Large N gauge theories and their confining strings
– what the lattice tells us.

Michael Teper (Oxford) - Strings 2024

- $SU(\infty)$ is close to $SU(3)$

- Confining flux tube – world sheet action

Is $SU(3)$ close to $SU(\infty)$?

calculate dimensionless ratios of physical quantities for various $SU(N \geq 2)$

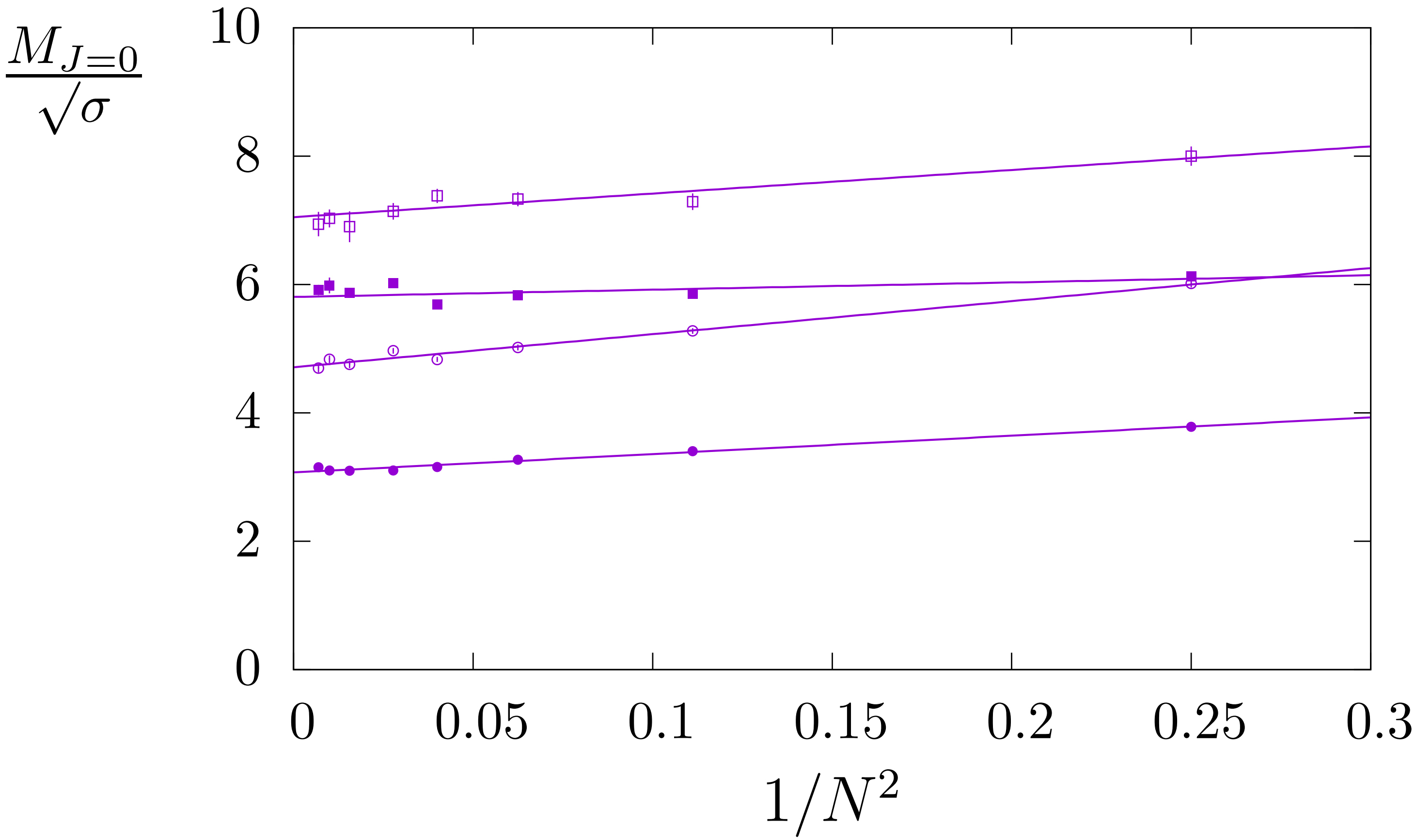
extrapolate to $SU(\infty)$ using the leading $O(1/N^2)$ correction:

$$\left.\frac{M_G}{\mu}\right|_N = \left.\frac{M_G}{\mu}\right|_\infty + \frac{c}{N^2}$$

e.g. M_G a glueball mass, and $\mu = \sqrt{\sigma}$ square root string tension

calculations from 2106.00364

some $N \to \infty$ extrapolations: scalar glueballs for $2 \leq N \leq 12$



$J^{PC} = 0^{++}$ ground (●) and first excited (■); 0^{-+} ground (○) and first excited (□) in units of the string tension. With extrapolations to $N = \infty$ from $N = 2 - 12$.

the above $M/\sqrt{\sigma}$ values are those of the continuum theories, i.e after taking the lattice spacing $a \to 0$ in physical units
e.g. $a\sqrt{\sigma} \to 0$

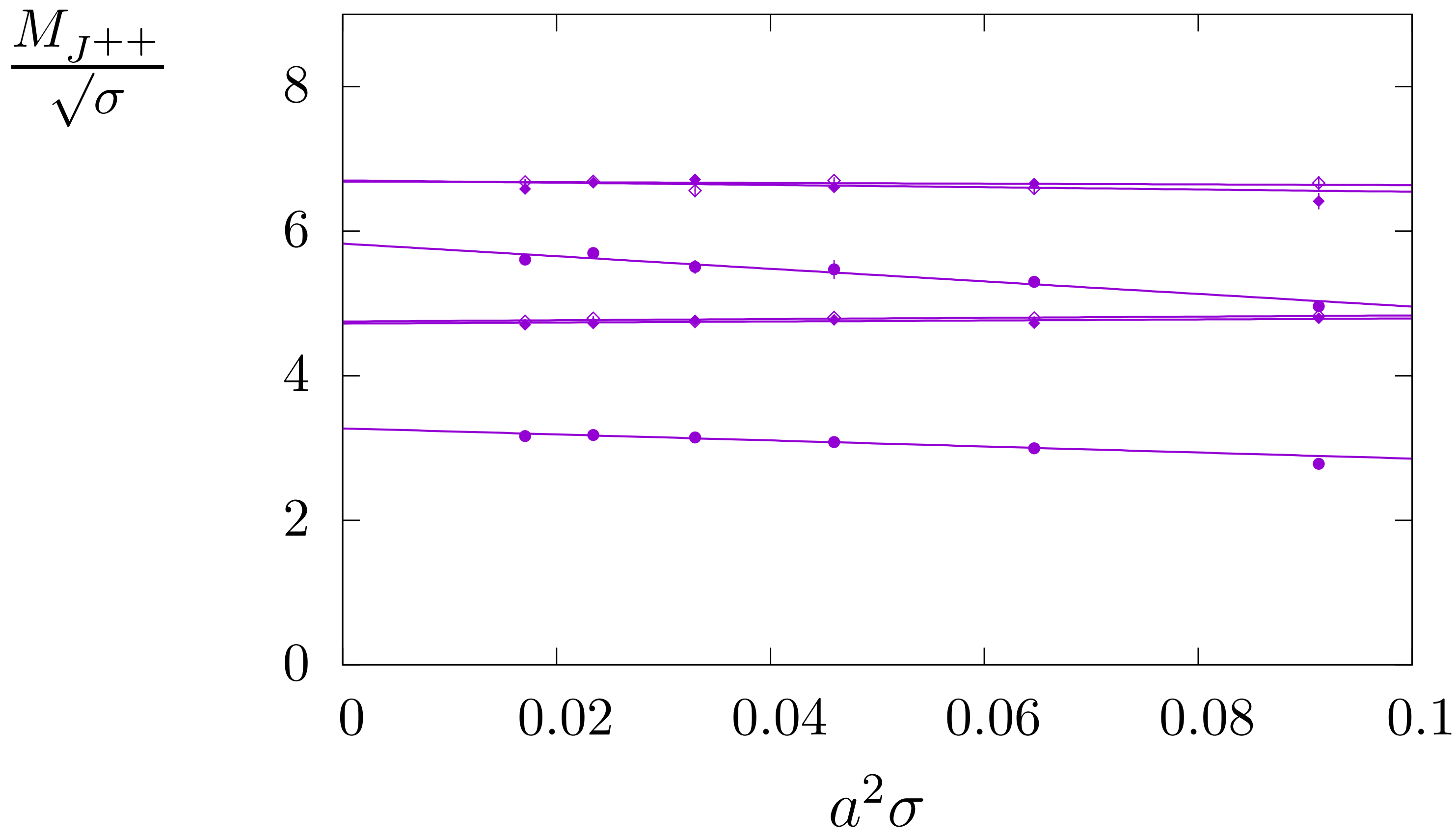
extrapolate to $a = 0$ using the leading $O(a^2)$ correction:

$$\frac{aM_G(a)}{a\sqrt{\sigma(a)}} = \frac{M_G(a)}{\sqrt{\sigma(a)}} = \frac{M_G(0)}{\sqrt{\sigma(0)}} + ca^2\sigma$$

where we vary the lattice spacing a by varying the value of g^2 in the lattice action, and calculate masses in lattice units from correlators:

$$\langle \phi^\dagger(t = an_t)\phi(0) \rangle = \sum_n |\langle vac|\phi|n\rangle|^2 \exp\{-aE_n n_t\}$$

e.g. SU(4): some continuum extrapolations



$A_1^{++} \to 0^{++}$ ($\bullet$), $E^{++} \to 2^{++}$ ($\blacklozenge$) and $T_2^{++} \to 2^{++}$ ($\lozenge$).

