

Aspects of de Sitter Quantum Gravity

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Consider Euclidean path integrals in the presence of a positive cosmological.

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- Include the effects of matter fields in the gravitational path integral.
- Direct evaluation in 4D gravity, in situations when there is a relation to 2D models.

$$\mathcal{Z} = \int_{\mathcal{M}} \underbrace{\mathcal{D}g}_{\text{Geometries}} \underbrace{\mathcal{D}\phi_i}_{\text{Fields}} e^{-\underbrace{S_E(g, \phi_i)}_{\text{Action for gravity+fields}}}$$

Focus

Consider Euclidean path integrals in the presence of a positive cosmological.

- Include the effects of matter fields in the gravitational path integral.
- Direct evaluation in 4D gravity, in situations when there is a relation to 2D models.

There will be no use of holography. Providing examples to argue that

- exact results for the path integral are possible in de Sitter,
- new features arise when quantum gravity is coupled to matter.

Outline

I. dS_3 quantum gravity coupled to matter

II. Quantum corrections for near-extremal black holes in dS_4



dS_3 quantum gravity coupled to matter

Advance methods in Chern-Simons to explore quantum gravity

Based on...

2001.09998

AC, Sabella-Garnier, Zukowski

2302.12281+2304.02668

AC, Coman, Fliss, Zukowski

2407.09608

Bourne, AC, Fliss



Einstein-Hilbert: Metric, curvature

$$Z_{matter}[g_{\mu\nu}] = \int D\phi e^{-S_{matter}[\phi, g_{\mu\nu}]} = \det(-\nabla^2 + m^2 \ell^2)^{-1/2}$$

$$\langle Z_{matter}[M] \rangle_{grav} = \int (Dg_{\mu\nu})_M e^{-S_{EH}[g_{\mu\nu}]} Z_{matter}[g_{\mu\nu}]$$

Today: massive scalar
field for simplicity

Today: Fixed topology, 3D gravity

Einstein-Hilbert: Metric, curvature

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Chern-Simons: Gauge connections

$$\log(Z_{matter}[g_{\mu\nu}]) = \frac{1}{4} \mathbb{W}_j[A_L, A_R]$$

$$\langle \mathbb{W}_j \rangle_{grav} = \int DA_{L/R} e^{ik_L S[A_L] + ik_R S[A_R]} \mathbb{W}_j[A_L, A_R]$$

Wilson Spool

dS₃ Quantum Gravity

$$\log(Z_{\text{scalar}}[g_{\mu\nu}]) = \frac{1}{4} \mathbb{W}_j[A_L, A_R]$$
$$\langle \mathbb{W}_j \rangle_{\text{grav}} = \int DA_{L/R} e^{ik_L S[A_L] + ik_R S[A_R]} \mathbb{W}_j[A_L, A_R]$$

Focus mainly on massive fields coupled to dS₃ gravity. **Why dS₃?**

- We can use the full power of SU(2) Chern-Simons theory.

[Carlip 1992; AC, Lashkari, Maloney 2011; Anninos, Denef, Law, Sun 2022]

- Make predictions for G_N corrections without the aid of holography.

How to couple matter to 3D gravity
via the Chern-Simons formulation?

Step 1: cast Euclidean dS_3 gravity in Chern-Simons language

Step 2: describe particles and fields in a group theoretic language

Step 3: incorporate step 2 into Chern-Simons path integral

Step 1: cast Euclidean dS₃ gravity in Chern-Simons language

- Gauge group: $SU(2) \times SU(2)$ leads to dS₃ Euclidean Gravity, $\Lambda = \frac{1}{\ell^2} > 0$
- Action: $-ik_L S_{CS}[A_L] - ik_R S_{CS}[A_R] = I_{EH}[g_{\mu\nu}] - i\delta I_{GCS}[g_{\mu\nu}]$
- Couplings: $k_L = \delta + i \frac{\ell}{4G_N}$, $k_R = \delta - i \frac{\ell}{4G_N}$
- Dictionary: $A_L = i \left(\omega^a + \frac{e^a}{\ell} \right) L_a$, $A_R = i \left(\omega^a - \frac{e^a}{\ell} \right) \bar{L}_a$

