Asymptotic Symmetries for Logarithmic Soft Theorems

2403.13053 & 2406.XXXXX WITH SANGMIN CHOI & ALOK LADDHA

EARLIER + ONGOING WORK WITH YORGO PANO, PRAHAR MITRA, EMILIO TREVISANI

ANDREA PUHM

UNIVERSITY OF AMSTERDAM

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What are the symmetries of Nature?

here: approximate our world by $\Lambda \approx 0$

Symmetries in the infrared

Gauge theory and gravity in $D = 4$ spacetime dimensions: rich infrared structure of ∞-dimensional symmetries underlying dynamical processes.

Infrared divergences in conventional S-matrix elements:
violation of the conservation laws associated with these symmetries.

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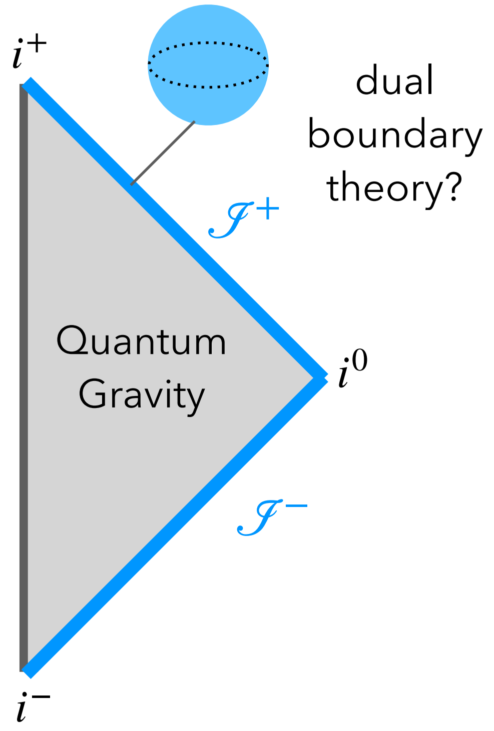
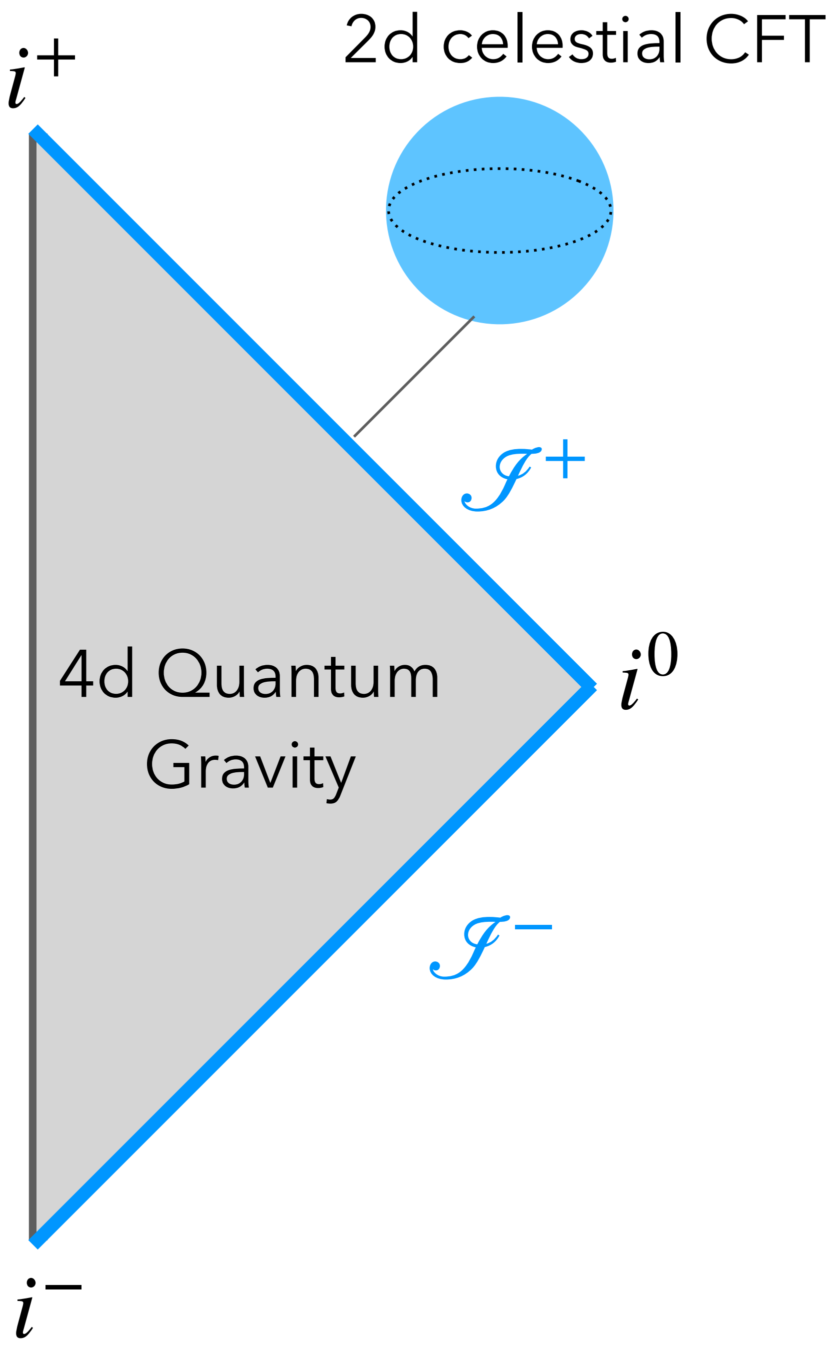
- Can we identify all **infrared = large-distance / low-energy** symmetries?

- Cornerstone for **holographic principle**!



Celestial holography



4d Quantum Gravity

in asymptotically flat spacetimes

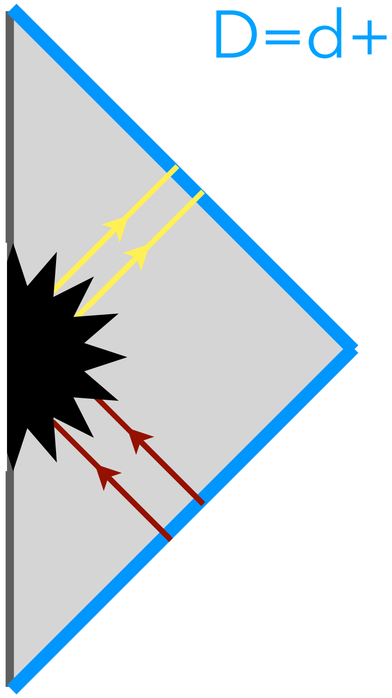
=

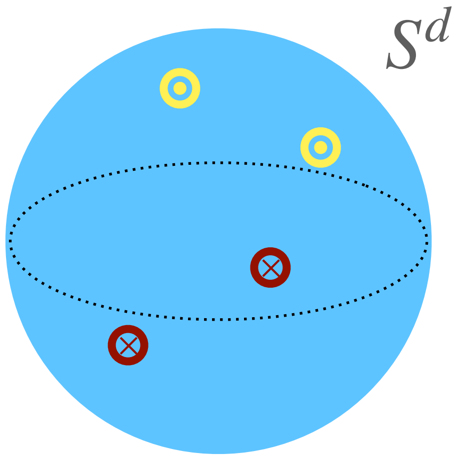
2d ''Celestial CFT''

Symmetry & observables

symmetry:

Lorentz group in D=d+2 dimensions $\cong$ Euclidean conformal group in d dimensions





Symmetry & observables

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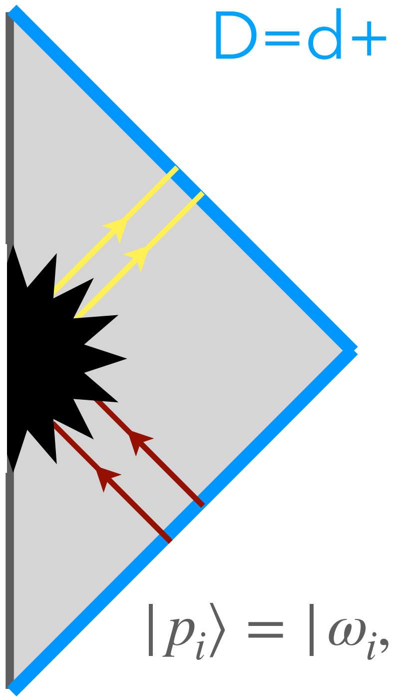
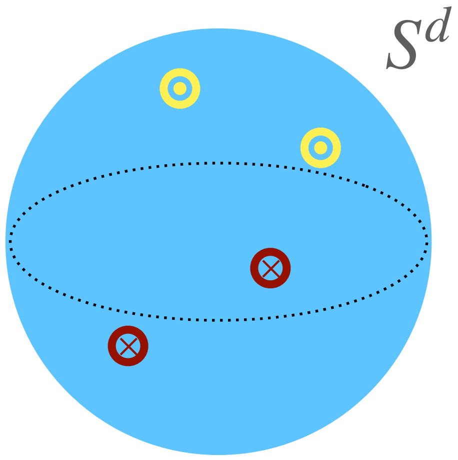
basic observables in flat space:

S-matrix



$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis



Standard amplitudes

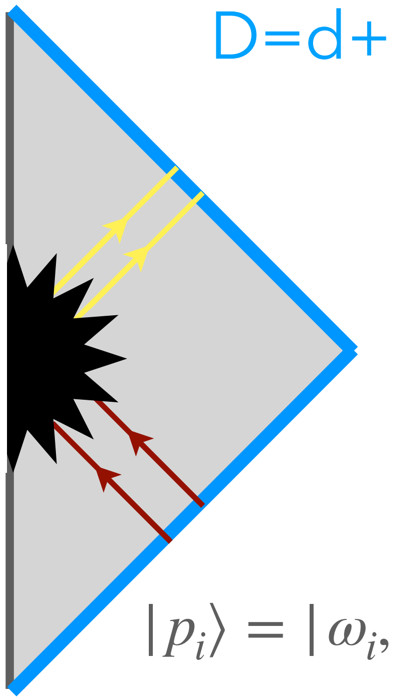
$$\langle p_n^{\text{out}} \ldots | \mathcal{S} | \ldots p_1^{\text{in}} \rangle$$

translation symmetry

Symmetry & observables

symmetry:

Lorentz group in D=d+2 dimensions $\cong$ Euclidean conformal group in d dimensions



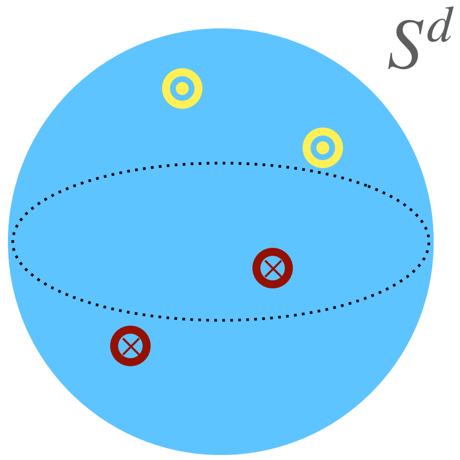
$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis

basic observables in flat space:

S-matrix

$$\xrightarrow[\int_0^\infty d\omega\, \omega^{\Delta-1}]{\mathscr{M}\text{ellin}}$$



$|\Delta_i, x_i\rangle$

boost-weight basis

Standard amplitudes

$$\langle p_n^{\text{out}} \ldots | \mathcal{S} | \ldots p_1^{\text{in}} \rangle$$

translation symmetry

Celestial amplitudes

$$\langle \mathcal{O}^-_{\Delta_1}(x_1) \ldots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$$

