UV constraints on gravitational EFTs

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Review talk, Strings 2025, Abu Dhabi

References are hyperlinks
[REF]

In this talk, I will assume that **the UV completion of gravity** obeys the standard rules of **quantum mechanics** and **causality**

$$S = \frac{1}{16\pi G_N}\int d^D x\sqrt{-g}\Big(R - 2\Lambda + \ldots\Big)$$

Goals:

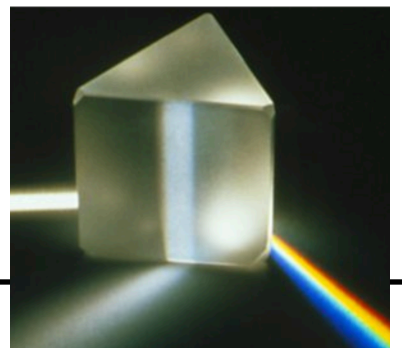
1. Motivate **twice-subtracted dispersion relations (2SDR)** in gravitational theories as an expression of **quantum causality**.
2. Review recent developments in deriving its consequences for the IR EFT.

Disclaimer: $\Lambda \leq 0$ ($AdS_{D\geq 4}$, $\mathbb{M}_{D>4}$).

[Algebraic approach panel]

Plan

1926-1927



Kronig
Kramers

UV gravity

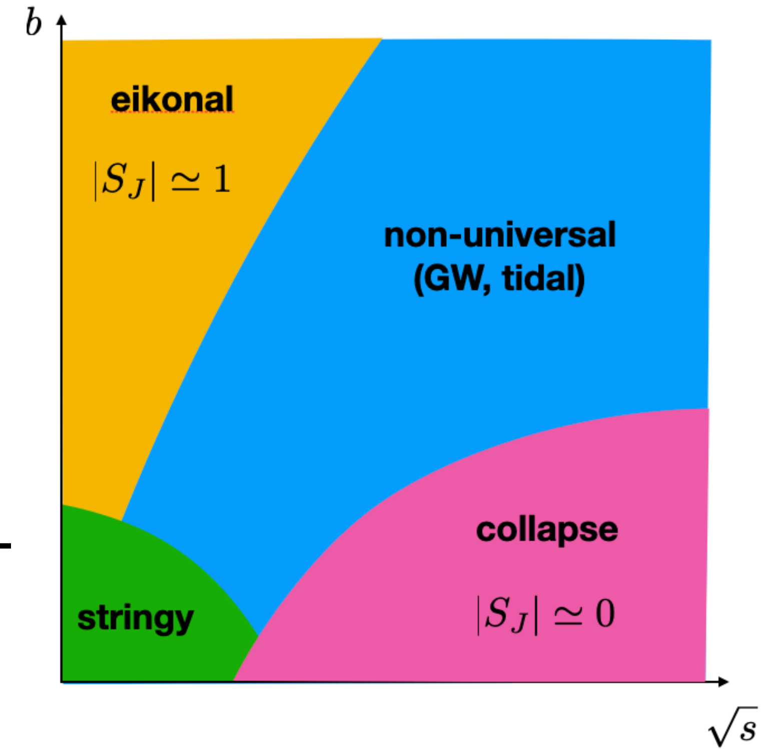
1972

Auberson
Khuri

1987

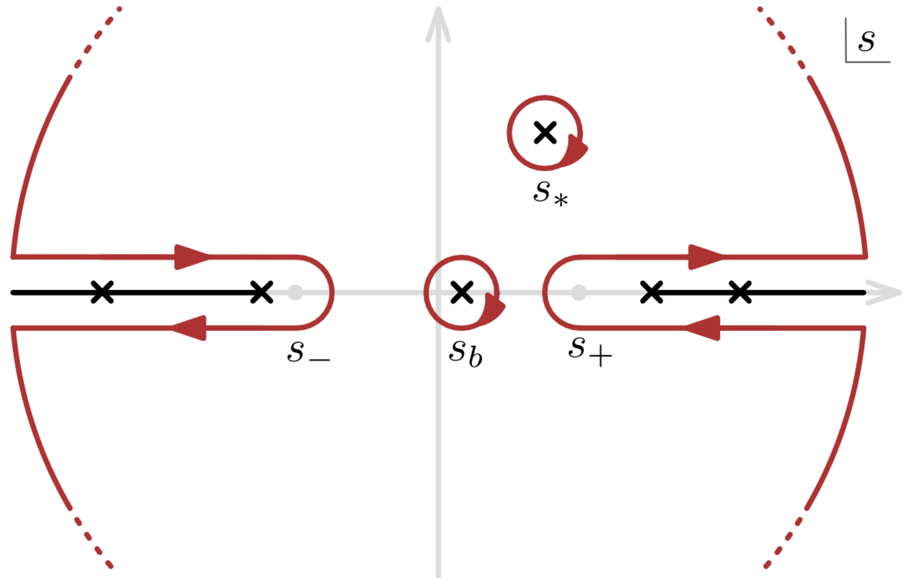
Gross
Mende
't Hooft

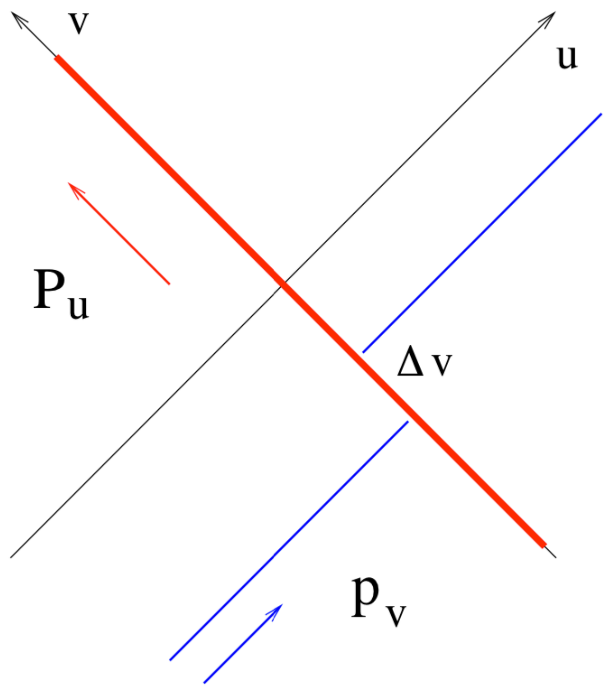
Amati
Ciafaloni
Veneziano



IR

$t = 0$	$t \neq 0$
1954	1956
Gell-Mann Goldberger Thirring	Bogolyubov
2006	2014
Adams et al.	CEMZ
$(\partial\phi)^4$	$\alpha_3(\text{Riem})^3$





2019
super convergence
AdS DR

2020
crossing (two-sided b)
higher moments

2021
graviton pole

2022
four gravitons where is M-theory?
WGC

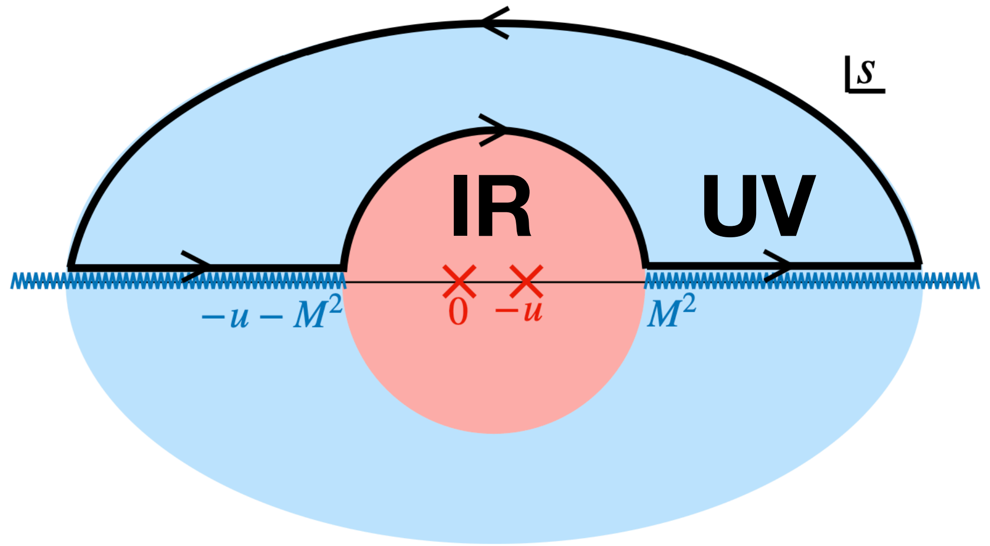
stringy amplitudes &correlators
where is tree-level string theory?

crossing-symmetric disp. rel
loops

2025
species bound/scale
Regge physics

...

Regge



gravitational dispersion relations: IR+UV=0

[Rastelli, Strings 2021]

Linear response DR

Titchmarsh theorem: A square-integrable function $S(t)$ is **causal**, i.e. zero for $t < 0$, iff its Fourier transform $S(\omega)$ satisfies **dispersion relations**.

[Toll]

$$f_{\text{out}}(t) = \int_{-\infty}^{t} dt' S(t-t') f_{\text{in}}(t)$$

$$S(\omega) \equiv \int_{0}^{\infty} dt e^{i\omega t} S(t)$$

$$\Longleftrightarrow$$

$$S(\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi i} \frac{S(\omega')}{\omega' - \omega}$$

$$\lim_{\omega \to \infty} S(\omega) = 0$$

No response occurs before the input signal

Analyticity in the upper-half plane $\mathrm{Im}\omega \geq 0$

Example: the Kramers-Kronig relations (a link between how a system absorbs and refracts light).

dispersion

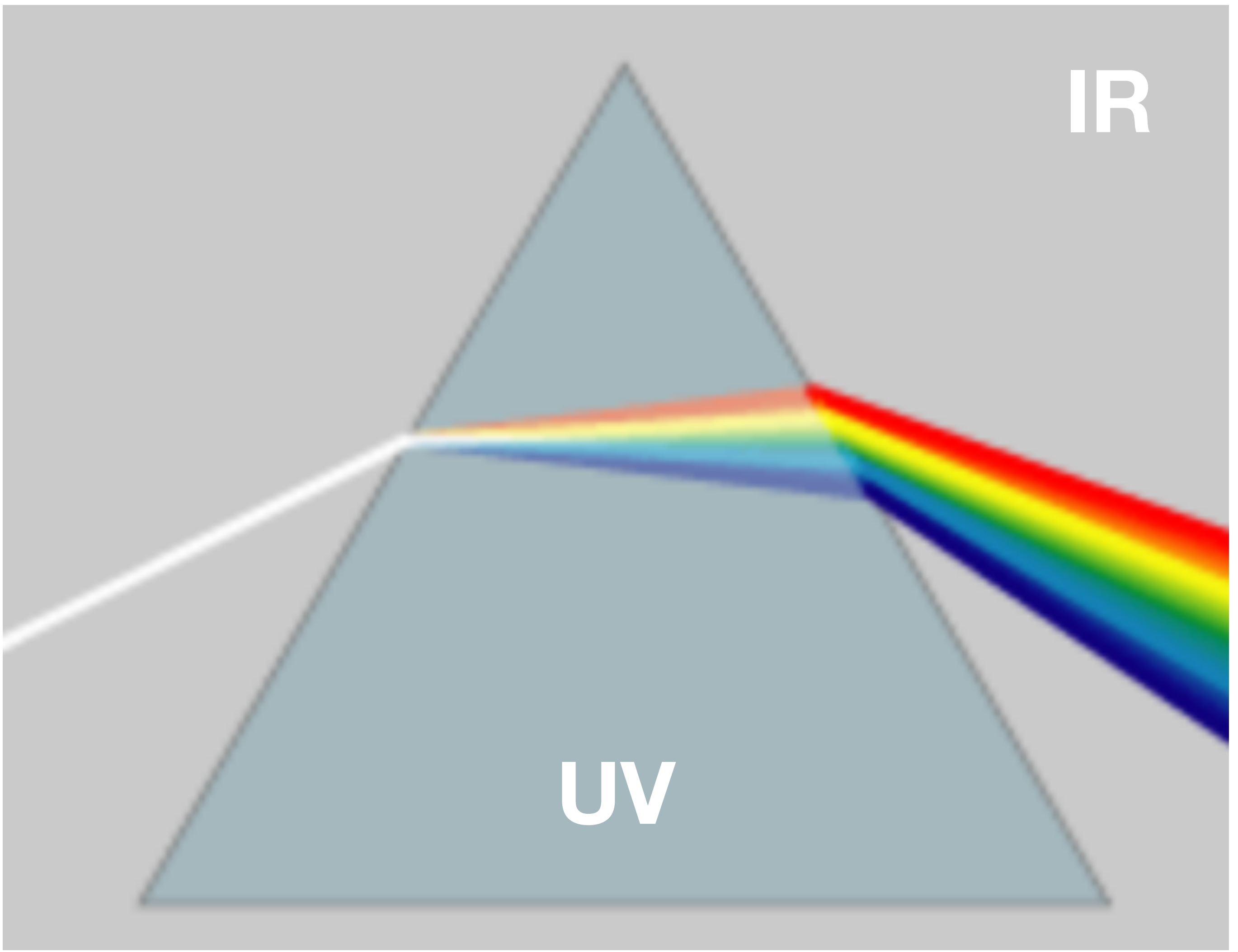
$$n(\omega) = 1 + \lim_{\epsilon \to 0} \int \frac{d\omega'}{\pi} \frac{\alpha(\omega')}{\omega'^2 - (\omega + i\epsilon)^2}$$

absorption

$$\alpha(\omega') \geq 0$$

$$= \left(1 + \underbrace{\int_{\omega_{\rm UV}}^{\infty} \frac{d\omega'}{\pi} \frac{\alpha(\omega')}{\omega'^2}}_{\geq 0}\right) + \omega^2 \underbrace{\int_{\omega_{\rm UV}}^{\infty} \frac{d\omega'}{\pi} \frac{\alpha(\omega')}{\omega'^4}}_{\geq 0} + \omega^4 \underbrace{\int_{\omega_{\rm UV}}^{\infty} \frac{d\omega'}{\pi} \frac{\alpha(\omega')}{\omega'^6}}_{\geq 0} + \ldots$$

causality+UV absorption: $n_{blue} > n_{green} > n_{red}$

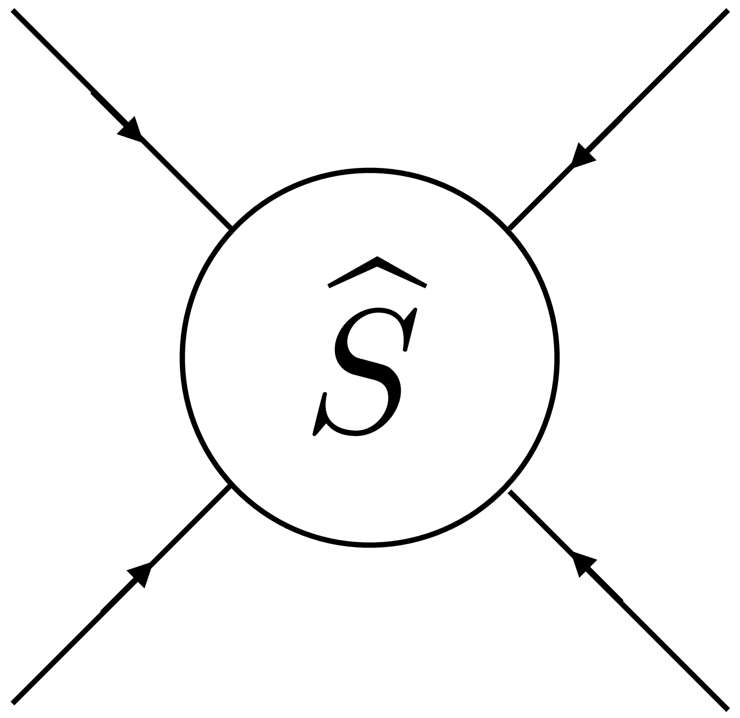


Positivity bounds on electromagnetic properties of media

[Creminelli, Janssen, Salehian, Senatore '24]

Gapped QFT DR

$s = -(p_1+p_2)^2$
$t = -(p_1+p_3)^2$



Scattering amplitude $T(s,t)$ obeys twice-subtracted dispersion relations (2SDR)

the derivation is subtle

1. LSZ+Wightman axioms (microcausality, temperedness)

$T(s,t) \sim \langle p_1 | [\mathcal{O}(x_2), \mathcal{O}(x_4)] | p_3 \rangle$

2. DR for unphysical masses

[Gell-Mann, Goldberger, Thirring '54]
[Bogolyubov '56]

3. Analytic completion (nontrivial for functions of 2 variables)

4. Unitarity (two subtractions)

$\widehat{S}\widehat{S}^\dagger = 1$ [Jin, Martin '64]

"...it is my profound opinion that that ***there must exist a representation of scattering amplitudes*** *such that the* ***dispersion relations appear in a much more transparent way****. Finding this representation is left as an a exercise to a student."* **Goldberger**

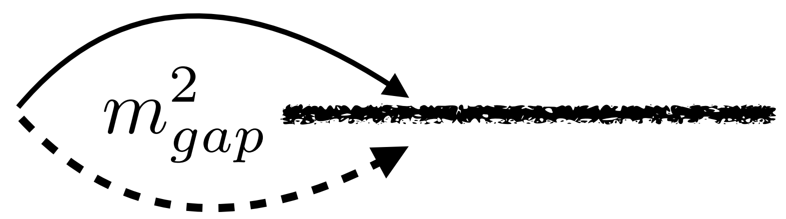
[UNIVERSE+: Positive Geometry in Particle Physics and Cosmology] [Simons collaboration on celestial holography]

discontinuity/imaginary part

Result (2SDR): $$T(s,t) = g(t) + \frac{1}{\pi}\int_{m_{gap}^2}^{\infty} ds' \frac{T_s(s',t)}{s'^2}\left(\frac{s^2}{s'-s} + \frac{u^2}{s'-u}\right)$$

subtraction term **2SDR** **$s-u$ crossing**

$$\lim_{|s|\to\infty} \frac{T(s,t)}{s^2} = 0$$



$$s+t+u=4m^2$$

Comment: s-u crossing is manifest. s-t crossing is non-manifest, this leads to extra equations (**null constraints**).

$$T(s,t) = T(u,t)$$ ✓

$$T(s,t) = T(t,s)$$ ✗ **extra sum rules**

[Roy '71]
[Auberson, Khuri '72]

[Tolley, Wang, Zhou '20]
[Caron-Huot, Van Duong '20]
[Arkani-Hamed, Huang, Huang '20]
[Sinha, Zahed '20]

QFT: example

Example: Sign of $(\partial\phi)^4$

[Adams,Arkani-Hamed,Dubovsky,Nicolis,Rattazzi '06]
[Komargodski, Schwimmer '11]

$$T(s,t) = g(0) + \int_{m^2_{gap}}^{\infty} \frac{ds'}{\pi} \frac{T_s(s',0)}{s'^2}(s^2+t^2+u^2) + \ldots \qquad T_s(s',0) \geq 0$$