Step 1: cast Euclidean dS_3 gravity in Chern-Simons language

Background S^3 connections

$$\begin{aligned}a_L &= i L_1 d\rho + i(\sin \rho L_2 - \cos \rho L_3)(d\varphi - d\tau) \\a_R &= -i \bar{L}_1 d\rho - i(\sin \rho \bar{L}_2 + \cos \rho \bar{L}_3)(d\varphi + d\tau)\end{aligned}$$

Holonomies

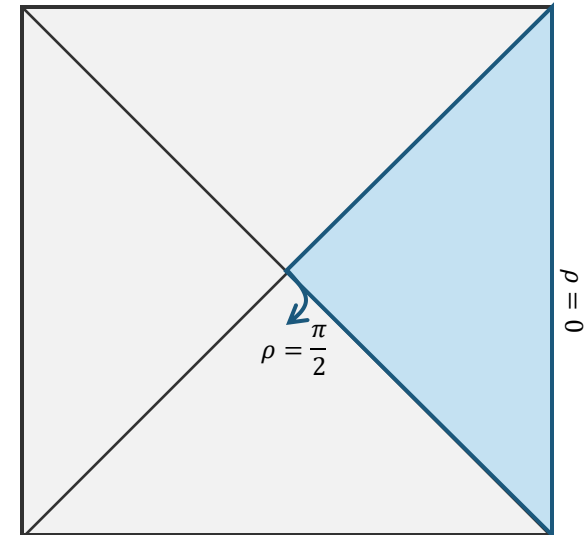
$$P \exp \oint_{\gamma} a_{L/R} \sim e^{2\pi i L_3 h_{L/R}}$$

For a cycle γ that wraps horizon at $\rho = \frac{\pi}{2}$

$$\begin{aligned}h_L &= 1 \\h_R &= -1\end{aligned}$$

Geometry: Static Patch

$$ds^2 = \ell^2 (\cos^2 \rho d\tau^2 + \sin^2 \rho d\varphi^2 + d\rho^2)$$



Step 2: describe particles and fields in a group theoretic language

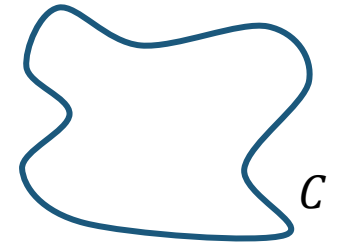
Step 2: describe particles and fields in a group theoretic language

Description of **particles** in CS language: Wilson lines/loops

$$W_R(C) = \underbrace{\text{Tr}_R \left(P \exp \oint_C A \right)}_{\text{Infinite dimensional representation of G. Encodes quantum numbers of the particle.}} = \underbrace{\int DU \exp(-S(U, A)_C)}_{\text{Path integral of a massive particle.}}$$

Infinite dimensional representation of G.
Encodes quantum numbers of the particle.

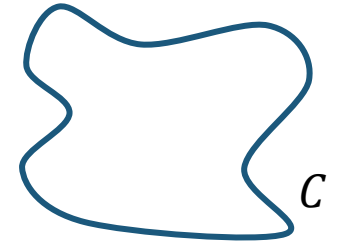
Path integral of a massive particle.



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For $SU(2)$ applied to dS_3 , these representations are non-standard (complex/real casimir).

Step 2: describe particles and fields in a group theoretic language

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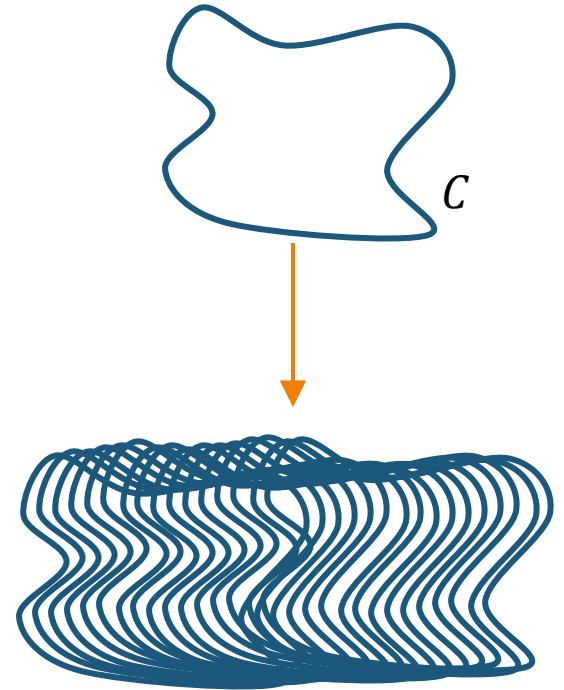
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Infinite dimensional representation of G.
Encodes quantum numbers of the particle.

