Nonperturbative Minimal (Super)string / Matrix Integral Duality

Strings 2024, CERN

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4 June 2024

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However, worldsheet perturbation theory around D-instantons is often ill-defined — the resulting answers contain undetermined constants that need to be fixed using certain assumptions, like duality. [Balthazar, Rodriguez, Yin]

[Kutasov, Okuyama, Park, Seiberg, Shih]

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They have been generalized to tackle various nonperturbative effects in critical 10-d superstring theory and its compactifications, leading to highly nontrivial checks of superstring dualities. [Alexandrov, Firat, Kim, Sen, Stefanski, Agmon, Balthazar, Cho, Rodriguez, Yin]

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- 2202.03448 [Eniceicu, RM, Murdia, Sen]
- 2206.13531[Eniceicu, RM, Murdia, Sen]
- 2210.11473 [Eniceicu, RM, Maity, Murdia, Sen]
- 2308.06320 [Eniceicu, RM, Murdia]
- Work in progress [Chakrabhavi, Eniceicu, RM, Murdia]

Why study minimal string theory?

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In the context of D-instanton perturbation theory, we note that an apparent mismatch found between 2d string theory and matrix quantum mechanics in a closed-string annulus one-point function was resolved by first analyzing the analogous problem in the minimal string/matrix integral setting. [Eniceicu, RM, Maity, Murdia, Sen].

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Yet, only the ij indices are responsible for the topological expansion. The $\mu, t, \mathbf{x}$ are not important for the existence of a planar expansion.

What is minimal string theory?

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The "path integral" in this setting is just an ordinary integral over one or more matrices and it strips down 't Hooft's large-N idea down to its bare bones, to just the color indices.

[Brezin, Itzykson, Parisi, Zuber]

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It is the earliest example of a duality between a gravitational and a non-gravitational system.

[Brezin, Kazakov, Kostov, Gross, Migdal, Douglas, Shenker, Moore, Seiberg, Staudacher, Knizhnik, Polyakov, Zamolodchikov, David, Distler, Kawai,...]

[Douglas, Klebanov, Kutasov, Maldacena, Martinec, McGreevy, Moore, Seiberg, Shih, Toumbas, Takayanagi, Verlinde,...]

Now I will introduce the two sides of this duality.

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Then I will describe the observable that we are computing.

Matrix integral (by example)

The observable we will study is the "partition function", or the value of the integral itself.

$$Z(N, t, g_2, g_4) := \int \frac{\mathrm{d}^{N^2} M}{\mathrm{vol}(U(N))} \exp\left(-\frac{N}{t} \operatorname{Tr} V(M)\right)$$

$$= \frac{1}{N!} \int \prod_{i=1}^{N} \frac{\mathrm{d}x_i}{2\pi} \Delta(x)^2 \exp\left(-\frac{N}{t} \sum_i V(x_i)\right)$$

$$V(x) = \frac{g_2}{2} x^2 + \frac{g_4}{4} x^4$$

Take $g_2 > 0$ and $g_4 < 0$

Double-scaling limit: $t = \frac{g_2^2}{-12g_4} - \varepsilon^2$, $N\varepsilon^{\frac{5}{2}} = \kappa$.

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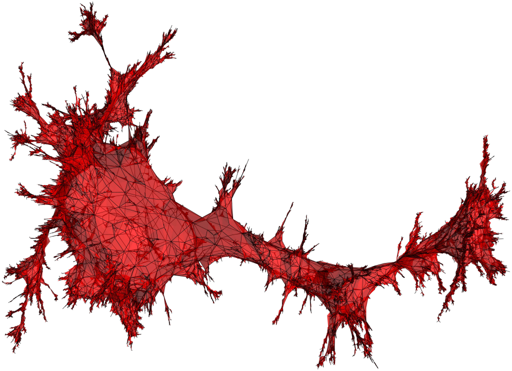
In this limit, Feynman diagrams with large number of vertices give the dominant contribution to Z, and a continuum limit can be taken so that the 2d surfaces become smooth. Now κ plays the role the genus counting parameter.



Picture Credit: Jeremie Bettinelli

The worldsheet CFT

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1. The $(2, p)$ minimal model, which is a (generally non-unitary) two-dimensional CFT. This can be thought of as the matter sector and has $c = 1 - \frac{6(2-p)^2}{2 \cdot p} < 1$.

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3. The *bc*-ghost CFT with $c = -26$.