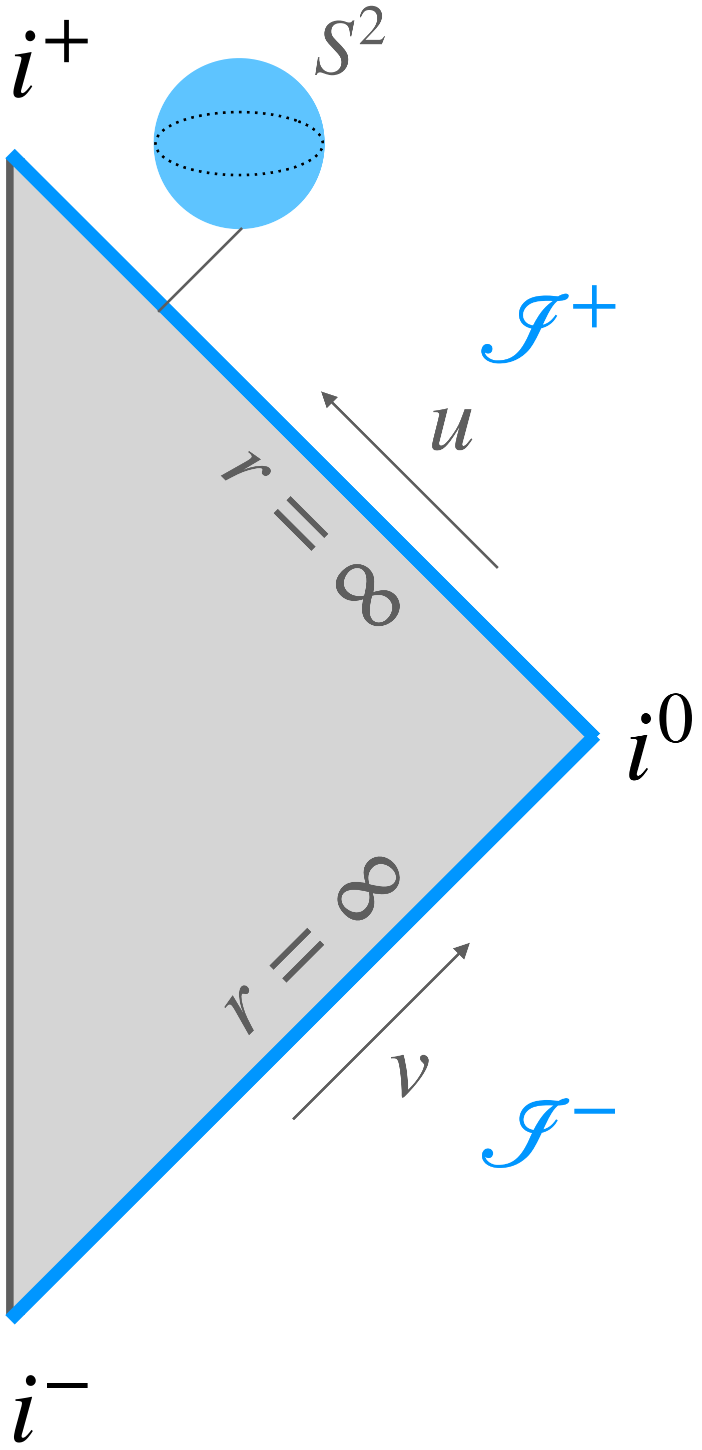
Lorentz symmetry

↕

global conformal symmetry

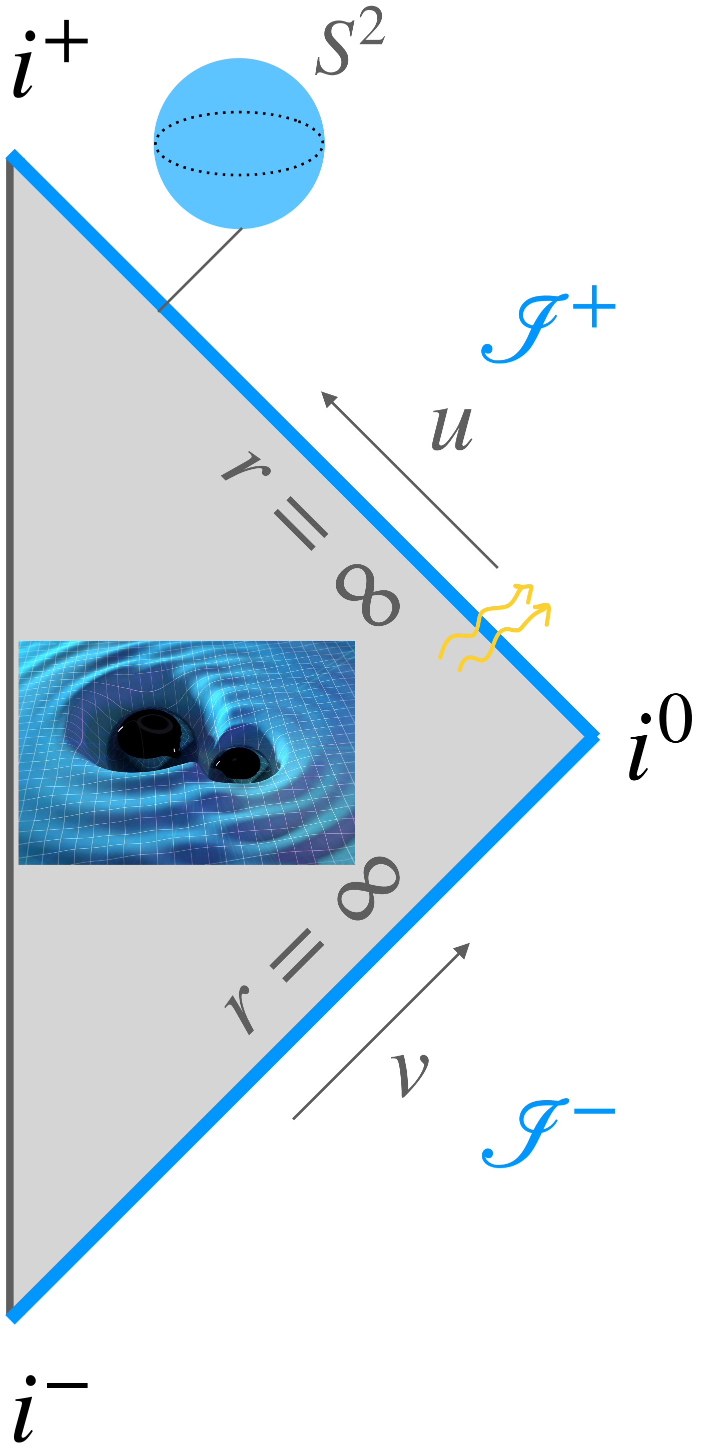
[de Boer, Solodukhin'03] [Pasterski,Shao,Strominger'17] [Pasterski,Shao'17]

Asymptotic symmetries



"Large" gauge/diffeo transformations that preserve the boundary conditions of the fields, i.e. their large-distance fall-offs.

Asymptotic symmetries



"Large" gauge/diffeo transformations that
preserve the boundary conditions of the fields,
i.e. their large-distance fall-offs.

gravity:

mass

$$ds^2 = -(1 + \underset{\uparrow}{\dots})du^2 - (2 + \dots)dudr$$

angular momentum

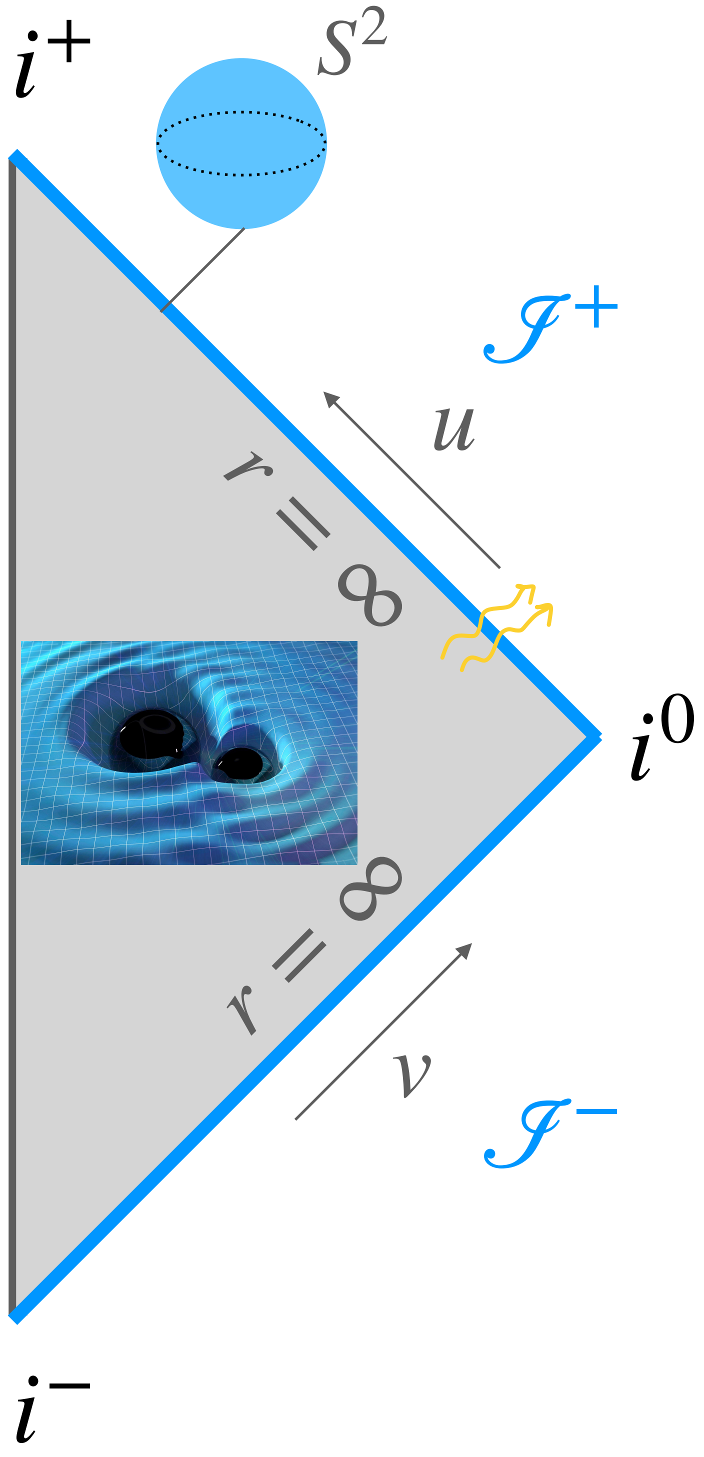
$$+(\dots)dudx^A \qquad A, B = z, \bar{z}$$

$$+(r^2\gamma_{AB} + r\,C_{AB} + \dots)dx^A dx^B$$

shear: gravitational waves

⇒ Bondi news $N_{AB} = \partial_u C_{AB}$

Asymptotic symmetries



"Large" gauge/diffeo transformations that preserve the boundary conditions of the fields, i.e. their large-distance fall-offs.

gravity:

Find ξ such that $\mathscr{L}_\xi g_{\mu\nu} \approx$ "0" as $r \to \infty$.

$\updownarrow$

$O(1/r^\#)$

Unlike gauge redundancies, asymptotic symmetries act non-trivially on the physical data → non-zero charges.

IR triangle

[He,Lysov,Mitra,Strominger'14]

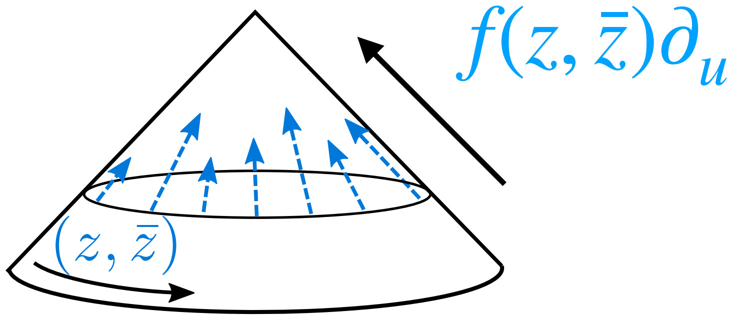
[Strominger,Zhiboedov'14]

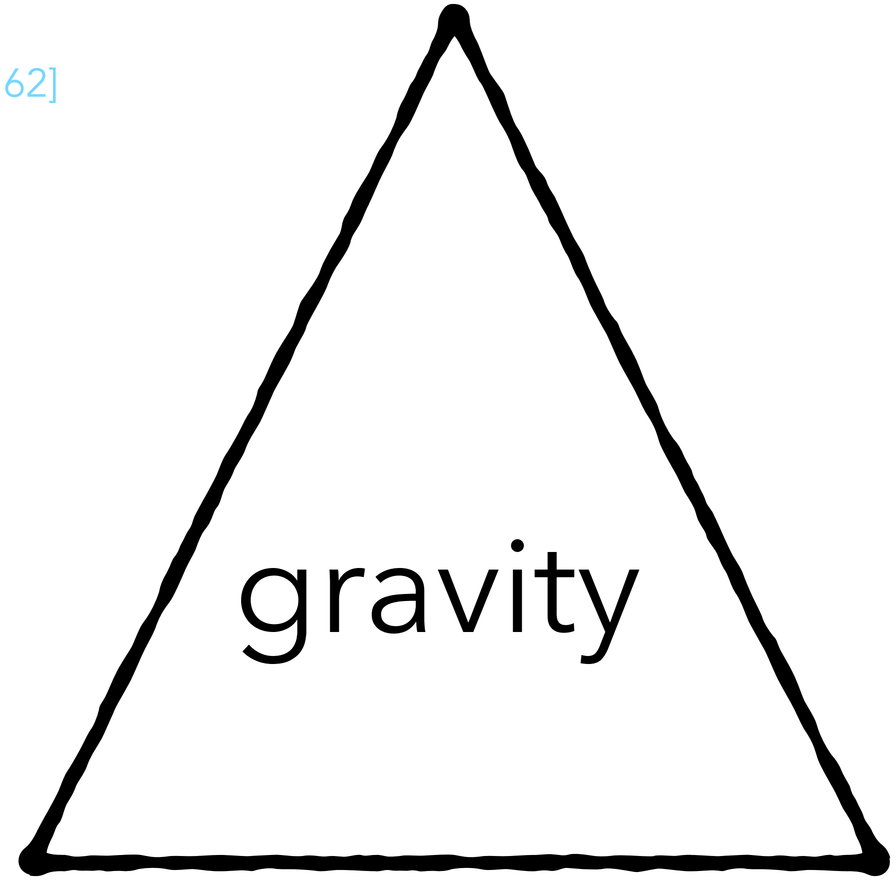
The symmetries of asymptotically
flat space are not just Poincaré
but an infinite extension!

[Bondi,van der Burg,Metzner'62] [Sachs'62]

BMS group

supertranslations

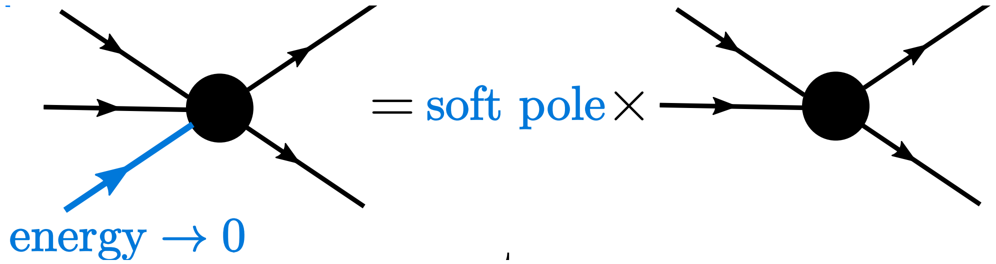




gravity

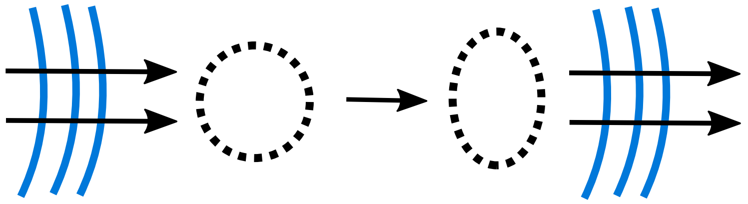
[Weinberg'65]

soft graviton theorem



[Zel'dovich,Polnarev'74 [Braginsky,Thorne'87]

displacement memory effect



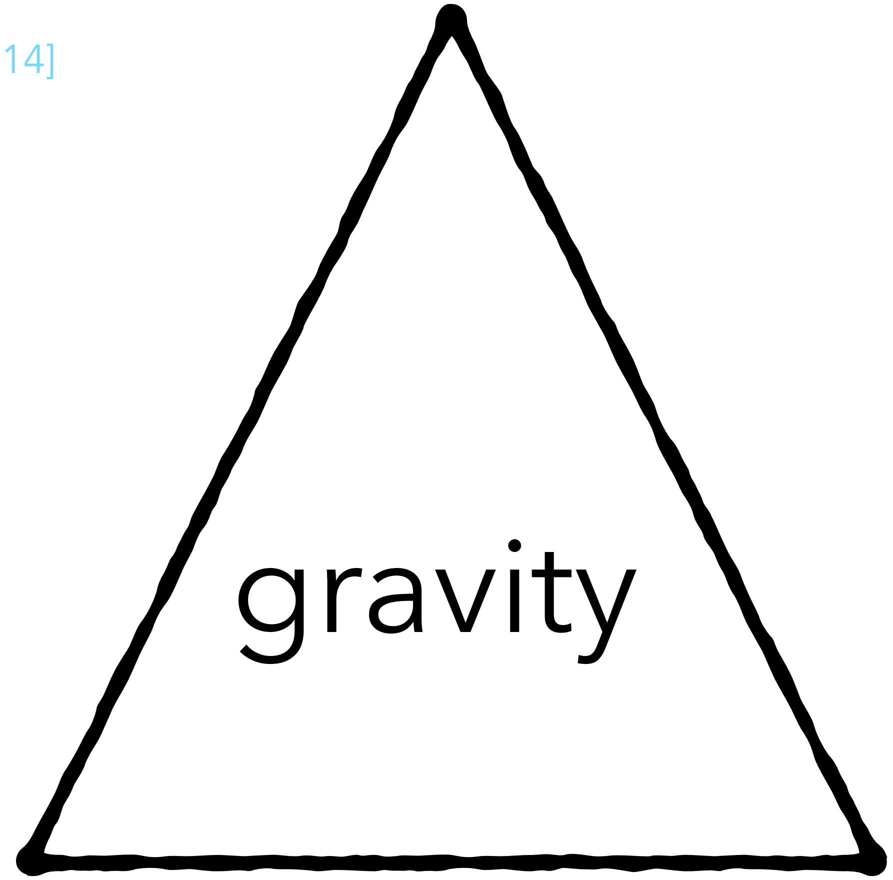
IR triangle

asymptotic symmetry

supertranslations,
superrotations, ...