NOTE: doublet E^{++} + triplet $T_2^{++} \longrightarrow$ five components of $J^{PC} = 2^{++}$ glueball lattice rotation irreps A_1, A_2, E, T_1, T_2 dimensions $1, 1, 2, 3, 3$

$\longrightarrow$ continuum rotation irreps J dimension $2J+1$

continuum $J \sim$ cubic R		
J		cubic R
0	$\sim$	A_1
1	$\sim$	T_1
2	$\sim$	$E + T_2$
3	$\sim$	$A_2 + T_1 + T_2$
4	$\sim$	$A_1 + E + T_1 + T_2$
5	$\sim$	$E + 2T_1 + T_2$
6	$\sim$	$A_1 + A_2 + E + T_1 + 2T_2$
7	$\sim$	$A_2 + E + 2T_1 + 2T_2$
8	$\sim$	$A_1 + 2E + 2T_1 + 2T_2$

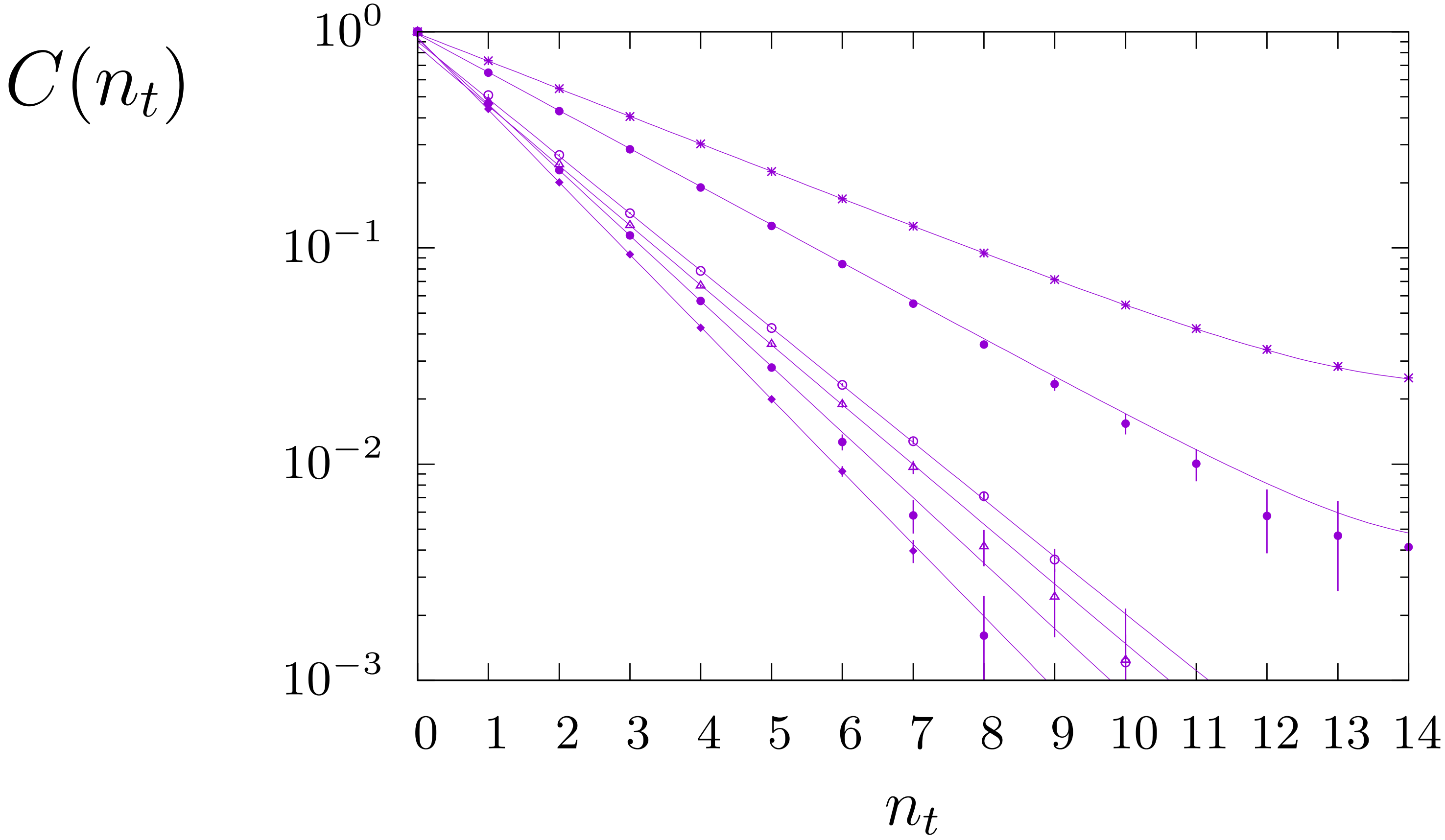
e.g. identifying some $J^{PC} = 0^{++}, 2^{++}$ states in $SU(3)$

continuum masses in units of string tension					
state	A_1^{++}	A_2^{++}	E^{++}	T_1^{++}	T_2^{++}
gs :	3.405(21)	7.705(85)	4.904(20)	7.698(80)	4.884(19)
ex1:	5.855(41)	8.81(20)	6.728(47)	7.72(11)	6.814(31)

A_1^{++} gs and $ex1$ no near-matching E, T_1, T_2 states $\Rightarrow J^{PC} = 0^{++}$

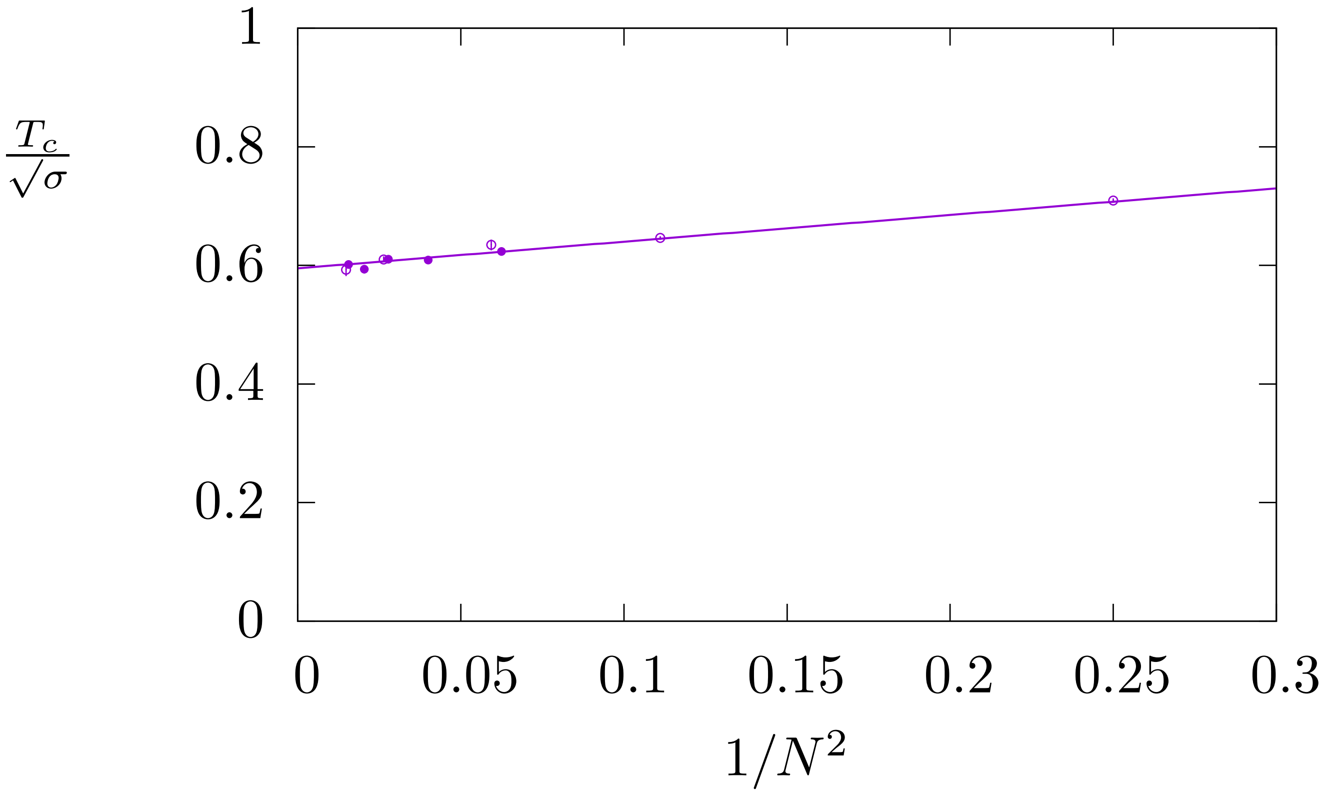
same for $E,^{++} T_2^{++}$ gs and $ex1 \Rightarrow J^{PC} = 2^{++}$

SU(8) , $20^3 30$, $a\sqrt{\sigma} = 0.1325$



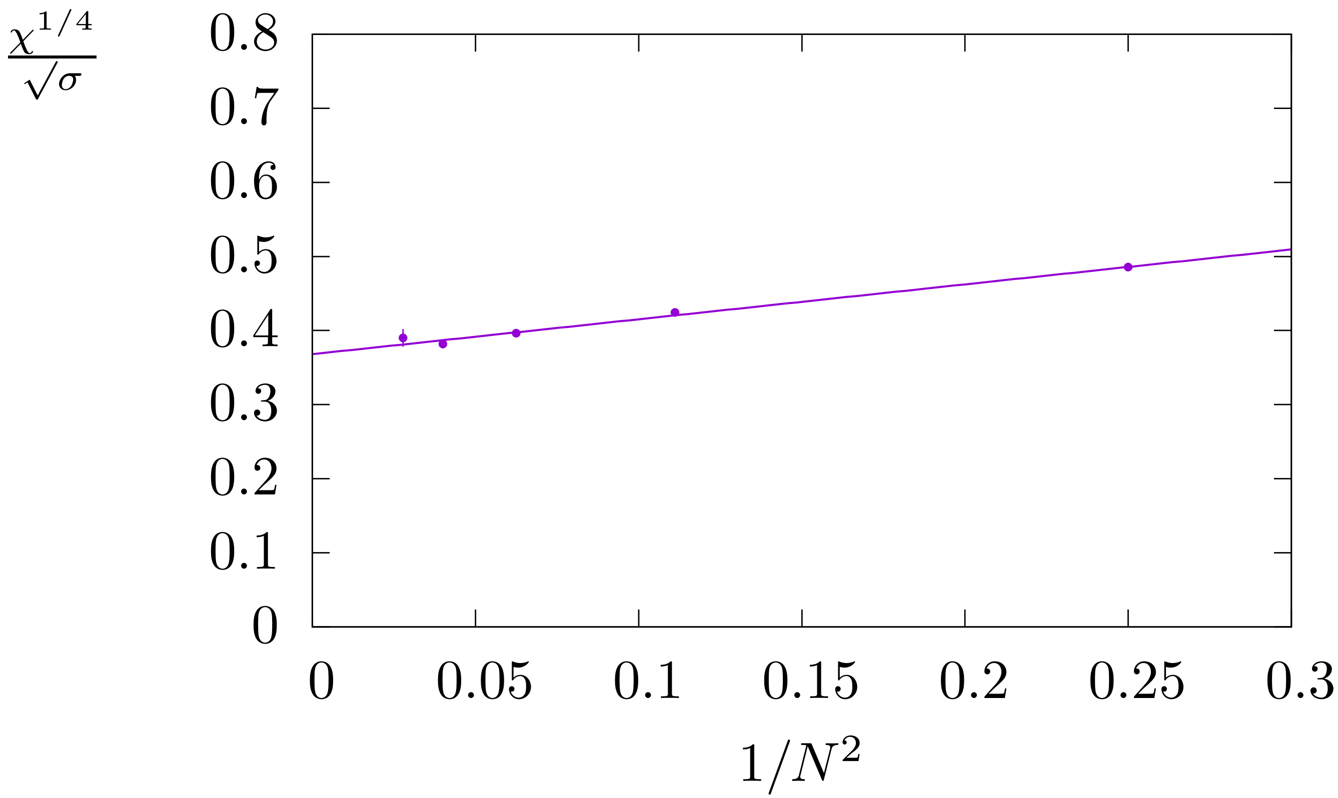
l_f $*$; $J^{PC} = 0^{++}(A_1^{++})$ ● ; $2^{++}(E^{++})$ ○ ; $0^{-+}(A_1^{-+})$ △ ; $1^{+-}(T_1^{+-})$ ♦

extrapolation of continuum $T_c/\sqrt{\sigma}$ to $N = \infty$:



$SU(2)$ 2nd order; $SU(3)$ weakly 1st order; $SU(N \geq 4)$ 1st order

data from 1202.6684, $\bullet$, and hep-lat/0502003, $\circ$ (slight shift for clarity) continuum topological susceptibility $\chi_L = \langle Q_L^2 \rangle$/volume vs N



$\chi|_{su3} \simeq (206(4)\text{`MeV'})^4 \quad ; \quad \chi|_{su\infty} \simeq (179(4)\text{`MeV'})^4$

So, is $SU(3)$ is 'close' to $SU(\infty)$?

lattice answer: Yes.

