Strings 2024, CERN

String Field Theory

Past, Present, and Future

Part 2

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The construction of closed and quantum SFT

Zwiebach '92: classical and quantum closed bosonic SFT

recipe - space of string fields

$$\hat{\mathcal{H}} \subset \mathcal{H}_{ws} = \mathcal{H}_{matter} \otimes \mathcal{H}_{bc}$$

Ψ obeys $b_0^- \Psi = L_0^- \Psi = 0$

$$b_0^\pm \equiv b_0 \pm \tilde{b}_0$$
$$L_0^\pm \equiv L_0 \pm \tilde{L}_0$$

- BV functional $S[\Psi] = \frac{1}{g_s^2}\left[\frac{1}{2}\langle \Psi | c_0^- Q_B | \Psi \rangle + \sum \frac{1}{n!}\{\Psi^{\otimes n}\}\right]$

obeys master equation that ensures quantum consistency with gauge invariance $(S, S) + 2\hbar \Delta S = 0$

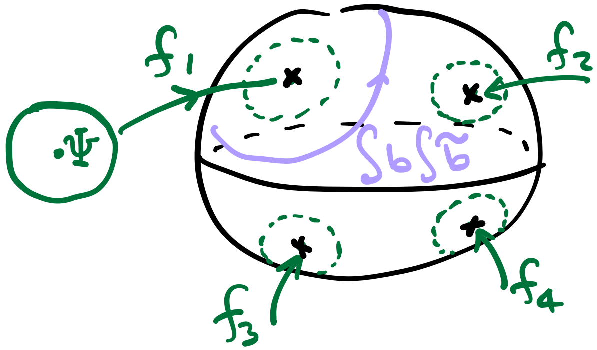
- string vertices $$\{\Psi^{\otimes n}\} = \frac{1}{(-2\pi i)^{n-3}} \int_{\Gamma_n} \langle e^{\mathcal{B}} \prod_{i=1}^{n} [\Psi(0)]^{(f_i)} \rangle$$

b ghost insertion

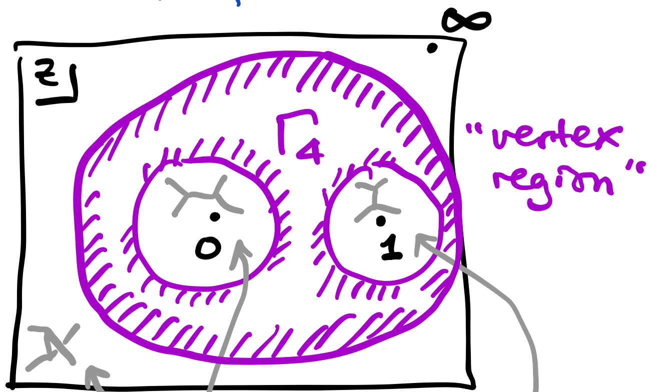
coordinate maps

a suitable domain in the moduli space of n-punctured Riemann surfaces

e.g. genus zero 4-point vertex is constructed from



fibered over



covered by Feynman diagrams

- string field path integral

$$\int \mathcal{D}\Psi \Big|_{\mathcal{L}} e^{-S[\Psi]}$$

a Lagrangian subspace of $\hat{\mathcal{H}}$ w.r.t. the Grassmann-odd BV symplectic str.

(BV gauge-fixing)

FAQ:

1. Is the action unique?

Far from, but different consistent choices of string vertices are related by field redefinition (i.e. "frame change")

Hata & Zwiebach '93

Similar to scheme dependence in renormalized Wilsonian conformal perturbation theory.

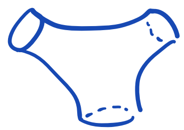
To list a few:

- lightcone vertices  Mandelstam, Kaku, Kikkawa, '70s

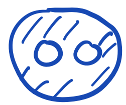
  (correlates geometry with lightcone momenta)

- minimal area metric vertices  Zwiebach '92

  ("efficient"?)