Perturbation expansion of the partition function

$$Z = \exp\left(c_0\,\kappa^2 + c_1 + c_2\kappa^{-2} + \ldots\right) \qquad \text{Matrix Integral}$$

$$= \exp\left(\text{sphere} + \text{torus} + \text{genus two} + \ldots\right) \qquad \text{String Theory}$$

In the string theory calculation, we sum over all closed Riemann surfaces, with no external vertex operators. At each genus, we need to do a moduli space integral.

Non-perturbative contributions to the partition function

$$Z^{(0)} = \exp\left(c_0\,\kappa^2 + c_1 + c_2\kappa^{-2} + \ldots\right) \qquad \text{Matrix Integral}$$

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$$Z = Z^{(0)} + Z^{(1)} + \ldots$$

$$\frac{Z^{(1)}}{Z^{(0)}} = \exp\left(d_0\,\kappa + d_1 + d_1' \log\kappa + \ldots\right) \qquad \text{Matrix Integral}$$

$$= \exp\left(\text{disk} + \text{annulus} + \ldots\right) \qquad \text{String Theory}$$

$$\frac{Z^{(1)}}{Z^{(0)}} = \exp\left(\text{disk} + \text{annulus} + \ldots\right)$$
$$= e^{-1/g_s}\, \mathcal{N}\left(1 + O(g_s)\right)$$

We want to compute the normalization prefactor $\mathcal{N}$, with the precise order one constant.

$$\boxed{\mathcal{N} = \exp(\text{annulus}) = \sqrt{g_s}\,\frac{\mathrm{i}}{\sqrt{8\pi}}\,\frac{\cot(\pi/p)}{\sqrt{p^2-4}}}$$

In this equation, the normalization of g_s is chosen so that the action or the tension of the instanton equals g_s^{-1}.

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To explain the basic puzzle and its resolution, we focus on the $(1, 1)$ ZZ brane.

The annulus diagram

We want to compute the annulus with both boundaries on a $(1, 1)$ ZZ brane:

$$A = \int_0^\infty \frac{\mathrm{d}t}{2t} \,\mathrm{Tr}_{\mathrm{open}} \left(e^{-2\pi t L_0} b_0 c_0\right)$$

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Put the known bootstrap answers for minimal model, Liouville and bc ghosts together [Zamolodchikov^2, Cardy, Rocha-Caridi, Martinec].

$$\mathrm{Tr}_{\mathrm{open}}\left(e^{-2\pi t L_0} b_0 c_0\right) =$$
$$= \left(e^{2\pi t} - 1\right) \sum_{k=-\infty}^{\infty} \left(e^{-2\pi t k(2pk+p-2)} - e^{-2\pi t(pk+1)(2k+1)}\right)$$

The problem with A

The problem is that A is ill-defined.

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We will see now that e^A is a better quantity to consider, and it is possible to make it well-defined.

Exponentiating the annulus

$$\mathrm{Tr}_{\mathrm{open}}\left(e^{-2\pi t L_0} b_0 c_0\right) =: \sum_b e^{-2\pi h_b t} - \sum_f e^{-2\pi \hat{h}_f t}$$

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$$\begin{aligned}\mathcal{N} := e^A = \left(\frac{\prod_f \hat{h}_f}{\prod_b h_b}\right)^{\frac{1}{2}} &= \frac{\prod_f' \hat{h}_f}{\prod_b h_b^{1/2}} \\ &= \int \prod_b \frac{d\phi_b}{\sqrt{2\pi}} \prod_f{}' dp_f dq_f\, \exp\left(-\frac{1}{2}\sum_b h_b \phi_b^2 - \sum_f{}' \hat{h}_f p_f q_f\right)\end{aligned}$$

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That is the meaning of the formula

$$\mathcal{N} = e^{A} = \int \prod_b \frac{d\phi_b}{\sqrt{2\pi}} \prod_f{}' dp_f dq_f \, \exp\left(-\frac{1}{2} \sum_b h_b \phi_b^2 - \sum_f{}' \hat{h}_f p_f q_f \right)$$

$$= \text{path integral of the D-instanton worldvolume theory}$$

$$= \text{path integral of the open string field theory}$$

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The tachyon field T is the coefficient of the $L_0 = -1$ component of the string field $T\, c_1|0\rangle$. So the worldvolume path integral contains the following integral over T

$$\int_C \frac{\mathrm{d}T}{\sqrt{2\pi}}\, e^{+\frac{1}{2}T^2}$$

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So for now we just do

$$\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}T}{\sqrt{2\pi}}\, e^{+\frac{1}{2}T^2} = \mathrm{i}$$