[Barnich,Troessaert'11] [Campiglia,Laddha'14]

extended / generalized
BMS group



soft theorem

ω^{-1} leading soft graviton,
ω^{0} subleading soft graviton, ...

[Cachazo,Strominger'14]

memory effect

displacement,
spin, ...

[Pasterski,Strominger,Zhiboedov'15]

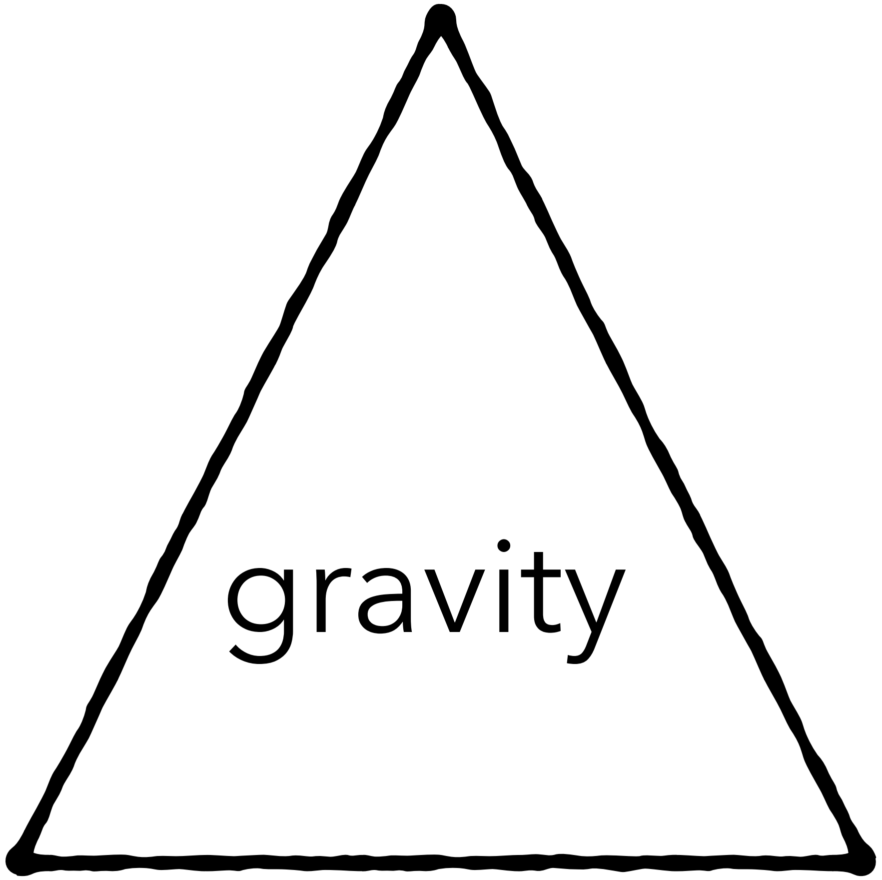
IR triangle

supertranslations,
superrotations, ...

[Barnich,Troessaert'11] ↕

local conformal
symmetry on S^2 !

just what we need
for CCFT :-)



asymptotic symmetry

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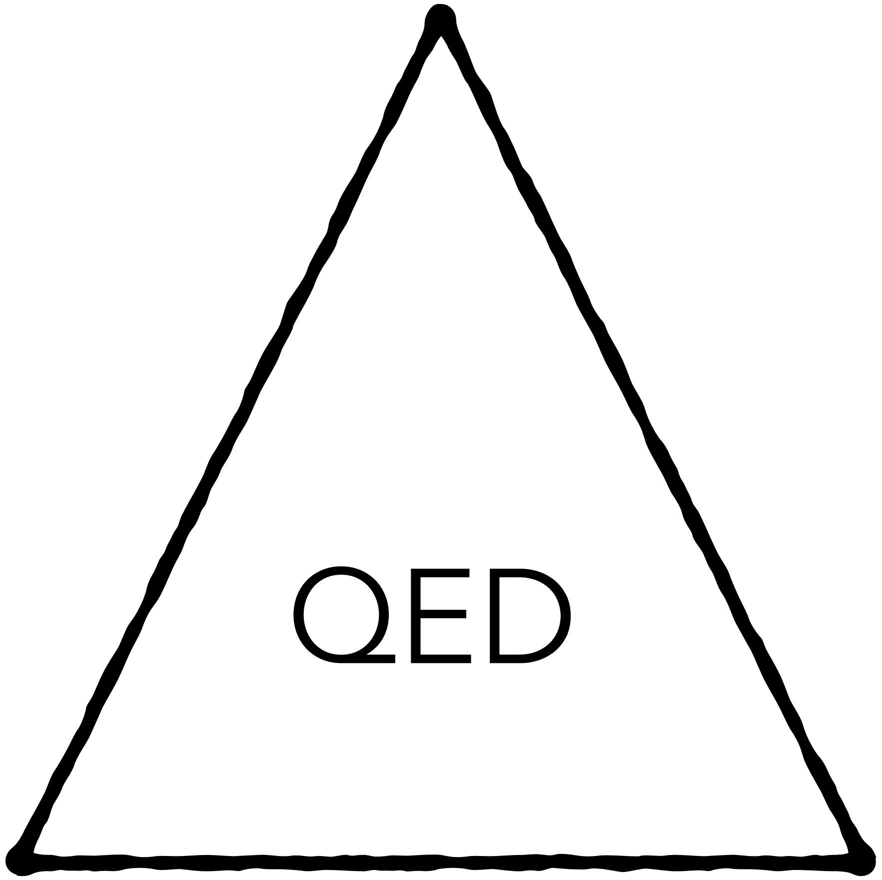


superphaserotation, ... **asymptotic symmetry**

QED

soft theorem

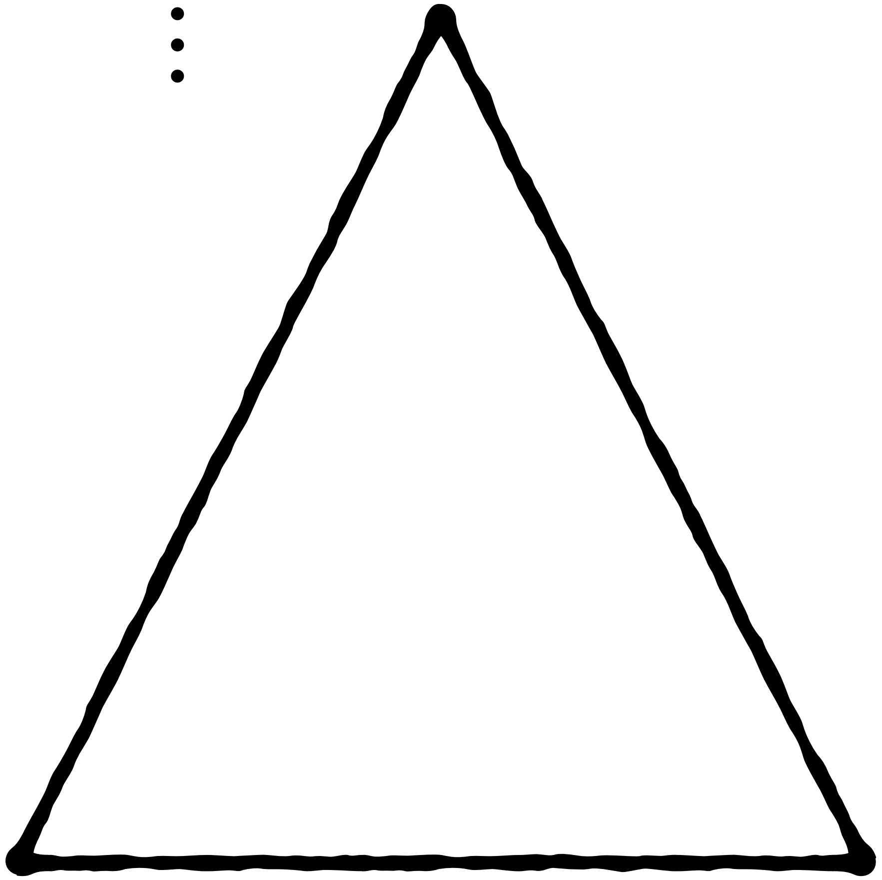
memory effect

ω^{-1} leading soft photon, ...

electromagnetic kick, ...

∞ IR triangle

(for projected S-matrix)



asymptotic symmetry

⋮

[Hamada,Shiu'18]
[Li,Lin,Zhang'18]

soft theorem

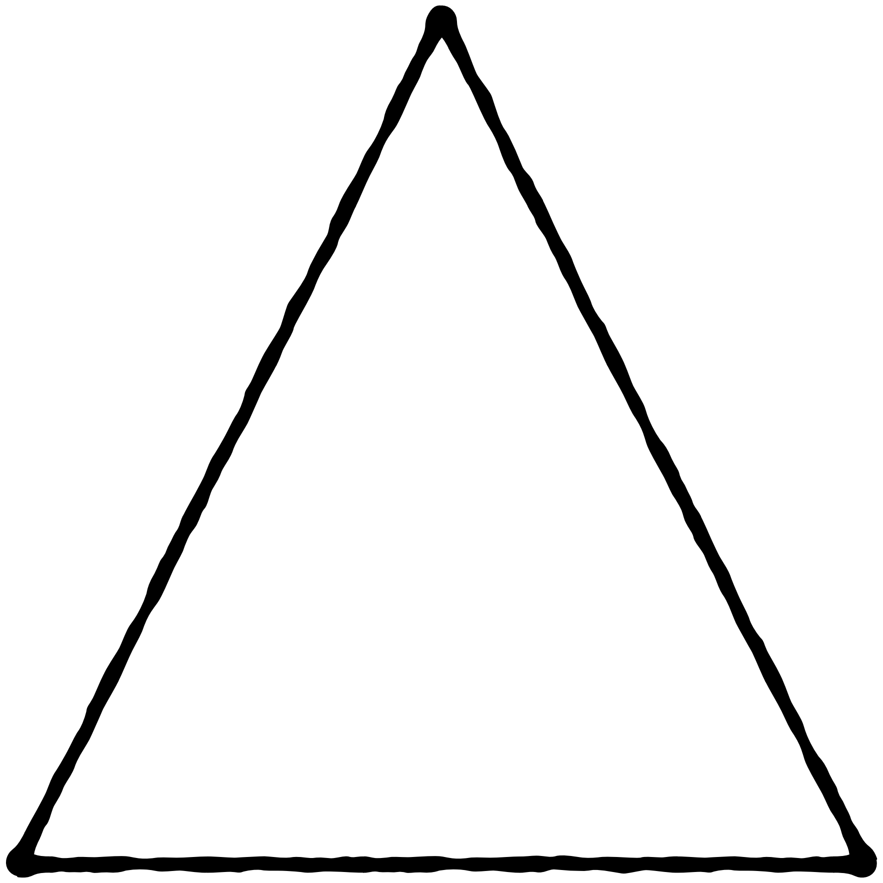
ω^n ⋮

$n = -1,0,1,\ldots$

memory effect

⋮

3 bases for the infrared



boost weight

Δ

asymptotic symmetry

soft theorem

energy

ω

memory effect

null time

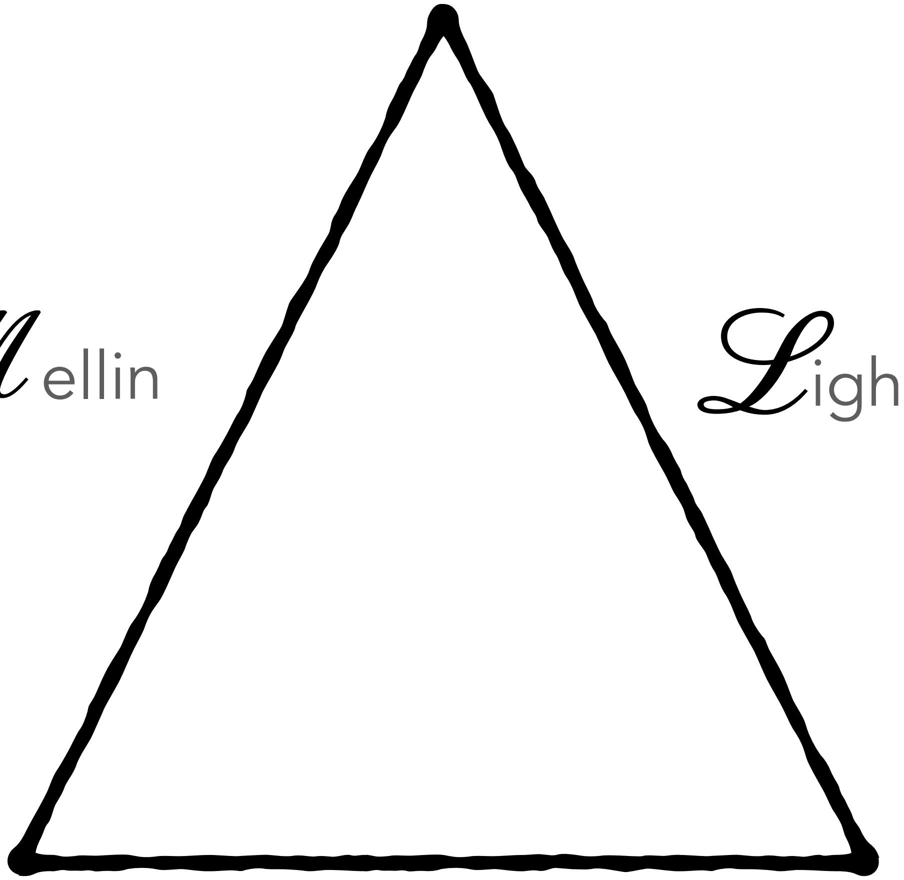
u

3 bases for the infrared

boost weight

Δ

asymptotic symmetry



$\mathscr{M}$ellin

$$\mathscr{M}(\,.\,) = \int_0^\infty d\omega \omega^{\Delta-1}(\,.\,)$$

[Pasterski,Shao,Strominger'17]