Path integral of a massive particle.

Description of **fields** in CS language: ...

$$\log \det(-\nabla^2 + m^2 \ell^2) \sim \sum_n \frac{1}{n} \text{Tr}_j \left(P e^{\frac{n}{2\pi} \oint A} \right)$$



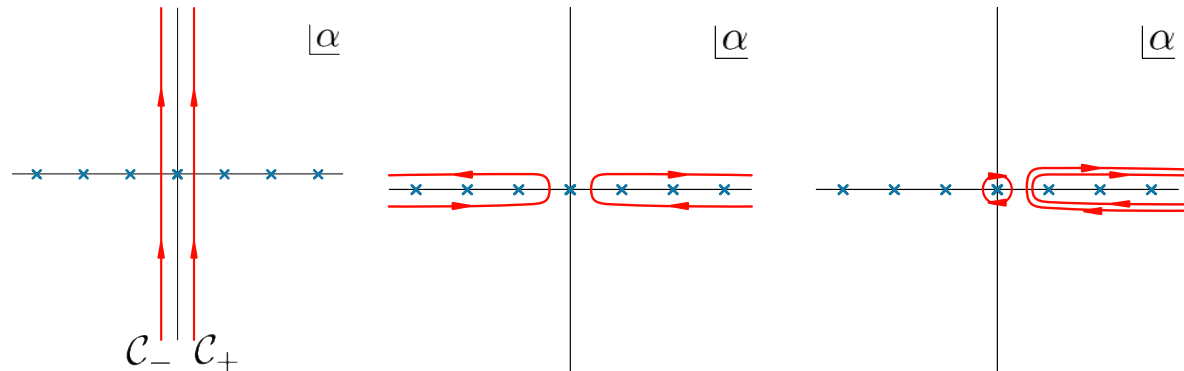
Step 2: describe particles and fields in a group theoretic language

Description of **fields** in CS language: [Wilson spool](#). Derivation relies on a group theoretic understanding of one-loop determinants.

For massive **scalar** fields

$$\mathbb{W}_j[A_L, A_R] = i \int_{\mathcal{C}} \frac{d\alpha}{\alpha} \frac{\cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} \text{Tr}_j(P e^{\frac{\alpha}{2\pi} \oint A_L}) \text{Tr}_j(P e^{-\frac{\alpha}{2\pi} \oint A_R})$$

where $\mathcal{C} = \mathcal{C}_+ \cup \mathcal{C}_-$



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This expression can be taken off-shell for the metric: A_L and A_R appear in a simple way!

Step 3: incorporate step 2 into Chern-Simons path integral

Einstein-Hilbert: Metric, curvature

$$Z_{matter}[g_{\mu\nu}] = \int D\phi e^{-S_{matter}[\phi, g_{\mu\nu}]} = \det(-\nabla^2 + m^2 \ell^2)^{-1/2}$$

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There are three things to keep in mind:

- Level is complex: $k = \delta - i \frac{\ell}{4G_N}$
- Background connection is not trivial.
- Assure that exact results are compatible with the non-standard representations

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We adapted exact methods to incorporate these tweaks:

- Abelianisation [Blau-Thompson]
- Supersymmetric Localization [Kapustin-Willet-Yaakov]

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- ✓ Path integral on S^3
- ✓ Wilson Loops on S^3
- ✓ Wilson Spool on S^3

Results

Massive spinning fields coupled to dS_3 quantum gravity

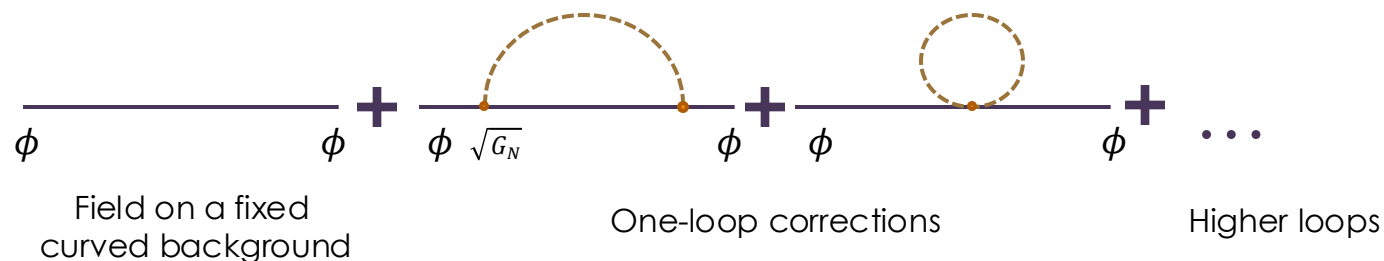
$$\frac{\langle \log Z_{\text{matter}}[S^3] \rangle_{\text{grav}}}{Z_{\text{grav}}[S^3]} = \log Z_{\text{matter}}[S^3] + \sum_{m=1}^{\infty} \left(\frac{G_N}{\ell} \right)^{2m} (\log Z)_{2m}$$