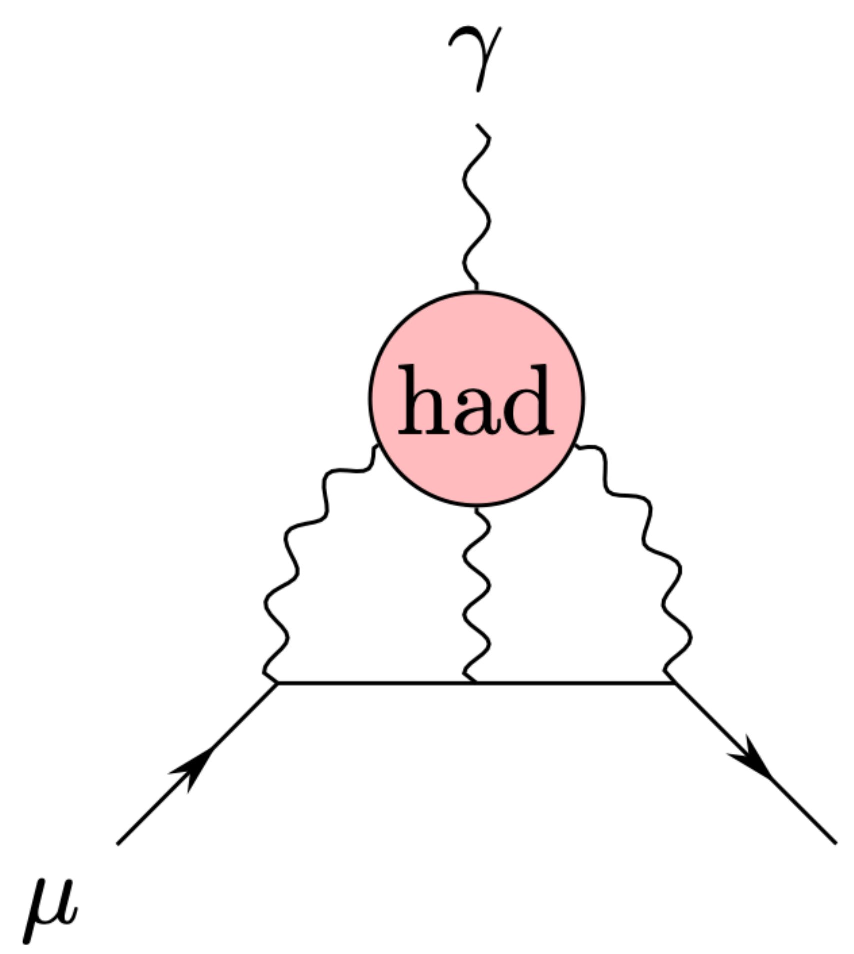
≥ 0

QM/unitarity

higher moments [Bellazzini, Miro, Rattazzi, Riembau, Riva '19]

Example: Light-by-light scattering for a_μ

[Colangelo,Hoferichter,Procura,Stoffer '17]
[Hoferichter,Stoffer,Zillinger '24]



$$\langle 0|T\left\{j^\mu_{\rm em}(x)j^\nu_{\rm em}(y)j^\lambda_{\rm em}(z)j^\sigma_{\rm em}(0)\right\}|0\rangle$$

$$a_\mu^{\rm HLbL} = 101.9(7.9)\times 10^{-11}.$$

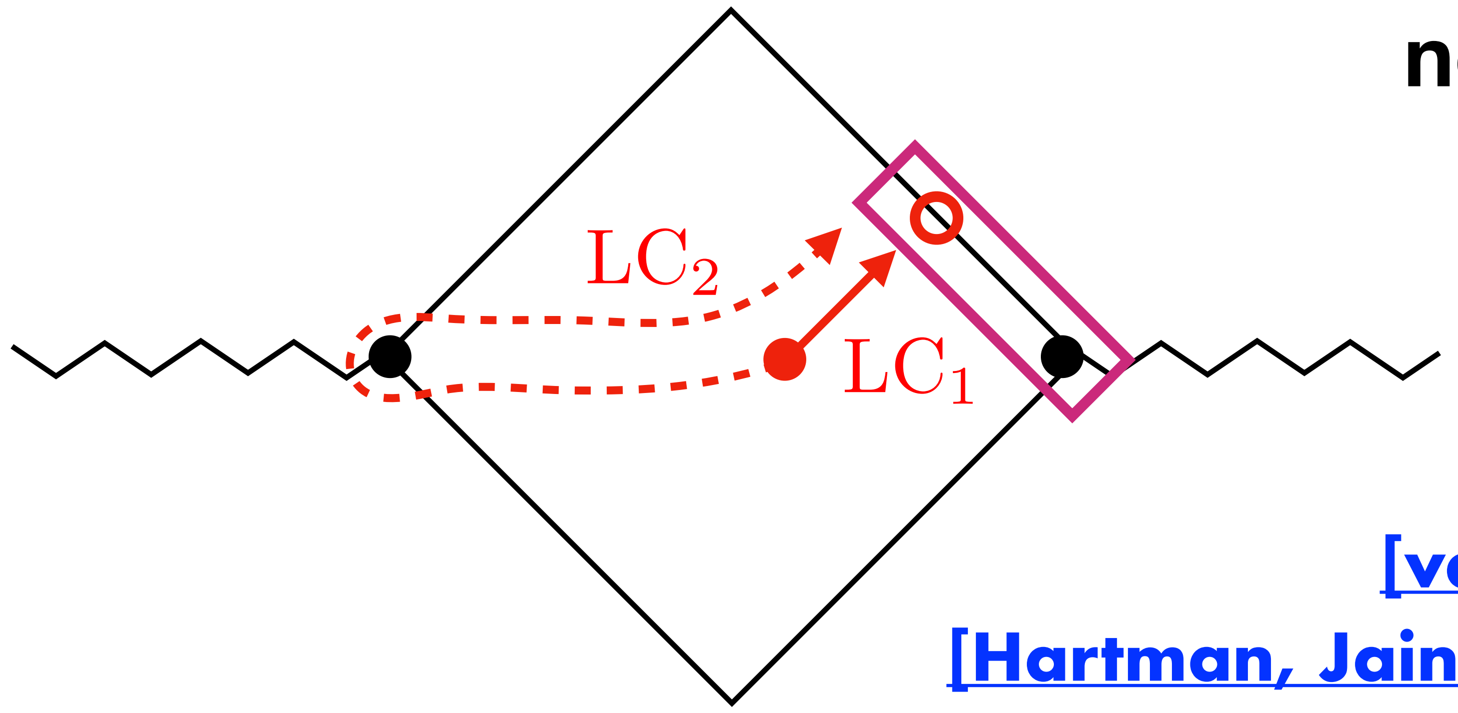
+combined with experimental data

CFT DR

CFT axioms + **LC OPE on the second sheet** ➡ CFT DR

[Kravchuk, Rychkov, Qiao '21]

not proven



[van Rees '24]
[Hartman, Jain, Kundu '16]

[Carmi, Caron-Huot '19]
[Mazac, Rastelli, Zhou '19]
[Penedones, Silva, AZ '19]

superconvergence

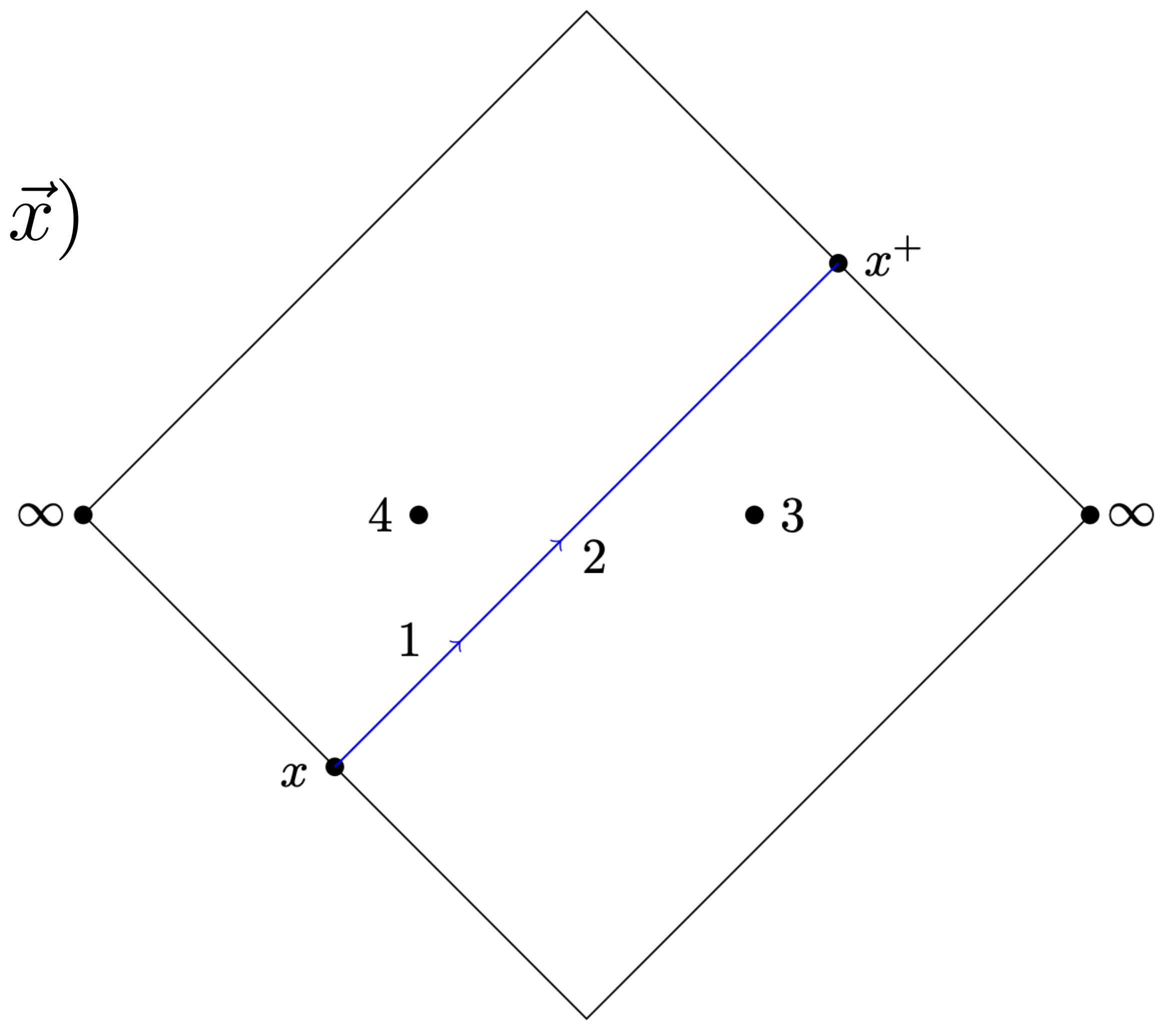
$$\langle \phi_4(x_4)[\mathcal{E}(\vec{x}_1),\mathcal{E}(\vec{x}_2)]\phi(x_3)\rangle = 0 \qquad \mathcal{E}(\vec{x}) = \int_{-\infty}^{\infty} dx^- T_{--}(0,x^-,\vec{x})$$

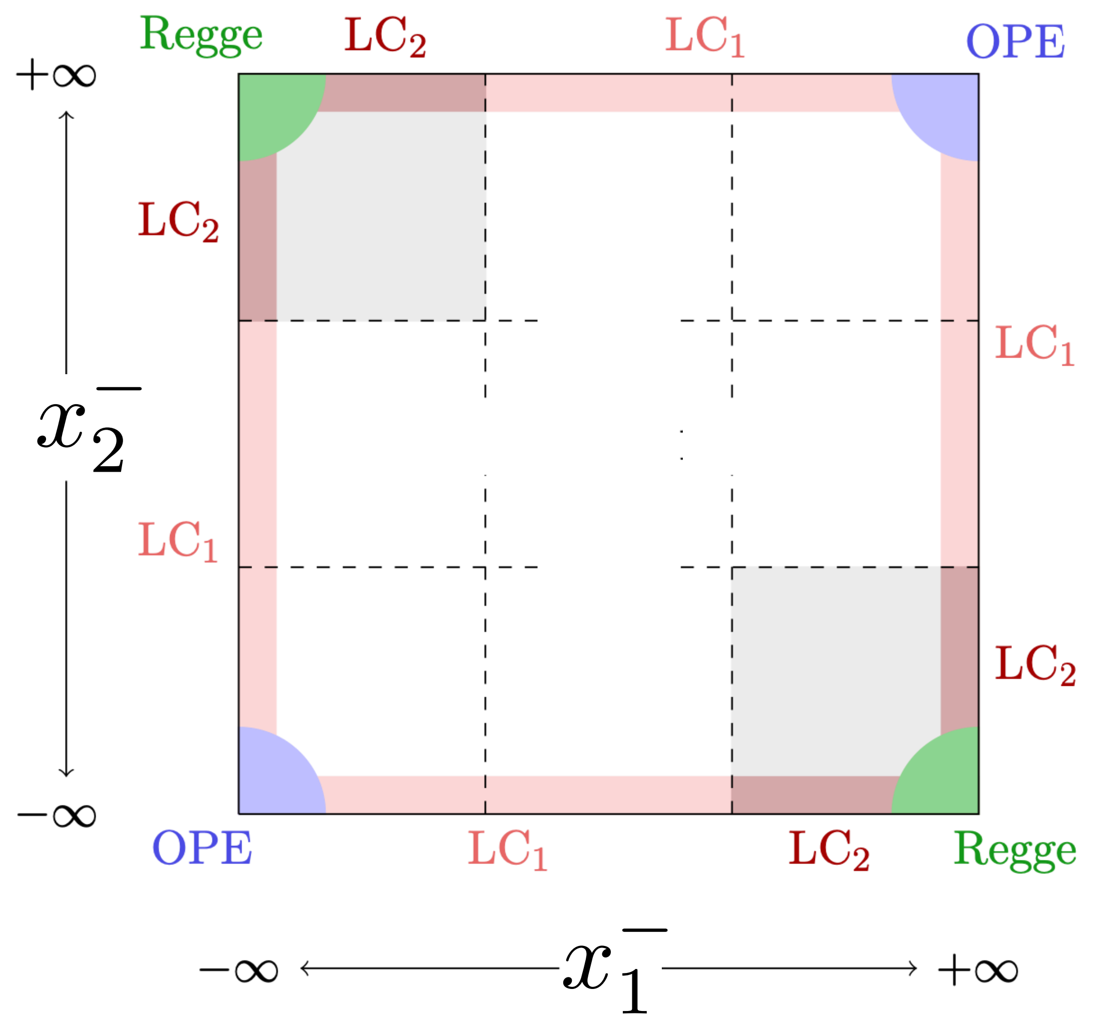
$$\langle \phi_4(x_4)\mathcal{E}(\vec{x}_1)\mathcal{E}(\vec{x}_2)\phi(x_3)\rangle = \langle [\phi_4(x_4),\mathcal{E}(\vec{x}_1)][\mathcal{E}(\vec{x}_2),\phi(x_3)]\rangle$$

$$-$$

double discontinuity

$$\langle \phi_4(x_4)\mathcal{E}(\vec{x}_2)\mathcal{E}(\vec{x}_1)\phi(x_3)\rangle = \langle [\phi_4(x_4),\mathcal{E}(\vec{x}_2)][\mathcal{E}(\vec{x}_1),\phi(x_3)]\rangle$$



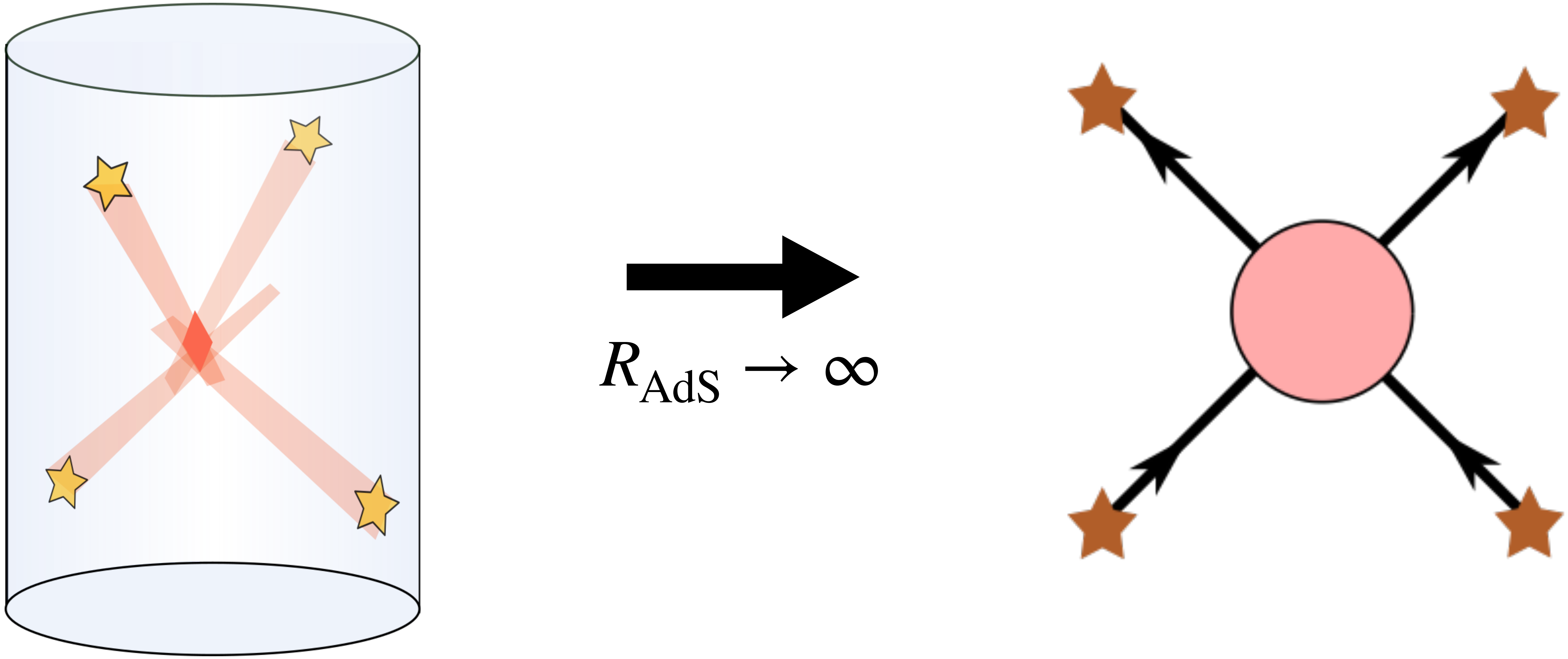


[Kologlu, Kravchuk, Simmons-Duffin, AZ '19] [Caron-Huot,Mazac,Rastelli,Simmons-Duffin '21]