$\mathscr{L}$ight-ray

$$\mathscr{L}(\,.\,) = \int_{-\infty}^{+\infty} du u^{-\Delta}(\,.\,)$$

[Pasterski,AP,Trevisani'21]

soft theorem

energy

ω

$\mathscr{F}$ourier

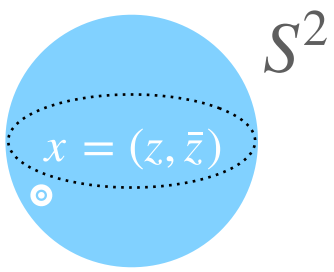
$$\mathscr{F}(\,.\,) = \int_{-\infty}^{+\infty} du e^{i\omega u}(\,.\,)$$

memory effect

null time

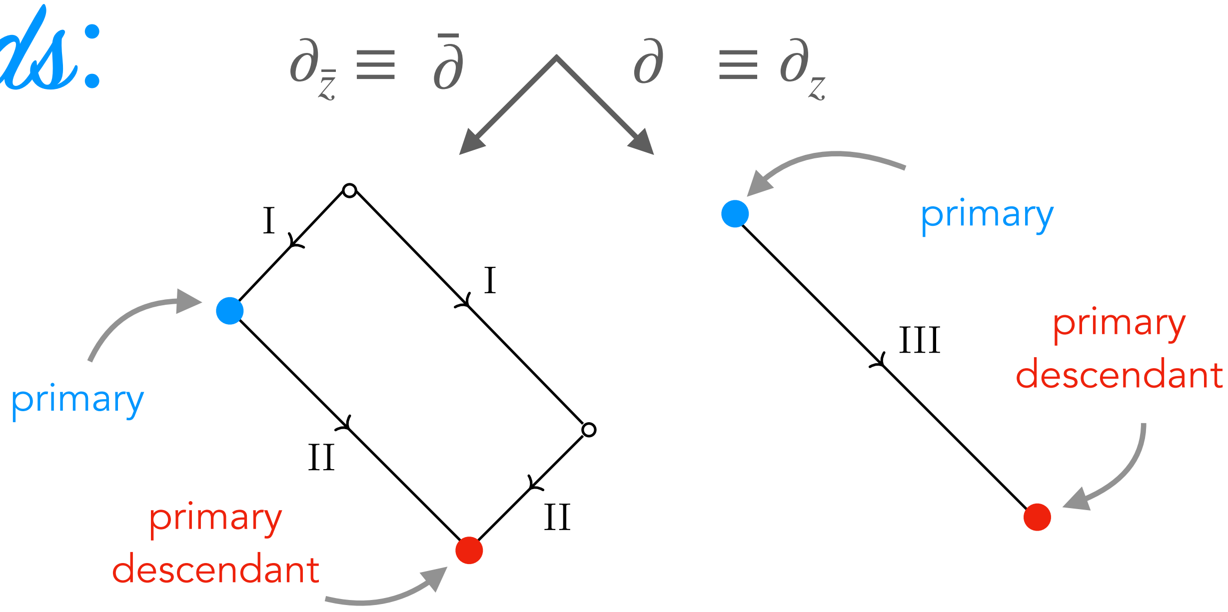
u

Celestial CFT$_2$



celestial diamonds:

[Pasterski,AP,Trevisani'21]



primary

primary

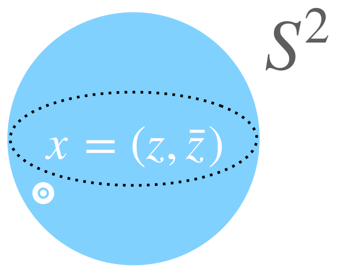
primary descendant

primary descendant

conservation equation $\bar{\partial}^{\#}\mathcal{O}(z, \bar{z}) = 0$

depends on   types I, II, III: spin of descendant >,<,= spin of parent primary

Celestial CFT$_2$



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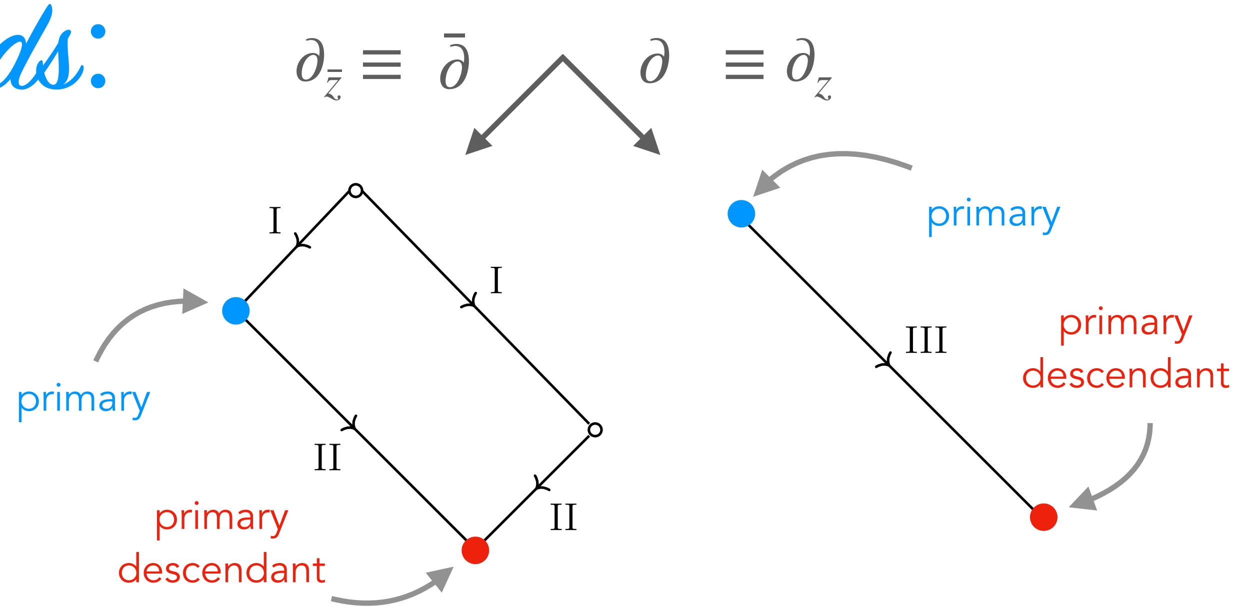


conservation equation $\bar{\partial}^{\#}\mathcal{O}(z, \bar{z}) = 0$

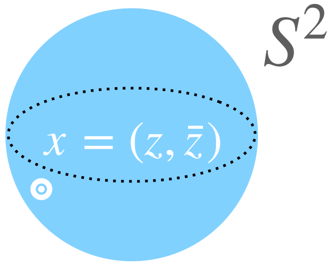
depends on types I, II, III: spin of descendant >,<,= spin of parent primary

gravity: $\mathcal{O}_\Delta(x)$ with $\Delta = 1{,}0$ generate extended BMS symmetries!

supertranslations superrotations: shadow of $\mathcal{O}_{\Delta=0}$ is CCFT$_2$ stress tensor

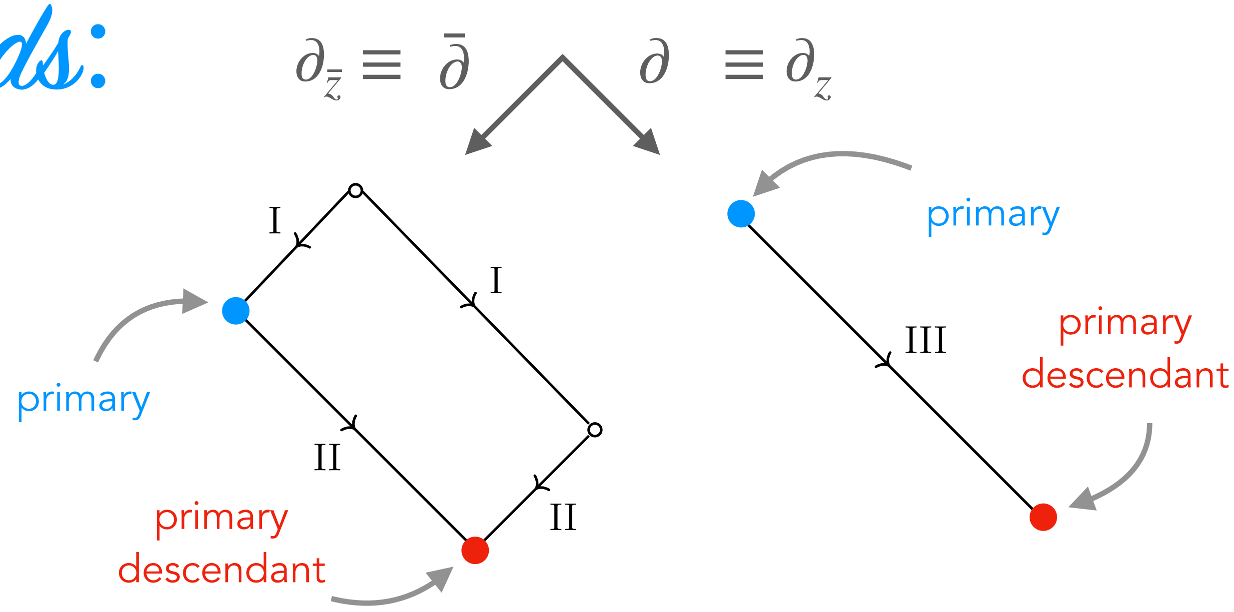
[Kapec,Mitra,Raclariu,Strominger'16]
[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]

Celestial CFT$_2$



celestial diamonds:

[Pasterski,AP,Trevisani'21]


$\partial_{\bar{z}} \equiv \bar{\partial}$

$\partial \equiv \partial_z$

primary

primary descendant

primary

primary descendant

conservation equation $\bar{\partial}^{\#}\mathcal{O}(z,\bar{z}) = 0$

↑ # depends on types I, II, III: spin of descendant >,<,= spin of parent primary

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[Kapec,Mitra,Raclariu,Strominger'16]
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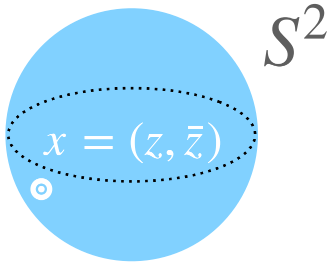
[Donnay,Pasterski,AP'20]

Classification of all celestial symmetries in CCFT$_{d\geq 2}$.

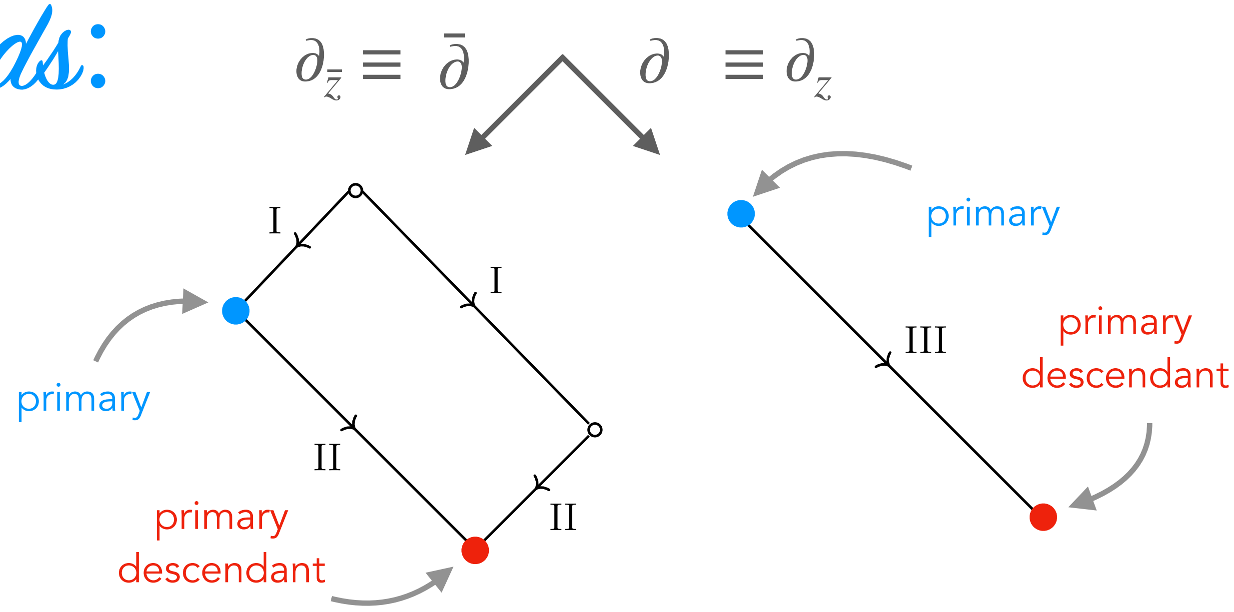
[Pasterski,AP,Trevisani'21] $d = 2$
[Pano,AP,Trevisani'23] $d > 2$

Celestial CFT$_2$



celestial diamonds:

[Pasterski,AP,Trevisani'21]



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Classification of all celestial symmetries in CCFT$_{d\geq 2}$.

[Pasterski,AP,Trevisani'21] $d = 2$
[Pano,AP,Trevisani'23] $d > 2$

$d = 2:$ $\mathcal{O}_\Delta(x)$ with $\Delta = 1, 0, -1, \ldots$ satisfy ∞ dimensional algebra! [Guevara,Himwich,Pate,Strominger'21]

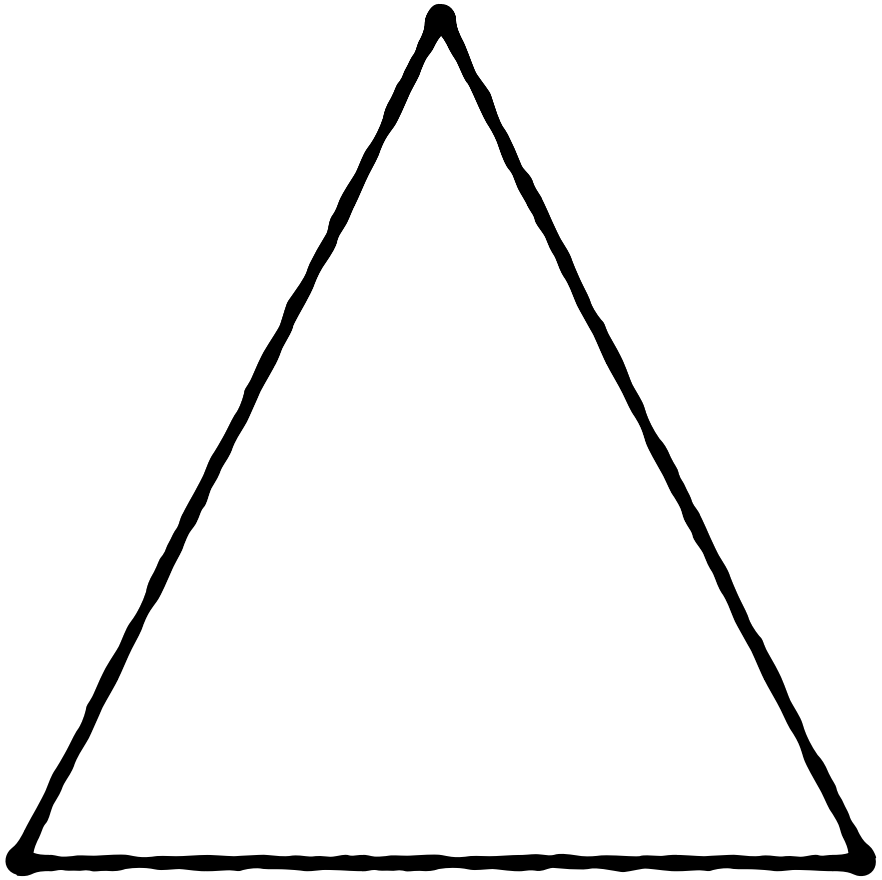
gravity: $w_{1+\infty}$

Arises in Penrose's twistor construction!