- hyperbolic vertices  Costello, Zwiebach '19

  (fully non-singular at quantum level ✓)

- flat asymmetric vertices  Mazel, Sandor, Wang, XY '24

  (simple for pert. theory, works only at the level of classical EOM)

2. Do different "background" worldsheet CFTs define equivalent SFTs?

They should! The resulting SFTs should be related by field redefinition.

$$\begin{array}{lccc} \text{CFT} & A & \dashrightarrow & B \\ \text{SFT} & \hat{\mathcal{H}}_A & \xrightarrow{F_{A,B}} & \hat{\mathcal{H}}_B \end{array}$$

classically $S_B[F_{A,B}[\Psi]] = S_A[\Psi]$

(quantumly, also change of measure)

Proven for infinitesimal marginal deformations

Sen & Zwiebach '93

3. Is CSFT more than perturbation theory?

Not obviously. But the pert. series may well converge for suitable observables at the classical level.

4. But bosonic CSFT is ill-defined at the quantum level?

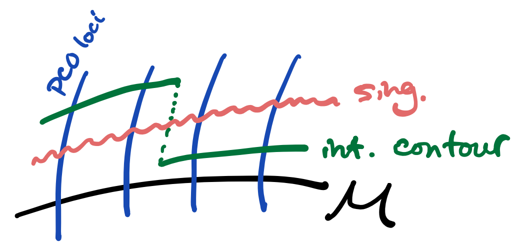
Unless the worldsheet matter CFT admits no normalizable relevant operator, as in c=1 string theory (and its deformations).

Sen 2014 : classical and quantum closed (and + open) super - SFT

new ingredients - String vertices are defined with choice of locations of Picture Changing Operators (in addition to coord. maps and domains of the moduli spaces)

- vertical integration Sen '14, Sen, Witten '15

a procedure of consistently integrating over the moduli space that evades spurious singularities



[equivalent to a specific way of breaking up the integral over super-moduli space Wang, XY '22]

space of string fields:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{NS,NS} \oplus \hat{\mathcal{H}}_{NS,R} \oplus \hat{\mathcal{H}}_{R,NS} \oplus \hat{\mathcal{H}}_{R,R}$$

Ψ obeys $b_0^- \Psi = L_0^- \Psi = 0$

assigned picture number $\begin{cases} -1 & \text{in NS sector} \\ -\frac{1}{2} & \text{in R sector} \end{cases}$

Note: auxiliary fields in $(-\frac{3}{2})$-picture also used to construct action

A complete formulation of superstring perturbation theory

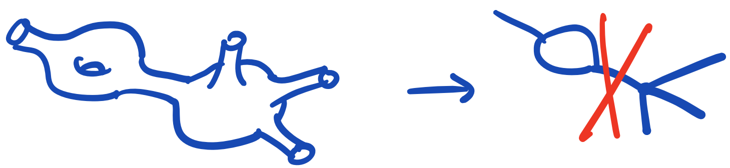
Sen '14 - '16 ; Reviews: de Lacroix, Erbin, Kashyap, Sen, Verma '17
Sen, Zwiebach '24

- by construction, finite vertices, free of UV divergences

  loop momentum integration contour specified by analytic conti.
  from that of Euclidean Green function Pius, Sen '16

- systematic treatment of mass renormalization Pius, Rudra, Sen '14

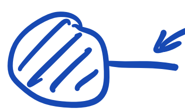
  e.g. to determine the mass and S-matrix elements
  for SO(32)-spinor particle in heterotic string theory



+ ...

  instead, work with amputated SFT diagrams as in perturbative QFT!

- systematic treatment of massless tadpoles



  ← massless field ϕ

  contributes to $V_{eff}(\phi)$ by $\propto \phi$,
  need to find new vacuum field configuration

  When massless tadpoles cancel (e.g. +), still need
  consistent cutoffs on different components of moduli space

(Further) Applications of SFT

I. Describing general string backgrounds

(a) superstring in RR flux backgrounds

no local worldsheet CFT in RNS formalism ?!