And if we add light quarks? Probably so, at least in quenched calculations, but need to do better. 1304.4437

but full QCD_N would be very useful for phenomenology and theory

Winding flux tube spectrum from string action

Spectrum of a flux tube wrapped around the x-torus, length l, propagating in (Euclidean) time $\Longrightarrow$ effective string action.
Light cone quantisation of bosonic string $\Longrightarrow$ Nambu-Goto/GGRT spectrum:

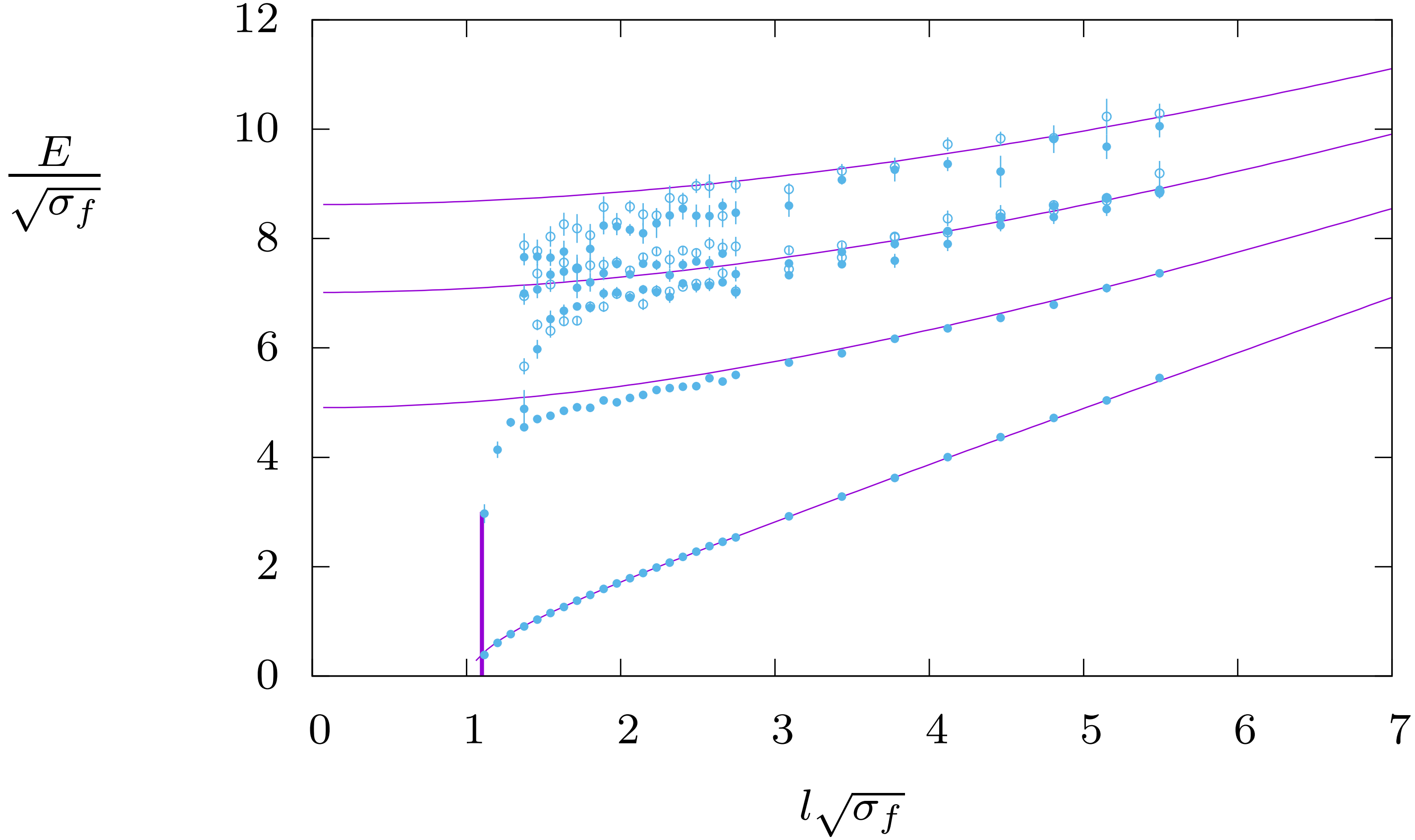
$$E_n(l) = \sigma l \left(1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{(D-2)}{24}\right)\right)^{\frac{1}{2}}$$

where $n = (N_L + N_R)/2 = N_R = N_L$.

Outside $D = 26$ (and $D = 3$) this LC quantisation leads to anomalous rotation commutators – at 'short' distances – so not whole story

Note also that the ground state becomes tachyonic for $\sigma l^2 \leq \pi(D-2)/3$

winding flux tube spectrum: $SU(6)$ in $D = 2 + 1$ 1602.07634



at $\beta = 171$ – corresponding to $a\sqrt{\sigma_f} \simeq 0.086$. Solid lines are NG(GGRT) string spectrum.

Universal part of spectrum

A flux tube wrapped around the x-torus (length l) propagating around the (Euclidean) time torus length τ) sweeps out a simple 2-torus surface if we are in the large-N limit where handles and higher genus surfaces are suppressed. Let $S_{eff}[S]$ be the world-sheet effective action, and $Z_{torus}(l,\tau)$ the path integral. Then

$$Z_{torus}(l,\tau) = \int_{T^2=l\times\tau} dS e^{-S_{eff}[S]} = \sum_{n,p} e^{-E_n(p,l)\tau}$$

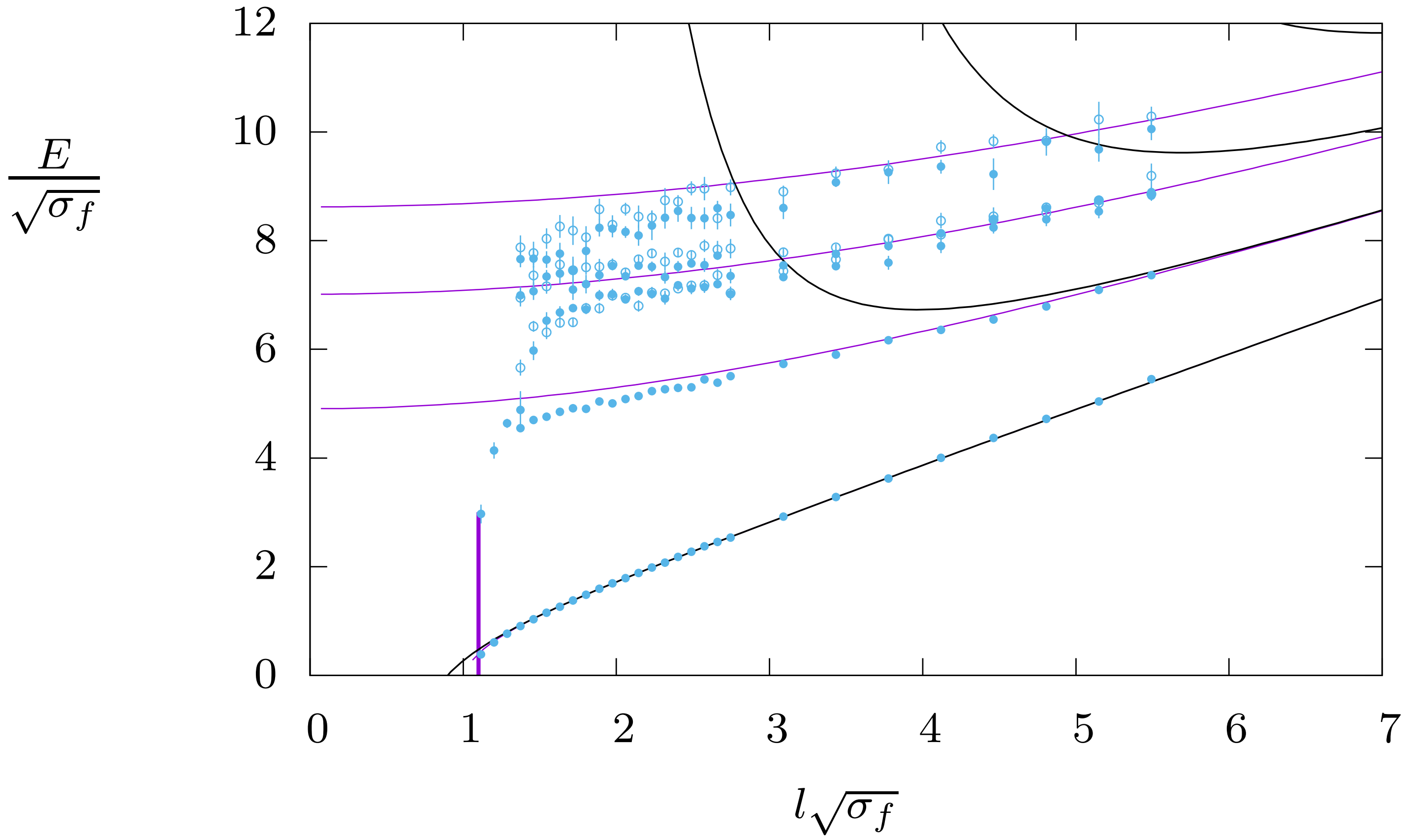
with $E_n(p,l)$ the energy of the n'th flux tube state of length l, momentum p.

The bulk symmetries constrain the $E_n(p,l)$ and hence the action $S_{eff}[S]$. This leads to some universal terms:

$$\frac{E_n(l)}{\sqrt{\sigma}} \stackrel{l\to\infty}{=} l\sqrt{\sigma} + \frac{c_1^{NG}}{l\sqrt{\sigma}} + \frac{c_2^{NG}}{(l\sqrt{\sigma})^3} + \frac{c_3^{NG}}{(l\sqrt{\sigma})^5} + O\left(\frac{1}{l^7}\right)$$

where c_i^{NG} are identical to those that arise in the expansion of E in powers of $1/l$ in the Nambu-Goto spectrum above.