[Strominger'21]

IR triangle

∞-dimensional
symmetry algebra
$\supset$ local conformal
symmetry on S^2 **!**



asymptotic symmetry

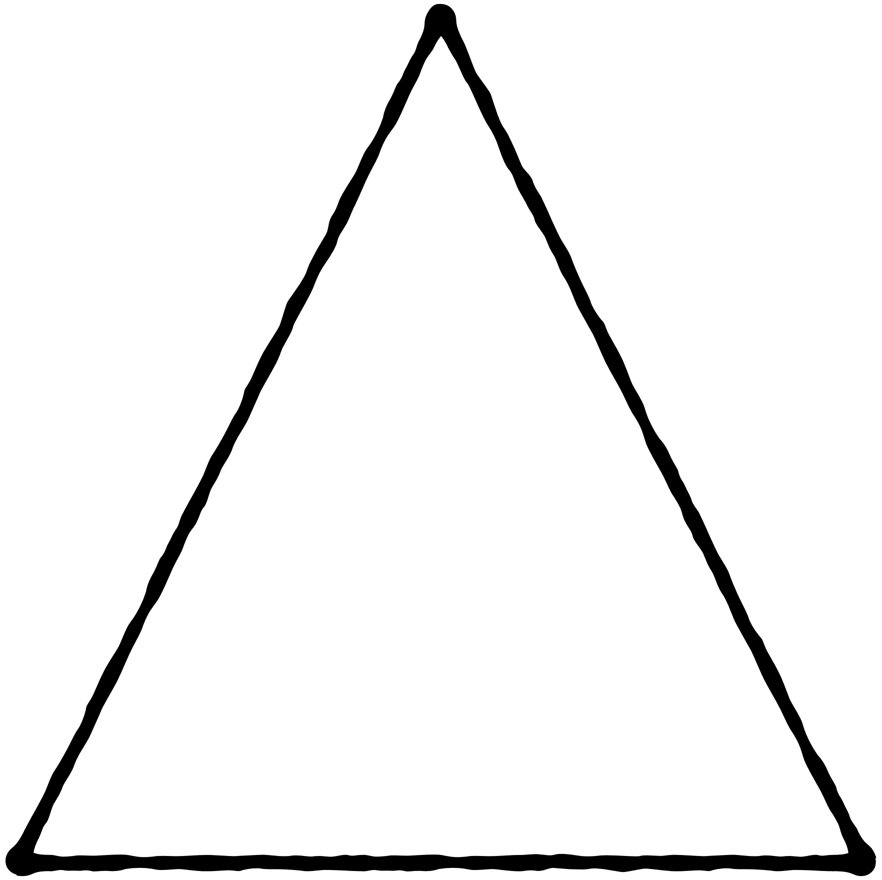
soft theorem

memory effect

IR triangle **@ tree !**

∞-dimensional
symmetry algebra

$\supset$ local conformal
symmetry on S^2 **!**



asymptotic symmetry

soft theorem

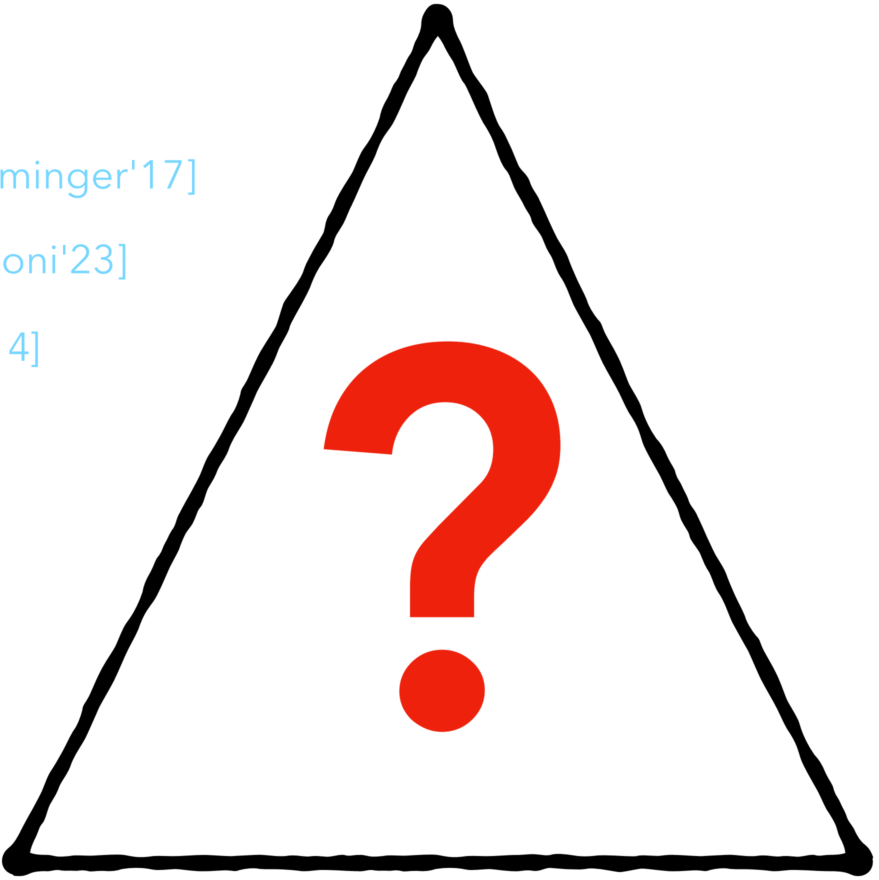
memory effect

IR triangle @ loop ?

∞-dimensional
symmetry algebra
⊃ local conformal
symmetry on S^2 ?

some progress in [He,Kapec,Raclariu,Strominger'17]
[Donnay,Nguyen,Ruzziconi'23]

based on 1-loop result [Bern,Davis,Nohle'14]



asymptotic symmetry

soft theorem

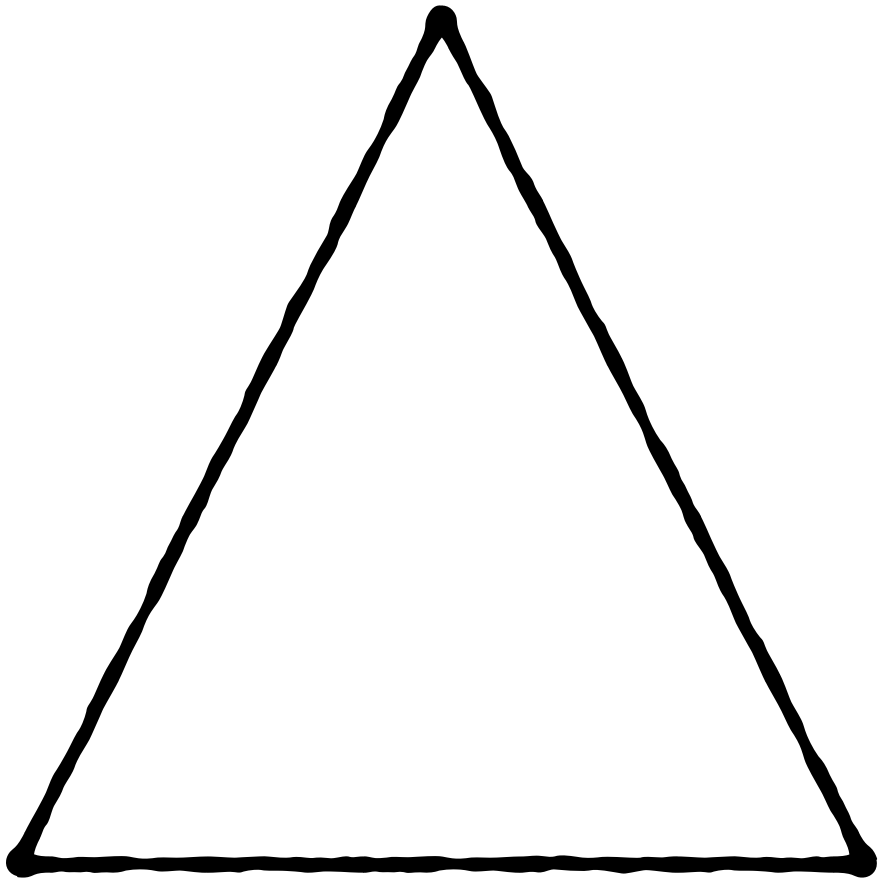
memory effect

IR triangle @ long-range IR effects

classical and quantum

∞-dimensional symmetry algebra

$\supset$ local conformal symmetry on S^2 ?



asymptotic ? symmetry

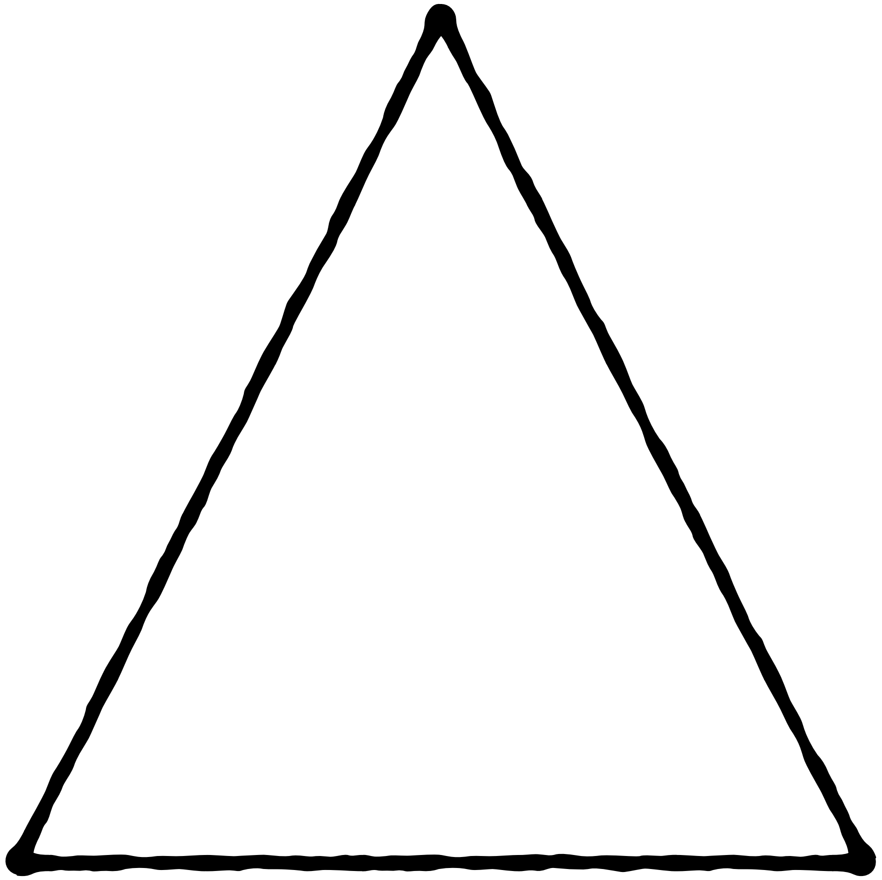
logarithmic soft theorem

tail memory effect

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

IR triangle @ long-range IR effects

classical and quantum



asymptotic ? symmetry

∞-dimensional
symmetry algebra
$\supset$ local conformal
symmetry on S^2 **?**

☠

logs render ambiguous
all subleading tree-level
soft theorems

↑

logarithmic soft theorem

tail memory effect

ω^{-1} leading soft
$\log \omega$ subleading soft
⋮

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

Power-law soft theorems

Tree-level amplitudes in (scalar) QED and gravity admit a soft expansion:

[Low'58] [Weinberg'65] [Cachazo,Strominger'14]
[Hamada'18] [Li'18]

hard momenta helicity

$$\mathcal{M}_{N+1}(p_1, \ldots, p_N; (\omega, q, \ell)) = \sum_{n=-1}^{\infty} \omega^n S_n(\{p_i\}, (q, \ell))\mathcal{M}_N(p_1, \ldots, p_N) + \ldots$$

soft momentum

$$p^\mu = \omega q^\mu(z, \bar{z})$$

⊃ non-universal *

$n = -1$	S_{-1}	Weinberg (leading) soft factor
$n = 0$	S_0	subleading tree soft factor
$n > 0$	$S_{n>0}$	sub^{n+1} leading tree soft factors

tree exact
& universal

* see [Elvang,Jones,Naculich'16] for classification in EFT of massless particles

Logarithmic Soft Theorems

Long-range effects yield **novel soft theorems**:

[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

hard momenta

helicity

$$\mathcal{M}_{N+1}(p_1,\ldots,p_N;(\omega,q,\ell)) = \sum_{n=-1}^{\infty} \omega^n (\ln\omega)^{n+1}\, S_n^{(\ln\omega)}(\{p_i\},(q,\ell))\mathcal{M}_N(p_1,\ldots,p_N) + \ldots$$

soft momentum

$$p^\mu = \omega q^\mu(z,\bar{z})$$

$\supset$ non-universal

$\sim \omega^n(\ln\omega)^{m\neq n+1}$

$n = -1$ $\quad S_{-1}^{(\ln\omega)} \equiv S_{-1}$ Weinberg (leading) soft factor

tree exact
& universal

$n = 0$ $\quad S_0^{(\ln\omega)} \neq S_0$ leading log soft factor

1-loop exact
& universal

$n > 0$ $\quad S_{n>0}^{(\ln\omega)} \neq S_{n>0}$ sub^n leading log soft factor

Is the **universality** of the loop exact logarithmic soft theorems a consequence of **asymptotic symmetries of the S-matrix** ?