AdS/CFT

In the flat space limit CFT DR produce the expected (smeared) 2SDR

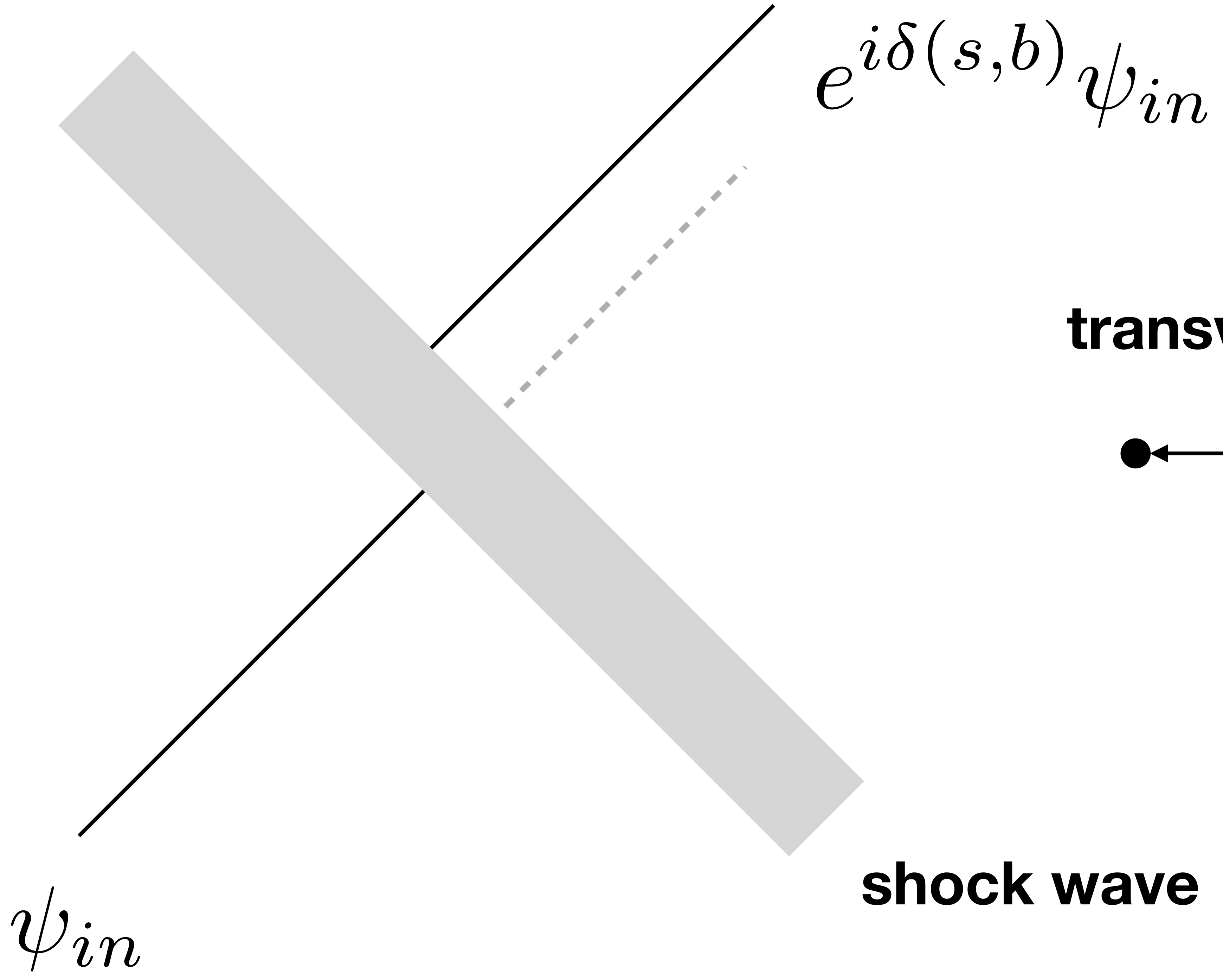
[Caron-Huot, Mazac, Rastelli, Simmons-Duffin '21]



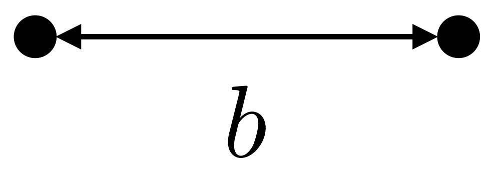
nonperturbative status of the flat space limit is not understood

[van Rees, Zhao '23]

Shockwave S-matrix DR



transverse plane



[Dray, 't Hooft '85]
['t Hooft '87]

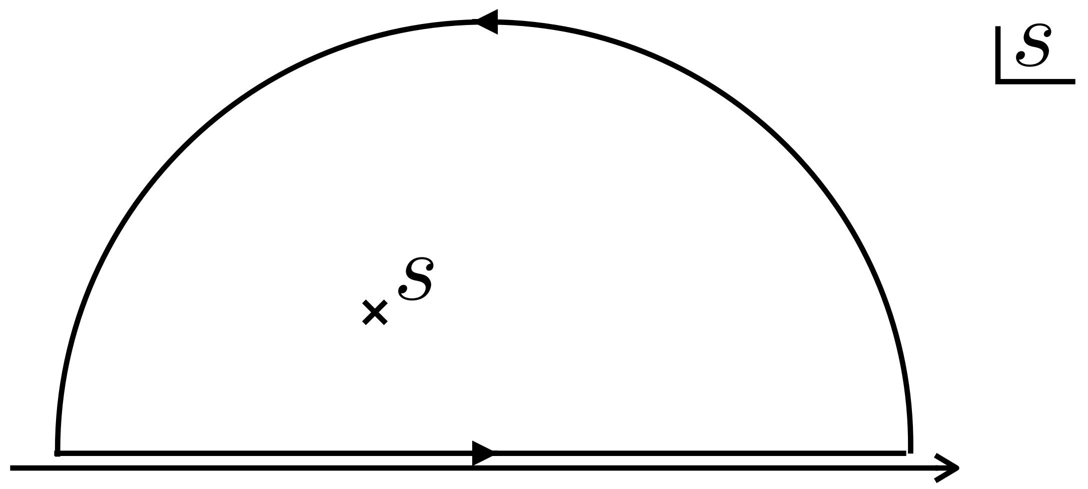
$$\delta(s,b) \sim \frac{G_N s}{b^{D-4}} > 0$$

Shapiro time delay/asymptotic causality

FT of the graviton pole

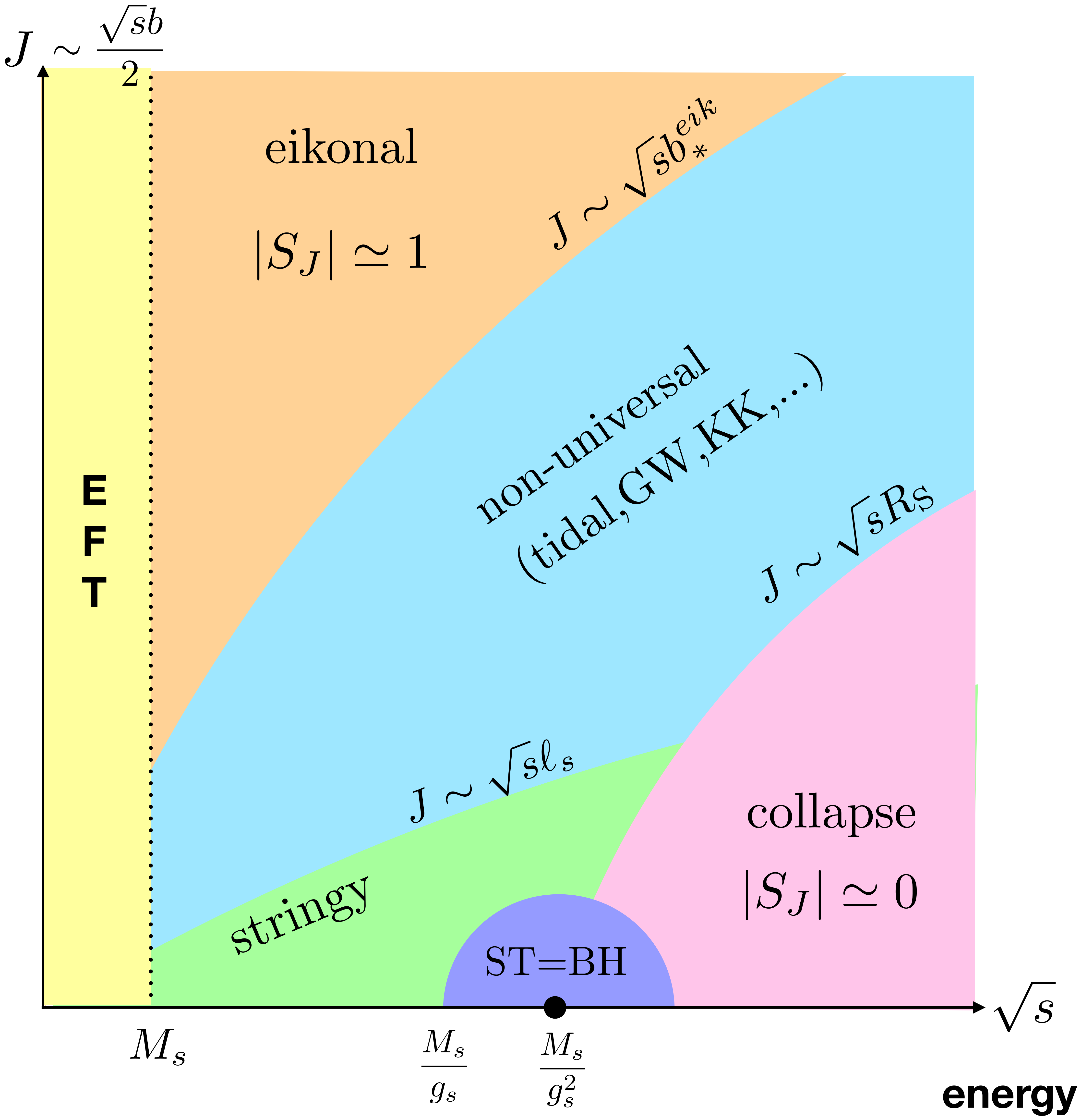
Phase shift dispersion relation

$$i(1 - e^{2i\delta(s,b)}) = \int_0^\infty \frac{ds'}{\pi} \frac{2\sin^2\delta(s',b)}{s'} \left(\frac{s}{s'-s} + \frac{s}{s'+s}\right)$$

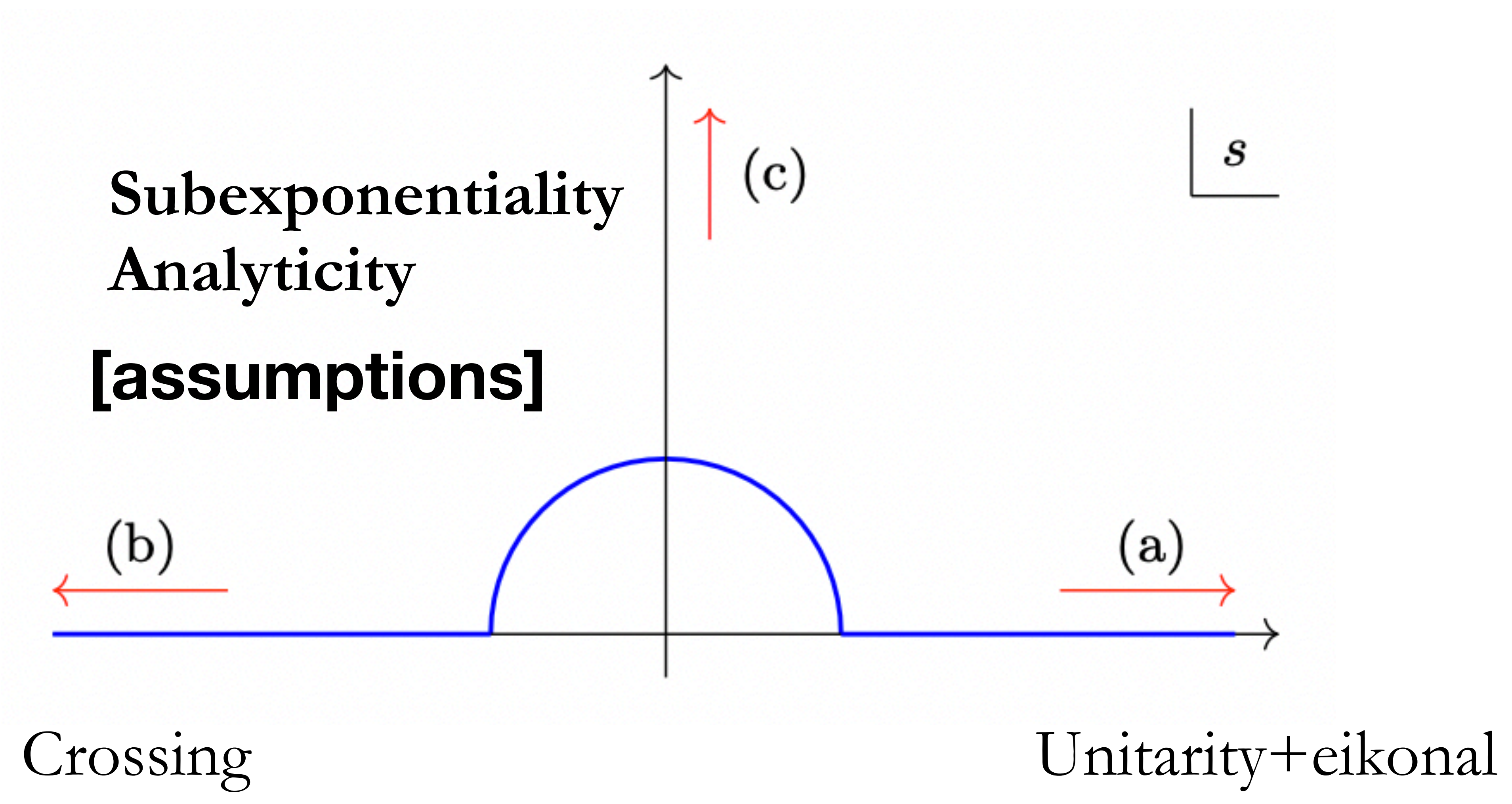


***(not true in dS, true in AdS)** [Bittermann, McLoughlin, Rosen '22]

Gravitational S-matrix



[Amati, Ciafaloni, Veneziano '87]
[di Vecchia, Heissenberg, Russo, Veneziano '23]
[Rothstein,Saavedra '24]



$$\lim_{|s|\to\infty} \frac{T(s,t)}{s^2} = 0 \qquad t < 0$$

➡ 2SDR in gravity

[Haring, AZ '22]

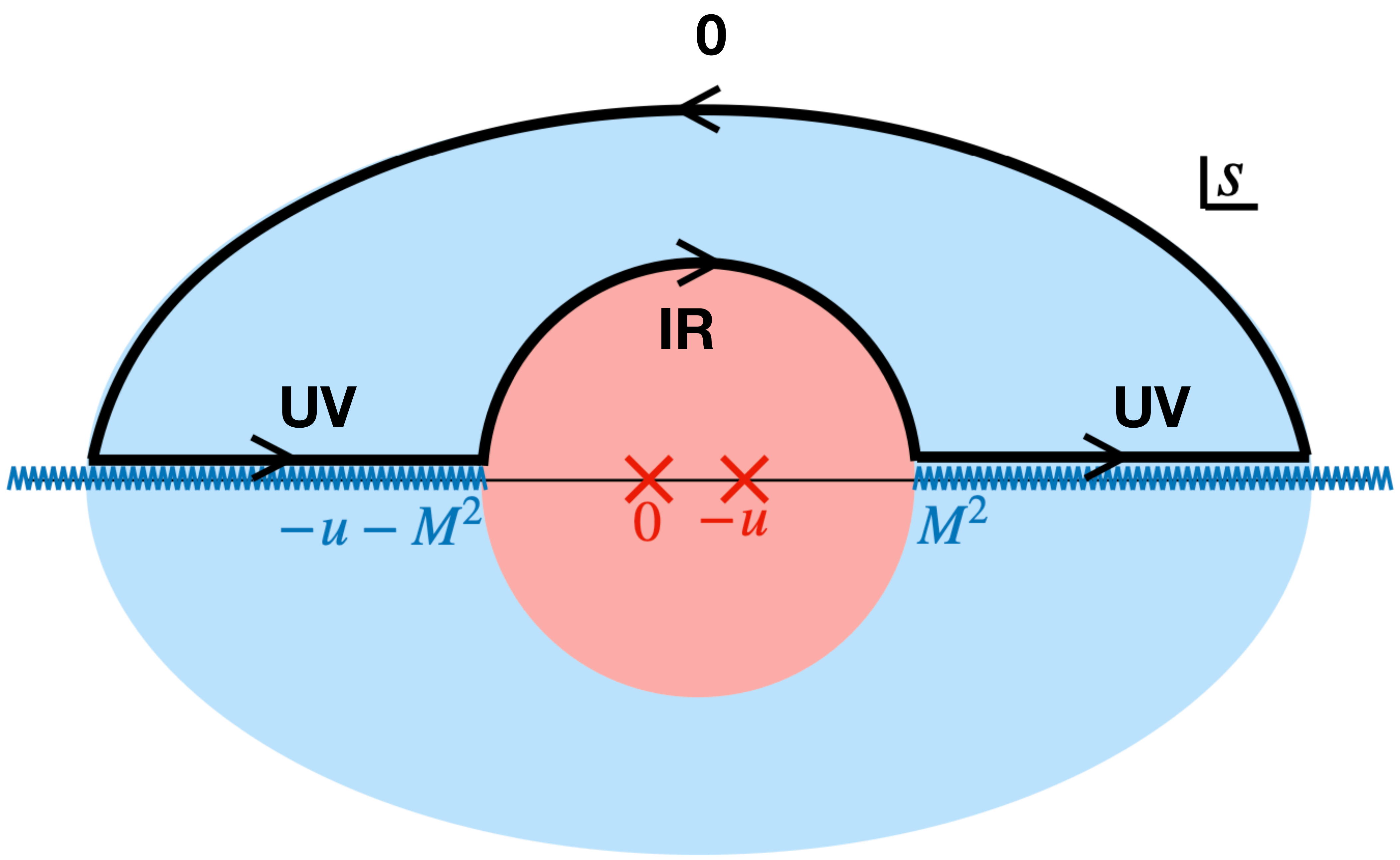
"We have found **no evidence for a lack of harmony** between gravity and the basic S-matrix properties." [Giddings, Porto '09]

Quantum causality (ACU) $\rightarrow$2SDR

ACU=analyticity+crossing+unitarity

not fully proven (yet)
no counterexamples

What are the **lessons** from quantum causality/dispersion relations?