[Alternative approach: pure spinor formalism]

begin with NSNS background, defines a closed SFT

E.O.M.

$$Q_B \Psi + \sum \frac{1}{n!} \mathcal{G} [\Psi^{\otimes n}] = 0$$

PCO

L_∞ string bracket, related to vertex $\{\Psi^{\otimes(n+1)}\}$

find solution Ψ describing turning on (perturbatively) RR flux, analyze fluctuations around Ψ to extract spectrum (and other observables e.g. amplitudes) in the new background

$AdS_3 \times S^3 \times M_4$ Cho, Collier, XY '18

$\mathbb{R}^{1,9} \to AdS_5 \times S^5$ See Minjae Cho's talk

GKP flux compactification Cho, Kim '23

(b) time-dependent background

Incomplete understanding of worldsheet CFT based on NLσM with Lorentzian target spacetime

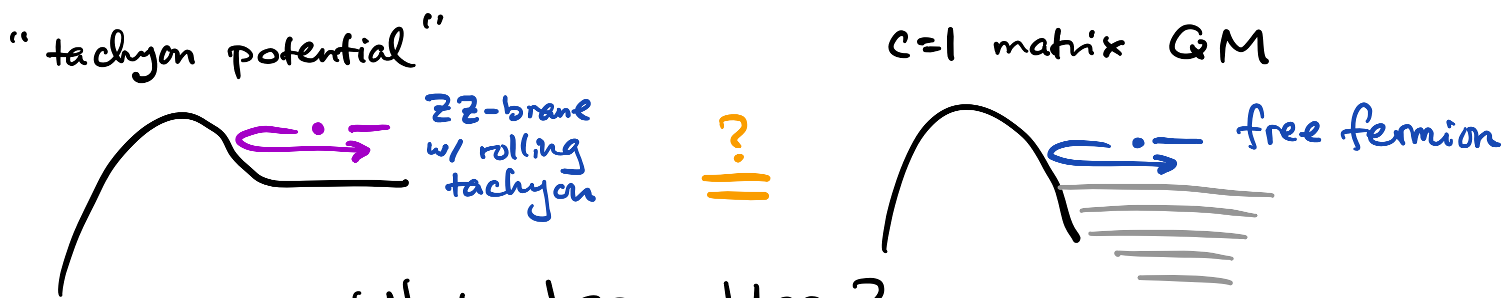
BCFT with time dependence   requires subtle analytic conti.

- open string rolling tachyon   Sen '02

In SFT, it is straightforward to describe nontrivial time-dependent string field solutions, at least perturbatively.

• The challenge is to identify and determine physical observables.   e.g. energy of rolling tachyon from covariant phase space of OSFT

Cho, Mazel, XY '23

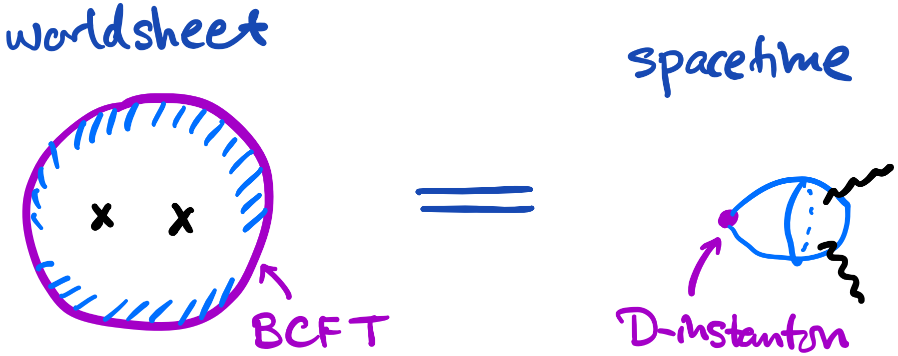


what observables ?