Luscher,Weisz hep-th/0406205; Aharony,Komargodski 1302.6257; Dubovsky,Flauger,Gorbenko 1404.0037 black lines are contribution of universal terms to spectrum



at $\beta = 171$ – corresponding to $a\sqrt{\sigma_f} \simeq 0.086$. Purple lines are NG spectrum.

Why?

expansion parameter is

$$\frac{8\pi}{\sigma l^2}\left(n - \frac{(D-2)}{24}\right)$$

and for $n > 0$ oscillating terms $\Longrightarrow$ universal terms 'blow up' for

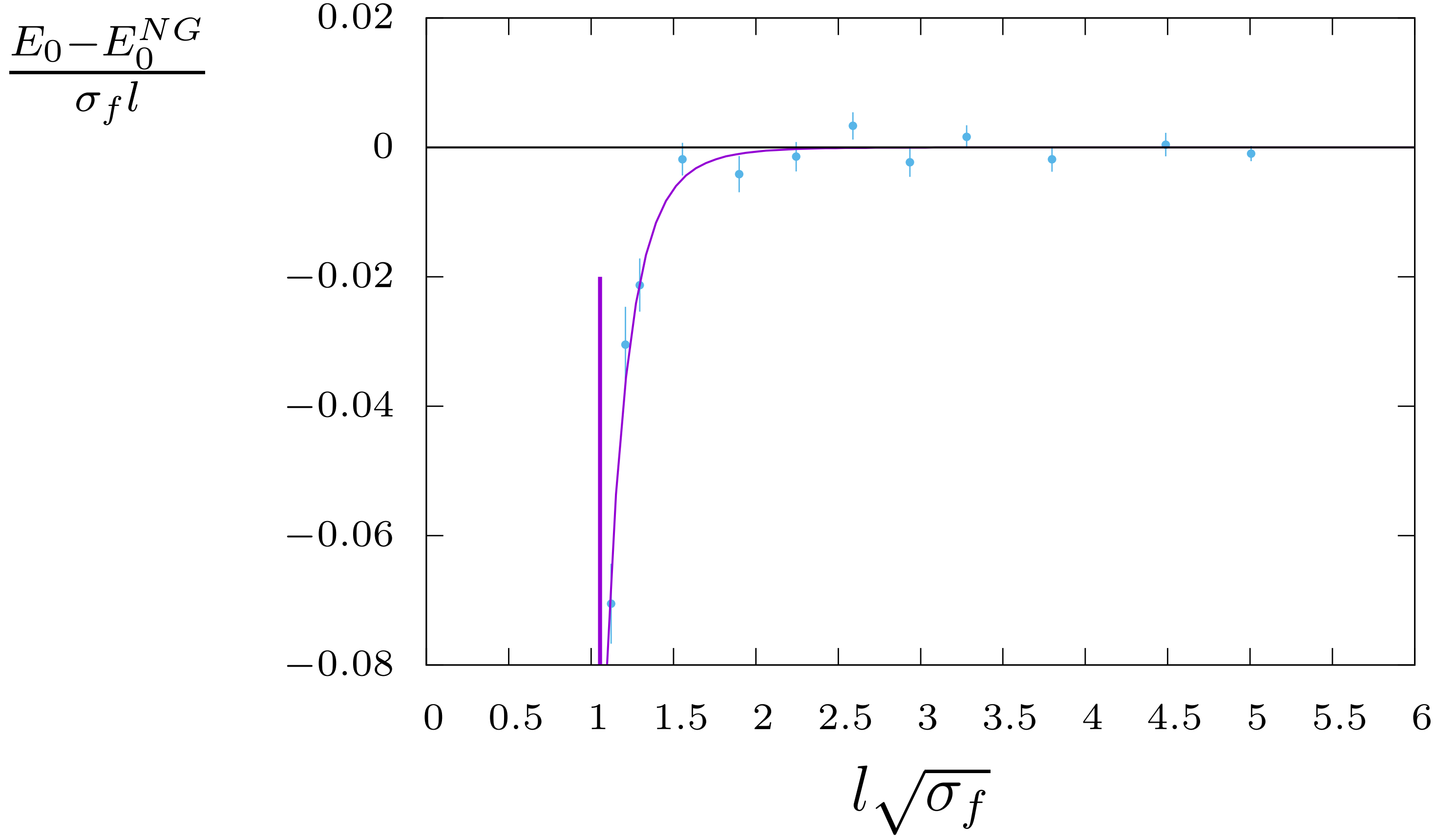
$$l\sqrt{\sigma} \lesssim \sqrt{(}8\pi(n - (D-2)/24) \overset{n>0}{\gg} 1$$

where $E_n(l)$ becomes large and we lose our lattice calculations

$\Longrightarrow$

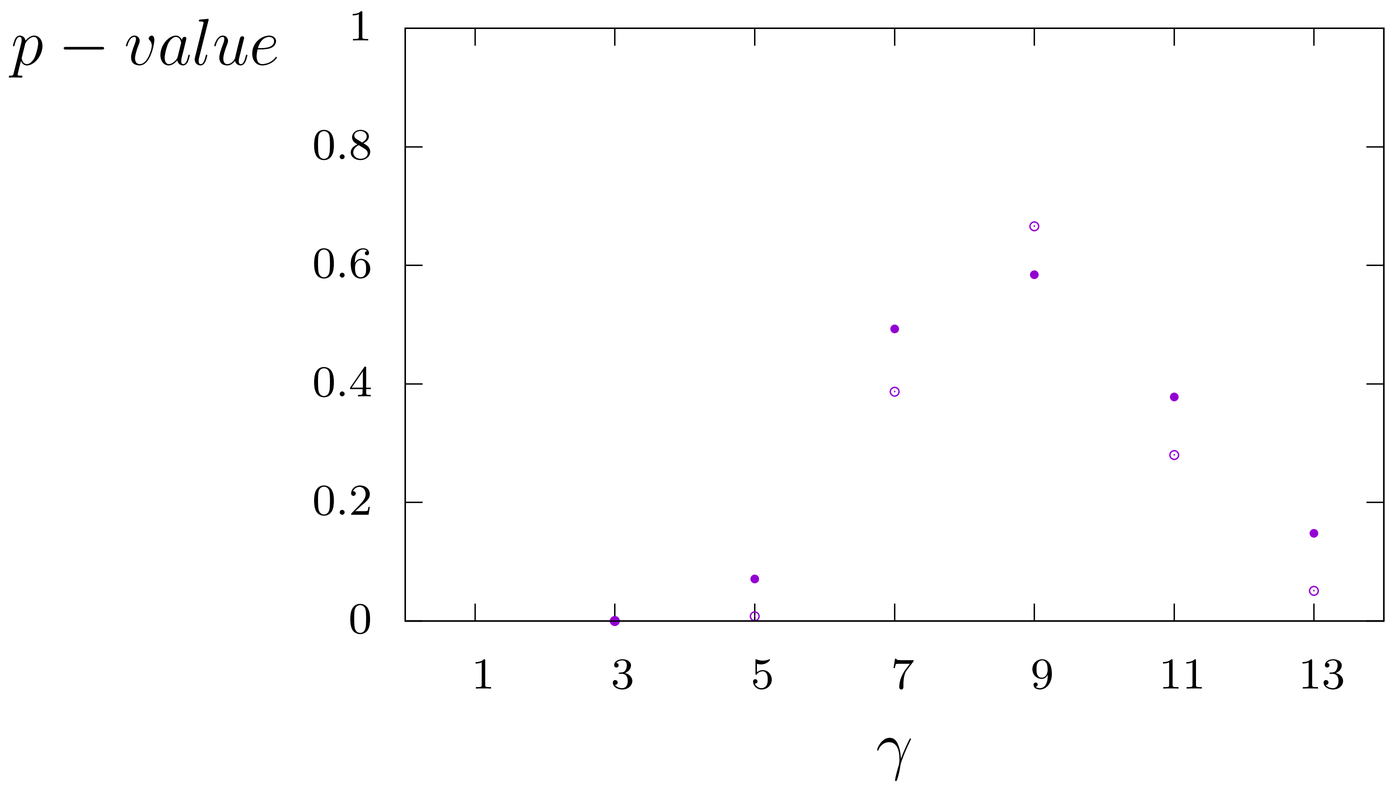
carry out test of universality fot $n = 0$, i.e. the ground state

Is the leading non-universal correction $O1/l^7)$ in $SU(4)$, $D = 2 + 1$? : 1602.07634



SU(4) $k = 1$ ground state energy: Nambu-Goto plus a $O(1/l^7)$. Vertical line ∼ deconfining transition.

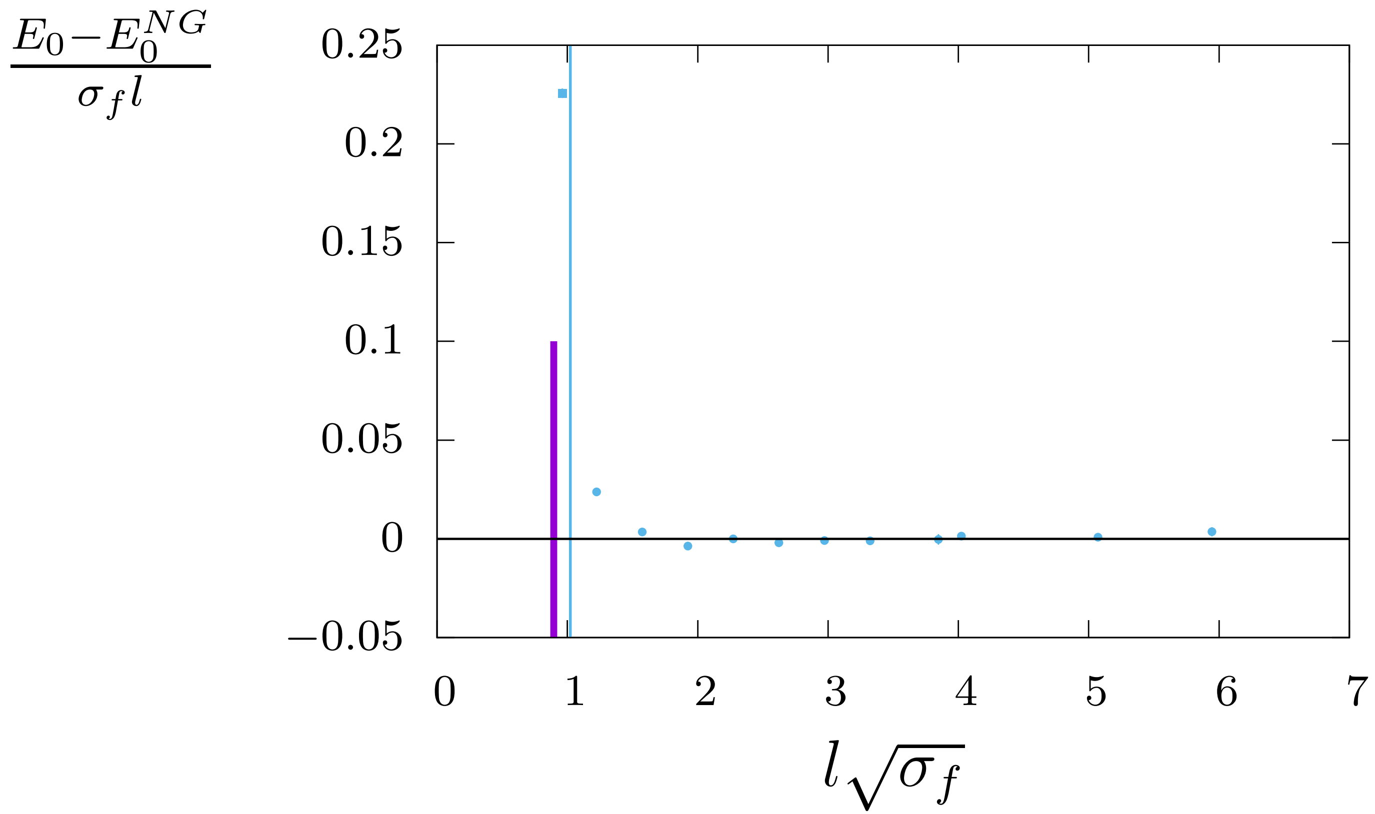
Flux tube ground state energy $= E_0^{NG}(l) + O(1/l^{\gamma}$: $SU(4)$ 1602.07634