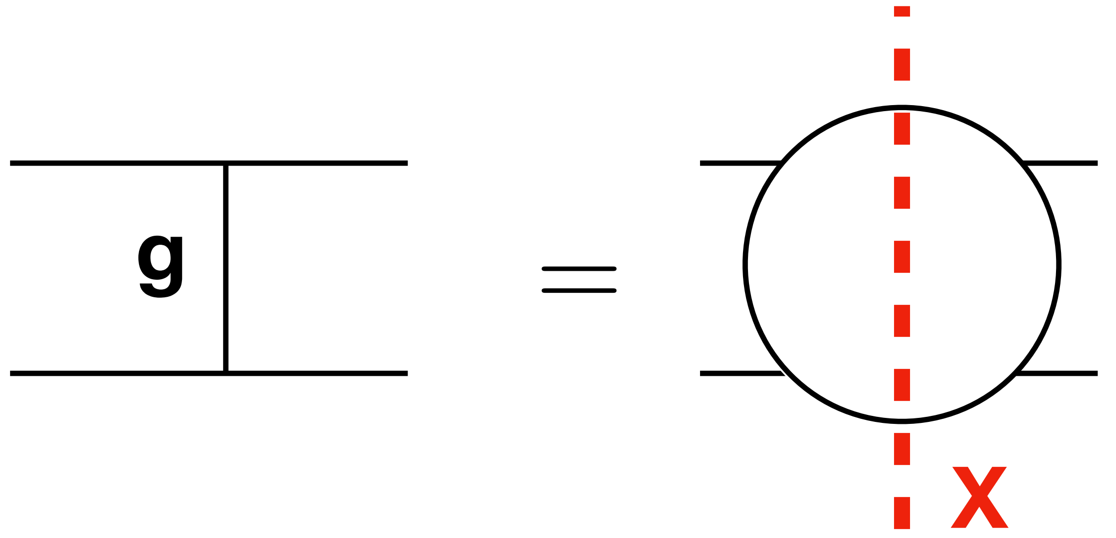
$$\oint_{\infty} \frac{ds}{2\pi i} f(s,t) T(s,t) = 0$$

IR+UV=0

$$S = \frac{1}{16\pi G_N} \int d^D x \sqrt{-g} \left(R - 2\Lambda + \ldots \right)$$

GN is the UV budget

The graviton pole comes from the UV



graviton has spin 2

2SDR

$$\frac{8\pi G_N s^2}{-t} + \cdots = s^2 \int_0^\infty \frac{ds'}{\pi} \frac{2T_s(s',t)}{(s')^3}, \qquad t < 0$$

Example: stringy graviton $\qquad T_s(s',t) \sim (s')^{2+\frac{\alpha' t}{2}} \qquad \alpha'|t| \gtrsim \frac{1}{\log 1/g_s^2}$

Example: eikonal $\qquad T_s(s, t = -\vec{q}^2) \sim s \int d^{D-2}\vec{b} e^{i\vec{b}\vec{q}} \sin^2 \delta(s,b) \qquad \alpha'|t| \lesssim \frac{1}{\log 1/g_s^2}$

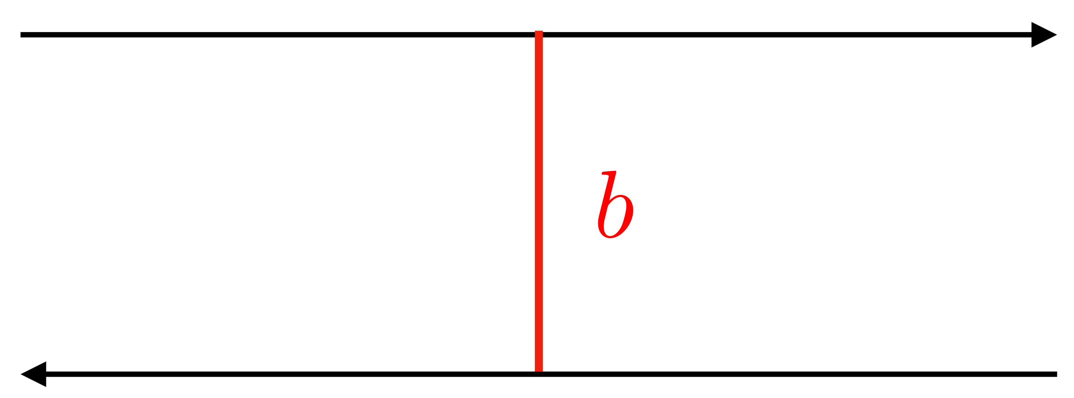
Unitarity: $\qquad T_s(m^2,t) = \sum_{J=0}^{\infty} \mathrm{Im}\, a_J(m^2) P_J\left(1 + \frac{2t}{m^2}\right), \quad 2 \geq \mathrm{Im}\, a_J(m^2) \geq 0\ .$

GN is the UV budget

The solution is to **smear** the amplitude

$$T_\psi(s) \equiv \int_0^{q_0} dq\,\psi(q)\,T(s,t=-q^2)$$

scattering experiment



$b \lesssim 1/q_0$

makes Legendres positive

$$\int_0^{q_0} dq\,\psi(q) P_J\left(1-\frac{2q^2}{m^2}\right) \geq 0$$

[Caron-Huot,Mazac,Rastelli,Simmons-Duffin '21]

In this way, we can get the following (schematic) equation for graviton scattering

$$G_N = \int_0^\infty \frac{ds'}{\pi} \text{nonnegative}_\psi(s')$$

discontinuity of the amplitude

G_N is the overall UV budget.

Example: For free matter, find $\psi(q)$ such that [Caron-Huot,Li '24]

$$G_N = c_s N_s G_N^2 \Lambda_{\mathrm{EFT}}^{d-2} + \int_{\Lambda_{\mathrm{EFT}}}^{\infty} ds' \, \# \quad \Rightarrow \quad c_s N_s G_N \Lambda_{\mathrm{EFT}}^{d-2} \leq 1$$

integrated out matter fields — **non-negative by unitarity** — **species bound** [Dvali '07]

Example: Graviton pole generation in string theory [Haring,AZ '24]

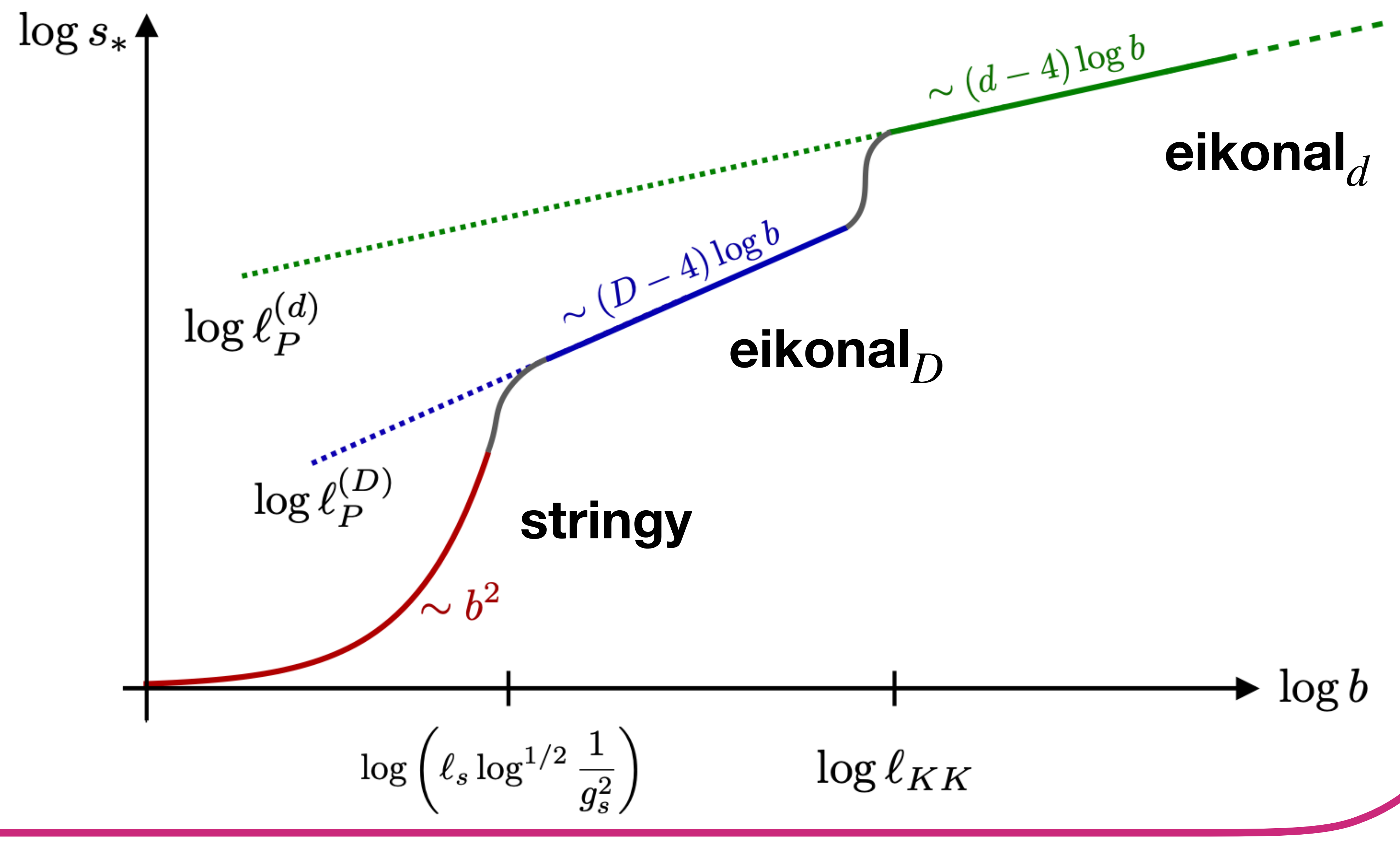
$$\int_0^\infty \frac{ds}{s^2} \, \mathrm{Im}\, a(s,b) \sim \frac{1}{2} \pi^{3-\frac{D}{2}} \Gamma\left(\frac{D-4}{2}\right) \frac{G_N}{b^{D-4}}$$

$$\frac{\int_0^{\Lambda_o} s^{-\frac{D}{2}} ds \, \mathrm{Im}\, a_J(s)}{\int_0^{\Lambda_o} s^{-\frac{D}{2}} ds \, \mathrm{Im}\, a_{J-2}(s)} \gtrsim \mathcal{O}(1)$$

species scale/higher-spin onset

[Dvali '07] [Caron-Huot,Li '24]

[van Beest,Calderon-Infante,Mirfendereski,Valenzuela '21]



Conjecture: QFT DoF cannot generate the graviton pole. [Weinberg, Witten '80]

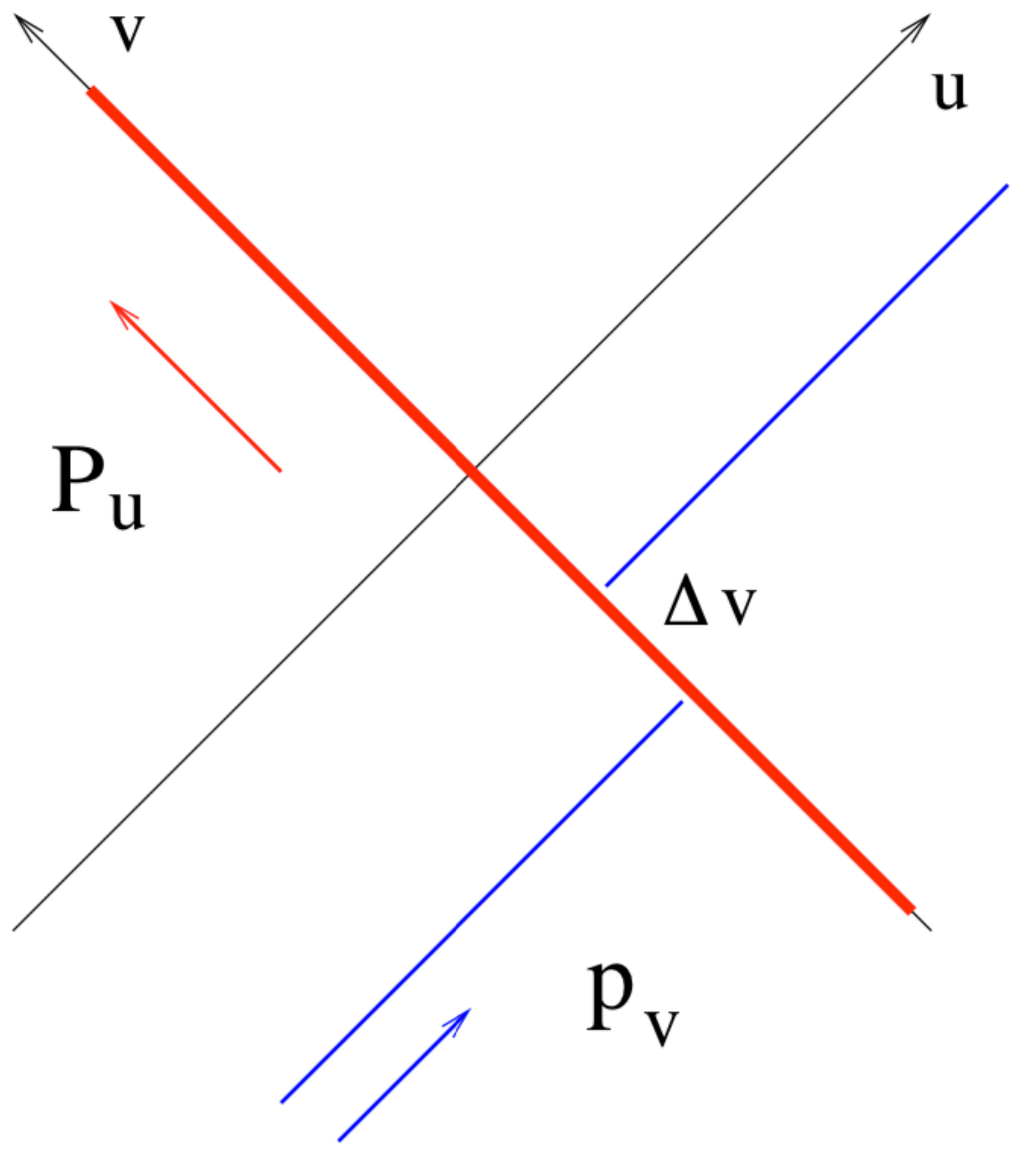
same story for $\Lambda < 0$

$$S = \frac{1}{16\pi G_N} \int d^D x \sqrt{-g} \left(R - 2\Lambda + \alpha_2 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \ldots \right)$$

Higher derivatives = massive higher spins

What controls corrections to general relativity?

$$S = \frac{1}{16\pi G_N}\int d^D x\sqrt{-g}\Big(R - 2\Lambda + \alpha_2 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \dots\Big)$$



$b^2 \sim |\alpha_2|$

$|\alpha_2| \lesssim \frac{1}{M_{\rm HS}^2}$

[Camanho, Edelstein, Maldacena, AZ '14]

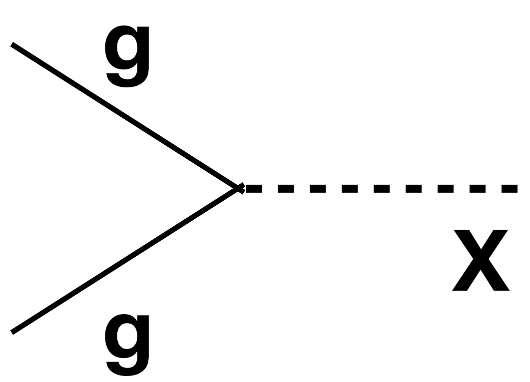
1. Tensor structures 4-point graviton amplitude

[Chowdhury, Gadde, Gopalka, Halder, Janagal, Minwalla '20]

10 parity-even tensor structures in $D \geq 8$

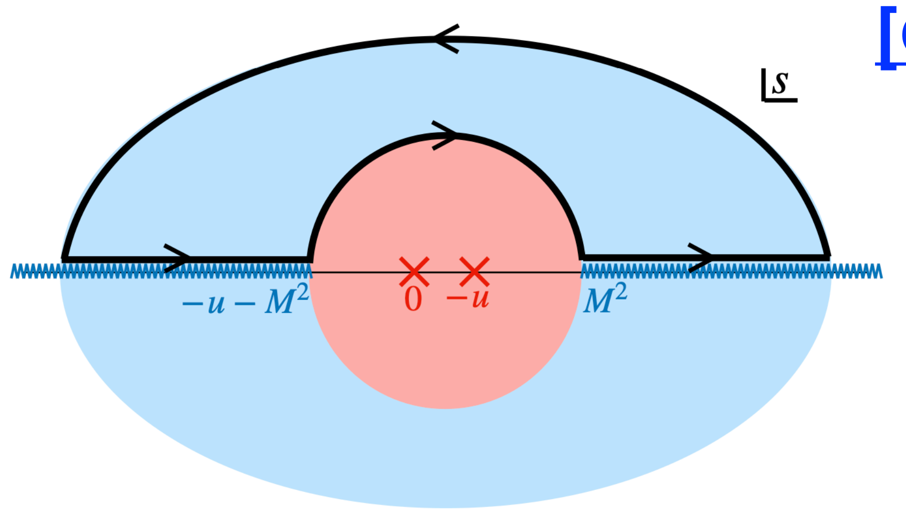
2. Partial wave expansion

20 graviton-graviton-X couplings



3. Dispersive sum rules/(Cauchy projections)

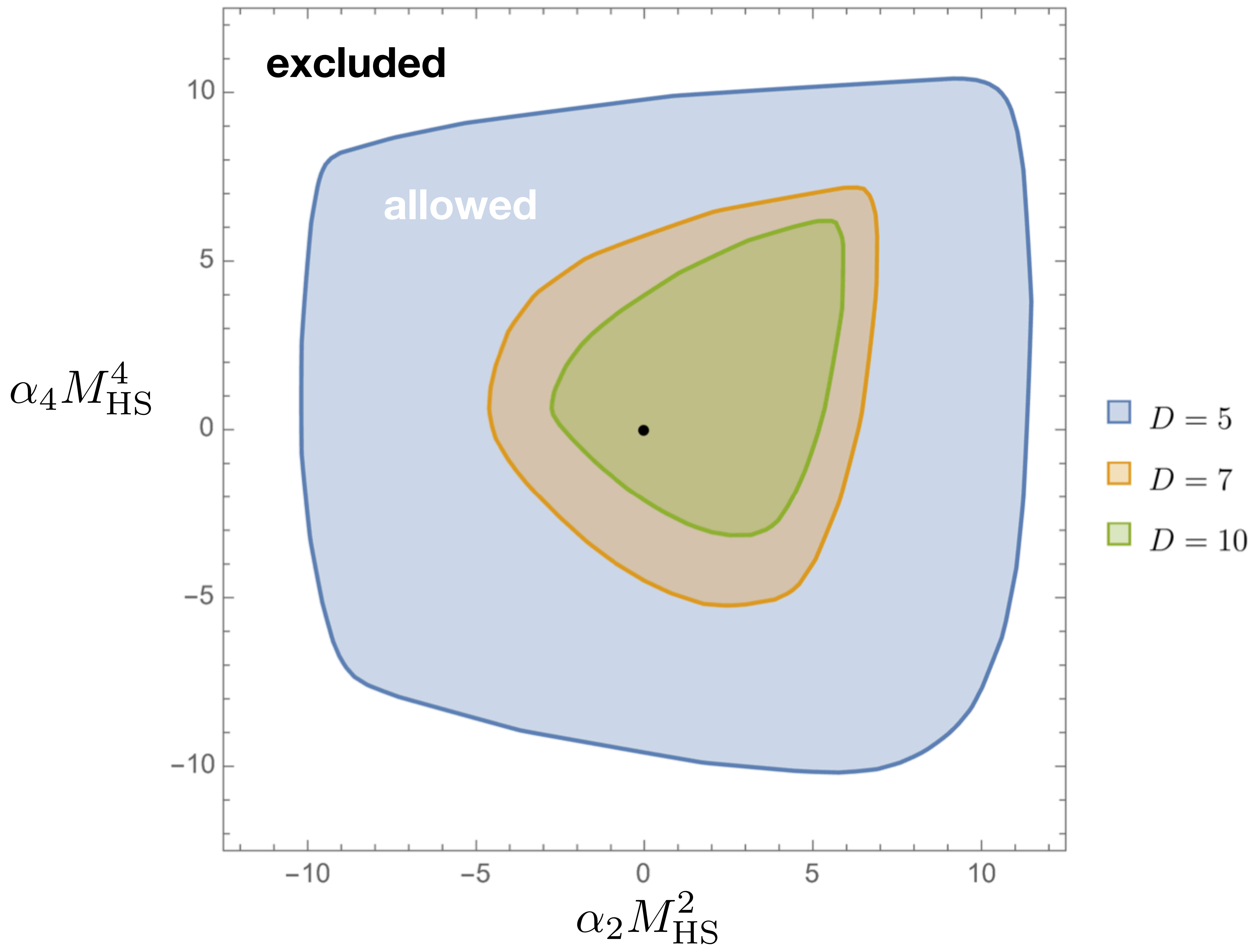
[Caron-Huot, Li, Parra-Martinez, Simmons-Duffin '22]



