Light ray operators, detectors, and energy correlators

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Two questions about Lorentzian QFT

- What can we measure at null infinity?
- How do correlators behave at large boost?

Both questions will lead us to the notion of a light-ray operator.

Kinematics of non-integer spin

Light-ray operators in CFT transform like primaries with non-integer spin.

- For a local operator, we can use index-free notation

$$\mathcal{O}(x,z) \equiv \mathcal{O}_{\mu_1\ldots\mu_J}(x) z^{\mu_1}\cdots z^{\mu_J} \quad \longleftarrow \quad \text{polynomial with degree } J$$

- To describe an operator with non-integer spin, we drop the polynomial requirement and allow general homogeneity

$$\mathbb{O}(x,\lambda z) = \lambda^{J_L}\mathbb{O}(x,z) \quad J_L \in \mathbb{C}$$

- $\mathbb{O}(x,z)$ is labeled by a spacetime point x and a null direction z.
- Every $\mathbb{O}$ has a "spin-shadow" related by $J_L \leftrightarrow 2-d-J_L$

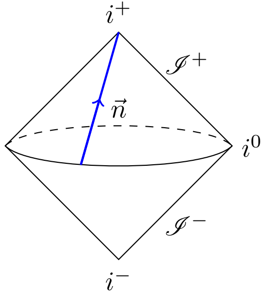
$$\mathbf{S}[\mathbb{O}](x,z) = \int D^{d-2}z'(-2z\cdot z')^{2-d-J_L}\mathbb{O}(x,z)$$

Example: light-transform [Kravchuk, DSD '18]

The light-transform of a local operator $\mathcal{O}$ is a null integral starting from x in the direction of z:

$$\mathbf{L}[\mathcal{O}](x,z) = \int_{-\infty}^{\infty} d\alpha (-\alpha)^{-\Delta-J} \mathcal{O}\left(x - \frac{z}{\alpha}, z\right)$$

- $\mathbf{L}[\mathcal{O}]$ transforms like a primary with $(\Delta_L, J_L) = (1-J, 1-\Delta)$.
- Setting $x = \infty$ with $z = (1, \vec{n})$ gives integral along $\mathscr{I}^+$, at a point $\vec{n}$ on the celestial sphere. Under the Lorentz group, it behaves like a primary on S^{d-1} with dimension $-J_L$. This is a kind of "detector."



The ANE(C) operator

- In flat-space CFT, the ANEC operator is the light-transform of the stress tensor $\mathbf{L}[T]$ [Hofman, Maldacena '08]
- Placing x at spatial infinity, we get an integral of T along $\mathscr{I}^+$ which measures the flux of energy in the direction $\vec{n}$

$$\mathcal{E}(\vec{n}) = 2\,\mathbf{L}[T](\infty, z)|_{z=(1,\vec{n})}$$

In a general QFT,

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int dt\, n^i T^0{}_i(t, r\vec{n})$$

- $\mathcal{E}(\vec{n})$ is a generator of a BMS algebra [Cordova, Shao '18]
- ANEC: $\mathbf{L}[T]$ is *positive*
  - Two proofs: quantum information [Faulkner, Leigh, Parrikar, Wang '16], causality [Hartman, Kundu, Tajdini '16]
  - Many applications: OPE bounds, operator dimension bounds, QNEC, a-theorem [Hofman, Maldacena '08; Cordova, Diab '17; Cordova, Maldacena, Turiaci '17; Ceyhan, Faulkner '18; Hartman, Mathys '23; ...]

