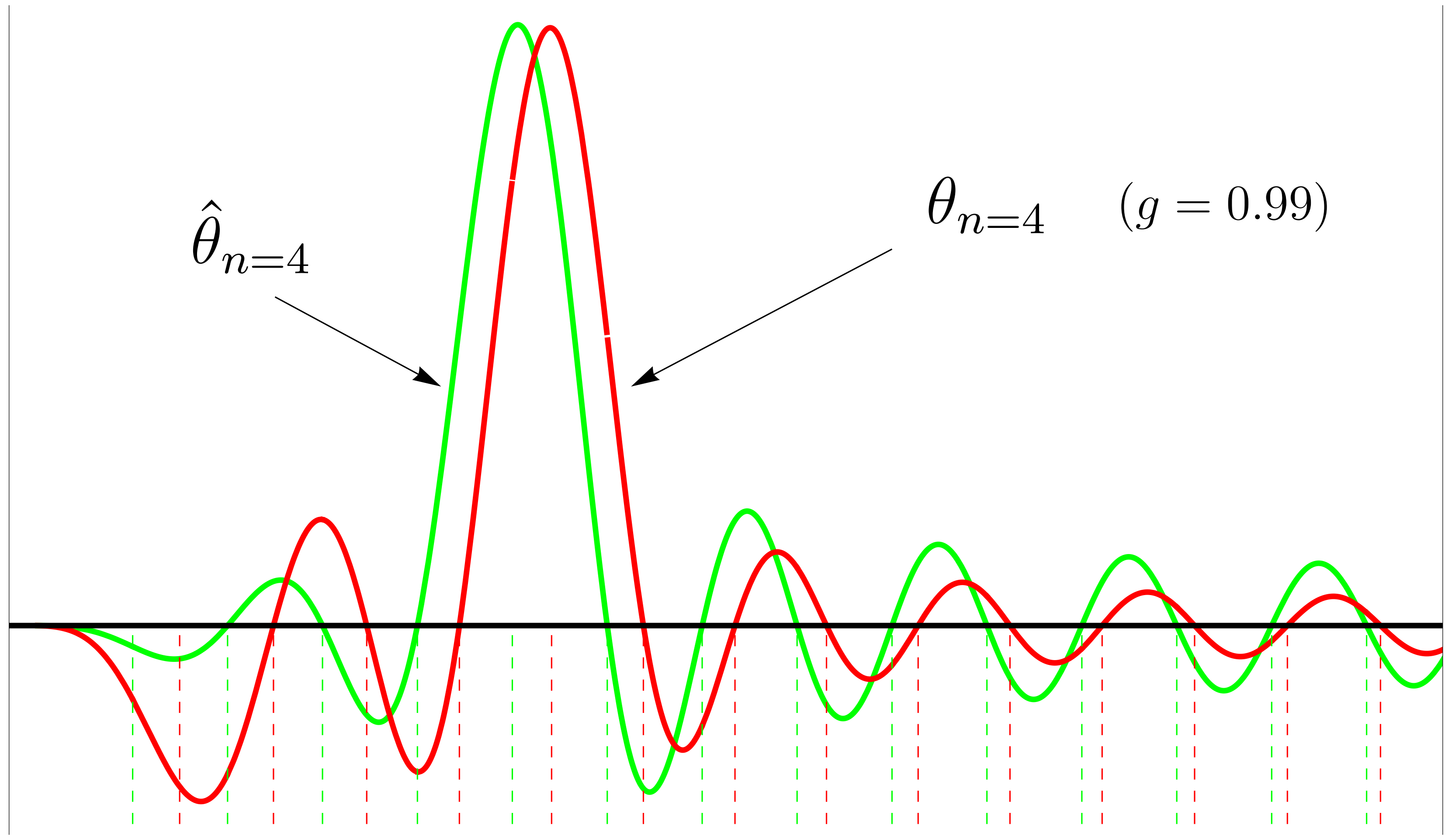
Explicit bases of functionals θ_n

dual to any $\Delta_n = 2\Delta_\phi + 2n + \gamma_n$

$\sim n^{-\epsilon}$ for large n

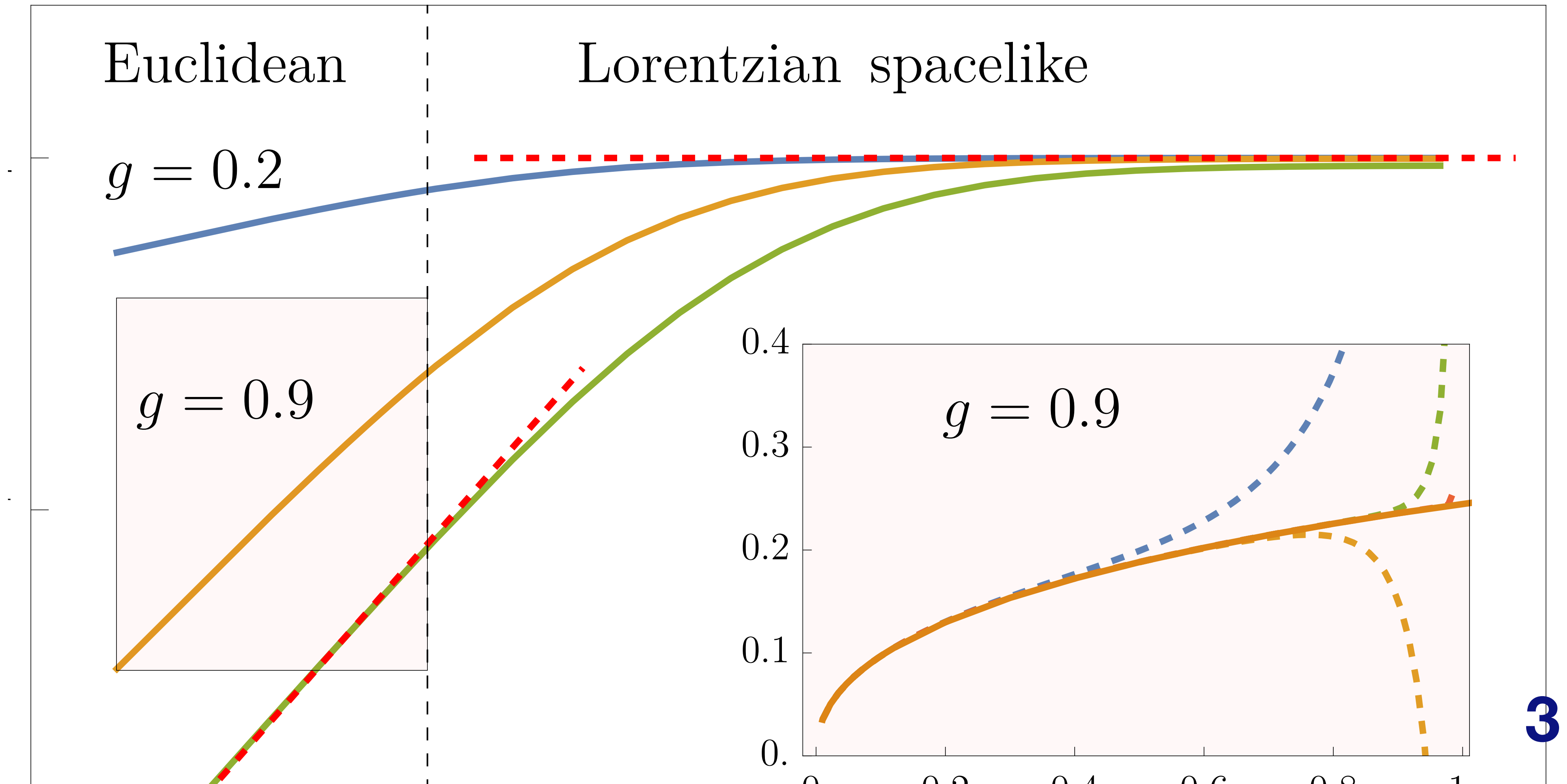
analytic for large n

- Explicit functional actions: $$\theta_n[g_\Delta] = \prod_{m\neq n}\left(\frac{\Delta - \Delta_m}{\Delta_n - \Delta_m}\right)$$



- Interacting "extremal" solutions:

$$F_\Delta^{\Delta_n} = g_\Delta - \sum_n \left[\prod_{m\neq n}\left(\frac{\Delta - \Delta_m}{\Delta_n - \Delta_m}\right)\right] g_{\Delta_n}$$



How?

$$\theta_n[F] = \oint dz\, f_n(z)\, F(z) = (f_n, \text{Disc}(F))$$

Dual

$$f_n \Subset \{\text{real-analytic on } (1,\infty)\}$$

test functions

$$\text{Disc}(F) \Subset \{\text{hyperfunctions on } (1,\infty)\}$$

distributions

How?