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Fields in the worldvolume gauge theory path integral before doing any Faddeev-Popov procedure are

$$A_\mu(k)\, c_1 \alpha^\mu_{-1}\, e^{\mathrm{i}k\cdot X}|0\rangle \; +$$
$$\psi(k)\, c_0 \; e^{\mathrm{i}k\cdot X}|0\rangle$$
$$k \in \mathbb{R}^{p+1}$$

The worldvolume gauge transformation is inferred from calculating $Q_{\mathrm{BRST}} \cdot \theta(k)\, e^{\mathrm{i}k\cdot X}|0\rangle$. This results in the following transformations:

$$A_\mu(k) \to A_\mu(k) + k_\mu\, \theta(k) \quad \text{AND}$$
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We can use this gauge freedom to set $\psi(k) = 0$. This introduces Fadeev-Popov ghost fields $C(k)$ and $B(k)$

$$C(k)\, e^{\mathrm{i}k\cdot X}|0\rangle +$$
$$B(k)\, c_1 c_{-1}\, e^{\mathrm{i}k\cdot X}|0\rangle$$

The Fock space states appearing above satisfy the Siegel gauge condition $b_0|\cdot\rangle = 0$.

Why D-instantons are special

Fadeev-Popov fields $C(k)$ and $B(k)$ from the previous slide

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Recall that

$$\mathrm{Tr}_{\text{open}}\left(e^{-2\pi t L_0} b_0 c_0\right) \overset{\text{worldsheet CFT}}{=} e^{2\pi t} - 2 + O(e^{-2\pi t})$$

The -2 comes from the C and B fields.

The expression for e^A contains the two-dimensional Grassmann integral (recall that $h_f = 0$ for these states)

$$\int \mathrm{d}B\mathrm{d}C\, e^{0\cdot BC} = 0\,.$$

What should we change?

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Further, recall the field that got set to zero for $p \geqslant 0$ and its gauge transformation:

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For the D-instanton case, there is no momentum available, and we have instead:

$$\psi\, c_0\, |0\rangle$$
$$\psi \to \psi$$

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So the remedy is to just "un Faddeev-Popov" the path integral

$$\int \mathrm{d}B\mathrm{d}C\, e^{0\cdot BC} \longrightarrow \frac{\int d\psi\, e^{-\psi^2}}{\int d\theta} = \frac{\sqrt{\pi}}{2\pi/g_o} \qquad \texttt{[Sen]}$$

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Convert g_o to the tension of the brane using $T = \dfrac{1}{2\pi^2 g_o^2}$ [Sen]

The final string theory result

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where the normalization of g_s is chosen so that $T = g_s^{-1}$.

Summary of string calculation

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String field theory helps us identify the culprit modes, and the new worldvolume path integral, with the $\psi \, c_0|0\rangle$ field produces a finite, unambiguous answer.

Comment

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The worldvolume path integral now includes the ψ field. So it will appear in Feynman diagrams on internal lines.

The matrix integral computation

The matrix computation

The matrix computation was worked out in the early 90s. The effects come from one-eigenvalue instantons, which are extrema of the effective potential felt by one eigenvalue

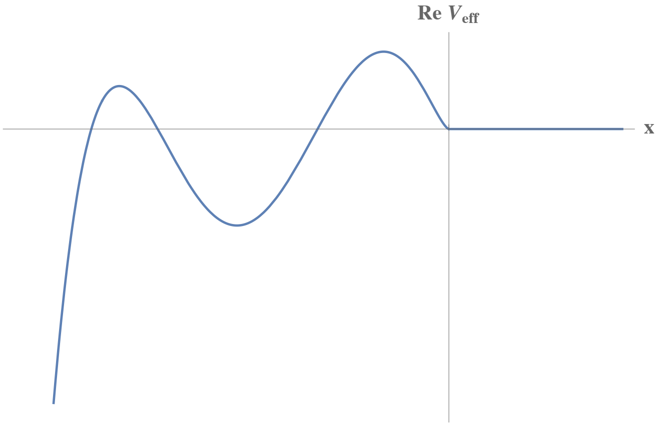
$$V_{\mathrm{eff}}(x,t,g_2,g_4) := V(x) - 2t\int_{-b}^{b} \mathrm{d}y\,\rho(y)\log(y-x)\,.$$

[David, Shenker, Ginsparg, Zinn-Justin; Marino, Schiappa, Weiss]

The effective potential

The double-scaled matrix integral dual to (the conformal background of) the $(2,p)$ minimal string has a specific form. [Moore, Seiberg, Staudacher]

For $p = 7$ it looks as follows



The matrix computation

We just need to compute the on-shell action and the one-loop Gaussian integral around the extrema shown on the previous graph.

$$\frac{Z^{(1)}}{Z^{(0)}} = e^{-\kappa V_{\mathrm{eff}}(x_n^*)} \, \frac{1}{\sqrt{\kappa}} \, d_1 \, \left(1 + O(\kappa^{-1})\right)$$

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I will not present the details, but the answer matches with the string theory computation.