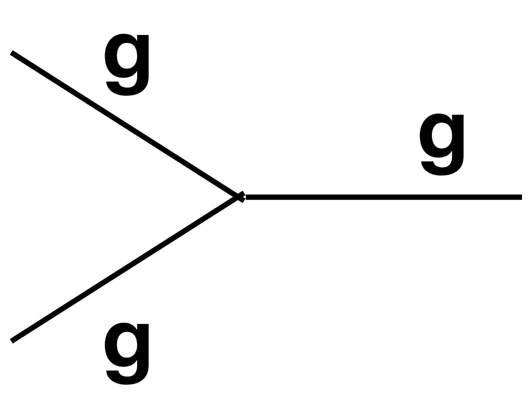
crossing-improved

[Caron-Huot, Li, Parra-Martinez, Simmons-Duffin '22]

$$S=\int \frac{d^Dx\sqrt{-g}}{16\pi G}\left(R+\frac{\alpha_2}{4}C^2+\frac{\alpha_4}{12}C^3+\frac{\alpha_4'}{6}C'^3+\dots\right)$$



three-graviton coupling



GR $\boxed{\bullet}\boxed{\bullet}$ $(e_1\cdot e_2)^2$

α_2 $\boxed{e_1}\boxed{e_2}\boxed{\bullet}\boxed{\bullet}$ $e_1\cdot e_2$

α_4 $\boxed{e_1}\boxed{e_1}\boxed{e_2}\boxed{e_2}\boxed{\bullet}\boxed{\bullet}$

$M_{\rm HS}$ mass of the lightest higher-spin state

$$\left|\frac{a-c}{c}\right|\leq\frac{23}{\Delta_{\rm HS}^2}+\mathcal{O}(1/\Delta_{\rm HS}^4)$$

rewriting the D=5 result in 4d CFT terms

[Heemskerk, Polchinski, Penedones, Sully '09]

HS states* = budget for higher derivatives**

* particles or loops of many species

**3pt coupling + contact terms

$$S = \frac{1}{16\pi G_N} \int d^D x \sqrt{-g} \Big(R + \alpha \ell_{Pl}^6 \; t_8 t_8 R^4 + \ldots \Big)$$

Minimal correction to GR

There are situations when $\alpha_2 = \alpha_3 = 0$ (maximal SUSY).

$$S = \frac{1}{16\pi G_N} \int d^D x \sqrt{-g} \Big(R + \alpha \ell_{Pl}^6 \, t_8 t_8 R^4 + \ldots \Big)$$

[Gross, Witten, Green, Vanhove, Gutperle, Russo, ...]

$$\frac{T(s,t,u)}{8\pi G_N} = s^4 \left(\frac{1}{stu} + \alpha\, \ell_P^6 + \ldots \right)$$

$$\alpha \ell_P^6 = \frac{2}{\pi} \int_0^\infty \frac{ds}{s^5} T_s(s,0)$$

$$\alpha \geq 0$$

$$\frac{T(s,t,u)}{8\pi G_N} = s^4 \Big(\underbrace{\frac{1}{stu}}_{\texttt{SUGRA}} + \underbrace{\prod_{A=s,t,u} (\rho_A + 1)^2 \sum_{a+b+c \leq N}{}' \alpha_{(abc)} \rho_s^a \rho_t^b \rho_u^c}_{\texttt{UV completion}} \Big)$$

[Paulos, Penedones, van Rees, Vieira '17]

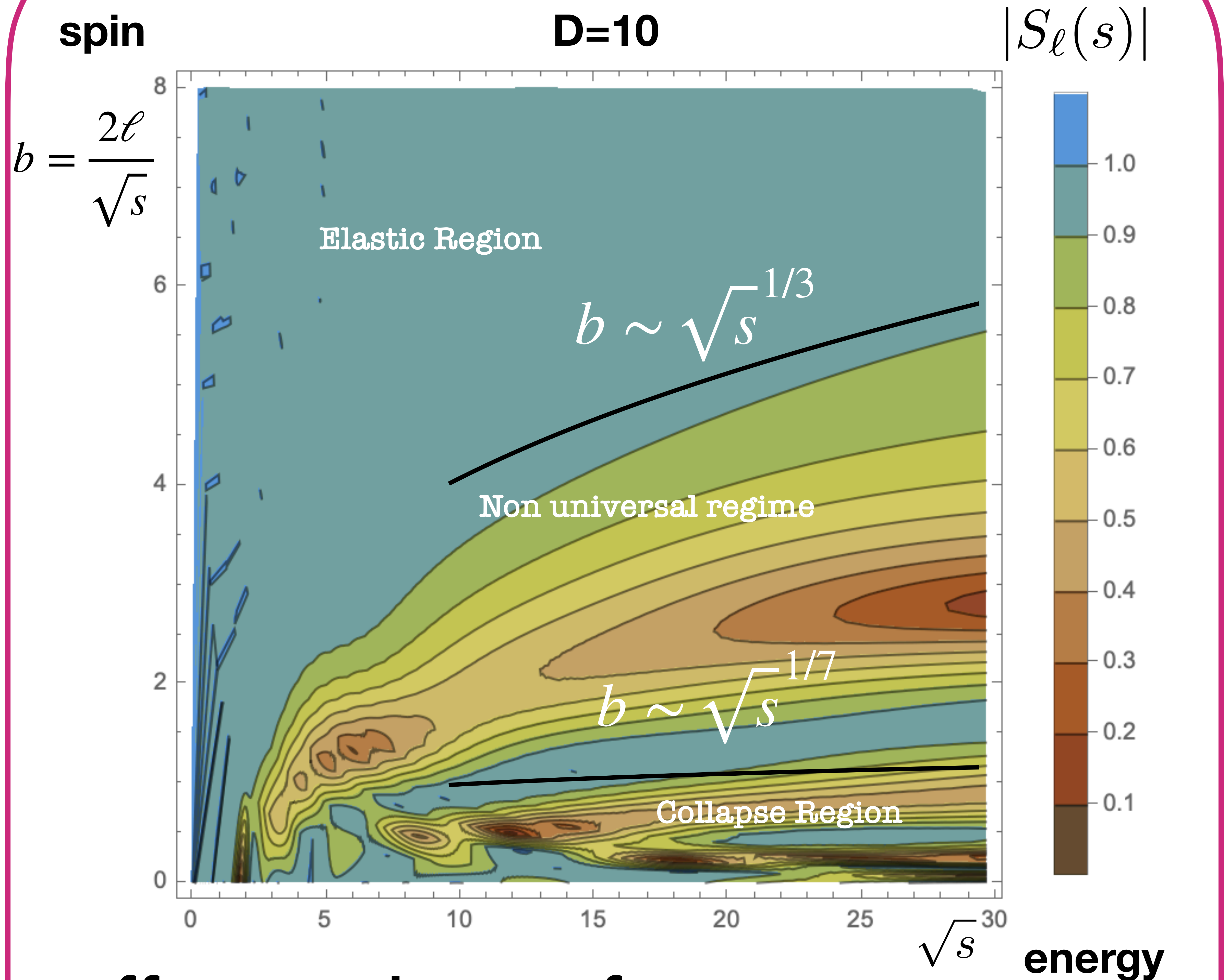
$$\alpha = \frac{(2\pi)^2}{3\ 2^7} \simeq 0.1028\,.$$

$$\alpha^{\text{IIB}} = \frac{1}{2^6} E_{\frac{3}{2}}(\tau, \bar{\tau}) \geq \frac{1}{2^6} E_{\frac{3}{2}}(e^{i\pi/3}, e^{-i\pi/3}) \approx 0.1389$$

$$\alpha^{\text{IIA}} = \frac{\zeta(3)}{32 g_s^{3/2}} + g_s^{1/2} \frac{\pi^2}{96} \geq \frac{\pi^{3/2} (\zeta(3))^{1/4}}{24\sqrt{3}} \approx 0.1403$$

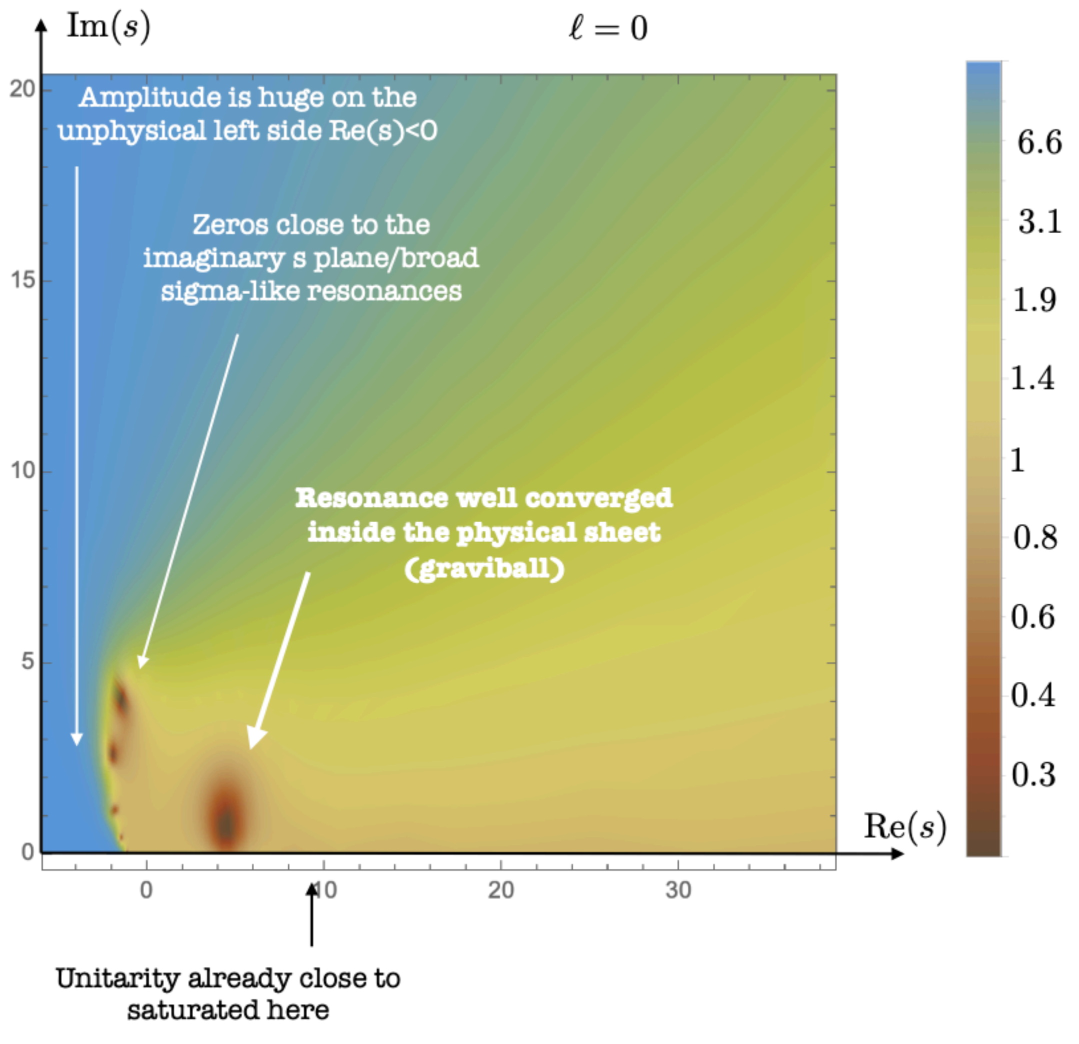
Dimension	Bootstrap	String/M–Theory
9	0.223 ± 0.002	0.241752
10	0.124 ± 0.003	0.138949
11	0.101 ± 0.005	0.102808

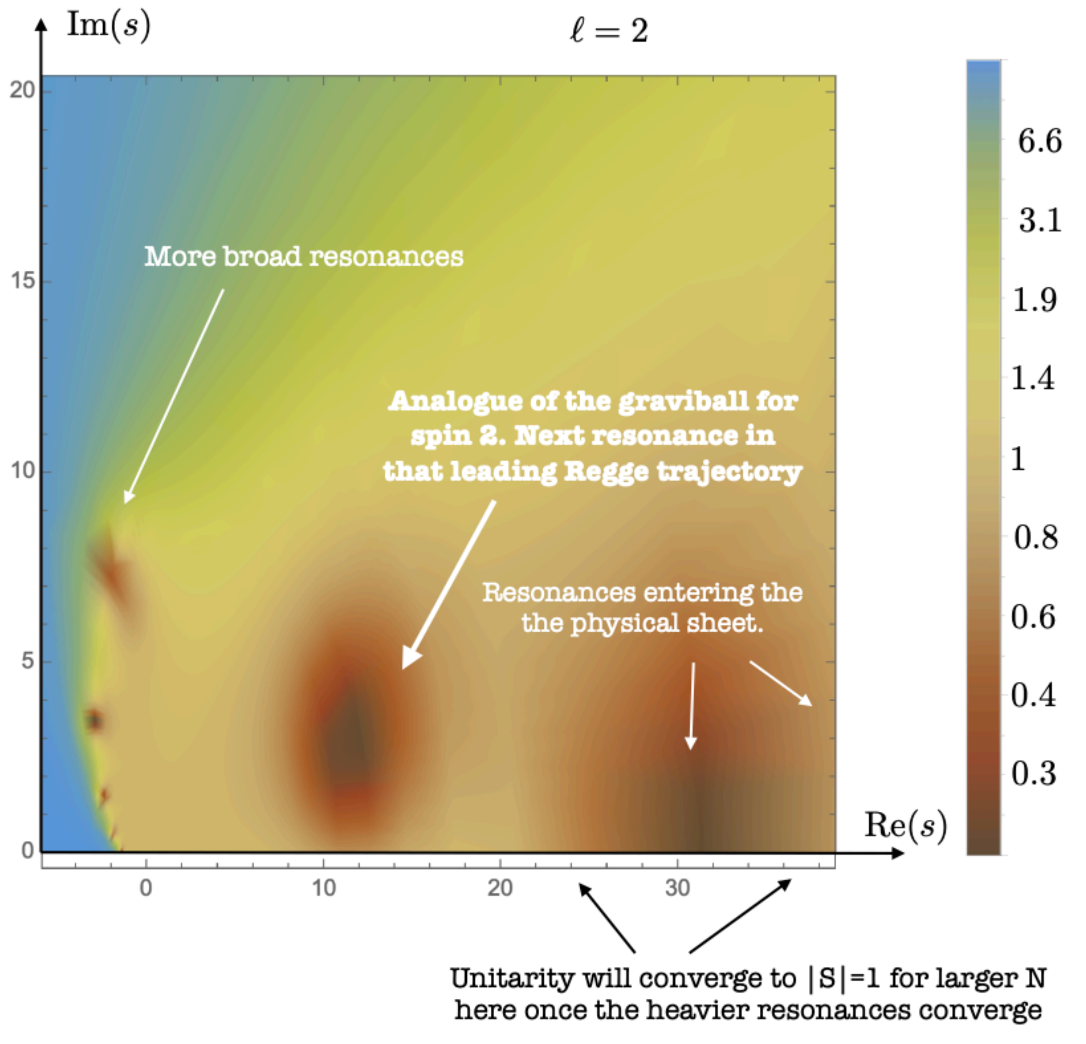
[Guerrieri, Murali, Penedones, Vieira '22]

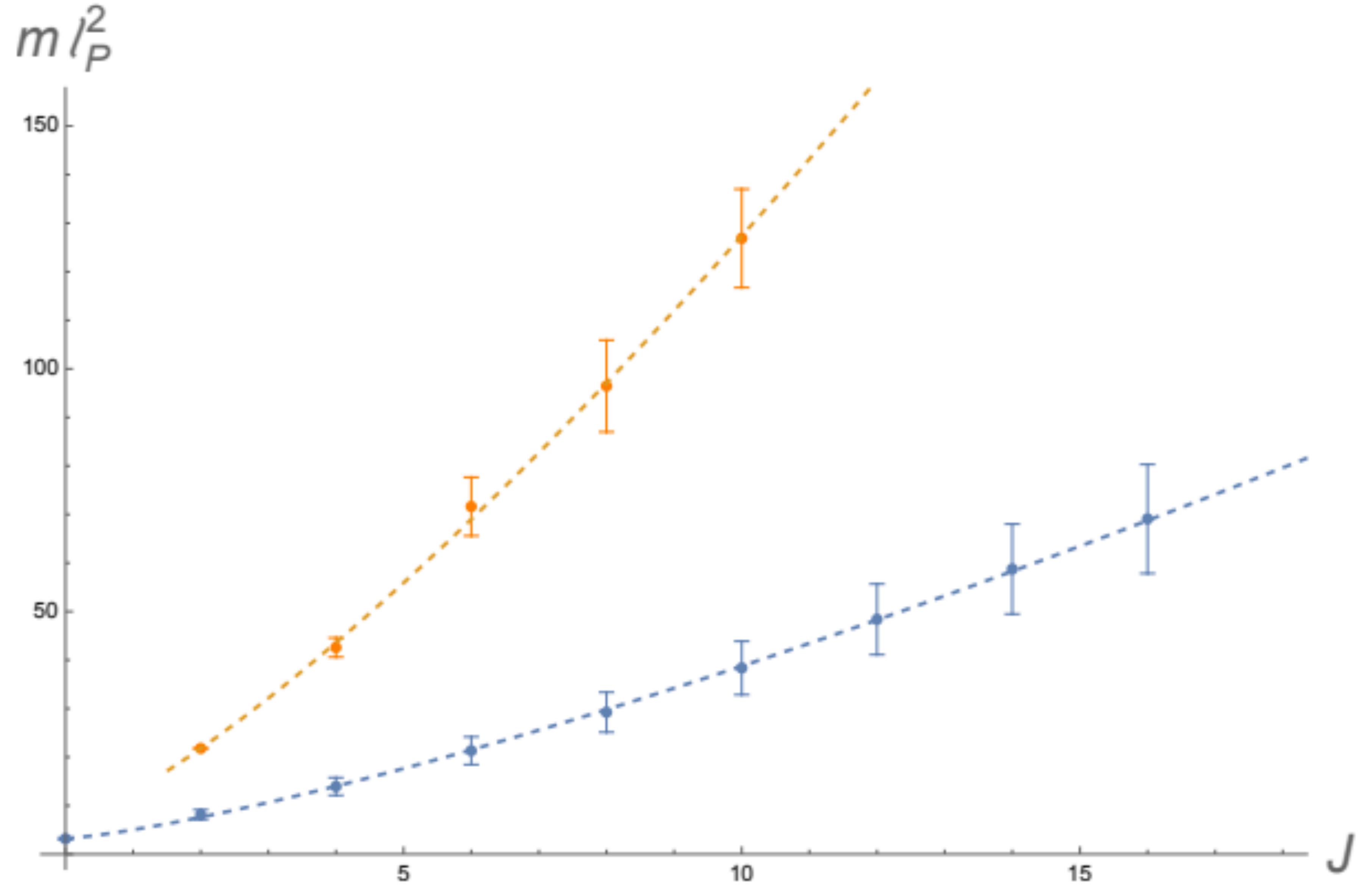


Different phase of gravity
(M-theory phase?)