Energy correlators [Basham, Brown, Ellis, Love '78]

$$\frac{\langle\Psi|\mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_k)|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \sum_{i_1,\ldots,i_k}\int d\sigma\prod_{j=1}^{k}E_{i_j}\delta(\vec{n}_j - \vec{p}_{i_j}/p^0_{i_j})$$

- Measures correlations between flux of energy in different directions $\vec{n}_j$ on the celestial sphere, in some state $|\Psi\rangle$.
- IR safe [Kinoshita '62; Lee, Nauenberg '64], under good theoretical control. Can be computed via amplitudes and/or correlation functions.
    - Calculations in QCD and $\mathcal{N}=4$ SYM [Hofman, Maldacena '08; Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov '13; Dixon, Luo, Shtabovenko, Yang, Zhu '18; Luo, Shtabovenko, Yang, Zhu '19; Henn, Sokatchev, Yan, Zhiboedov '19; Chen, Luo, Moult, Yang, Zhang, Zhu '20; Chicherin, Korchemsky, Sokatchev, Zhiboedov '23 ...]
    - (Related calculations in classical gravity [Kosower, Maybee, O'Connell '18; ...])
- Experimentally measurable. Can cleanly access lots of different physics: jet substructure, top mass, QGP... [Komiske, Moult, Thaler, Zhu '22; Holguin, Moult, Pathak, Procura '22; Chen, Moult, Thaler, Zhu '22; Lee, Mecaj, Moult '22, ...], and...

CMS determination of α_s from $\langle\mathcal{EEE}\rangle/\langle\mathcal{EE}\rangle$ [CMS '24]

CERN-EP-2024-010
2024/02/22

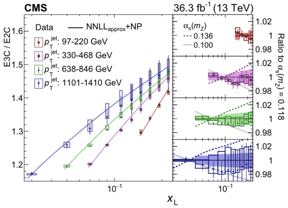
CMS-SMP-22-015

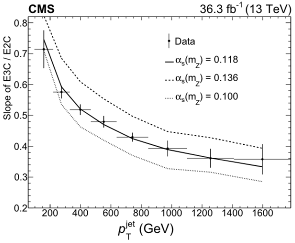
Measurement of energy correlators inside jets and determination of the strong coupling $\alpha_S(m_Z)$

The CMS Collaboration*

Abstract

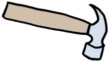
Energy correlators that describe energy-weighted distances between two or three particles in a jet are measured using an event sample of $\sqrt{s} = 13\,\text{TeV}$ proton-proton collisions collected by the CMS experiment and corresponding to an integrated luminosity of $36.3\,\text{fb}^{-1}$. The measured distributions reveal two key features of the strong interaction: confinement and asymptotic freedom. By comparing the ratio of the two measured distributions with theoretical calculations that resum collinear emissions at approximate next-to-next-to-leading logarithmic accuracy matched to a next-to-leading order calculation, the strong coupling is determined at the Z boson mass: $\alpha_S(m_Z) = 0.1229^{+0.0040}_{-0.0050}$, the most precise $\alpha_S(m_Z)$ value obtained using jet substructure observables.





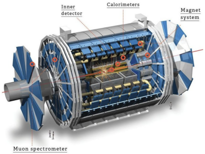
What can we measure at null infinity?

- $\mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_k)$ is a kind of multi-point light-ray/light-cone operator
- What else can we measure? What kinds of detectors $\mathcal{D}$ exist?
- Can we understand the space of detectors in terms of basic components, like we understand local operators in CFT?

 $= \sum_i h_i \mathcal{O}_i$

 $= \sum_j c_j \mathcal{D}_j$

- Can we decompose $\mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_k)$ and other detectors into basic components (light-ray OPE)?



$\stackrel{?}{=} \sum_j a_j \mathcal{D}_j$

What can we measure at null infinity?

Interesting example: CFT coupled to gravity.

- This theory does not have local correlation functions *or* scattering amplitudes.
- What are the observables?
- It should have detectors whose event shapes we can measure $\langle \mathrm{BH\text{-}BH}|\mathcal{D}|\mathrm{BH\text{-}BH}\rangle$.
- A holographic theory of flat space should know about these detectors.