Mass renormalization: $\ell^2 m^2 = (s-1)^2 + \mu^2$

$$\mu_R = \mu + \left(-\frac{48}{5} \mu^3 + \left(\frac{24}{\pi} - \frac{16\pi}{3} + 32\pi^2 s^2 \right) \mu^2 \right) e^{-2\pi\mu} \left(\frac{G_N}{\ell} \right)^2 + \dots$$

Presenting $\mu \gg 1$ limit.
Exact results in references.

Concrete predictive statement about how dynamical gravity renormalizes field



Results

Massive spinning fields coupled to dS_3 quantum gravity

$$\frac{\langle Z_{matter}[S^3] \rangle_{grav}}{Z_{matter}[S^3]} = Z_{grav}[S^3] + \sum_{m=1}^{\infty} \left(\frac{G_N}{\ell} \right)^{2m} (Z_{grav})_{2m}$$

Renormalization G_N : integrate out massive field

$$G_N = G_{N,R} \left(1 + \left(\frac{16\pi}{5} \mu^5 - \left(8 - \frac{16\pi^2}{9} + \frac{32\pi^2}{3} s^2 \right) \mu^4 \right) e^{-2\pi\mu} \left(\frac{G_{N,R}}{\ell} \right)^2 + \dots \right)$$

Presenting $\mu \gg 1$ limit.
Exact results in references.

We have introduced a new object: [the Wilson spool](#).

- Allows us to incorporate matter fields in the Chern-Simons formulation of 3D gravity.
- Tested at $G_N \rightarrow 0$, where the Wilson spool reproduces the one-loop determinant of massive spinning fields.

$$\begin{aligned}\log(Z_{matter}[S^3]) &= \log \det(-\nabla^2 + m^2 \ell^2)^{-\frac{1}{2}} \\ &= \frac{1}{4} \mathbb{W}_j[a_L, a_R]\end{aligned}$$

- We can also make predictions for quantum corrections, without the aid of holography.



Overview

dS₃ gravity

- ✓ Massive spinning fields in S^3 via CS theory
- ✂ One-loop determinants on Lens Spaces via CS theory [wip Bourne, AC, Fliss, Law]
- ✂ Mass renormalization in metric formulation (2-loops)

AdS₃ gravity

- ✓ One-loop determinants on rotating BTZ via CS theory
- ✂ One-loop determinants on handlebodies via CS theory [wip Bourne, Fliss, Knighton]