[plots A. Guerrieri]





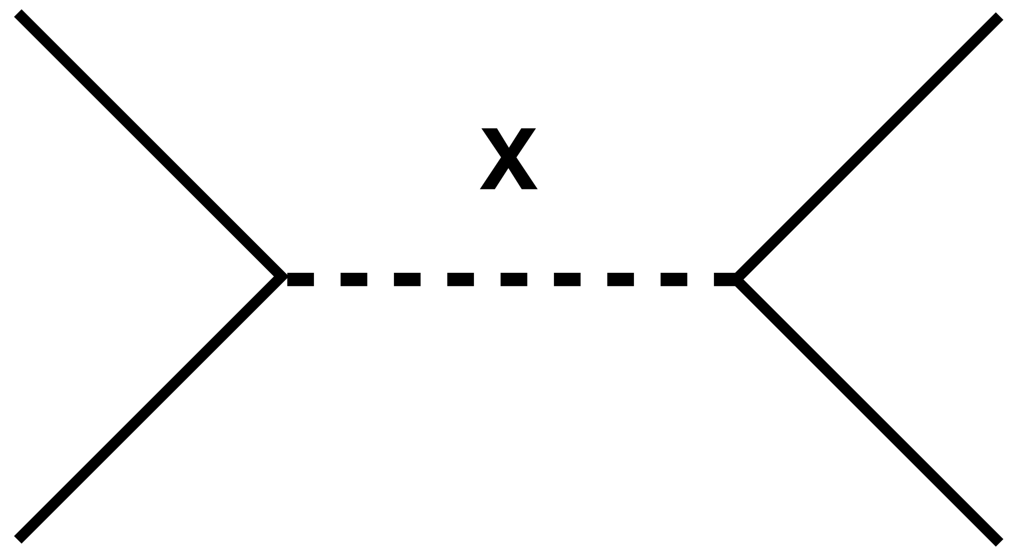


further higher-derivative corrections can and do exhibit intricate multi-scale structure (QFT corrections from integrating out particles, KK modes, genuine QG corrections)

[Calderon-Infante,Castellano,Herraez ’25, to appear]

Space of weakly coupled massive higher-spin amplitudes:

what are the options and
where is tree-level string theory?



Space of massive HS amplitudes

In the full theory **asymptotically**

CFT

CFT Regge intercept

flat

$$\lim_{|s|\to\infty} \frac{T(s,t)}{s^2} = 0 \qquad t<0$$

$$J_{\rm np} \leq 1$$

[Muzinich,Soldate '88]

[Caron-Huot '17]

eikonal: $T(s,t) \sim s^{2-\frac{D-4}{2(D-3)}}$

eikonal: $J_{\rm np} = 1$

At **intermediate** energies/tree-level we instead have

CFT

CRG conjecture:
[TIFR I, '20]

$$T_{\rm tree}(s,t) \leq s^2 \longleftarrow J_{\rm pl} \leq 2$$

[TIFR II, '20]

[MSS '15]

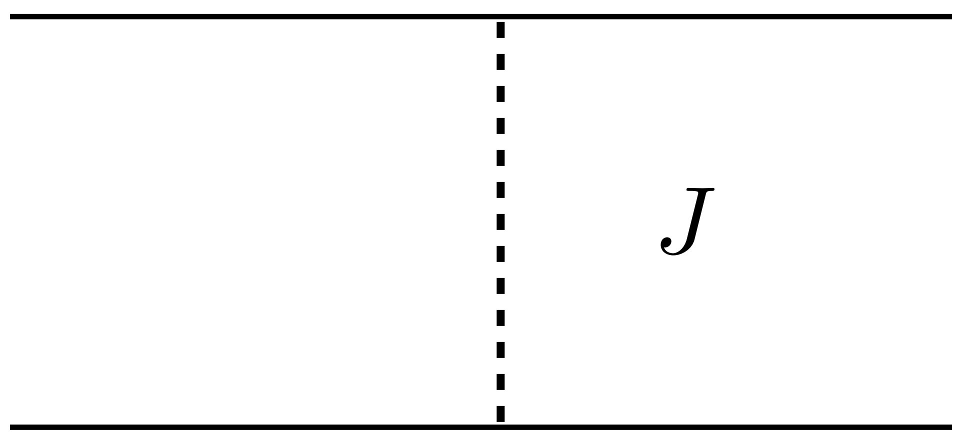
2SDR [Haring, AZ '22]

CHSRG conjecture:
[Albert, Knop, Rastelli '24]

$$T^{\rm HS}_{\rm tree}(s,t) < s^2$$

$$J^{\rm HS}_{\rm pl} < 2$$

Single massive HS particle exchange ($J > 2$) implies infinitely many of them.



$$\sim s^J > s^2$$

[Chiang, Huang, Li, Rodina, Weng '21]
[Berman, Geiser '24]

Example: tree-level string theory

$$T(s,t) = -8\pi G_N \frac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}$$

[Talk by Y-t.Huang]

can be deformed at the level of 2-2

[Arkani-Hamed, Huang, Huang '20]
[Haring, AZ '23]

What is the space of such amplitudes?

Surprise 1: accumulation points

[Caron-Huot, Van Duong '20]

$$T(s,t) = \frac{8\pi G_N}{stu} + \frac{\chi}{(M^2-s)(M^2-t)(M^2-u)}$$

infinitely many particles
with $s = M^2$

strings in AdS
[Maldacena, Remmen '22]

Surprise 2: single Regge trajectory amplitude

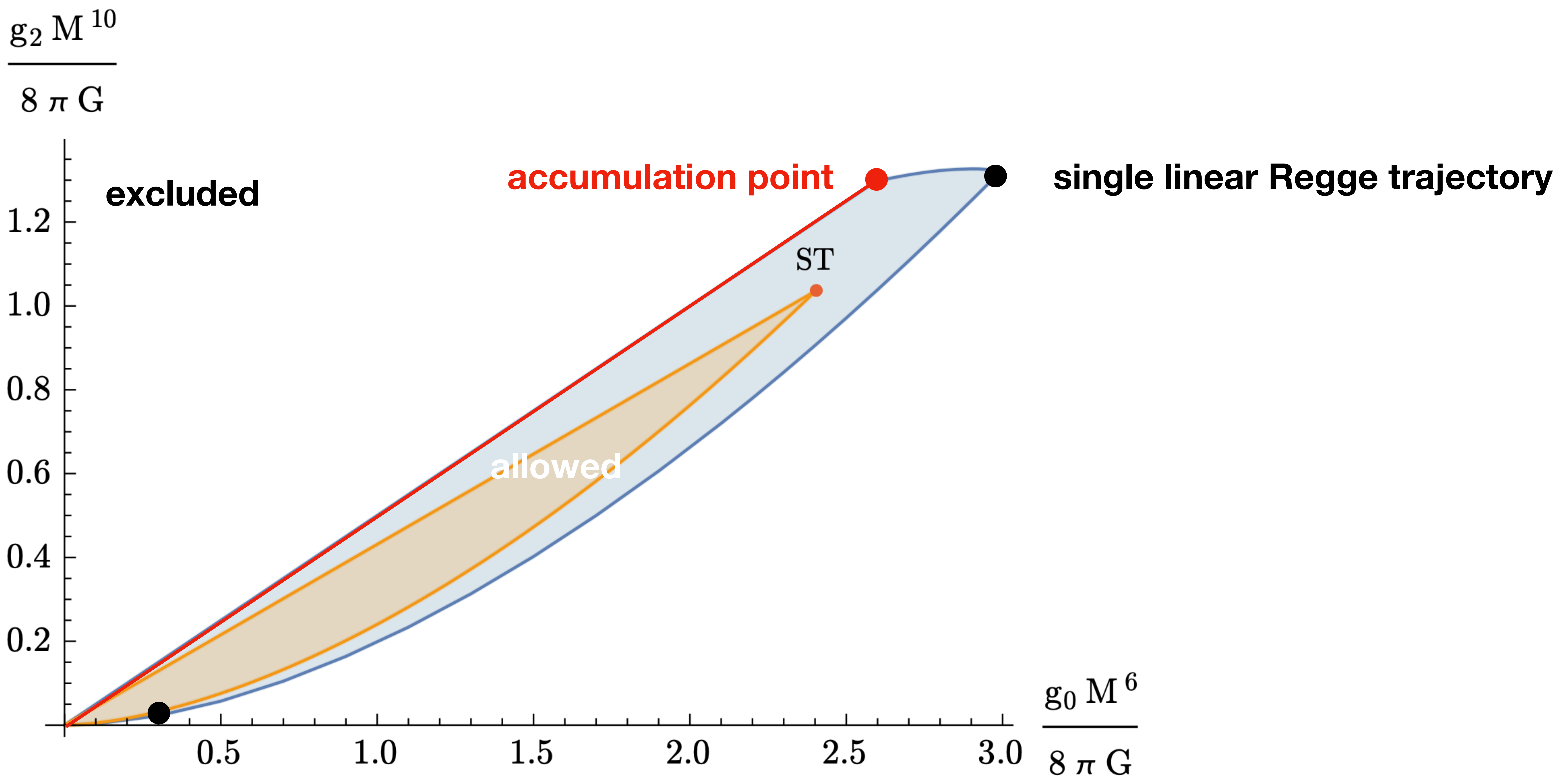
[Albert,Henriksson,Rastelli,Vichi '23]
[Albert, Knop, Rastelli '24]

- In AdS a single Regge trajectory solution: O(N)/Vasiliev [Klebanov, Polyakov '02]

weakly coupled conformal large N flux tube [Klebanov,Maldacena,Thorn '06]

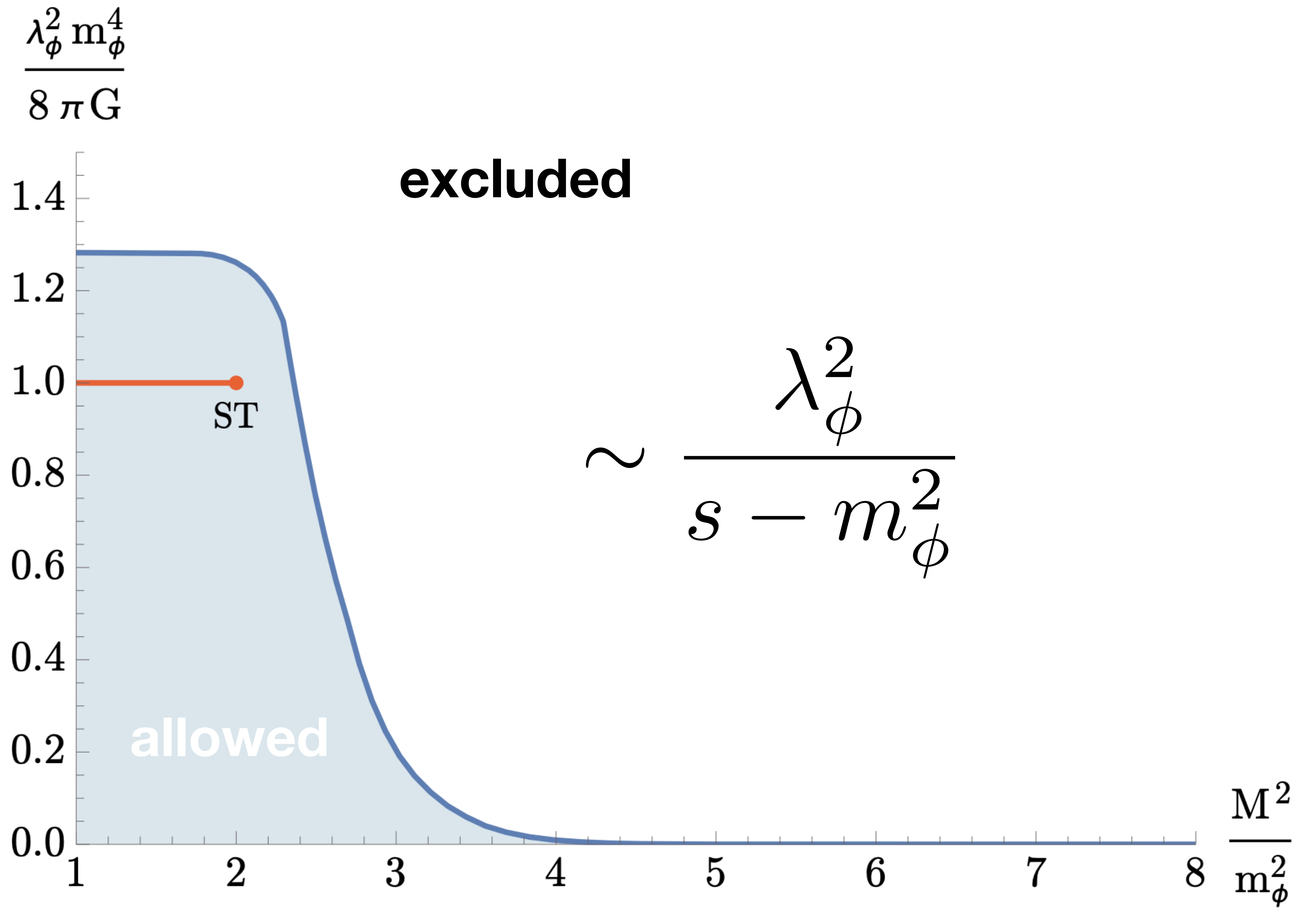
- In flat space, general argument for ∞ many trajectories [Eckner, Figueroa, Tourkine '24]

$$T(s,t) = \frac{8\pi G_N}{stu} + g_0 + g_2(s^2+t^2+u^2) + \ldots$$

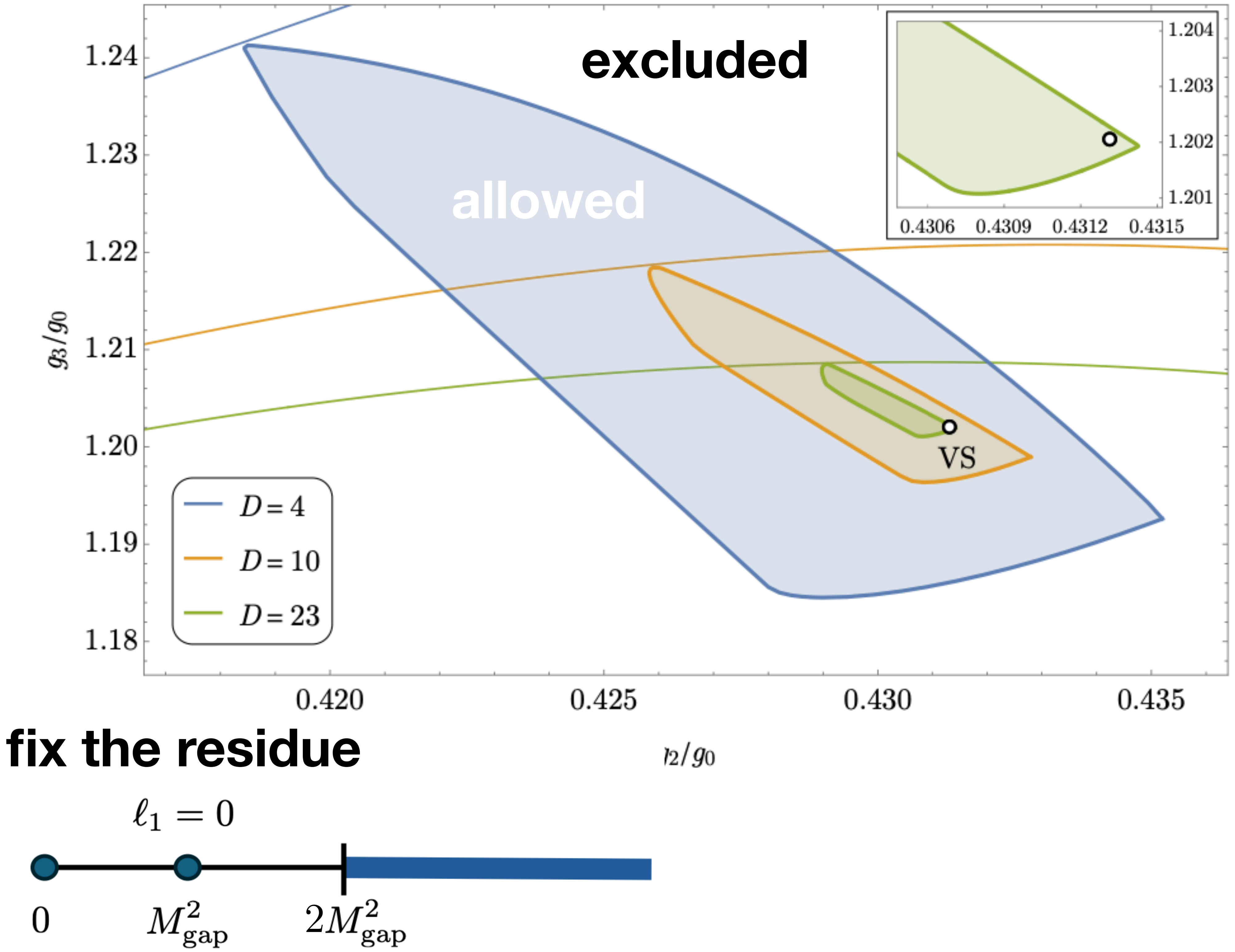


[Albert, Knop, Rastelli '24]

extra input is very constraining
(residues, gaps, Wilson coefficients)



[Albert, Knop, Rastelli '24]