Detectors in free scalar theory

- In the free scalar theory,

$$\mathcal{E}(z) = \mathbf{L}[T](\infty, z) = \int_0^\infty dE\, E^{d-2} a^\dagger(Ez) a(Ez)$$

  It counts particles weighted by E.

- More generally, we can measure

$$\mathcal{E}_J(z) = \int_0^\infty dE\, E^{d+J-4} a^\dagger(Ez) a(Ez),$$

  which counts particles weighted by E^{J-1}.

- For integer J, $\mathcal{E}_J = \mathbf{L}[\mathcal{O}_J]$ with $\mathcal{O}_J = \phi \partial^J \phi$.
- But since E is positive, we can also let $J \in \mathbb{C}$. This gives the leading Regge trajectory of the free theory.
- Can write $\mathcal{E}_J$ as a bilocal integral along a null ray

$$\mathcal{E}_J(z) = \frac{1}{\Gamma(-J)} \int d\alpha_1 d\alpha_2 \frac{1}{|\alpha_1 - \alpha_2|^{J+1}} \phi(\alpha_1; z) \phi(\alpha_2; z)$$

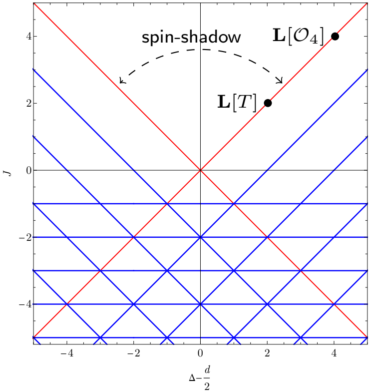
Renormalizing detectors

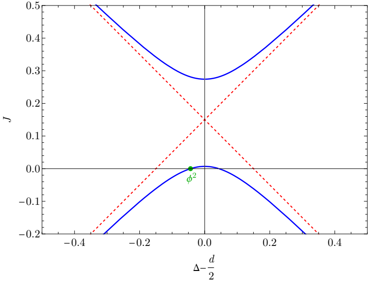
- When we turn on interactions, $\mathcal{E}_J$ is no longer IR/collinear safe. Splitting conserves E, but not E^{J-1}.
- Manifests as IR/collinear divergences in perturbation theory.
- The theory is telling us that the bare $\mathcal{E}_J$ is not a "good" observable. Need to renormalize it to find out what the "good" observables are.

local operator	detector
"measure at a point"	"measure in cross-sections"
UV divergence	IR divergence
need to renormalize	need to renormalize
theory-dependent	theory-dependent
OPE	light-ray OPE
radial quantization	?

- Renormalized detectors give an *operator definition* of IR safe weighted cross-sections. (Can we do the same for amplitudes?)

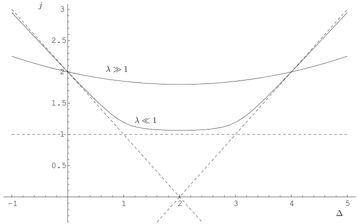
Chew-Frautschi plot in the Wilson-Fisher theory [Caron-Huot, Koloğlu, Kravchuk, Meltzer, DSD '22]

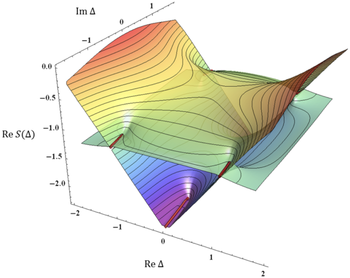




- Turn on interactions: $\mathcal{E}_J$ mixes/recombines with its shadow!
- It turns out there is no invariant distinction between "light-ray" and "light-cone" operators.
- $\gamma_{\phi\partial^J\phi} = -\frac{\epsilon^2}{9J(J+1)}$ (2-loop) vs. $\gamma_{\phi^2} = \frac{2\epsilon}{3}$ (1-loop) [Caron-Huot; Alday, Henriksson, van Loon '17]

CF plot in planar $\mathcal{N} = 4$ SYM [Brower, Polchinski, Strassler, Tan '06]