II. D-instantons

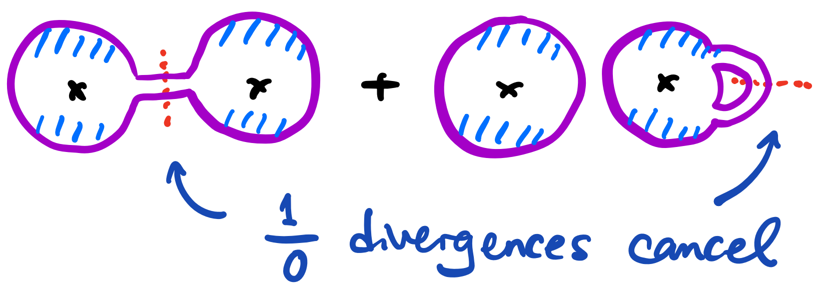
- Non-perturbative effects in string theory $\sim \mathcal{O}(e^{-\frac{1}{g_s}})$ captured by BCFT localized in target spacetime

Polchinski '94, Green, Gutperle '97



- Naive on-shell worldsheet approach suffers from ambiguities Balthazar, Rodriguez, XY '19

e.g. ZZ-instanton contribution to $1\to 1$ amplitude in $c=1$ string theory



need consistent cutoffs on two disconnected components of the moduli space to fix the finite part

Sen '20 : first-principle approach to D-instantons based on open + closed SFT

Key ingredients - integrating out open string fields on D-instantons produces a correction to closed string eff. action

$$e^{-\Gamma[\Psi_c]}\Big|_{\text{D-inst}} = e^{-\frac{C}{g_s}} \int D\Psi_o\Big|_{\mathcal{L}} \, e^{-S_{o\text{-}c}[\Psi_o,\, \Psi_c]}$$

- Siegel gauge singular, replace with Sen gauge

nontrivial checks • against $c=1$ MQM Sen '20

and • against soft dilaton relations in IIB Agmon, Balthazar, Cho, Rodriguez, XY '22

- "massless" open string field functional integral (replacing the naive D-instanton moduli integral) must be performed non-pert.ly

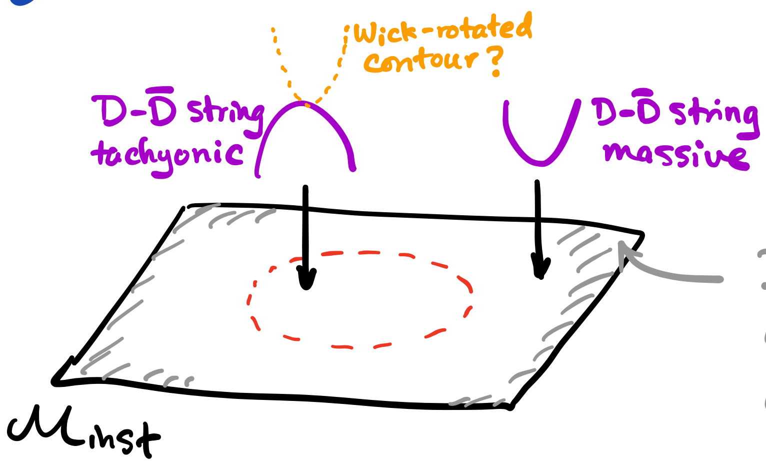
SFT approach to D-instantons has been further applied to :

- multi-D-instantons in IIB at R^4 order — Sen '21
- wrapped D-instantons on Calabi-Yau spaces — Alexandrov, Sen, Stefanski '21
- D-instantons in orientifold — Alexandrov, Firat, Kim, Sen, Stefanski '22
- ZZ-instantons in minimal string theory — Eniceicu, Mahajan, Murdia, Sen '22

A major challenge that remains is to understand the open string field integration contour in general, and how to piece together the perturbative expansions around different D-instantons.