Near-extremal BHs in dS_4

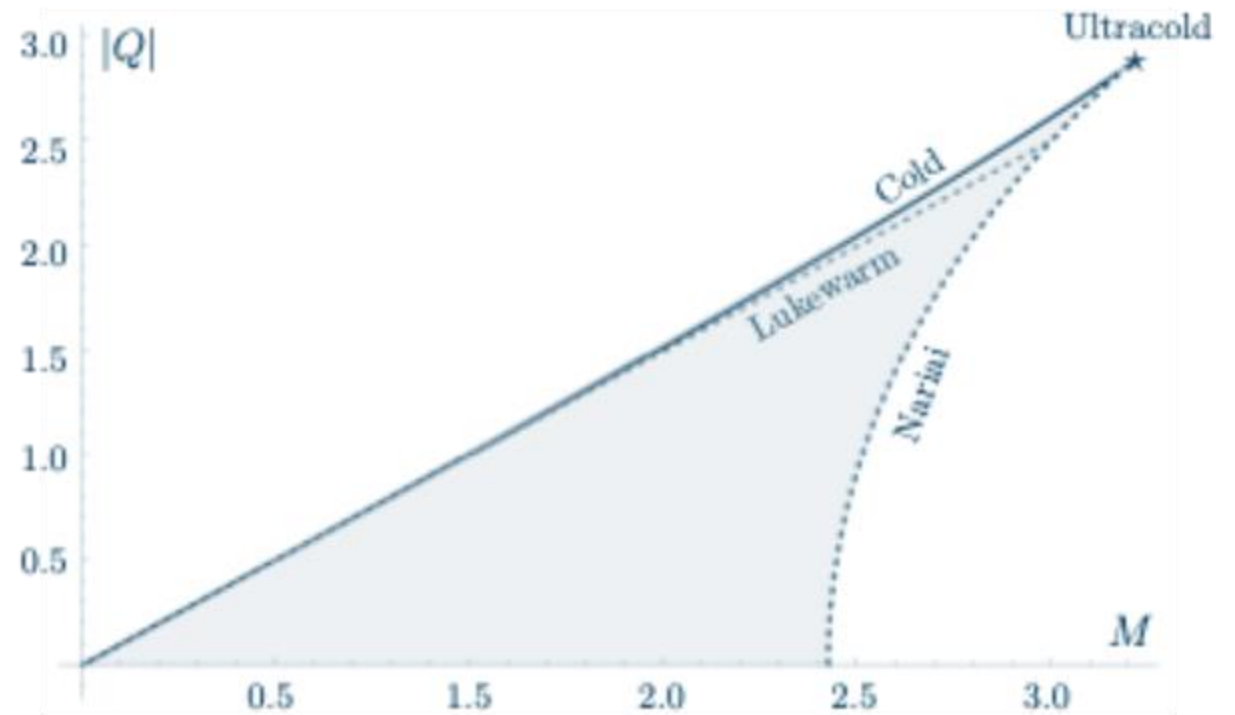
Limitations of 2D models and pathologies of de Sitter
[to appear M. Blacker, AC, W. Sybesma, C. Toldo]

Charged BHs in dS_4

Einstein-Maxwell + positive c.c.

$$S_E = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} (R - 2\Lambda - F_{\mu\nu}F^{\mu\nu})$$