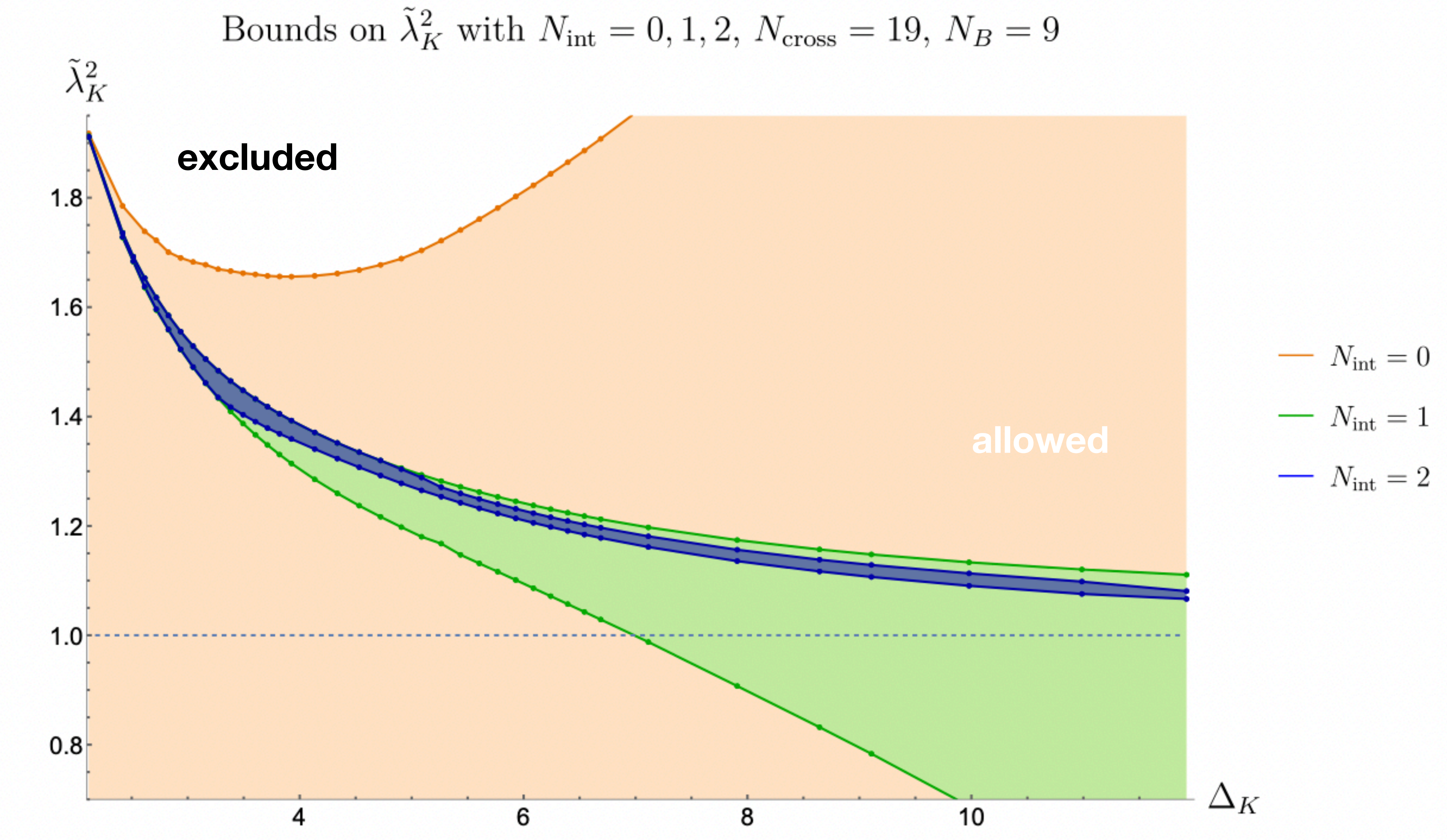
[Berman, Elvang, Geiser, Lin '24]

- multi-point factorization, unitarity, and the Hagedorn growth

[Gross '69][Bianchi, di Vecchia '20][Arkani-Hamed, Cheung, Figueiredo, Remmen '23]

- strings in AdS (integrability, single-valuedness, SUSY localization)

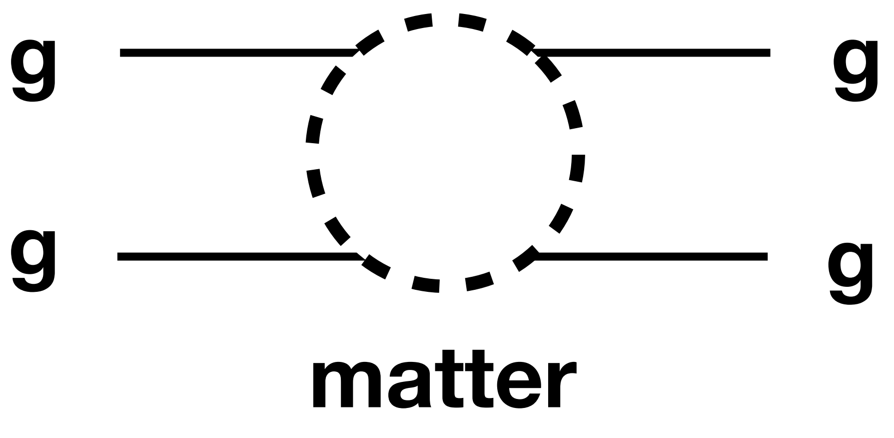
[Alday, Hansen '23][Cavaglia, Gromov, Preti '23]
[Chester,Dempsey,Pufu '23] ...

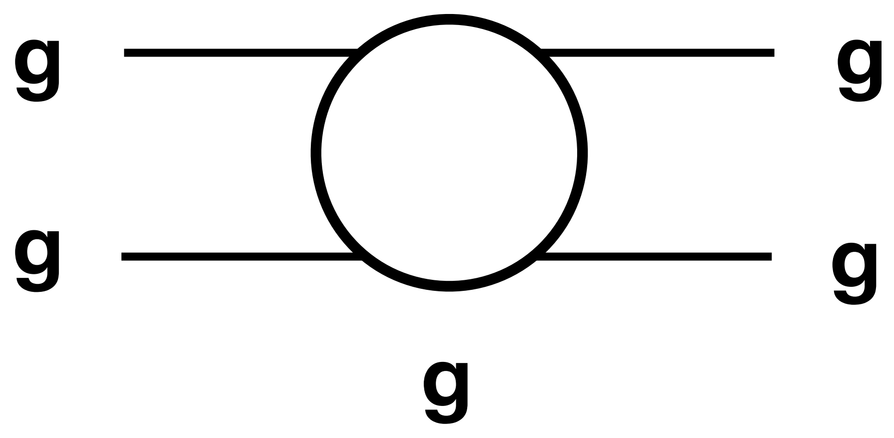


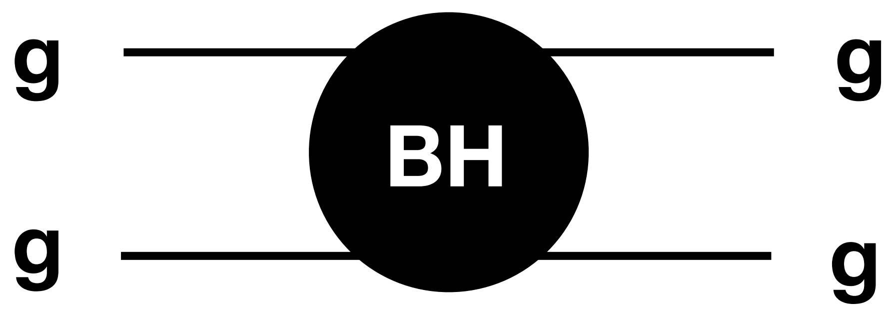
extra constraints from SUSY localization

[Caron-Huot, Coronado, Zahraee '24]

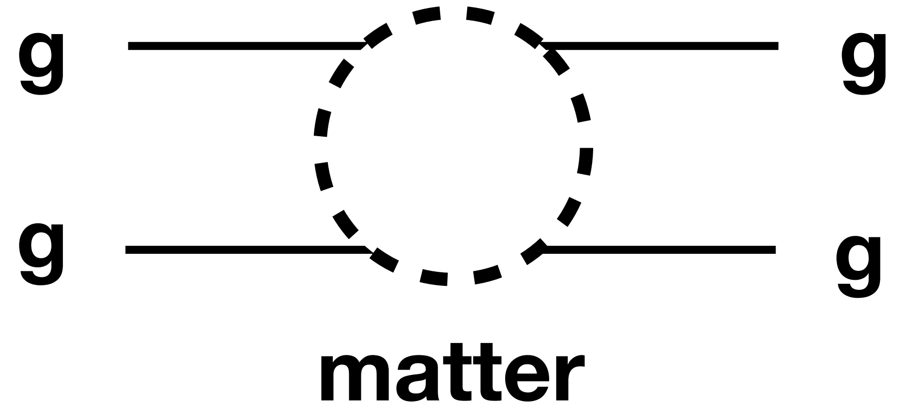
Loops







Matter loops



Integrating out known particles (neutrinos, electrons,…) induces higher-derivative corrections in the gravitational effective action.

[Drummond, Hathrell '80]
[Bellazzini, Isabella, Lewandowski, Sgarlata '22]

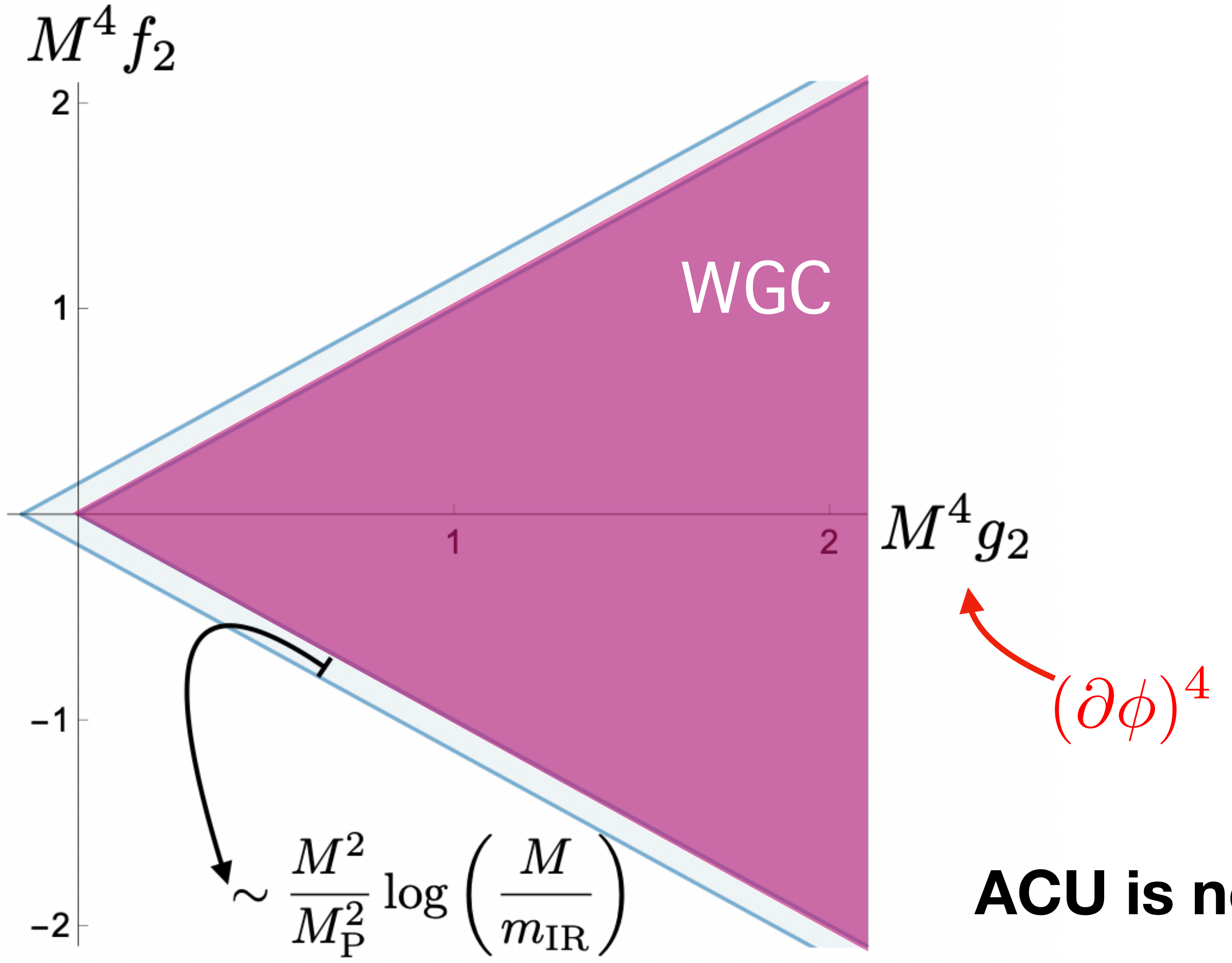
$$\mathcal{L} = \sqrt{-g}\Big(\frac{M_{\rm P}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_1(F_{\mu\nu}F^{\mu\nu})^2 + \alpha_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \beta W_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \dots\Big)\,.$$

BH WGC $\quad \frac{Q}{M} \geq \frac{Q}{M}\Big|_{\rm extr}\ :\quad 16\alpha_{1,2} = g_2 \pm f_2 \qquad g_2 \geq |f_2| \geq 0$

[Kats, Motl, Padi '06]
[Arkani-Hamed,Motl,Nicolis,Vafa '06]

It follows from dispersion relations at $G_N = 0$.

[Cheung, Remmen '14] [Hamada,Noumi,Shiu '18]



[Henriksson, McPeak, Russo, Vichi '22]

- 4×4 helicity matrix

$$\mathbf{M} = \begin{pmatrix} \mathcal{M}^{++--} & \mathcal{M}^{++-+} & \boxed{\mathcal{M}^{+++-}} & \mathcal{M}^{++++} \\ \mathcal{M}^{+---} & \mathcal{M}^{+--+} & \mathcal{M}^{+-+-} & \mathcal{M}^{+-++} \\ \mathcal{M}^{-+--} & \mathcal{M}^{-+-+} & \mathcal{M}^{-++-} & \mathcal{M}^{-+++} \\ \mathcal{M}^{----} & \mathcal{M}^{---+} & \mathcal{M}^{--+-} & \mathcal{M}^{--++} \end{pmatrix}$$

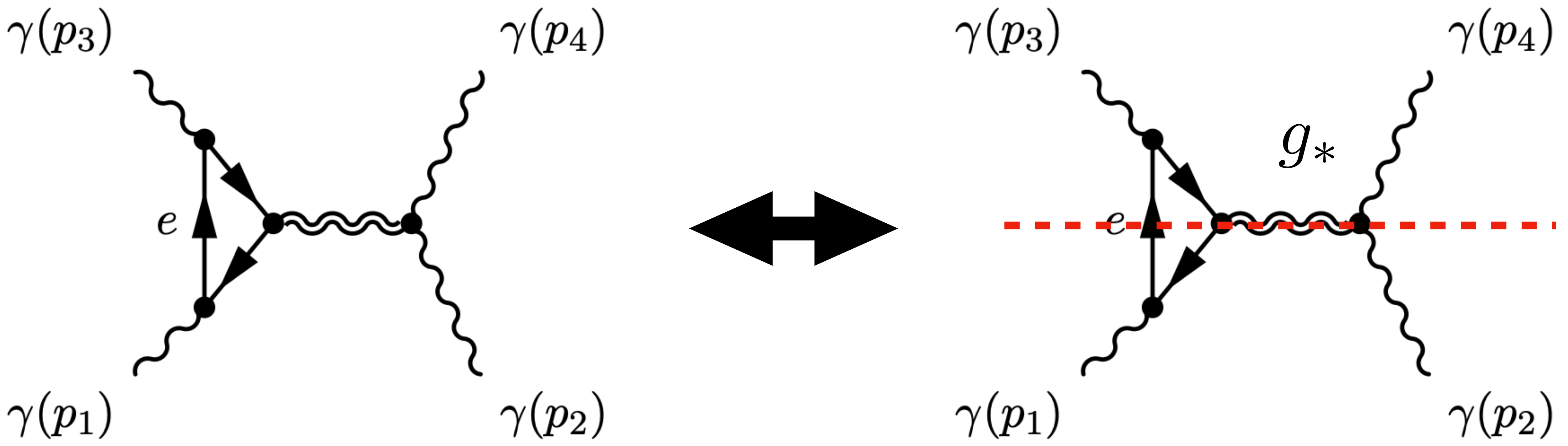
- graviton pole treatment

ACU is not enough!

[Caron-Huot, Tokuda '24]

- negative contribution & 2SDR

$$g_2 = -\frac{11}{90\pi^2}\frac{e^2}{M_{Pl}^2 m_e^2}$$



$$T(s,t) \sim f(t)s^{\alpha(t)}$$

correction to $\partial_t f(t)$

Graviton loops

Graviton loops make some of the techniques inapplicable.