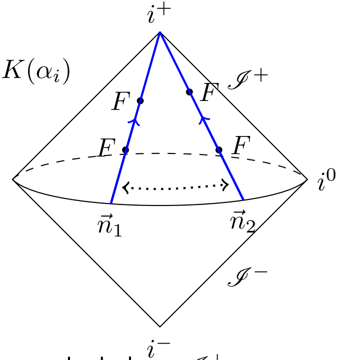




- "Twist-2" sector is closed in the planar limit.
- When interactions are turned on, the 45° DGLAP trajectory mixes with the horizontal BFKL trajectory, forming a smooth Riemann surface. [Kuraev, Lipatov, Fadin '77; Balitsky, Lipatov '78; Braun, Korchemsky, Mueller '03; ...]
- BPST understood the surface at strong coupling by constructing a Regge trajectory of vertex operators in the bulk string theory.
- Shape now known exactly at finite λ from integrability [Alfimov, Gromov, Kazakov '15; Gromov, Kazakov, Leurent, Volin '15; Gromov, Levkovich-Maslyuk, Sizov '17]

What is a BFKL trajectory?

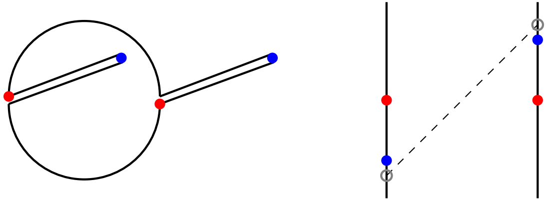
$$\int d\vec{n}_1 d\vec{n}_2 \left(\frac{1-\vec{n}_1\cdot\vec{n}_2}{(1-\vec{n}_1\cdot\vec{n})(1-\vec{n}_2\cdot\vec{n})}\right)^{\frac{\Delta-1}{2}} \int d\alpha_i K(\alpha_i)$$



- Integral of $F_{\mu\nu}$'s along Wilson lines stretched along $\mathscr{I}^+$.
- $\int d\vec{n}_1 d\vec{n}_2(\cdots)$ projects onto Lorentz irrep with spin $J_L = 1-\Delta$.
- $\Delta_L = 1 - J = 0$ is fixed (at tree level). Varying Δ, we trace a horizontal trajectory on the Chew-Frautschi plot at $J = 1$.
- Can add more Wilson lines to get infinitely more trajectories at $J = 1$. When we turn on the coupling, they mix in an intricate way (Balitsky-JIMWLK evolution). [Mueller, Patel, Balitsky, Kovchegov, Jalilian-Marian, Kovner, McLerran, Weigert, Leonidov, Iacnu, ...; Caron-Huot '13]

Large boost: "light rays" in 1d

$$\mathrm{OTOC}: \langle W(\tfrac{\beta}{2}+it)V(\tfrac{\beta}{2})W(it)V(0)\rangle$$



The scramblon/Pomeron is the fastest-growing part at large t. It is an intrinsically 2-sided operator associated to the gray points (1d "light ray")

$$e^{Kt}\{W_L W_R\} \sim e^{\lambda_\star t}\mathbb{S} + \dots$$

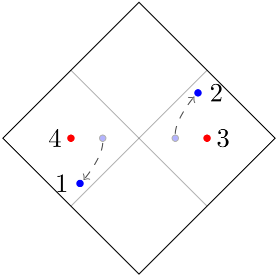
"Inversion formula":

$$\mathbb{O}_\lambda \equiv \int_0^\infty dt\, e^{-\lambda t} e^{Kt}\{W_L W_R\} \quad \Longrightarrow \quad \mathbb{O}_\lambda \sim \frac{1}{\lambda - \lambda_\star}\mathbb{S} + \dots$$

OTOC $\to$ a matrix element of $\mathbb{S}$: [Kitaev; Maldacena, Qi '18; Stanford, Lin '23, ...]