Best fits to SU(4) $k = 1$ ground state energy using Nambu-Goto with a $O(1/l^{\gamma})$ correction: p-value for all $l \in [13, 60]$, $\bullet$, and for $l \in [13, 18]$, $\circ$, versus γ.

$SU(2), D = 2+1$: NG tachyonic transition not shielded by deconfinement

1602.07634



at $\beta = 16$ – left of light vertical line E_0^{NG} is tachyonic (for ■ we have set it to zero). Thick vertical line locates the deconfining transition.

why does the GGRT spectrum work so well – and what are corrections to it?
1404.0037

$$E_{\rm GGRT}(N_L, N_R) = \sqrt{\frac{4\pi^2(N_L - N_R)^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N_L + N_R - \frac{D-2}{12}\right)}\,,$$

no particle production, and $2 \to 2$ phonon phase shift: $2\delta(s) = sl_s^2/4$
$\Longrightarrow$ momenta and pseudoenergies (TBA):

$$p_{li}R + \sum_j 2\delta_{a_i a_j}\left(p_{li}, p_{rj}\right) - i\sum_b \int_0^\infty \frac{dq}{2\pi}\frac{d2\delta_{a_i b}\left(ip_{li}, q\right)}{dq}\ln\left(1 - e^{-R\epsilon_r^b(q)}\right) = 2\pi N_{li}\,,$$

$$\epsilon_l^a(q) = q + \frac{i}{R}\sum_i 2\delta_{ab_i}\left(q, -ip_{ri}\right) + \frac{1}{2\pi R}\sum_b \int_0^\infty dq'\,\frac{d2\delta_{ab}\left(q, q'\right)}{dq'}\ln\left(1 - e^{-R\epsilon_r^b(q')}\right)\,,$$

$\Longrightarrow$ state energy

$$\Delta E = \sum_i p_{li} + \sum_i p_{ri} + \frac{1}{2\pi}\sum_a \int_0^\infty dq\,\ln\left(1 - e^{-R\epsilon_l^a(q)}\right) + \frac{1}{2\pi}\sum_a \int_0^\infty dq\,\ln\left(1 - e^{-R\epsilon_r^a(q)}\right)\,.$$

$SU(6)$, $k=1$, parity=+ flux tube

spectrum: $\Delta E(R) = E(R) - \sigma R, \ \sigma = 1/l_s^2$

$\Longrightarrow$

lines GGRT; deviations at low R; some degeneracy breaking

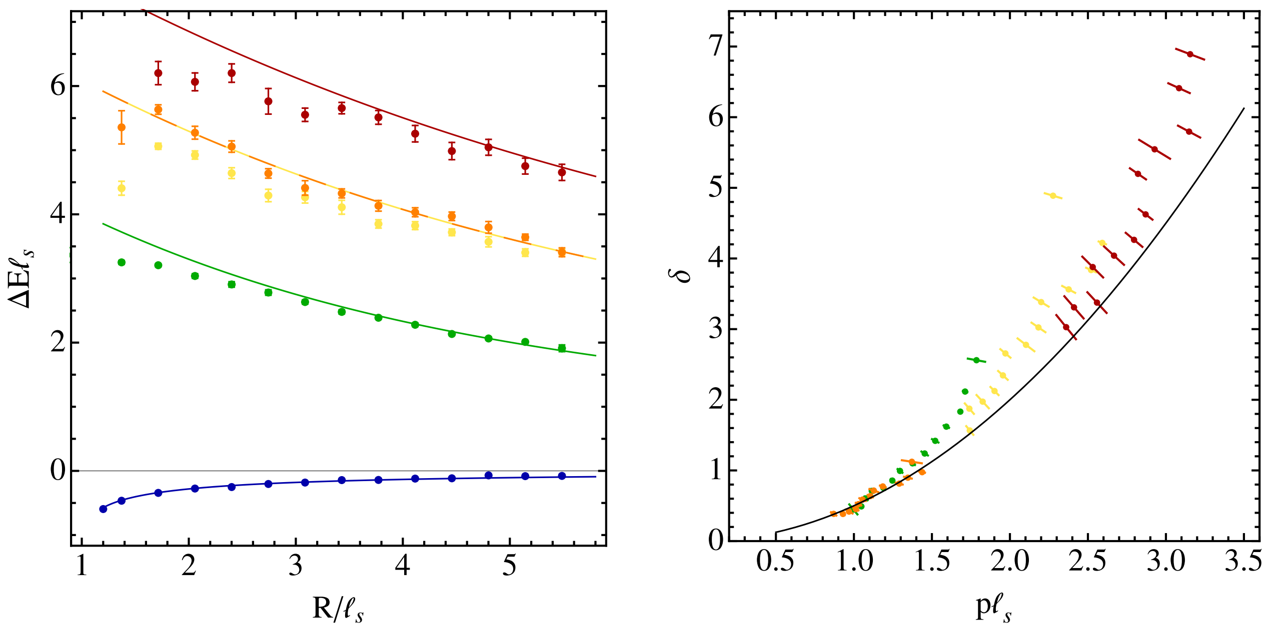
energies $\xrightarrow{TBA}$ phonon-phonon phase shifts δ

close to $2\delta_{GGRT} = l_s^2 s/4$ but some deviation

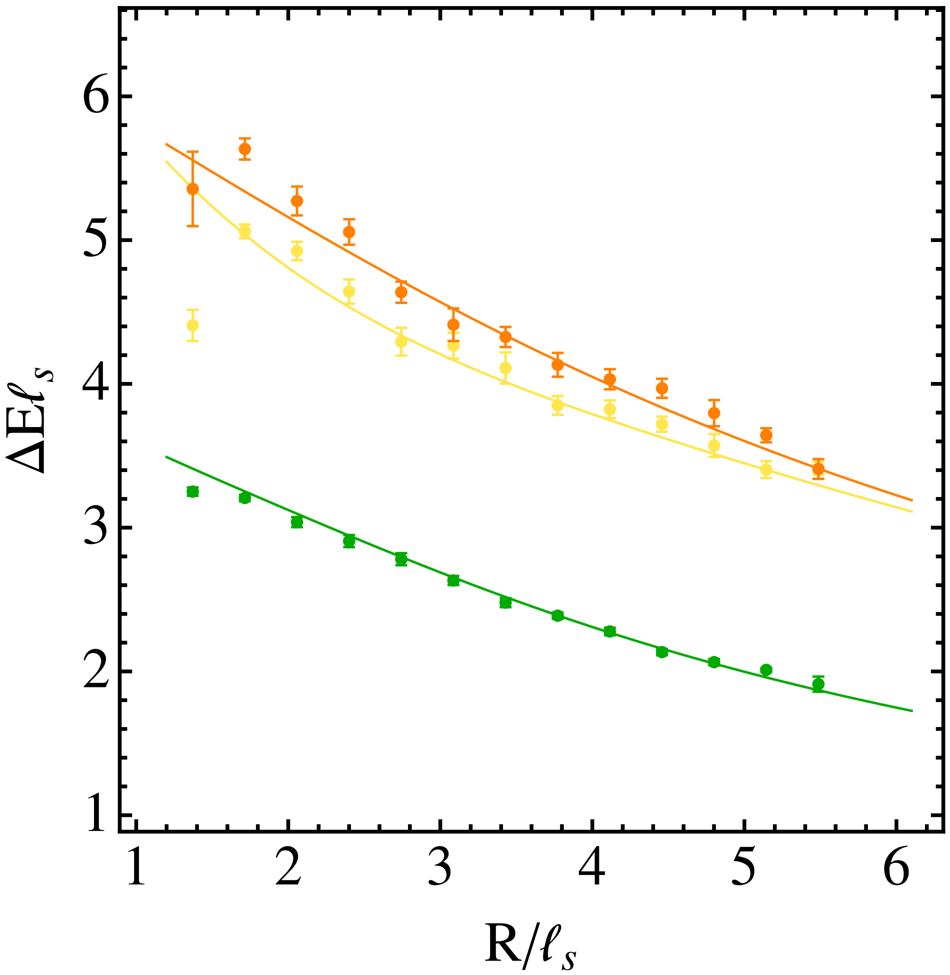
use modified phase shift $\delta = \delta_{GGRT} + \gamma_3 l_s^6 s^3$ in TBA with lattice