Asymptotic symmetries
for
logarithmic soft theorems

Charge conservation law

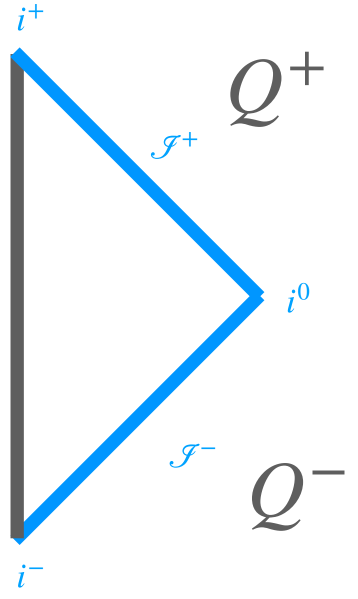
To establish a symmetry interpretation for a soft theorem from first principles: for asymptotic symmetry transformations δ compute charges $Q^{\pm}$ from the symplectic structure

$$\Omega_{i^{\pm} \cup \mathscr{I}^{\pm}}(\delta, \delta') = \delta' Q^{\pm}$$

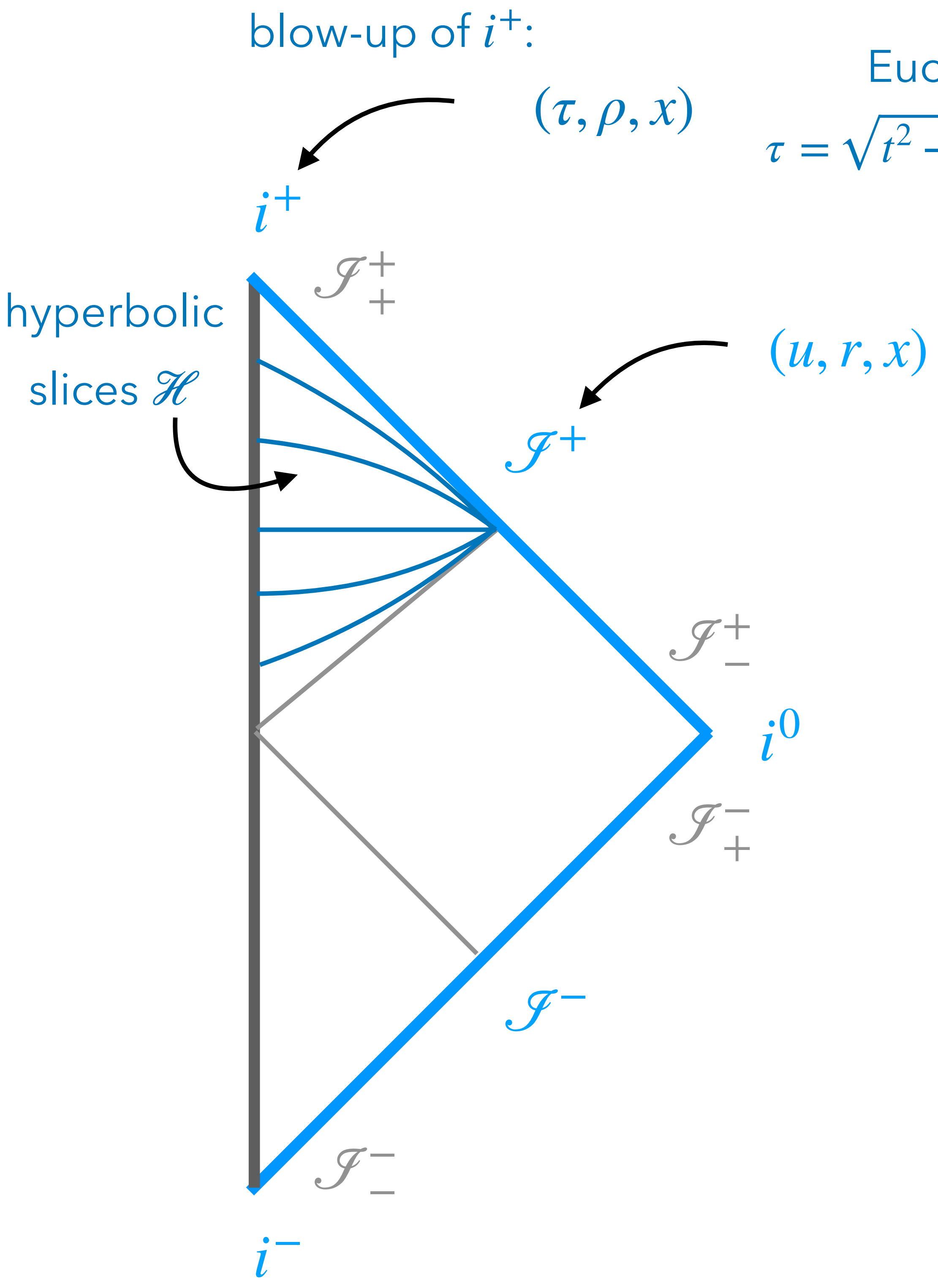
in the **covariant phase space formalism** and show that the **charge conservation law**

$$Q^{+} = Q^{-}$$

corresponds to the **soft theorem**.



Structure at ∞



Euclidean AdS$_3$ coordinates

$\tau = \sqrt{t^2 - r^2}$ $\rho = \frac{r}{\sqrt{t^2 - r^2}}$

retarded Bondi coordinates

$u = t - r$

$$\Omega_{i^+ \cup \mathscr{I}^+}(\delta, \delta') = \delta' Q^+$$

limit $\tau \to \infty$ at fixed ρ

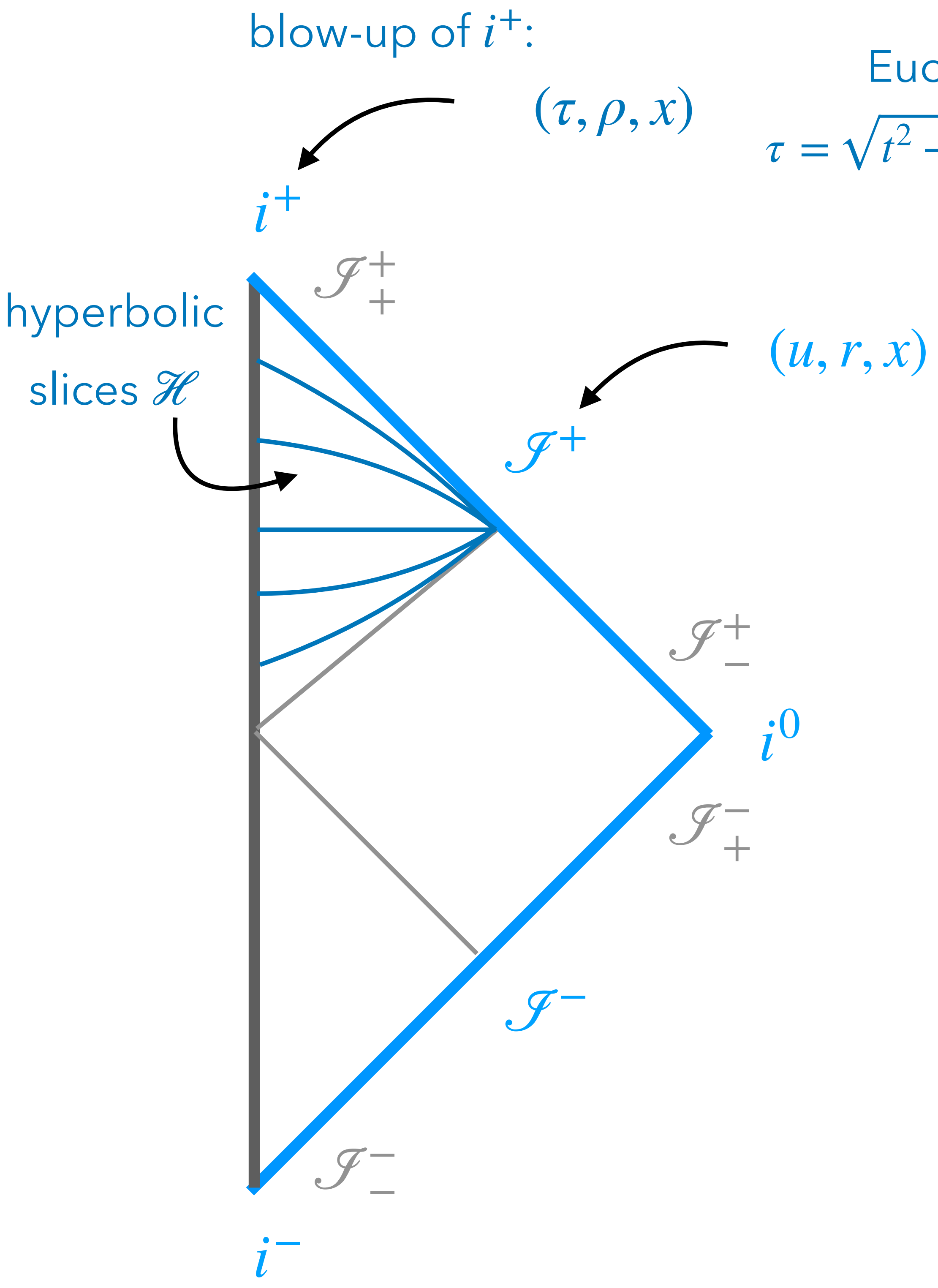
limit $r \to \infty$ at fixed u

$$Q^+_H + Q^+_S = Q^+$$

hard charge

soft charge

Structure at ∞



Euclidean AdS$_3$ coordinates

$$\tau = \sqrt{t^2 - r^2} \qquad \rho = \frac{r}{\sqrt{t^2 - r^2}}$$

retarded Bondi coordinates

$$u = t - r$$

$$\Omega_{i^+ \cup \mathscr{I}^+}(\delta, \delta') = \delta' Q^+$$

limit $\tau \to \infty$ at fixed ρ

limit $r \to \infty$ at fixed u

$$Q^+_H + Q^+_S = Q^+$$

hard charge

soft charge

in soft theorem

action on hard matter

soft insertion

Log symmetry for gravity

Effective quantum theory of linear metric perturbations

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad \kappa = \sqrt{32\pi G_N}$$

+ minimally coupled massive real scalar field φ

Soft graviton factor

[Weinberg'65]

[Cachazo,Strominger'14]

Leading soft factor $\sim \frac{1}{\omega}$:

$$S_{-1} = \frac{\kappa}{2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{q \cdot p_i}$$

p_i^μ... hard momenta

$p^\mu = \omega q^\mu$... soft momentum

$\varepsilon^{\mu\nu}$...soft graviton polarization

Subleading soft factor $\sim \omega^0$:

$$S_0 = -\frac{i\kappa}{2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\mu q_\lambda J_i^{\lambda\nu}}{q \cdot p_i}$$

ambiguous if long-range IR effects

Soft graviton factor

[Weinberg'65]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

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Subleading soft factor $\sim \ln \omega$:

$$S_0^{(\ln \omega)} = S_{0,\text{classical}}^{(\ln \omega)} + S_{0,\text{quantum}}^{(\ln \omega)}$$

classical: late time gravitational radiation from particle acceleration via long-range gravitational interaction

$$S_{0,\text{classical}}^{(\ln \omega)} = \frac{i(\frac{\kappa}{2})^3}{8\pi} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{q \cdot p_i} \sum_{j \neq i, \eta_i \eta_j = 1} \frac{(p_i \cdot p_j)\left[p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right]\left[2(p_j \cdot p_j)^2 - 3 p_i^2 p_j^2\right]}{\left[(p_i \cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}}$$

quantum: $\omega_{\text{soft}} \ll \omega_{\text{loop}} \ll \omega_{\text{hard}}$
(1-loop)

$$S_{0,\text{quantum}}^{(\ln \omega)} = -\frac{(\frac{\kappa}{2})^3}{16\pi^2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\rho} p_i^\rho q_\nu}{q \cdot p_i} \left(p_i^\mu \partial_{p_i}^\nu - p_i^\nu \partial_{p_i}^\mu\right) \sum_{j \neq i} \frac{2(p_i \cdot p_j)^2 - p_i^2 p_j^2}{\sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2)}} \ln \left(\frac{p_i \cdot p_j + \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}{p_i \cdot p_j - \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}} \right)$$

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[Weinberg'65]
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+ drag

quantum: $\omega_{\mathrm{soft}} \ll \omega_{\mathrm{loop}} \ll \omega_{\mathrm{hard}}$

(1-loop)

$\omega_{\mathrm{IR}} \ll \omega_{\mathrm{loop}} \ll \omega_{\mathrm{soft}}$

$$S_{0,\mathrm{quantum}}^{(\ln\omega)} = -\frac{(\frac{\kappa}{2})^3}{16\pi^2}\sum_{i=1}^{N}\frac{\varepsilon_{\mu\rho}p_i^\rho q_\nu}{q\cdot p_i}\left(p_i^\mu\partial_{p_i}^\nu - p_i^\nu\partial_{p_i}^\mu\right)\sum_{j\neq i}\frac{2(p_i\cdot p_j)^2 - p_i^2p_j^2}{\sqrt{(p_i\cdot p_j)^2 - p_i^2p_j^2)}}\ln\left(\frac{p_i\cdot p_j + \sqrt{(p_i\cdot p_j)^2 - p_i^2p_j^2}}{p_i\cdot p_j - \sqrt{(p_i\cdot p_j)^2 - p_i^2p_j^2}}\right)$$