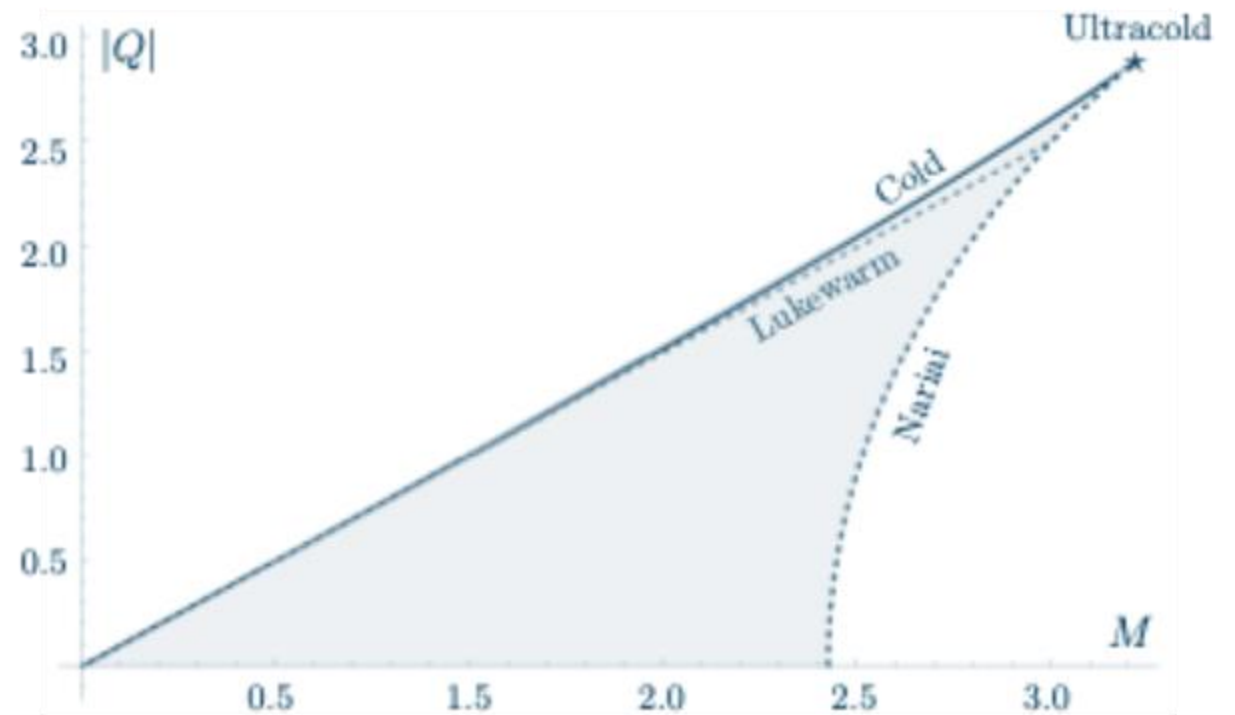
M: "mass"
Q: charge



Charged BHs in dS_4

Extremal limits:

- Cold branch, $r_+ = r_-$, $AdS_2 \times S^2$
- Nariai, $r_c = r_+$, $dS_2 \times S^2$
- Ultracold, $r_c = r_+ = r_-$, $Mink_2 \times S^2$

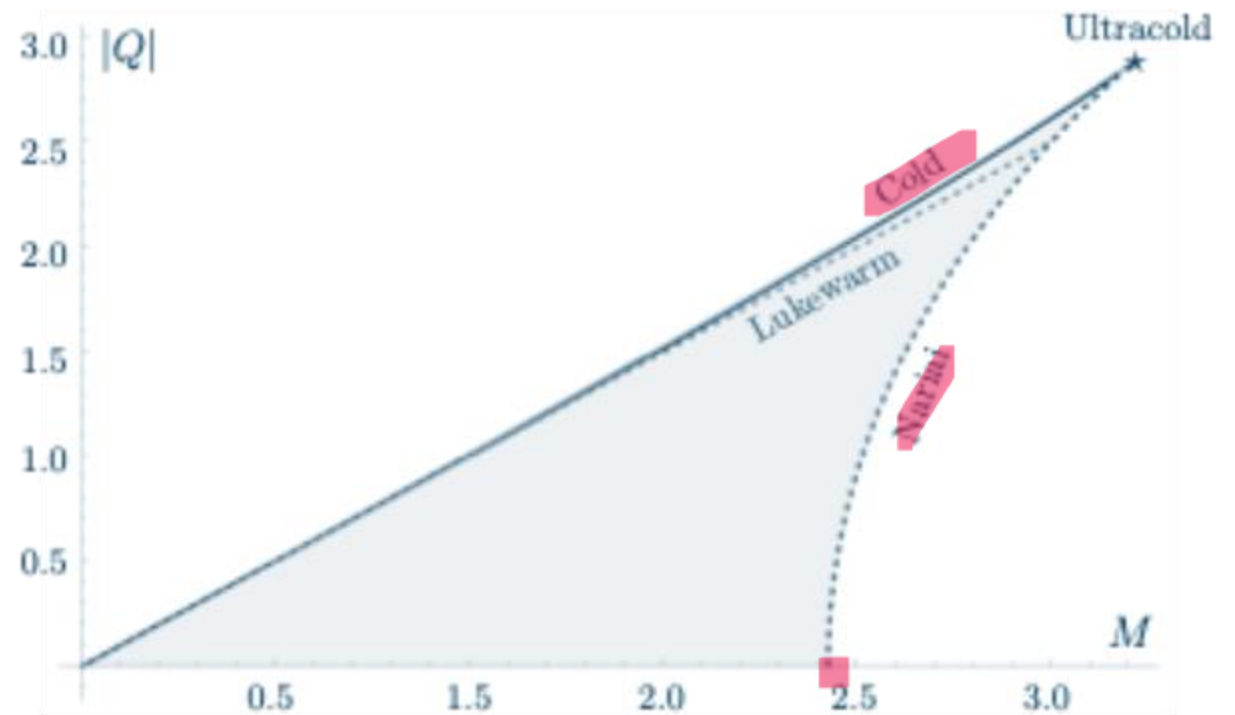


More details, see e.g., 2212.14356 [AC, Mariani,Toldo]

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Near-extremal GPI

$$\mathcal{Z} = \int_{\mathcal{M}} \mathcal{D}g \mathcal{D}A e^{-S_E(g,A)}$$

Near-extremal GPI

$$\mathcal{Z} = \int_{\mathcal{M}} \mathcal{D}g \mathcal{D}A e^{-S_E(g,A)} = e^{-I(g_e, A_e)} \underbrace{\int_{\mathcal{M}} \mathcal{D}h \mathcal{D}a e^{-S_{(1)}(h,a) + \dots}}_{\text{One-loop determinants}} + \dots$$