$E_i \longrightarrow \gamma_3 \simeq 0.7(1)/(2\pi)^2$

1404.0037

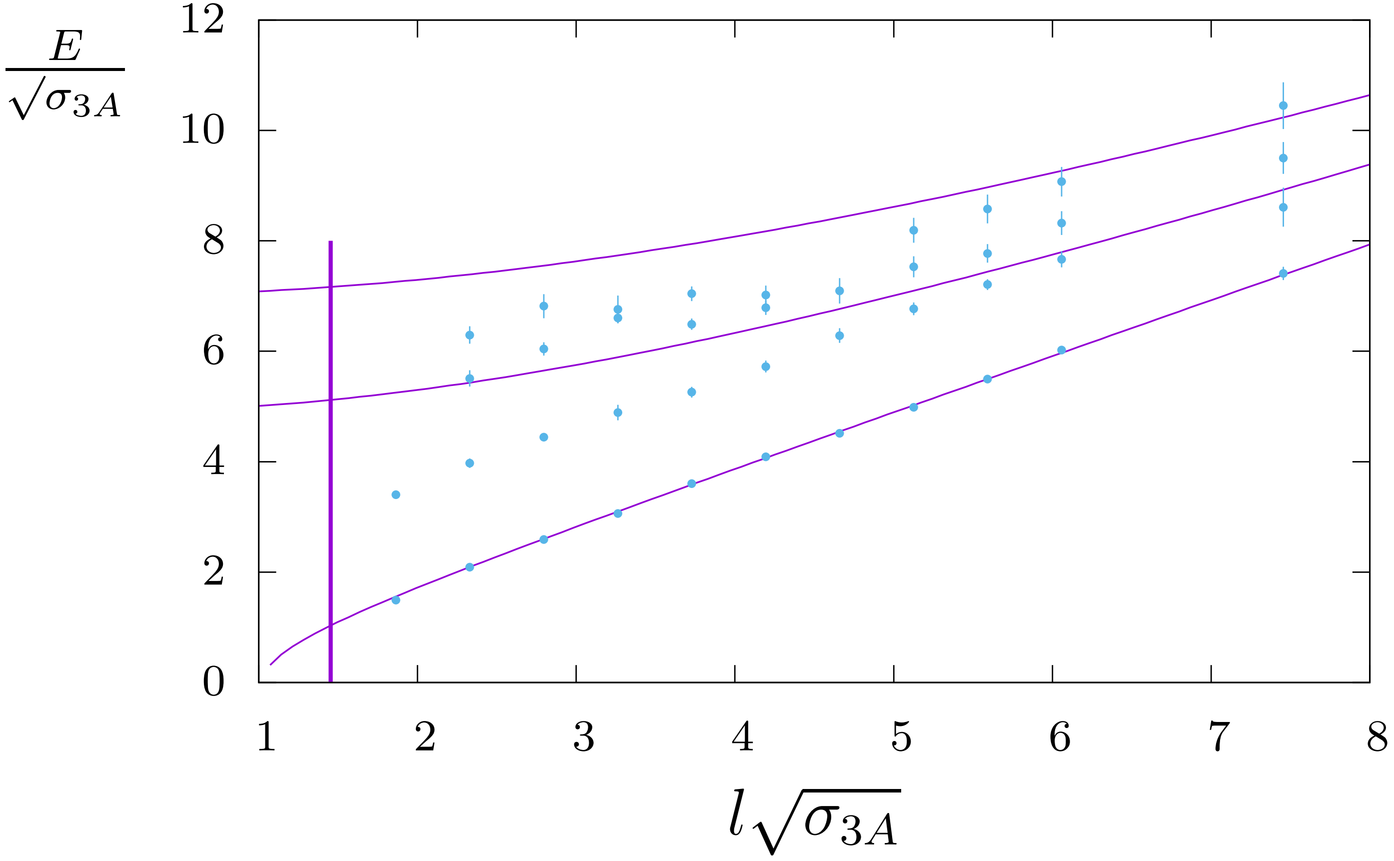


ΔEℓ_s
R/ℓ_s
δ
pℓ_s



ΔEℓ_s
R/ℓ_s
1
2
3
4
5
6
1
2
3
4
5
6

Something different: $k = 3A$ flux tube in $SU(6)$: massive mode? 1602.07634



a bulk or a world-sheet excitation?

apply same technology to $SU(6)$, $k = 3A$, parity=+ flux tube spectrum, where we saw evidence of a massive mode