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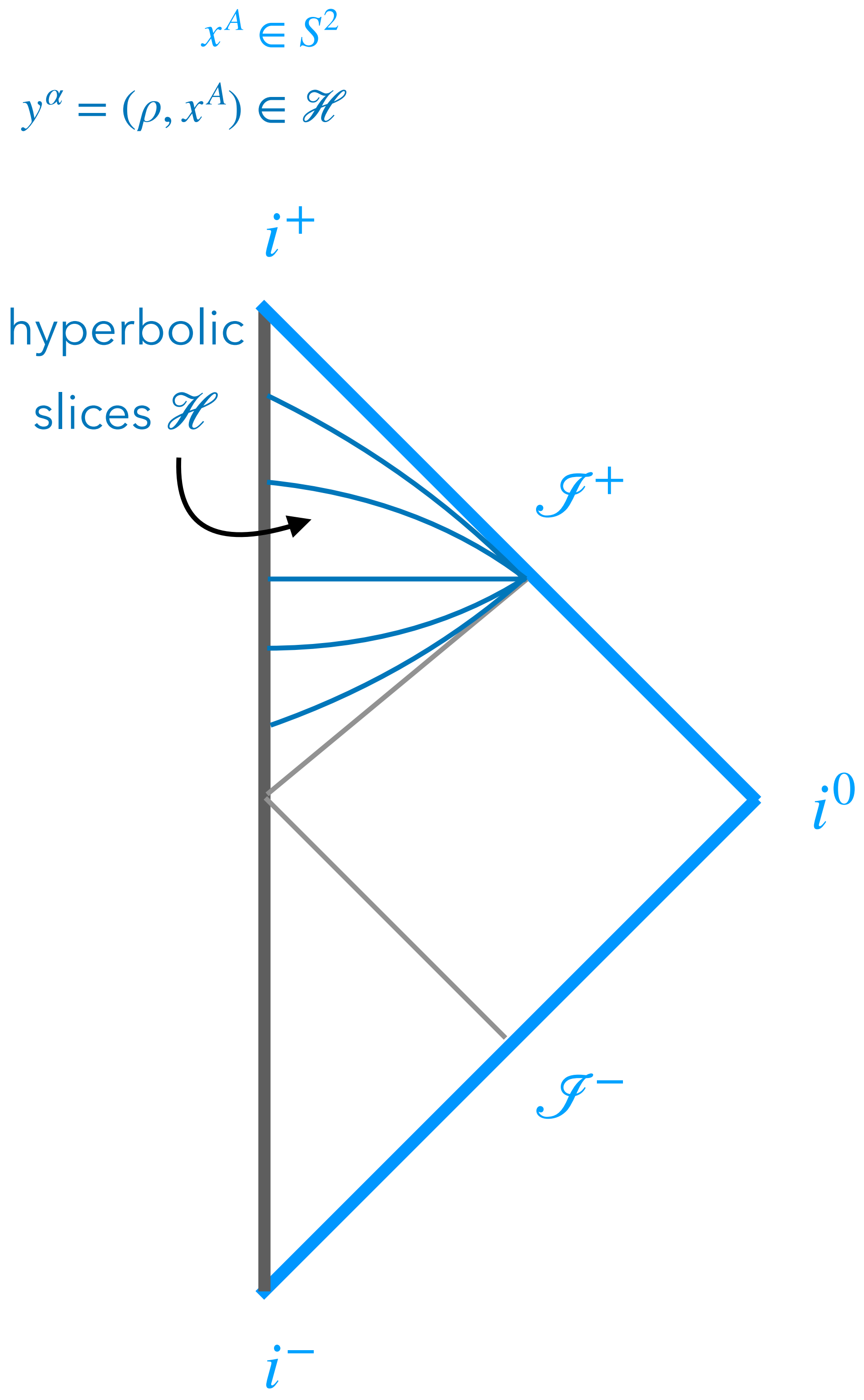
Symmetry @ $i^+ \cup \mathscr{I}^+$



Superrotation across $i^+ \cup \mathscr{I}^+$

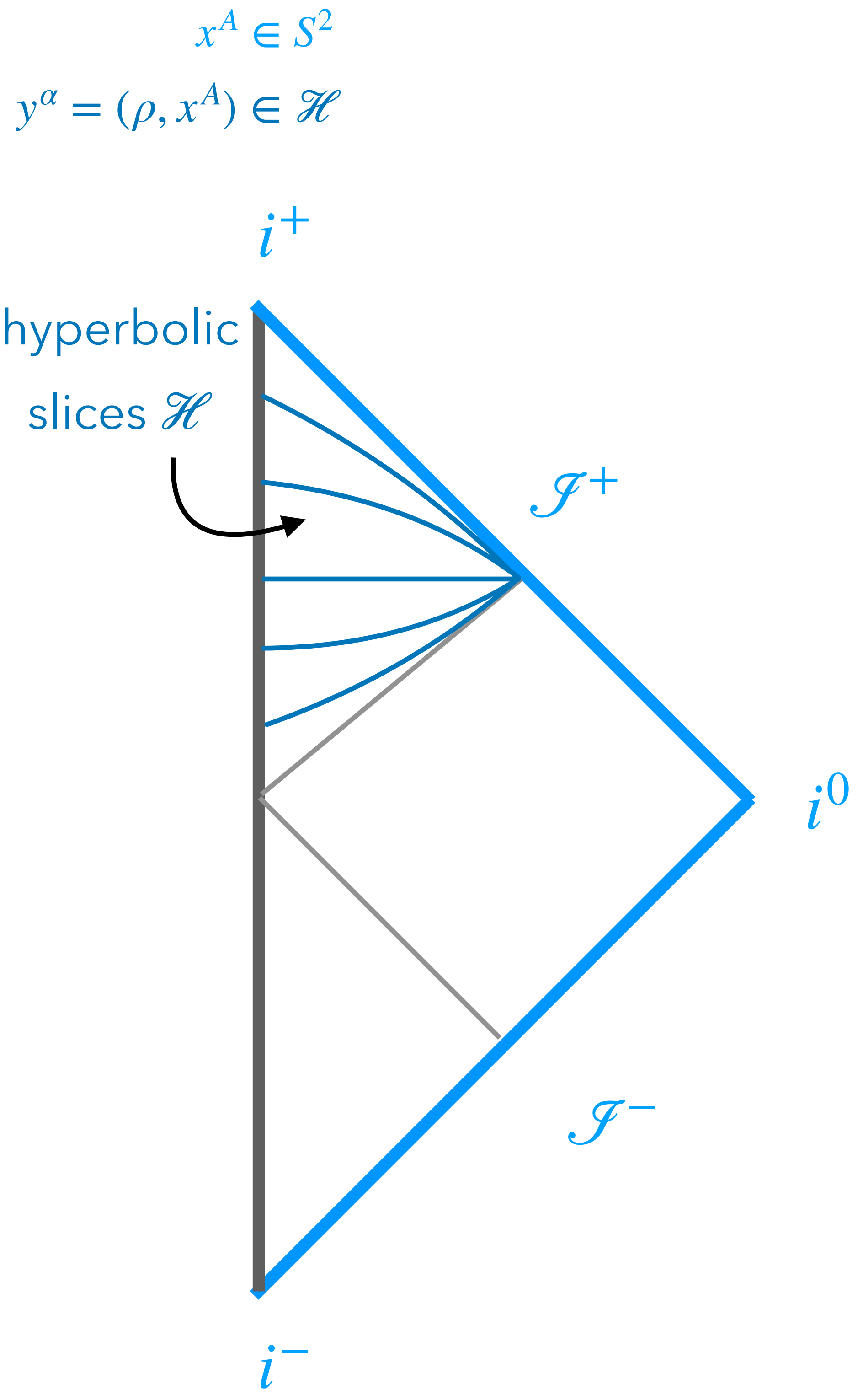
$\mathscr{I}^+$: $\boxed{Y^A(x)}$ $\qquad \delta g_{\mu\nu} = \mathscr{L}_{Y^A} g_{\mu\nu}$

vector Green's function

i^+ : $\boxed{\bar{Y}^\alpha(y)} = \int_{S^2} d^2x\, G^\alpha_A(y;x) Y^A(x)$ $\qquad \delta\varphi = \bar{Y}^\alpha \partial_\alpha \varphi$

$\Rightarrow$ Superrotation vector field extends smoothly across $i^+ \cup \mathscr{I}^+$.

Phase space @ $i^+ \cup \mathscr{I}^+$



Asymptotic phase space on $i^+ \cup \mathscr{I}^+$

$$\mathscr{I}^+ : \quad C_{AB}(u,x) \overset{u\to\infty}{=} C^{+(0)}_{AB}(x) + \boxed{\frac{1}{u} C^{+(1)}_{AB}(x)} + \dots$$

$$= \lim_{r\to\infty} \frac{1}{r} h_{AB}(u,r,x)$$

displacement memory ↑ $C^{+(0)}_{AB}(x)$

tail ↑ $\frac{1}{u} C^{+(1)}_{AB}(x)$

$$i^+ : \quad h_{\tau\tau}(\tau, y) \overset{\tau\to\infty}{=} \boxed{\frac{1}{\tau} \overset{1}{h}_{\tau\tau}(y)} + \dots$$

sourced by matter stress tensor

$$\varphi(\tau, y) = \frac{\sqrt{m}}{2(2\pi)^{3/2}} \sum_{n=0}^{\infty} \frac{e^{-im\tau}}{\tau^{\frac{3}{2}+n}} \left(\overset{\ln}{b}_n(y) \boxed{\ln \tau} + b_n(y) \right) + h.c. + \dots$$

$\overset{\ln}{b}_n$ and b_{n+1} for $n \geq 0$ fixed by $b \equiv b_0$

⇒ Tails and logs from long-range interactions.

Gravity: hard and soft charges

[Choi,Laddha,AP'24]

$$\Omega_{i^+\cup\mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$$

$$Q = Q_H + Q_S$$

hard charge

soft charge

$$\Omega_{i^\pm\cup\mathscr{I}^\pm}(\delta, \delta') = \delta' Q_\pm$$

superrotation

Gravity: hard and soft charges

[Choi,Laddha,AP'24]

diverges logarithmically as $\tau \to \infty, u \to \infty$ →

$$\Omega_{i^+\cup\mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$$

$$\downarrow \qquad \downarrow$$

$$Q = Q_H + Q_S$$

hard charge   soft charge

$$\Omega_{i^\pm\cup\mathscr{I}^\pm}(\delta, \delta') = \delta' Q_\pm$$

$$\downarrow$$

superrotation

Gravity: hard and soft charges

[Choi,Laddha,AP'24]

diverges logarithmically as $\tau \to \infty, u \to \infty$

regulate via late-time cutoff Λ^{-1}

$$\Omega_{i^+ \cup \mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$$

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Gravity: hard and soft charges

[Choi,Laddha,AP'24]

$$\begin{array}{rcccc} \Omega_{i^+\cup\mathscr{I}^+} = & \Omega_{i^+} & + & \Omega_{\mathscr{I}^+} \\ & \downarrow & & \downarrow \\ Q = & Q_H & + & Q_S \end{array}$$

hard charge soft charge

$$\Omega_{i^\pm\cup\mathscr{I}^\pm}(\underset{\downarrow}{\delta}, \delta') = \delta' Q_\pm$$

superrotation

Regularized Noether charge:

$$\boxed{Q^\Lambda = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \ldots}$$

$$Q_H^{(\ln)}[\bar{Y}] = \frac{\kappa m^3}{4(2\pi)^3} \int_{i^+} d^3y\, \bar{Y}^\alpha b^\dagger b \partial_\alpha \overset{1}{h}_{\tau\tau}$$

$$Q_S^{(\ln)}[Y] = -\frac{2}{\kappa} \int_{\mathscr{I}^+} du\, d^2x\, D_z^3 Y^z \partial_u u^2 \partial_u C^{zz} + \text{h.c.}$$

$$Q_H^{(0)}[\bar{Y}] = \frac{i m^2}{4(2\pi)^3} \int_{i^+} d^3y\, \bar{Y}^\alpha \left[(\partial_\alpha b^\dagger) b - b^\dagger \partial_\alpha b \right]$$

$$Q_S^{(0)}[Y] = -\frac{2}{\kappa} \int_{\mathscr{I}^+} du\, d^2x\, \underset{\uparrow}{D_z^3} Y^z u \partial_u C^{zz} + \text{h.c.}$$

D_A is the covariant derivative on S^2

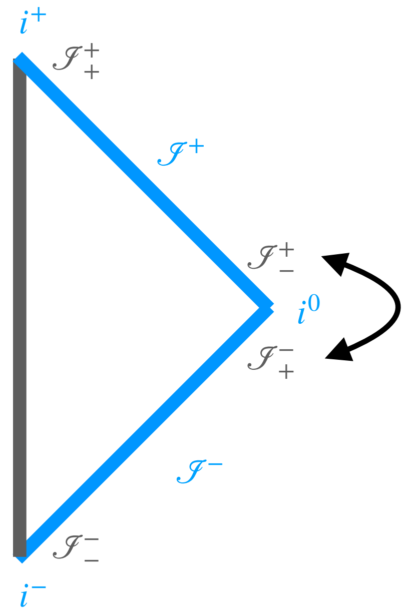
Finite charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \underbrace{\left(Q_H^{(0)} + Q_S^{(0)} \right)}_{\equiv Q^{(0)}} + \dots$$

Conservation law:

$$Q_+^{(0)} = Q_-^{(0)}$$

Upon identifying the fields and gauge parameter antipodally:



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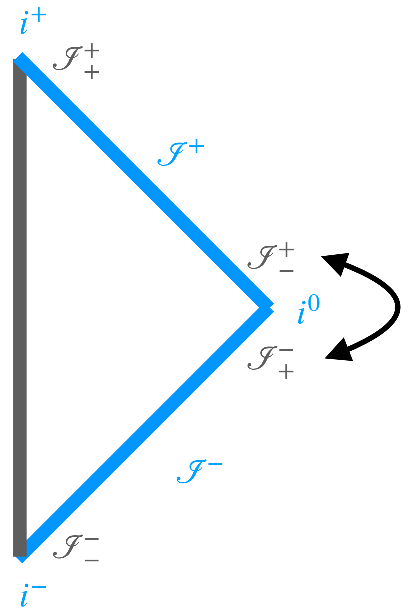
[Kapec,Lysov,Pasterski,Strominger'14]



judicious choice of
superrotation $Y^A(x)$

$$\downarrow$$

Tree-level subleading soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left(\omega^{-1} S_{-1} + \omega^0 S_0 \right) \mathscr{M}_N + \ldots$$

tree-level soft expansion

[Cachazo,Strominger'14]

Finite charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \underbrace{\left(Q_H^{(0)} + Q_S^{(0)} \right)}_{\equiv Q^{(0)}} + \dots$$

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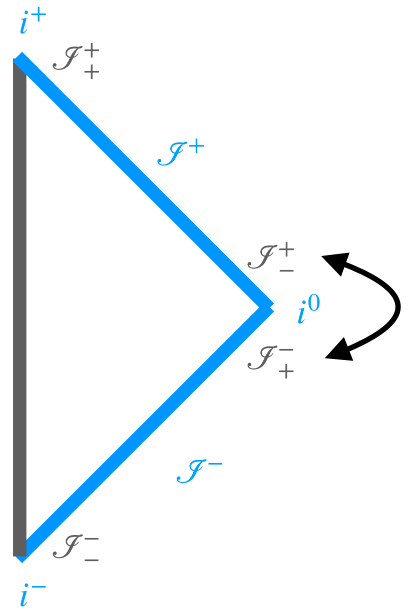

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tree-level soft expansion

[Cachazo,Strominger'14]

Recall: IR effects render subleading soft theorem at tree-level ambiguous.