e.g. D-$\bar{D}$ instanton contribution to graviton amplitude in IIB

Agmon, Balthazar, Cho, Rodriguez, Yin '22



D^6R^4 term receives contribution only from asymptotic region

III. Quantum string vacua

Generally, quantum effects can modify string vacua in a way that is not captured by a worldsheet CFT (which is insensitive to the background dilaton value / string coupling)

Example: $SO(16) \times SO(16)$ heterotic string   Alvarez-Gaumé, Ginsparg, Moore, Vafa '86

cf. Miguel Montero's review talk

defined by a worldsheet CFT that involves $\mathbb{R}^{1,9}$ target spacetime, but the 10d effective theory receives a string 1-loop contribution to the vacuum energy density. Consequently the true vacuum configuration is not described by 10d Minkowskian spacetime, but rather a cosmological solution of the eff. theory driven by a runaway potential.

In the SFT framework, such vacua can be described by solutions to the quantum E.O.M. (deduced from the 1PI string field eff. action Sen '14)

Explicit SFT constructions of (perturbative) vacuum solutions that capture quantum effects :

SO(32) het string on Calabi-Yau with standard embedding

1-loop Fayet-Iliopoulos term — Dine, Seiberg, Witten '87; Dine, Ichinose, Seiberg '87, Atick, Sen '88

$$V_{eff} = \frac{1}{2}(\phi^* \phi - c\, g_s^2)^2$$

$\rightarrow$ shifted vacuum SFT solution (restoring susy) — Sen '15

Avenues for future investigation :

SFT solutions that describe flux compactification with D-instanton effects taken into account via integrating out D-inst. open string fields — cf. Minjae Cho's talk

Can SFT capture other non-perturbative effects ?

e.g. NS5-brane instantons as non-pert. saddles ?

gaugino condensate ? KKLT '03

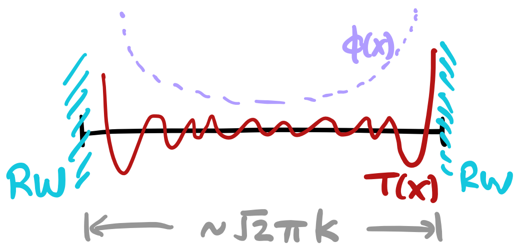
Some additional future prospects

- Completion of classical closed SFT ?

even though closed SFT has thus far been defined perturbatively, the construction of perturbative solutions that describe (worldsheet) CFT deformations indicates that it ought to be possible to sum up the perturbative series in the string field, at least at the level of physical observables.

Mukherji, Sen '91
Scheinpflug, Schnabl '23

e.g. "tachyon-dilaton eschatology solution"



$=$ A_k minimal model

Mazel, Sandor, Wang, XY '24

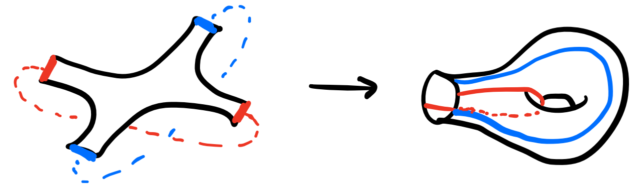
"what is the space of admissible string fields?"

The realization of diffeomorphism as an (L_∞-) gauge transf. in SFT should play a key role in solutions that describe topology change and horizons.

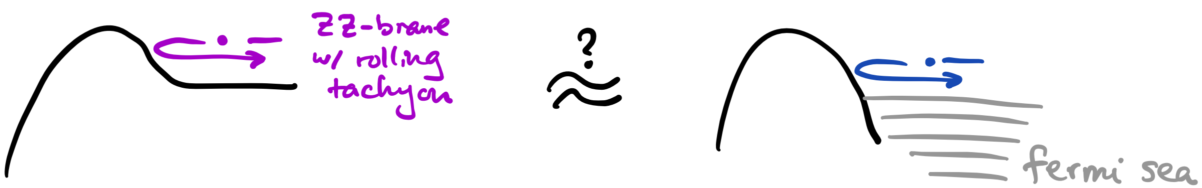
Ghoshal, Sen '92
Mazel, Wang, XY w.I.P.