$$\mathrm{OTOC} = e^{\lambda_* t}\langle V_L|\mathbb{S}|V_R\rangle + \dots$$

Conformal Regge theory [Costa, Goncalves, Penedones '12; Caron-Huot '17, Kravchuk, DSD '18]



In the Regge limit, the correlator becomes a matrix element of a "Pomeron" operator

$$\langle \mathcal{O}_4 \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle \sim e^{(J_\star - 1)t} \langle \mathcal{O}_4 | \mathrm{Res}_{J=J_\star} \mathbb{O}_{\frac{d}{2},J} | \mathcal{O}_3 \rangle.$$

The Pomeron can be extracted from

$$\mathbb{O}_{\Delta,J}(x,z) = \int dx_1 dx_2 K_{\Delta,J}(x_1,x_2,x,z) \mathcal{O}_1 \mathcal{O}_2$$

Residue localizes the integral to a neighborhood of a null plane. Intuitively, the Pomeron is a 2-sided operator in "angular/Rindler quantization." [Agia, Jafferis '22]

The light-ray kernel [Kravchuk, DSD '18]

(Some? all?) light-ray operators are packaged together by

$$\mathbb{O}_{\Delta,J}(x,z) = \int dx_1 dx_2 K_{\Delta,J}(x_1,x_2,x,z)\mathcal{O}_1\mathcal{O}_2$$

- $\mathbb{O}_{\Delta,J}$ transforms like a primary with $(\Delta_L, J_L) = (1-J, 1-\Delta)$, but Δ and J can be complex.
- Conjecturally, the poles occur at a Riemann surface in the Δ-J plane, with residues being light-ray operators.

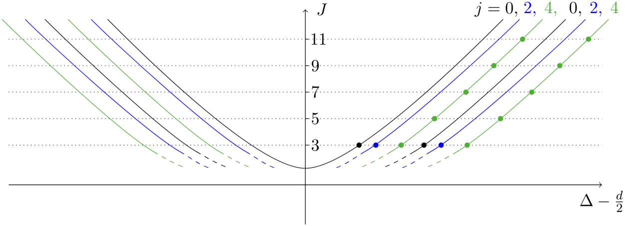
$$\mathbb{O}_{\Delta,J} \sim \sum_i \frac{\mathbb{O}_{i,J}}{\Delta - \Delta_i(J)}.$$

- The kernel is constructed so that at integer J, $\mathbb{O}_{i,J} = \mathbf{L}[\mathcal{O}_{\Delta_i,J}]$.
- The light-ray operator at $\Delta = \frac{d}{2}$ with largest J is the Pomeron.
- Setting $x = \infty$ gives a class of detectors $\mathcal{D}_{i,J}(z) = \mathbb{O}_{i,J}(\infty, z)$.

The light-ray OPE

- Light-ray operators appear in $\mathcal{E}(\vec{n}_1) \times \mathcal{E}(\vec{n}_2)$ OPE ($\vec{n}_1 \to \vec{n}_2$). [Hofman, Maldacena '08]
- General statement in CFT [Chang, Koloğlu, Kravchuk, DSD, Zhiboedov '20]

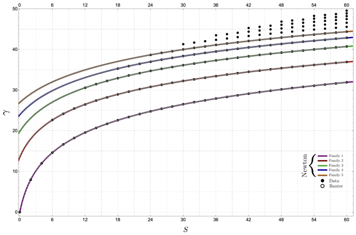
$$\mathcal{E} \times \mathcal{E} = \sum_i \sum_{j=0.2.4} \mathbb{O}^+_{i,J=3,j} + \sum_{n.i} \mathcal{D}_{2n} \mathbb{O}_{i,J=3+2n,j=4}$$

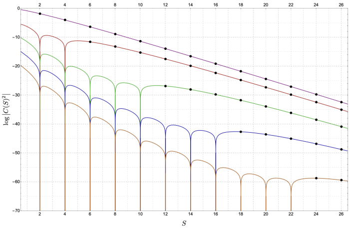


Proof at the level of kernels: relating $\mathbf{L}[\cdots]\mathbf{L}[\cdots]$ to $K_{\Delta,J}$. [NB: Cannot do $\mathcal{O}_1 \times \mathcal{O}_2$ OPE inside the integral — it doesn't converge.]