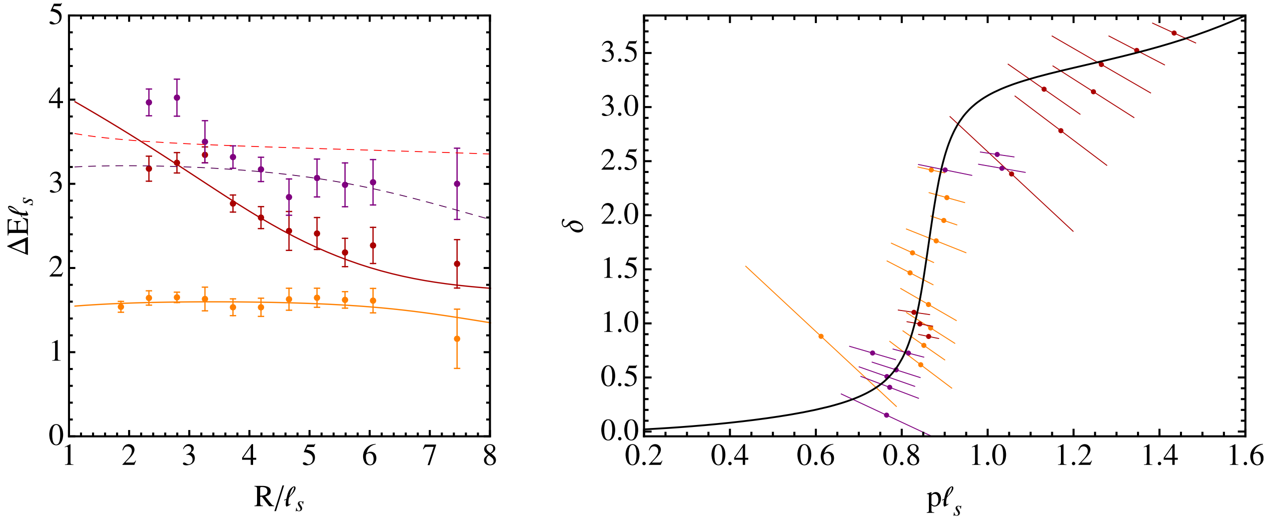
$\Longrightarrow$

phase shift has classic resonance shape

massive resonance: $m = 1.74/l_s^{3A}, \ \Gamma = 0.16/l_s^{3A}$

about half the bulk mass gap, and similar to $D = 3 + 1$ axion mass

1404.0037



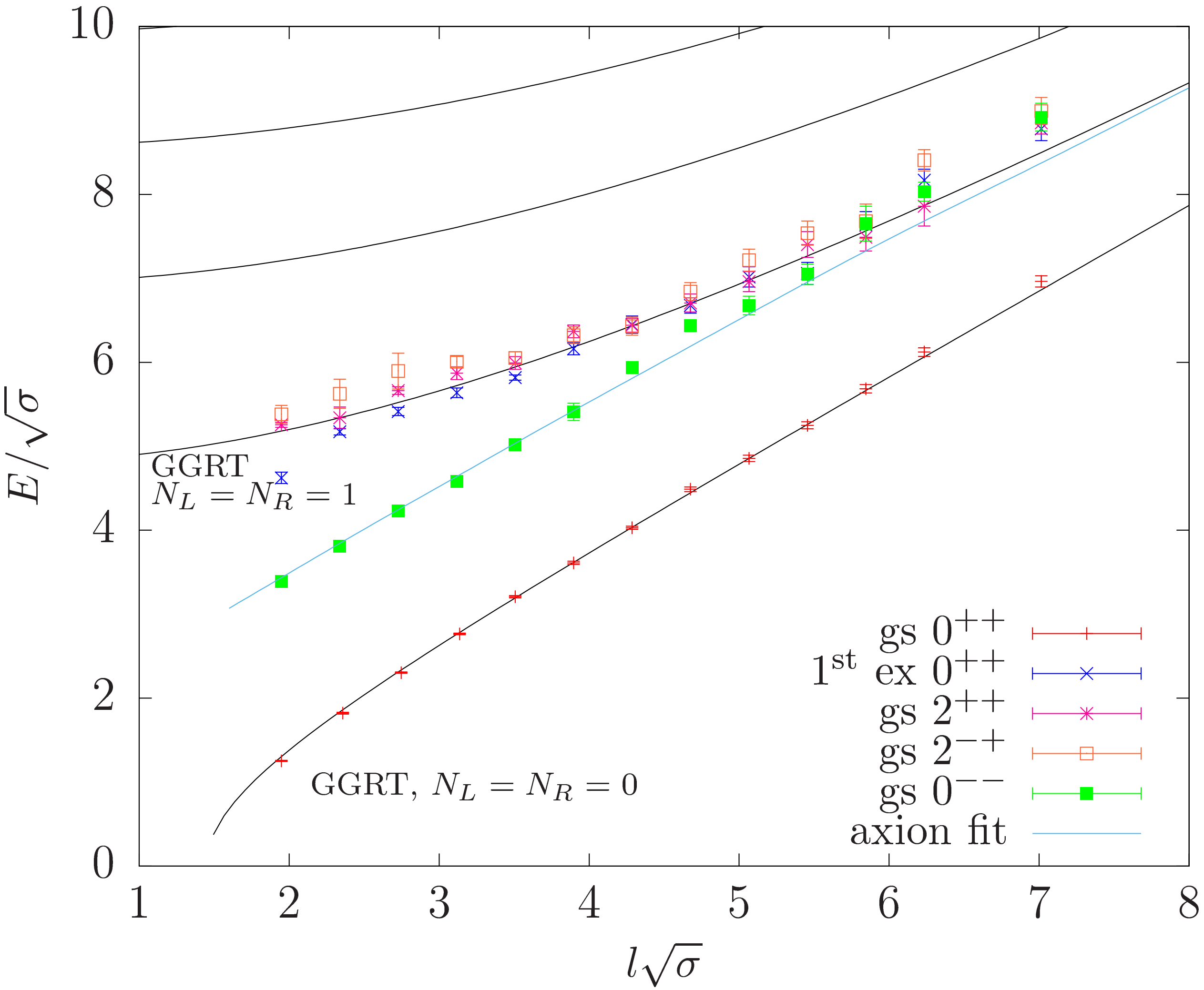
ΔEℓ_s
R/ℓ_s
δ
pℓ_s

$$D = 2 + 1 \quad \longrightarrow \quad D = 3 + 1$$

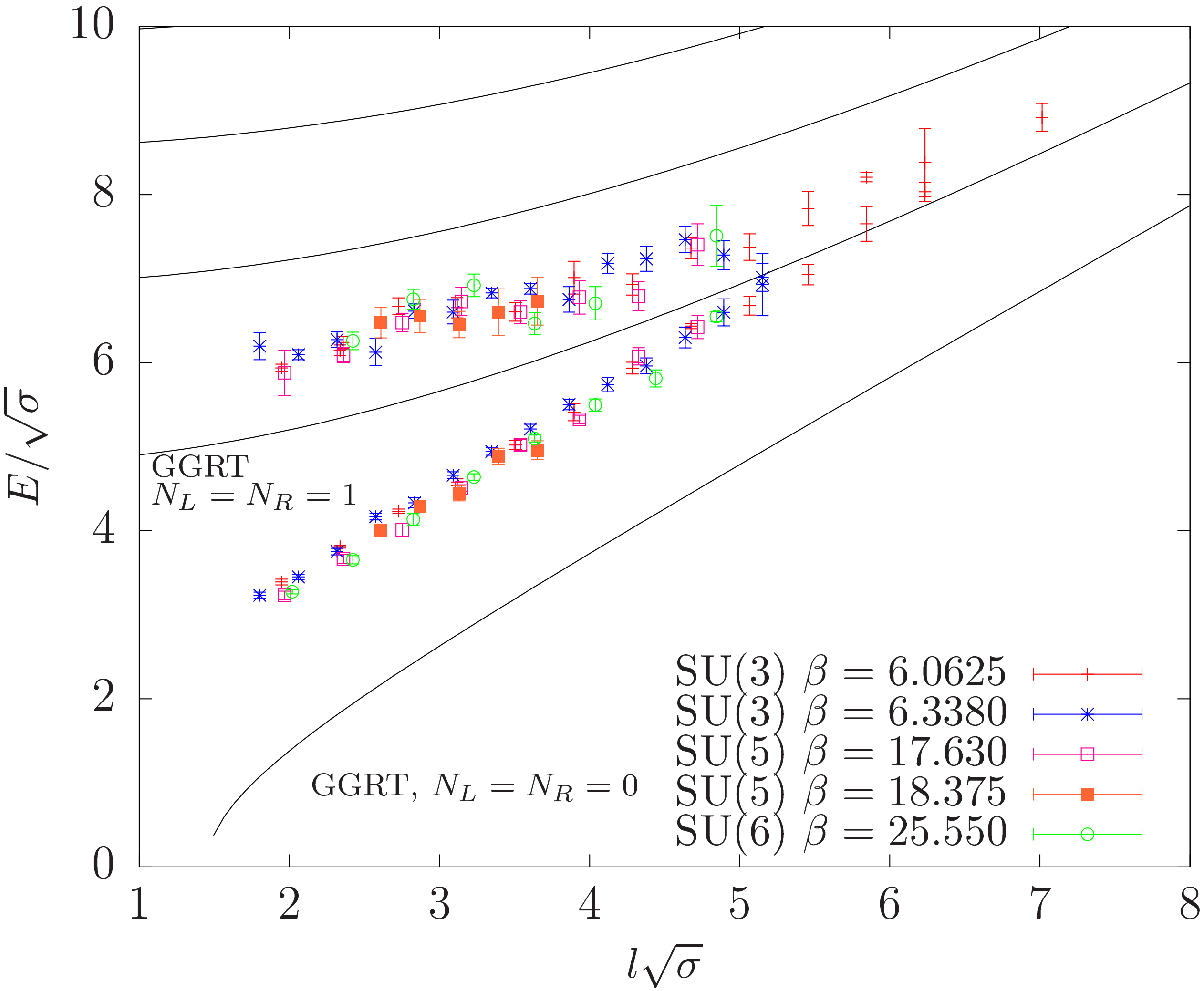
more quantum numbers; spin around axis

GGRT spectrum very good approximation *except* for the 0^{--}

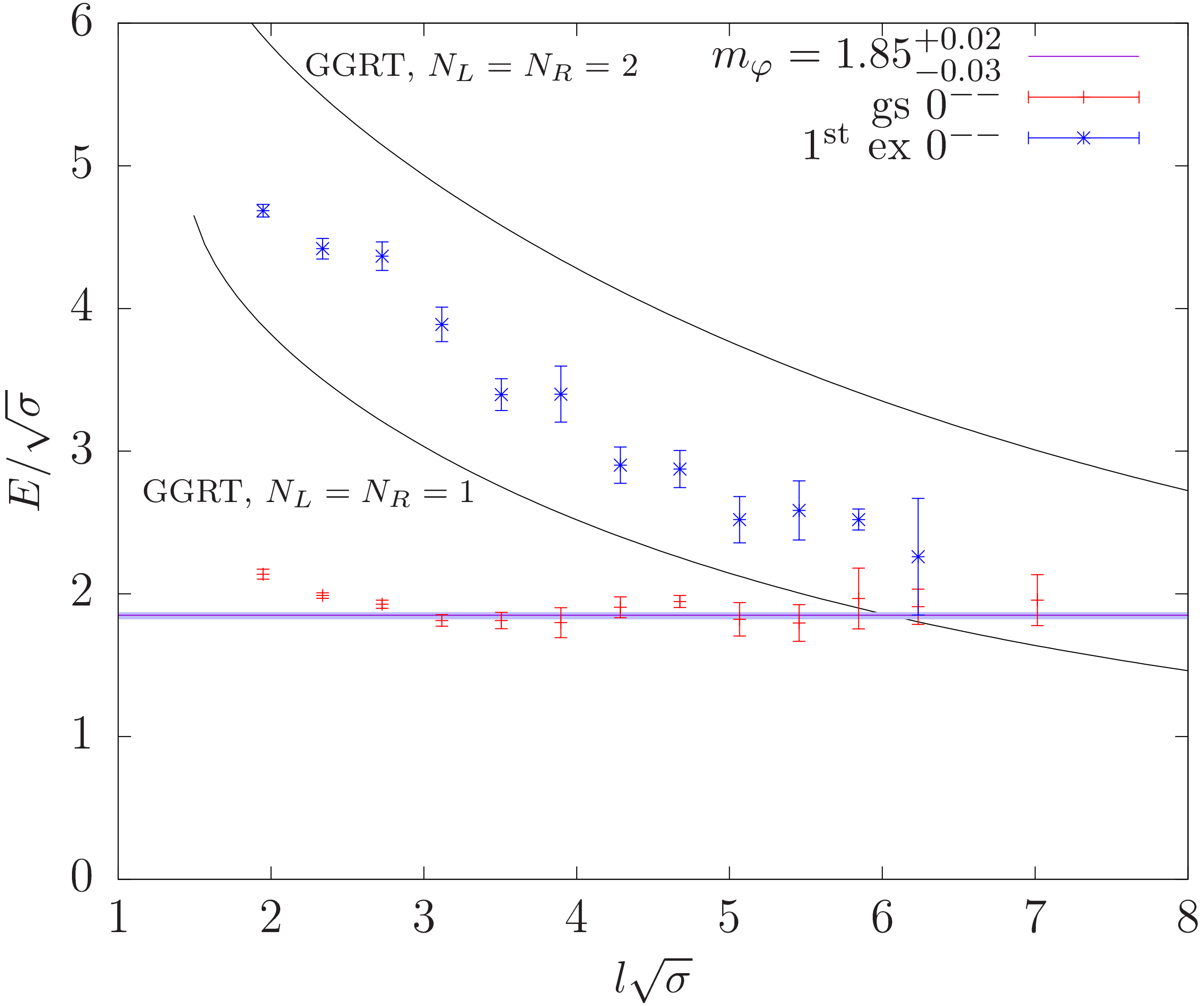
1702.03717



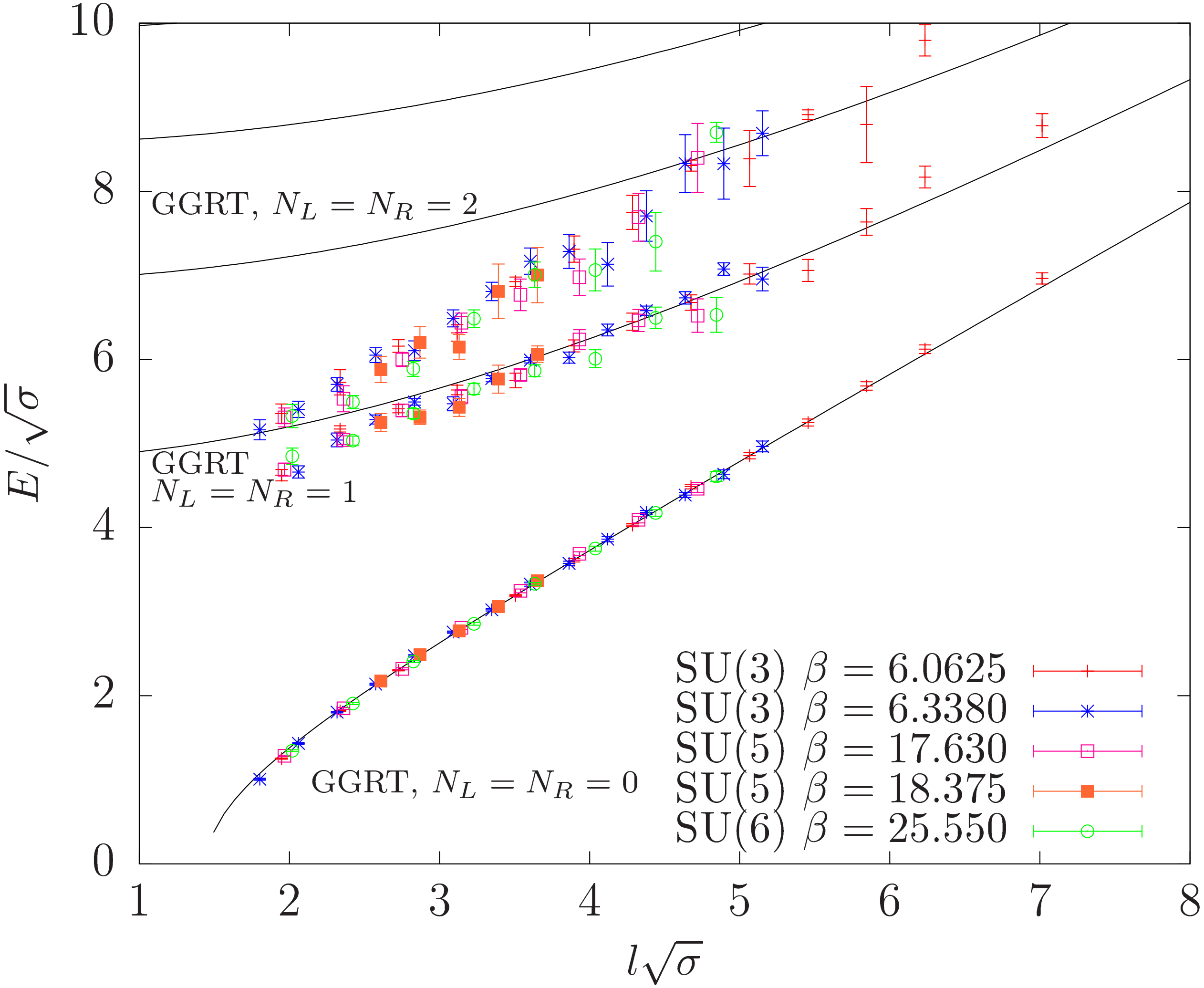
GGRT
$N_L = N_R = 1$
GGRT, $N_L = N_R = 0$
gs 0^{++}
1^{st} ex 0^{++}
gs 2^{++}
gs 2^{-+}
gs 0^{--}
axion fit
$E/\sqrt{\sigma}$
$l\sqrt{\sigma}$



10
8
6
4
2
0
1
2
3
4
5
6
7
8
$E/\sqrt{\sigma}$
$l\sqrt{\sigma}$
GGRT
$N_L = N_R = 1$
GGRT, $N_L = N_R = 0$
SU(3) $\beta = 6.0625$
SU(3) $\beta = 6.3380$
SU(5) $\beta = 17.630$
SU(5) $\beta = 18.375$
SU(6) $\beta = 25.550$