- Does OSFT make sense at the quantum and non-perturbative level?

  - D-instanton effects may be understood via non-pert. saddles
  - open string loops can cover moduli space of Riemann surfaces with boundary



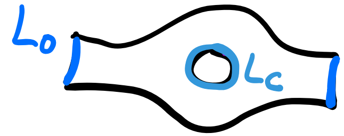
Perhaps the simplest test case is Witten's cubic bosonic OSFT on ZZ-brane in c=1 string theory

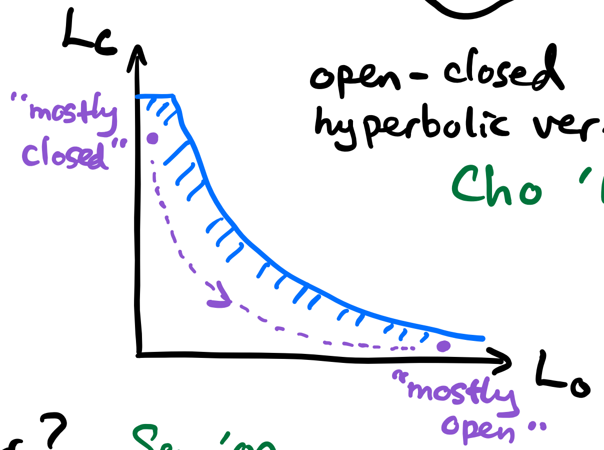


OSFT knows the fermi surface ... and other fermions too?

- Time-dependent perturbation theory around the rolling tachyon?
- Define the OSFT path integral non-perturbatively?

- Open-closed duality may be understood as a field redefinition in open + closed SFT



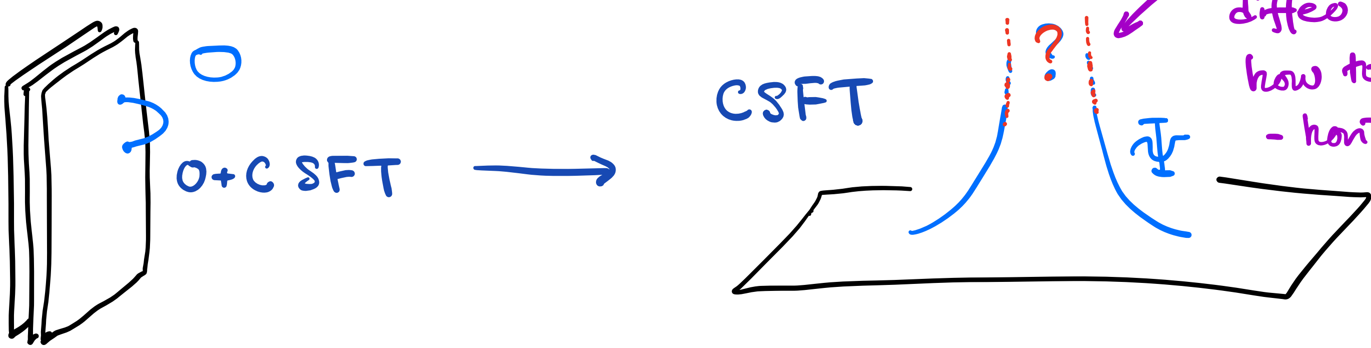


open-closed hyperbolic vertices Cho '19

- Can this idea be realized in tachyon matter? Sen '02

open string rolling tachyon $\longleftrightarrow$ closed string radiation

- Derive holography by I. open-closed field redefinition
  II. decoupling limit ?



need to better understand diffeo in SFT and how to take the "near-horizon" limit !