$$\theta_n[F] = \oint dz\, f_n(z)\, F(z) = (f_n, \mathrm{Disc}(F))$$

Dual

$$f_n \qquad\qquad\qquad\qquad\qquad\qquad \mathrm{Disc}(F)$$

$$\Cap \qquad\qquad\qquad\qquad\qquad\qquad\qquad \Cap$$

$$\{\text{real-analytic on } (1,\infty)\} \quad \subset \quad L^2 \quad \subset \quad \{\text{hyperfunctions on } (1,\infty)\}$$

test functions ***distributions***

Dual

Constructed Schauder bases $\{f_n\}$ ⟷ $\{\mathrm{Disc}(g_{\Delta_n})\}$

Paley-Wiener theorem: $\theta_n(\Delta) := \theta_n[g_\Delta]$ entire function

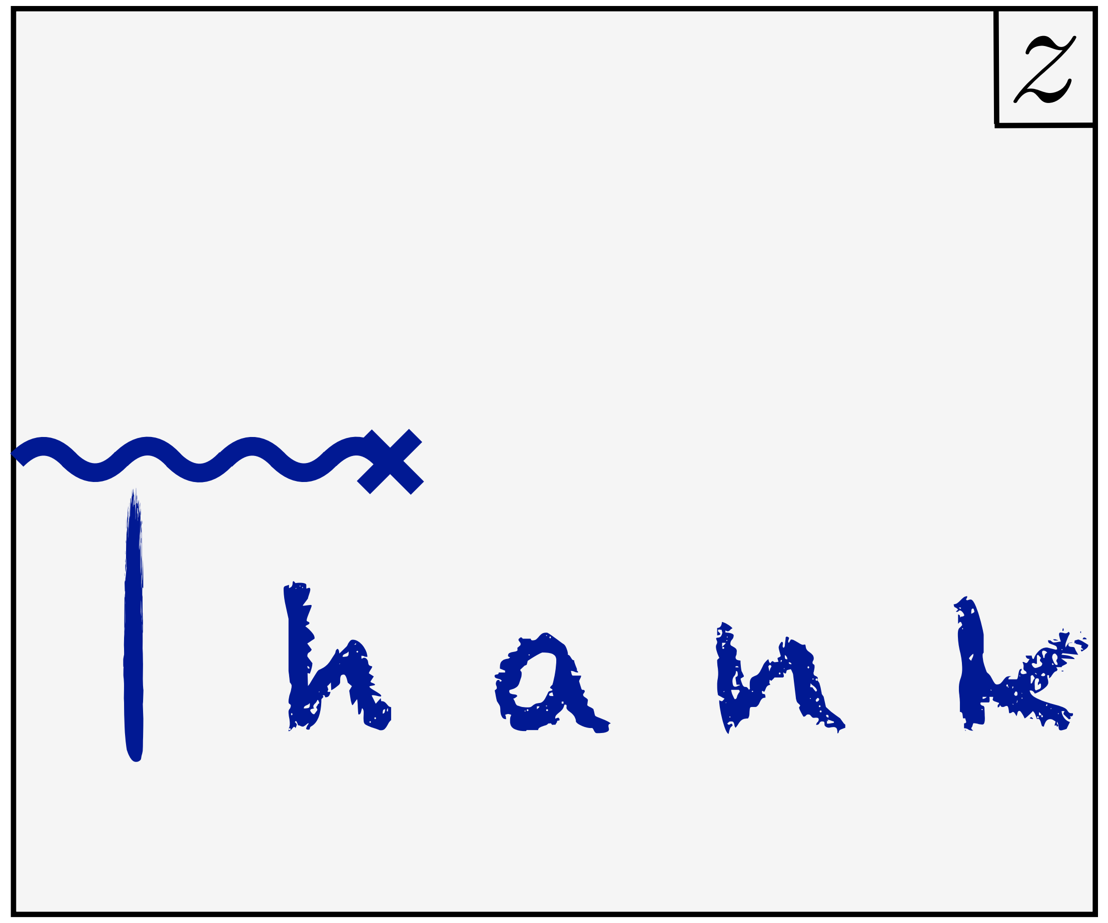
$$= \prod_{m \neq n} \left(\frac{\Delta - \Delta_m}{\Delta_n - \Delta_m} \right)$$

Future directions…

1. *Numerics: Crossing+Locality*

2. *Analytic extremal solutions of 1d crossing equation?*

3. *Modified locality with gauge or gravitational dressing*



Thank you!