$m_\varphi = 1.85^{+0.02}_{-0.03}$
gs 0^{--}
$1^{\rm st}$ ex 0^{--}
GGRT, $N_L = N_R = 2$
GGRT, $N_L = N_R = 1$
$E/\sqrt{\sigma}$
$l\sqrt{\sigma}$


GGRT, $N_L = N_R = 2$
GGRT
$N_L = N_R = 1$
GGRT, $N_L = N_R = 0$
SU(3) $\beta = 6.0625$
SU(3) $\beta = 6.3380$
SU(5) $\beta = 17.630$
SU(5) $\beta = 18.375$
SU(6) $\beta = 25.550$
$E/\sqrt{\sigma}$
$l\sqrt{\sigma}$
0
2
4
6
8
10
1
2
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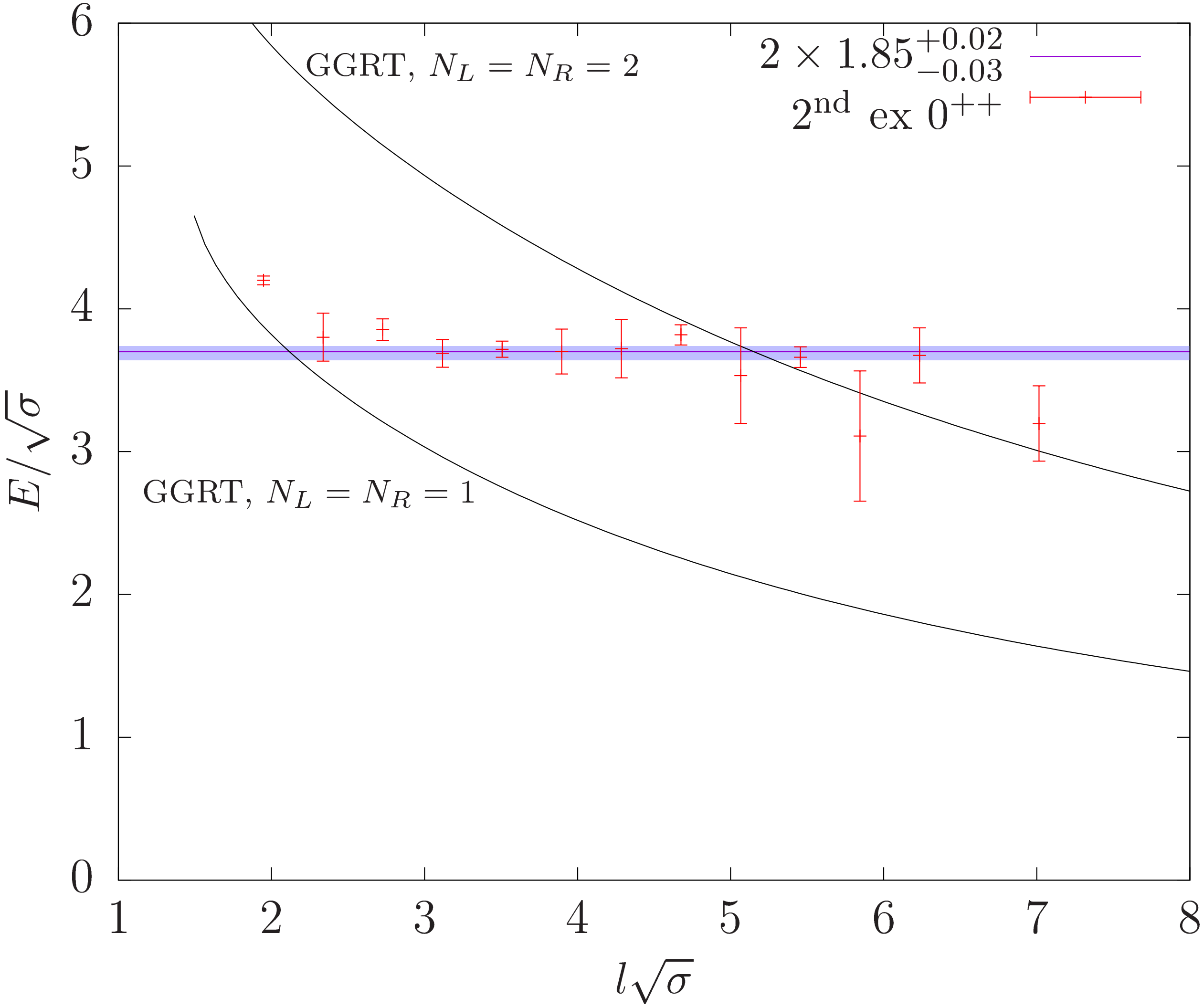
5
6
7
8



6
5
4
3
2
1
0
1 2 3 4 5 6 7 8
$E/\sqrt{\sigma}$
$l\sqrt{\sigma}$
GGRT, $N_L = N_R = 2$
GGRT, $N_L = N_R = 1$
$2 \times 1.85^{+0.02}_{-0.03}$
$2^{\rm nd}$ ex 0^{++}

0^{--} massive mode $\longrightarrow$ 'axion' on world sheet

Add to world sheet action an axion ϕ:

$$S_\phi = \int d^2\sigma\sqrt{-h}\left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{Q_\phi}{4}h^{\alpha\beta}\epsilon_{\mu\nu\lambda\rho}\partial_\alpha t^{\mu\nu}\partial_\beta t^{\lambda\rho}\phi\right) ,$$

with

$$t^{\mu\nu} = \frac{\epsilon^{\alpha\beta}}{\sqrt{-h}}\partial_\alpha X^\mu \partial_\beta X^\nu .$$

i.e. the axion is coupled to the self-intersection number of the world sheet
1404.0037

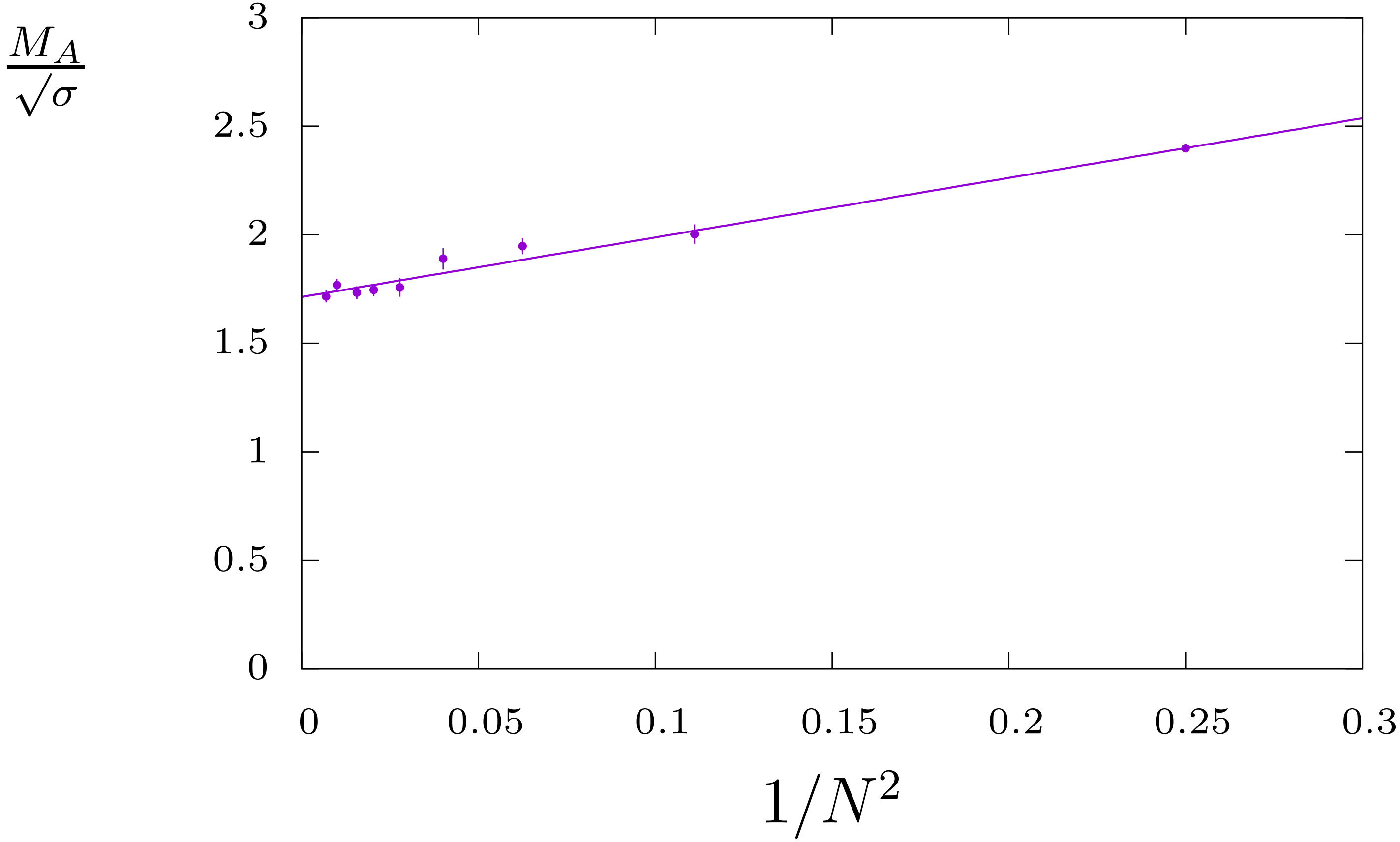
Integrability?

In $D = 4$ Goldstone bosons + massless axion with coupling
$Q_\phi = \sqrt{7/16\pi} \simeq 0.373 \quad \Longrightarrow$
world-sheet integrability 1511.01908

Our axion: $Q_\phi = 0.38(4)$ *but* $m \simeq 1.85(3) l_s^{-1}$

maybe $m \to 0$ as $N \to \infty$?
No: remains massive as $N \to \infty$: $SU(N), N \in [2, 12], D = 3 + 1$
1702.03717

Axion remains massive as $N \to \infty$: $SU(N), N \in [2, 12], D = 3 + 1$ 1702.03717



Some Conclusions

- physical quantities of the $SU(\infty)$ gauge theory can be calculated quite precisely – and indeed $N = 3$ is 'close to' $N = \infty$

- confining flux tubes are well described by the Nambu-Goto string action in both $D = 2 + 1$ and $D = 3 + 1$ $SU(N)$ gauge theories – except for the interesting presence of a massive world-sheet axion particle in $D = 3 + 1$ – with the deconfinement of the gauge theory protecting the theory from becoming tachyonic at very small lengths

- the parallel theoretical work on the universal terms of the world-sheet action, especially relevant for long strings, and of the corrections to Nambu-Goto confining flux tubes, using the TBA formalism, especially relevant to shorter strings, provides a very nice example of a positive symbiosis between hep-lat and hep-th.