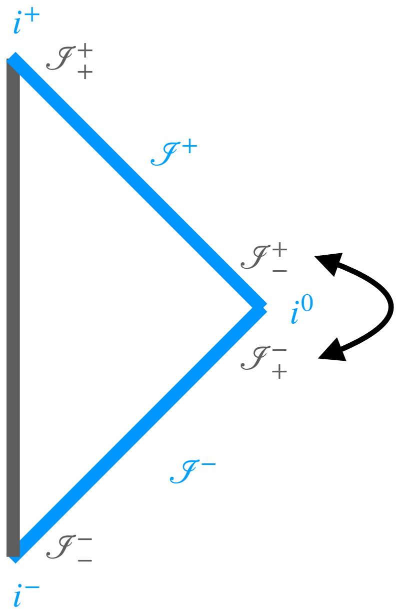
Logarithmic charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \underbrace{\left(Q_H^{(\ln)} + Q_S^{(\ln)} \right)}_{\equiv Q^{(\ln)}} + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \dots$$

Conservation law:

$$Q_+^{(\ln)} = Q_-^{(\ln)}$$

Upon identifying the fields and gauge parameter antipodally:



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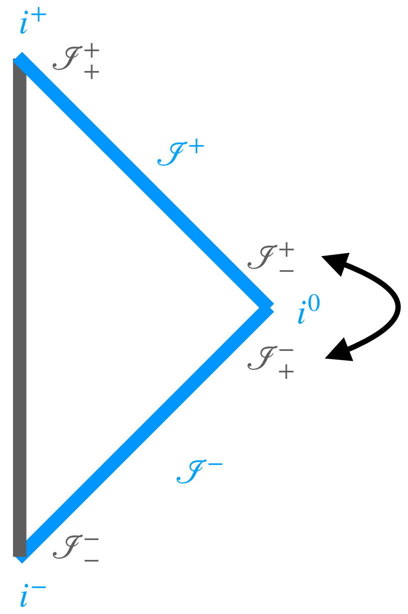


judicious choice of **superrotation** $Y^A(x)$

$$\downarrow$$

Logarithmic soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left(\omega^{-1} S_{-1} + \omega^0 \ln \omega \, S_0^{(\ln \omega)} \right) \mathscr{M}_N + \ldots$$

log soft expansion

[Sahoo,Sen'18] [Saha,Sahoo,Sen'19]

This establishes the symmetry interpretation of the classical logarithmic soft graviton theorem. [Choi,Laddha,AP'24]

[Choi,Laddha,AP - to appear]

Comment on projectors

Comparing logarithmic and subleading tree-level soft charges:

subleading tree: $$Q_S^{(0)}[Y] = -\frac{2}{\kappa}\int_{\mathscr{I}^+} du\, d^2x\, D_z^3 Y^z\, u\partial_u C^{zz} + \text{h.c.}$$

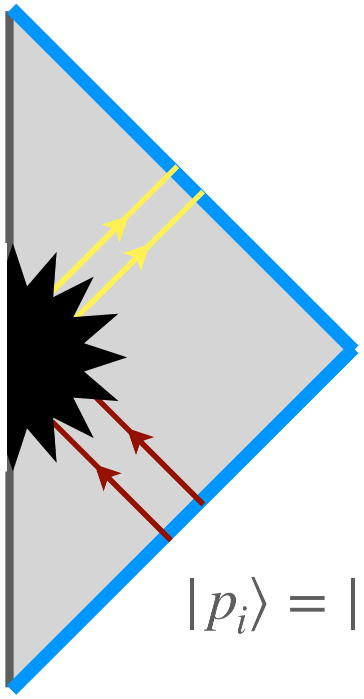
log: $$Q_S^{(\ln)}[Y] = -\frac{2}{\kappa}\int_{\mathscr{I}^+} du\, d^2x\, D_z^3 Y^z\, \partial_u u^2 \partial_u C^{zz} + \text{h.c.}$$

Different projectors acting on soft insertion!

In the Ward identity $\langle out \,|\, Q_S^+ S - S Q_S^- \,|\, in \rangle = -\langle out \,|\, Q_H^+ S - S Q_H^- \,|\, in \rangle$ the projectors in Q_S pick out the tree-level $O(\omega^0)$ vs IR corrected $O(\ln \omega)$ soft terms, while the the action of Q_H on the hard particles gives the soft factors.

Soft tower

S-matrix



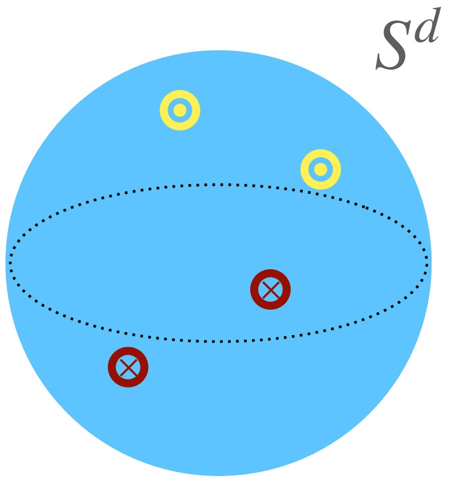
$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis

$$\langle p_n^{\text{out}} \ldots | \mathcal{S} | \ldots p_1^{\text{in}} \rangle$$

translation symmetry

$$\xrightarrow[\int_0^\infty d\omega \omega^{\Delta-1}]{\mathcal{M}\text{ellin}}$$



$|\Delta_i, x_i\rangle$

boost-weight basis

$$\langle \mathcal{O}^-_{\Delta_1}(x_1) \ldots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$$

Lorentz symmetry

energetically soft expansion:		``conformally soft” poles:	
$\frac{1}{\omega}, 1, \omega, ..$	**tree-level**	$\Delta = 1, 0, -1, \ldots$	simple poles
$\log \omega, \ldots$	**loops**	$\Delta = 0, \ldots$	higher poles

Same as tree-level subleading symmetry operators!

Gravity is a drag

Extra term in soft graviton factor from gravitational drag on the soft graviton due to the other finite energy particles:

$$\Delta_{\rm drag} S^{(\ln \omega)}_{0,\rm classical} = \frac{i}{4\pi}(\log \omega^{-1} + \log r^{-1}) \sum_{j,\eta_j=-1} (q \cdot p_j)\, S_{-1}$$

[Saha,Sahoo,Sen'19]

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[Saha,Sahoo,Sen'19]

The resulting time delay to reach a detector at distance r can be captured by defining the retarded time at the detector:

$$u = t - r + \log r \times f_{\text{matter}}$$

$$f_{\text{matter}} = 2G \sum_{j=1}^{N} p_j \cdot n$$

$$n^{\mu} = (1, \frac{\vec{x}}{r})$$

↑

effect of the long range gravitational force on the gravitational wave as it travels from the scattering center to the detector

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In our covariant phase space approach the drag is captured by an infrared divergence in the radiative symplectic structure via (⋆) which cannot be regularized away since it depends on the matter content at i^+.

[Choi,Laddha,AP - to appear]

Log symmetry for scalar QED

Gauge field $\mathscr{A}_\mu$ + minimally coupled complex massive scalar field ϕ

$$\delta \mathscr{A}_\mu = \partial_\mu \epsilon^{\mathscr{I}^+} \qquad \delta\phi = ie\epsilon^{i^+}\phi$$

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divergent
superphaserotation:

$$r\epsilon(x) + \frac{u}{2}(2 + \triangle)\epsilon(x) + \dots \qquad \tau\bar{\epsilon}(y) + \dots$$

$$\bar{\epsilon}(y) = \int_{S^2} d^2\hat{x}\, G(y;x)\epsilon(x)$$

$$\tau\rho = r$$

$$r\epsilon(x) + \dots$$

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$$\tau\rho = r$$

$$r\epsilon(x) + \ldots$$

↓

linear and log divergence as $\tau \to \infty$, $u \to \infty$ →

$$\Omega_{i^+ \cup \mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$$

↓

subtract corner term & regulate via late-time cutoff Λ^{-1}

$$Q^\Lambda = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \ldots$$

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$$Q_+^{(\ln)} = Q_-^{(\ln)} \qquad\qquad Q_+^{(0)} = Q_-^{(0)}$$

[Choi,Laddha,AP'24]

matches logarithmic soft photon theorem
[Sahoo,Sen'18] [Saha,Sahoo,Sen'19]

matches subleading tree-level soft photon theorem
[Low'58]

Log Soft → Symmetry !

Noether charge from long-range IR effects:

$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \ldots$$

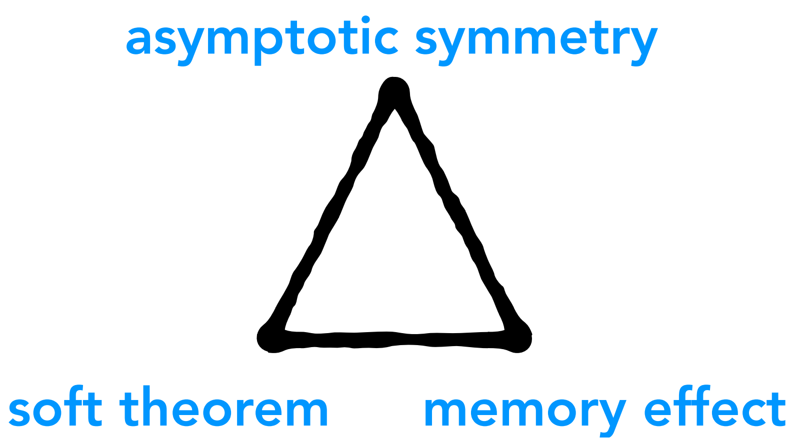
physical IR scale

Our covariant phase space approach achieves:

- first principles derivation of the conserved charge Q
- clear split between tree-level and logarithmic charges
- regulator Λ^{-1} arises from the relevant infrared scale: large $|\tau|$ & $|u|$
- Λ can be removed in the end since $Q_{\pm}^{(\ln)}$ are finite.

Symmetry interpretation for classical log soft theorem. ✓

Summary



- Asymptotic / soft symmetries encode universal large-distance /low-energy physics.

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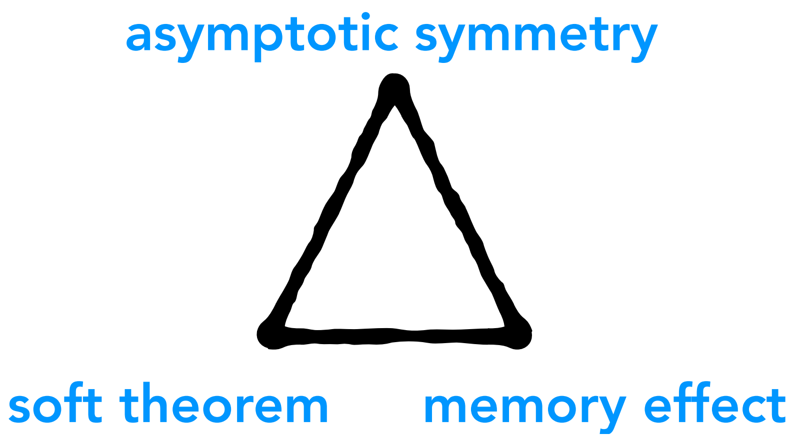

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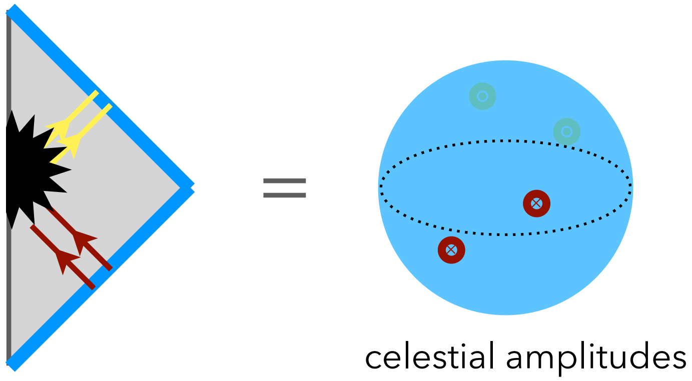

- Celestial boost basis makes some aspects of symmetries manifest.

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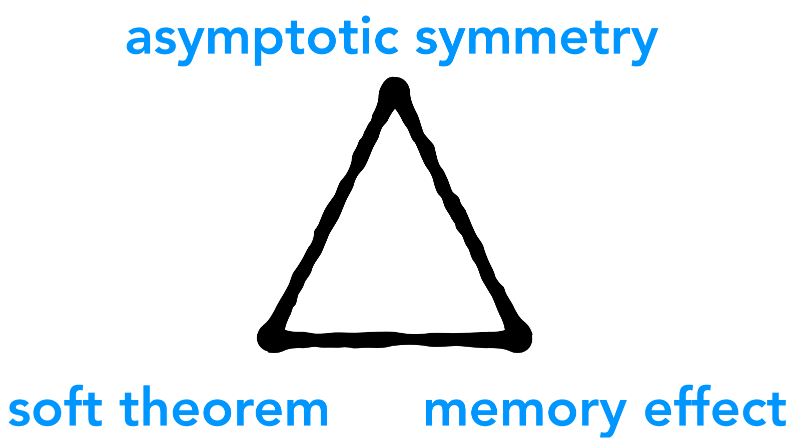

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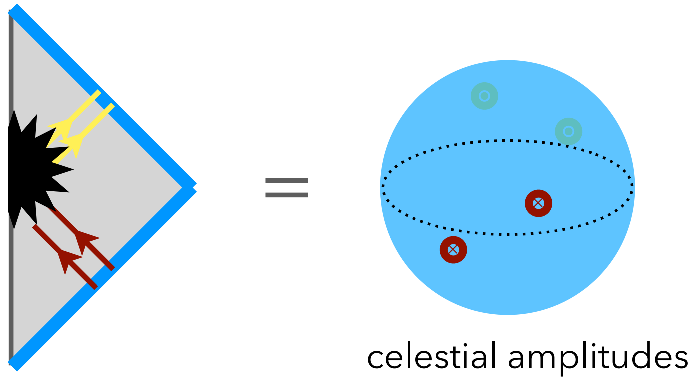

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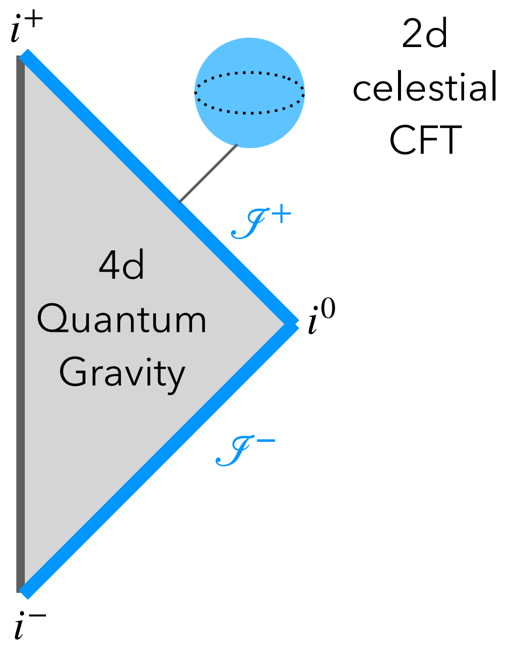

- Identifying the symmetries is key for a holographic principle in asymptotically flat space. We have established the celestial conformal symmetry in the presence of long-range IR effects.

Outlook: ∞ log towers

We expect the methods we developed to be applicable to the infinite tower of subleading logarithmic soft theorems.

$$\frac{\mathcal{M}_{N+1}(p_1,\ldots,p_N;(\omega,q,\ell))}{\mathcal{M}_N(p_1,\ldots,p_N)} = \sum_{n=-1}^{\infty} \omega^n(\ln\omega)^{n+1}\,S_n^{(\ln\omega)}(\{p_i\},(q,\ell)) + \ldots$$

How many universal all-loop exact log symmetries ?

Thank you!