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The duality between the matrix integral and string theory holds for perturbation theory in κ^{-1} around each saddle point.

We need to specify a defining contour for the eigenvalues of the matrix integral that will pick out a particular linear combination of Lefshetz thimbles to sum over.

String theory also needs a corresponding defining contour in the complex plane of the open-string tachyon.

Generalizations

1. Two-matrix integrals that are dual to the (p', p) minimal string [2206.13531]

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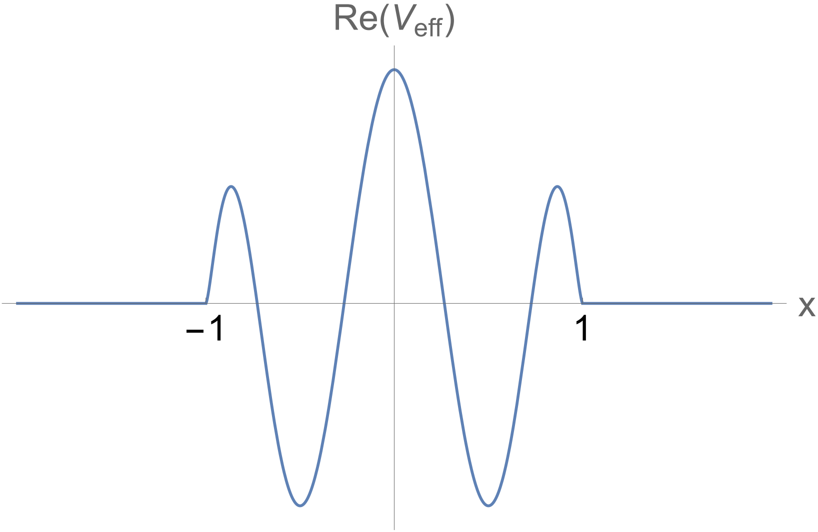
4. Type 0B $(2, 4k)$ minimal superstring [To appear soon] [Klebanov, Maldacena, Seiberg, Chakravarty, Sen]

   - Gapped, two-cut phase
   - Edge-less phase (the leading nonperturbative effect is from two instantons)

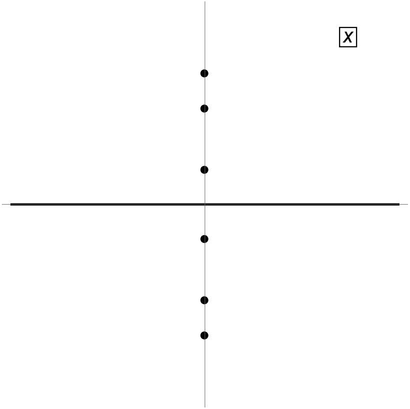
The culprit states are the following NS sector states

$$\beta_{-\frac{1}{2}} c_1 e^{-\phi} |0\rangle, \quad \gamma_{-\frac{1}{2}} c_1 e^{-\phi} |0\rangle$$

Type 0B minimal superstring - Two-cut phase



Type 0B minimal superstring - Edgeless phase



"Wrong-sheet instantons" or "ghost instantons" [Marino, Schiappa, Schwick] are crucial in this phase [Eniceicu, RM, Murdia, 2308.06320].

Some future questions

1. A deeper understanding of the role of “ghost instantons”?
   [Marino, Schiappa, Schwick]

2. A deeper understanding of string theory description of hole states in $c = 1$ matrix quantum mechanics?

Summary

1. Worldsheet description of string observables by itself is inadequate in the presence of D-instantons.

2. We discussed in this talk how insights from string field theory help us compute a finite, unambiguous answer for the normalization of ZZ instanton amplitudes in the minimal string. The answers match perfectly with the dual matrix integrals.

Chitraang Murdia, Postdoc @ UPenn

Dan Stefan Eniceicu, PhD student @ Stanford