- Derivations of leading term [Dixon, Moult, Zhu '19; Korchemsky '19]. Light-ray OPE $\implies$ "factorization theorem" for $\langle \mathcal{E}\mathcal{E} \rangle$ [Chen '23]
- General OPE for $\mathbb{O}_1 \times \mathbb{O}_2$ currently unknown... Perturbative explorations: [Chen, Moult, Sandor, Zhu '22; Chang, DSD '22; Yan, Zhang '22; Chicherin, Sokatchev, Yan, Zhu '24...]

Mysteries of the Chew-Frautschi plot: Higher twist

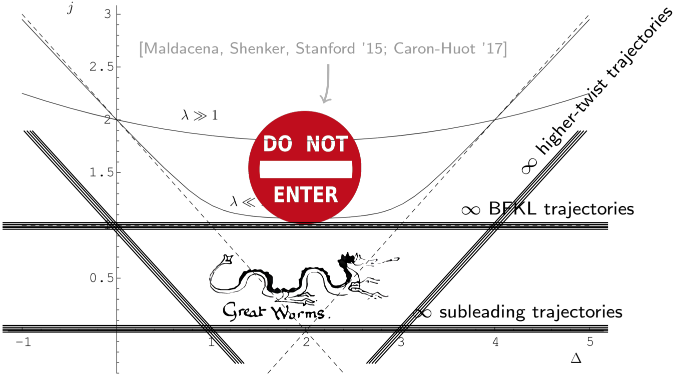




- Twist > 2 operators: number of local operators grows with spin.
- Infinite number of smooth Regge trajectories, but their matrix elements develop zeros in precise pattern. [Homrich, DSD, Vieira '22; Klabbers, Preti, Szécsényi '23]
- General mechanism explained in [Henriksson, Kravchuk, Oertel '23]. Involves an interesting light-ray "two-point function" [Caron-Huot '13]

$$\langle T\{\mathbb{O}_1 \mathbb{O}_2\}\rangle$$

Mysteries of the Chew-Frautschi plot: finite N



- What is this structure supposed to look like at finite λ and finite N? Do Regge trajectories unify into a single surface?
- Leading $\log N/N$ correction to $\langle \mathcal{E}\mathcal{E} \rangle$ at large λ: [Chen, Karlsson, Zhiboedov '24]

Some open questions

- How do you measure a general detector operator at a collider?
- Can we measure detectors in a condensed matter system?
- Can we formulate EFT running and matching for detectors? (Could help organize understanding of confinement effects in $\langle \mathcal{E} \cdots \mathcal{E} \rangle$ [Jaarsma, Li, Moult, Waalewijn, Zhu '23; Csaki, Ismail '24].)
- What does the Chew-Frautschi plot look like at finite λ and finite N?
- Can we find positivity/rigidity conditions for light-ray operators? Can we formulate bootstrap conditions?
- What behavior in the deep Regge limit is possible? Transparency vs. chaos? [Stanford '15; Murugan, Stanford, Witten '17; Caron-Huot, Gobeil, Zahree '20]
- Are other Lorentzian singularities described by other types of operators?
- Does "factorization theorem" $=$ OPE? [Chen '23]
- What is the general form of the light-ray OPE?
- How are light-ray operators and conformal line defects related?
- Do light-ray operators participate in interesting algebras? [Casini, Teste, Torroba '17; Cordova, Shao '18; Korchemsky, Sokatchev, Zhiboedov '21; Korchemsky, Zhiboedov '21; Faulkner, Speranza '24]

Thanks!