\downarrow

$$\begin{aligned} g &= g_{\text{extremal}} + h \\ A &= A_{\text{extremal}} + a \end{aligned}$$

Around near-horizon of extremal black holes, infinitely many normalizable zero modes (h^0, a^0) :

$$\int_{\mathcal{M}} \mathcal{D}h^0 \mathcal{D}a^0 = \infty$$

Near-extremal GPI

$$\mathcal{Z} = \int_{\mathcal{M}} \mathcal{D}g \mathcal{D}A e^{-S_E(g,A)} = e^{-I(g_e + \delta g, A_e + \delta A)} \int_{\mathcal{M}} \mathcal{D}h \mathcal{D}a e^{-S_{(1)}(h,a) + \dots} + \dots$$



$$\begin{aligned} g &= g_{\text{extremal}} + \delta g + h^0 + \delta h_T^0 \\ A &= A_{\text{extremal}} + \delta A + a^0 + \delta a_T^0 \end{aligned}$$

with $\delta g \sim O(T)$, $\delta a \sim O(T)$

Around near-horizon of **near**-extremal black holes, zero modes are lifted:

$$\int_{\mathcal{M}} \mathcal{D}h^0 \mathcal{D}a^0 e^{-\lambda_h(T) \langle h^0 | h^0 \rangle} e^{-\lambda_a(T) \langle a^0 | a^0 \rangle} \sim T^\#$$

with $\lambda_{h,a}(T) \sim O(T)$

Results

Extremal dS_4 BH	Tensor modes		Vector modes		U(1) modes		\mathcal{Z}_{low-T}
	$\langle h^0 h^0 \rangle$	$\lambda_h(T)$	$\langle h^0 h^0 \rangle$	$\lambda_h(T)$	$\langle a^0 a^0 \rangle$	$\lambda_a(T)$	
$AdS_2 \times S^2$ (+,+,+,+)							
$dS_2 \times S^2$ (-,-,+,+)							
$dS_2 \times S^2$ (-,-,-,-)							
$dS_2 \times S^2$ Q=0							

dS_2 JT sector

Results

Extremal dS ₄ BH	Tensor modes		Vector modes		U(1) modes		\mathcal{Z}_{low-T}
	$\langle h^0 h^0 \rangle$	$\lambda_h(T)$	$\langle h^0 h^0 \rangle$	$\lambda_h(T)$	$\langle a^0 a^0 \rangle$	$\lambda_a(T)$	
AdS ₂ × S ² (+,+,+,+)	+	+	+	+	+	+	$\sim T^{7/2}$
dS ₂ × S ² (-,-,+,+)							
dS ₂ × S ² (-,-,-,-)							
dS ₂ × S ² Q=0							

$$\mathcal{Z}_{low-T} \sim \int_{\mathcal{M}} \mathcal{D}h^0 \mathcal{D}a^0 e^{-\lambda_h(T)\langle h^0|h^0 \rangle} e^{-\lambda_a(T)\langle a^0|a^0 \rangle} + \dots$$

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dS ₂ × S ² (-,-,-,-)	+	+	+	+	-	-	...
dS ₂ × S ² Q=0	+	-	-	-			...

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Overview

Near-extremal black holes in dS_4 provide a sharp lab to connect with toy models in 2D and quantify the role of extra dimensions (matter).

- Cold branch: $AdS_2 \times S^2$
- Nariai: $dS_2 \times S^2$
- Ultracold: $Mink_2 \times S^2$

Quantum corrections of dS_4 BHs display pathologies that are not present for BHs in $Mink_4/AdS_4$. Resolving them could be important for the interpretation of the Euclidean GPI in de Sitter.

Conclusions



Future Directions

I. dS_3 quantum gravity coupled to matter

Higher form symmetries in QG

$\langle \log Z_{\text{scalar}} \rangle$ versus $\log \langle Z_{\text{scalar}} \rangle$

Microscopic interpretation: edge modes within

Quantum corrections to Wilson spool in AdS_3

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II. Quantum corrections for near-extremal black holes in dS_4

Microscopic interpretation: analytic continuations

Ultracold limit and its ties to $Mink_2$ gravity.

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Thank you!