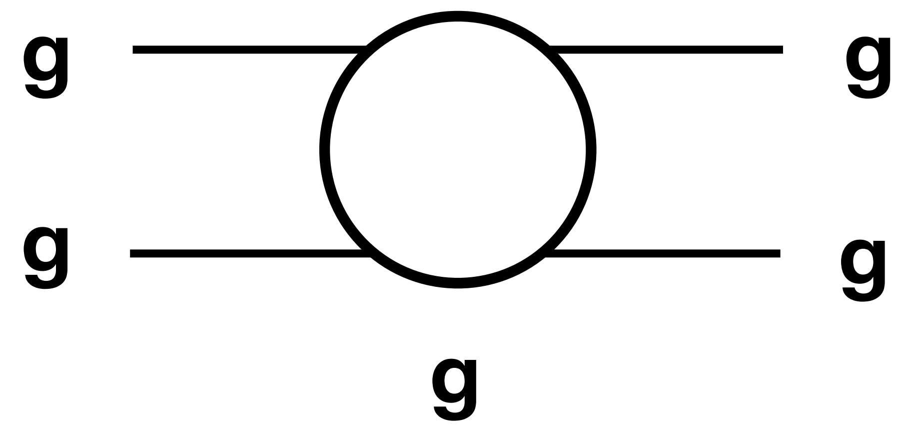
Dispersive crossing: $$T(s,t) = T(t,s)$$

$$T(s,t) \sim g_4 (s^2 + t^2 + u^2)^2$$

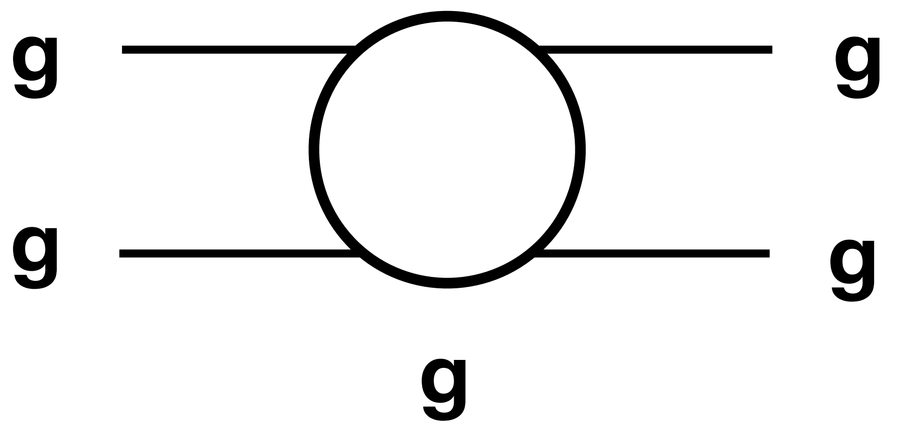
Null constraint (expanding around t=0): $$96 g_4 = \partial_s^4 T(s,t)\Big|_{t=0} = 2\partial_s^3 \partial_t T(s,t)\Big|_{t=0}$$

Cannot be used at the loop level ($t = 0$ is singular):

$$T(s,t) \sim G_N^2 s^2 \log(-s)(-t)^{\frac{D-4}{2}}$$

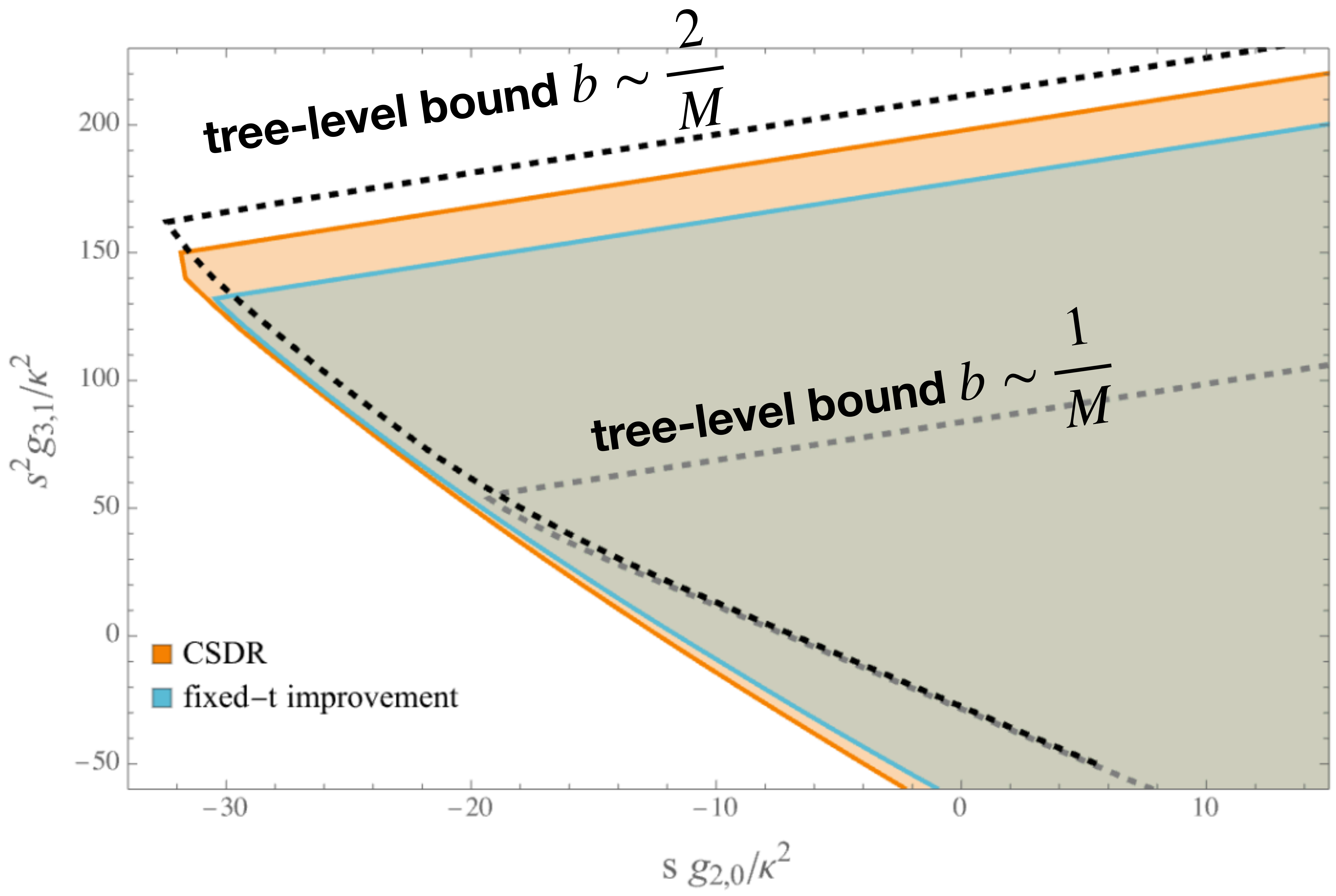


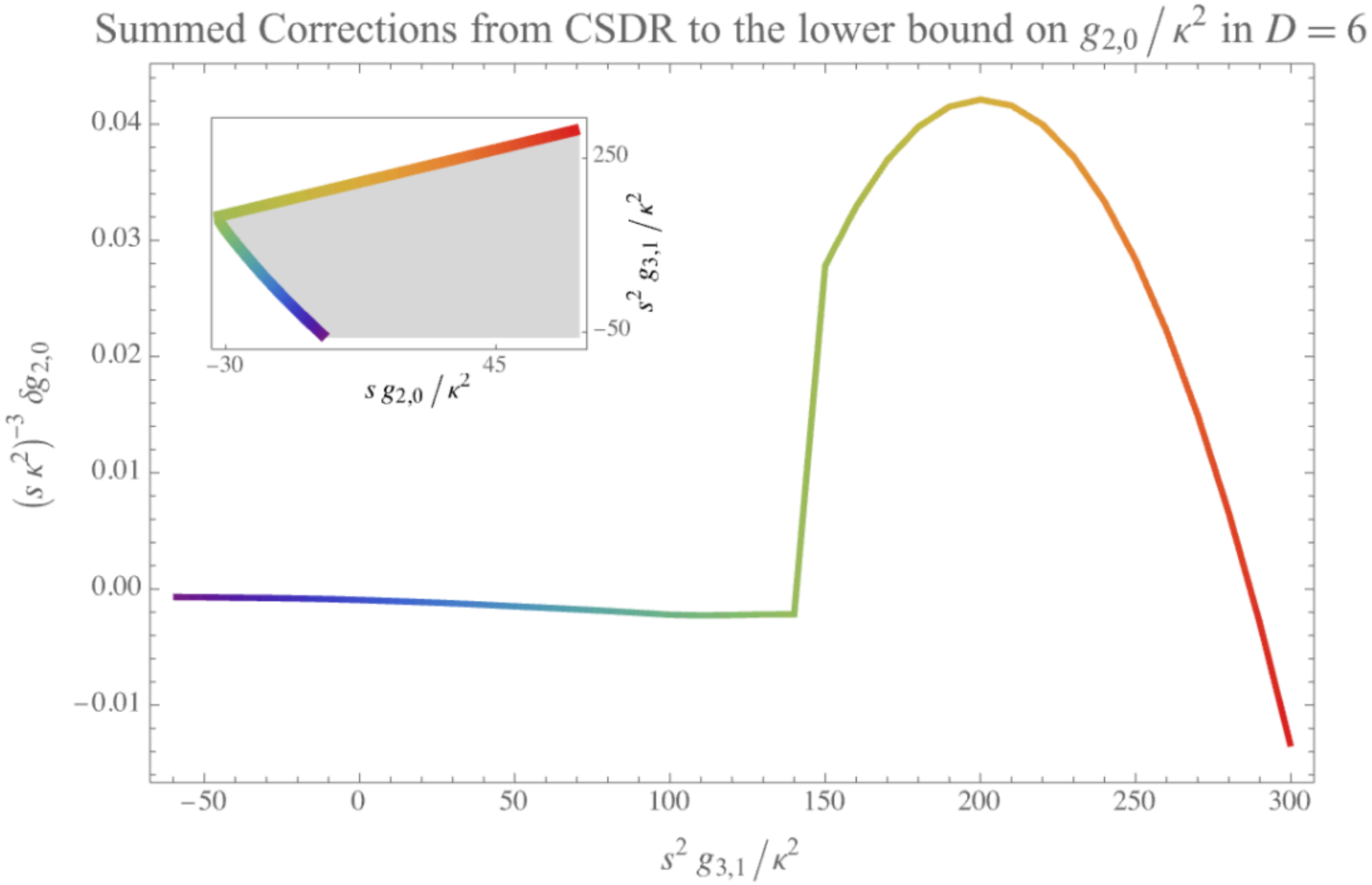
Graviton loops



[Li '23]
[Beadle, Isabella, Perrone, Ricossa, Riva '24]

One can impose crossing away from $t = 0$!



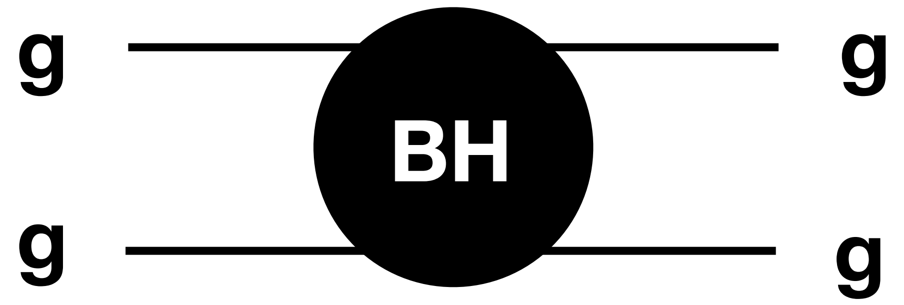


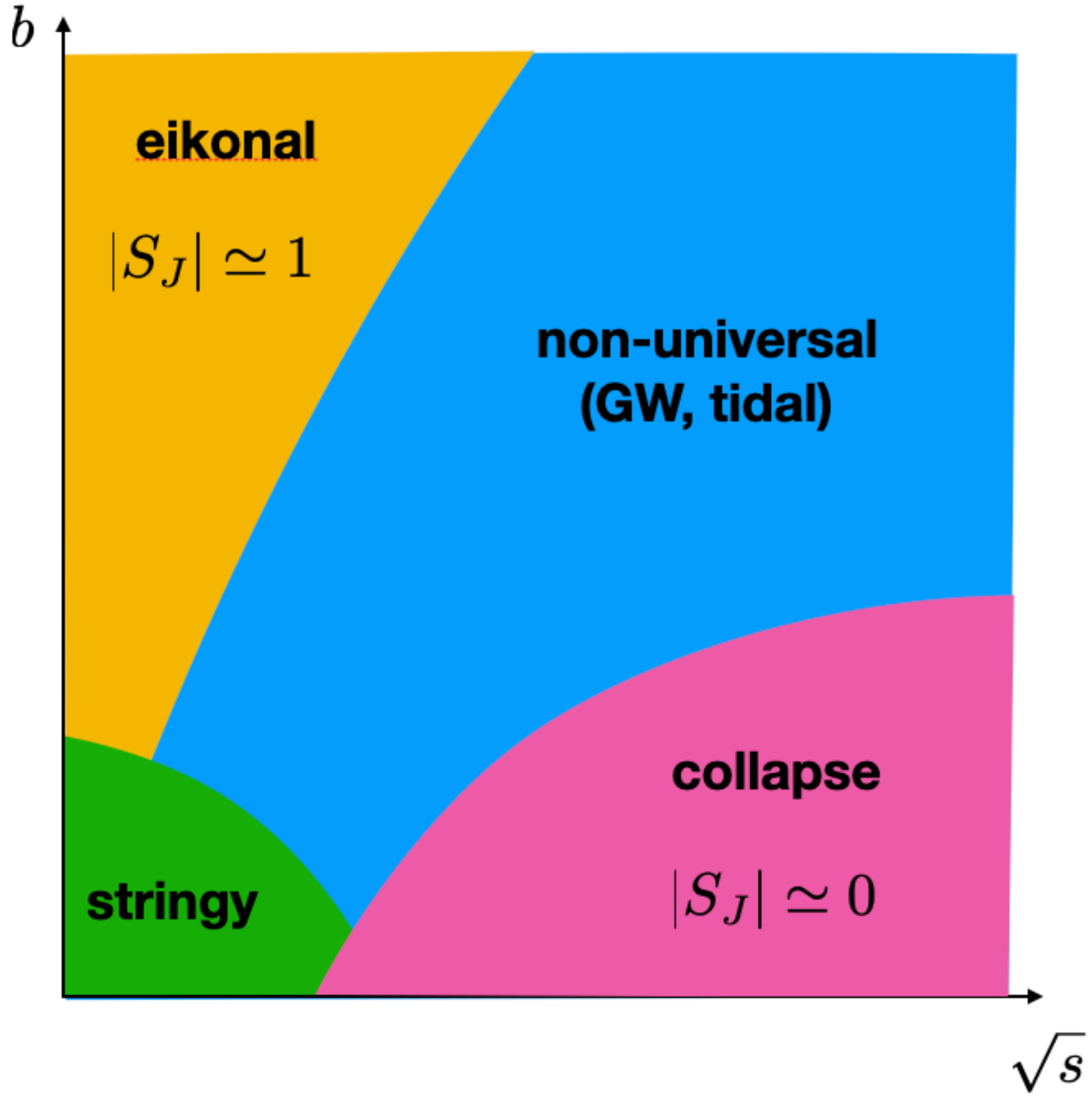
[Beadle, Isabella, Perrone, Ricossa, Riva '25, to appear]

Black hole loops

It is also interesting to consider the contribution of black holes to the dispersion relations (black disc)

$$\left(R_{\text{Sch}}(s)\right)^{D-3} = \frac{16\pi G_N\sqrt{s}}{(D-2)\Omega_{D-2}}$$





Using 2SDR and $a(s,b) \simeq i$ we get

$$S(s,b) = 1 + ia(s,b) \simeq 0$$

$$T^{\text{bd}}(s,t) \sim G_N^2(s^2+u^2)(-t)^{\frac{D-4}{2}}$$

s-channel black holes produce a **one-loop** effect non-analytic in t via 2SDR.

[Haring,AZ '24]

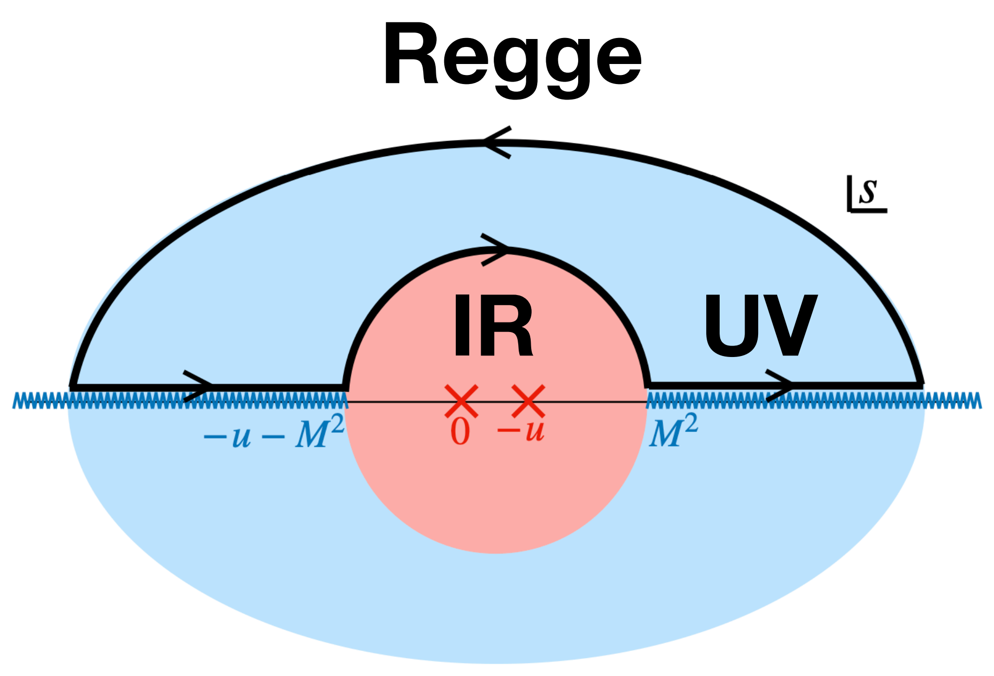
Summary

In relativistic quantum (gravity) theory, causality naturally gives rise to the twice-subtracted dispersion relations.

Dispersion relations offer a nonperturbative and quantitative method to explore the space of theories.

They become particularly powerful when combined with other data (experimental, integrability, susy, …).

- 2019: super convergence; AdS DR
- 2020: crossing (two-sided b); higher moments
- 2021: graviton pole
- 2022: four gravitons where is M-theory?; WGC
- stringy amplitudes &correlators; where is tree-level string theory?
- crossing-symmetric disp. rel; loops
- species bound/scale; Regge physics
- 2025: …



gravitational dispersion relations: IR+UV=0

[Rastelli, Strings 2021]

Future directions

complete 4-graviton bootstrap (w loops)

[Elias Miro, Guerrieri, Gumus '22] [Chang, Landau, Simmons-Duffin '23]
[TTTT 3d Ising] [Chester, Dempsey, Pufu '24]

tree-level

mixed amplitudes/correlators

[Hong,Wang,Zhou '23] [Xu,Hong,Wang,Zhou '24]

beyond 2-2

[Guerrieri, Homrich, Vieira '24] [Carillo Gonzales, de Rham, Jaitly, Pozsgay, Tokareva '23]
[Caron-Huot, Giroux, Hannesdottir, Mizera '23] [Gross '69]

polynomial boundedness/ LC OPE on the second sheet

[van Rees '24] [Buoninfante, Shao, Tokareva '24]

solve theory/find new theories (string theory, M-theory)

good observables in D=4 and dS

[Prabhu,Satishchandran,Wald '22]

fixed angle/BHs/swampland [I.Valenzuela]

chaos in scattering

[Polchinski '15] [Gross, Rosenhaus '21]

exp data/BSM/cosmology [F.Riva]

[Creminelli, Janssen, Senatore '22] [Hui, Kourkoulou, Nicolis, Podo, Zhou '23]
[Bellazzini, Isabella, Ricossa, Riva '23]

meromorphic primal /nonperturbative dual

[Guerrieri, Sever '21]

[see also the Strings 2021 S-matrix bootstrap discussion]

Thank you for your attention!

Acknowledgments: Thanks to Yu-tin Huang, Shiraz Minwalla, Francesco Riva, Irene Valenzuela for the joint preparation of the session. Thanks to Jose Calderon-Infante, Riccardo Gonzo, Andrea Guerrieri, Kelian Häring, Dalimil Mazac, Balt Van Rees, Gabriele Veneziano, Leonardo Rastelli for useful discussions.

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CFT DR

Mellin space: $T(s,t) \to M(s,t)$

Coordinate space:

$$\frac{z\bar{z}}{(1-z)(1-\bar{z})}\mathcal{G}(z,\bar{z}) = \int_0^1 dw d\bar{w}\ K(z,\bar{z},w,\bar{w})\ \mathrm{dDisc}\left[\frac{w\bar{w}}{(1-w)(1-\bar{w})}\mathcal{G}(w,\bar{w})\right]$$
$$+\int_0^1 dw d\bar{w}\ K(\tfrac{z}{z-1},\tfrac{\bar{z}}{\bar{z}-1},w,\bar{w})\ \mathrm{dDisc}\Big[w\bar{w}\mathcal{G}'(w,\bar{w})\Big],$$

Analytic functionals: double twists (n,ℓ)

$$S = \frac{1}{16\pi G_N} \int d^D x \sqrt{-g} \Big(R - 2\Lambda + \dots \Big)$$

string theory

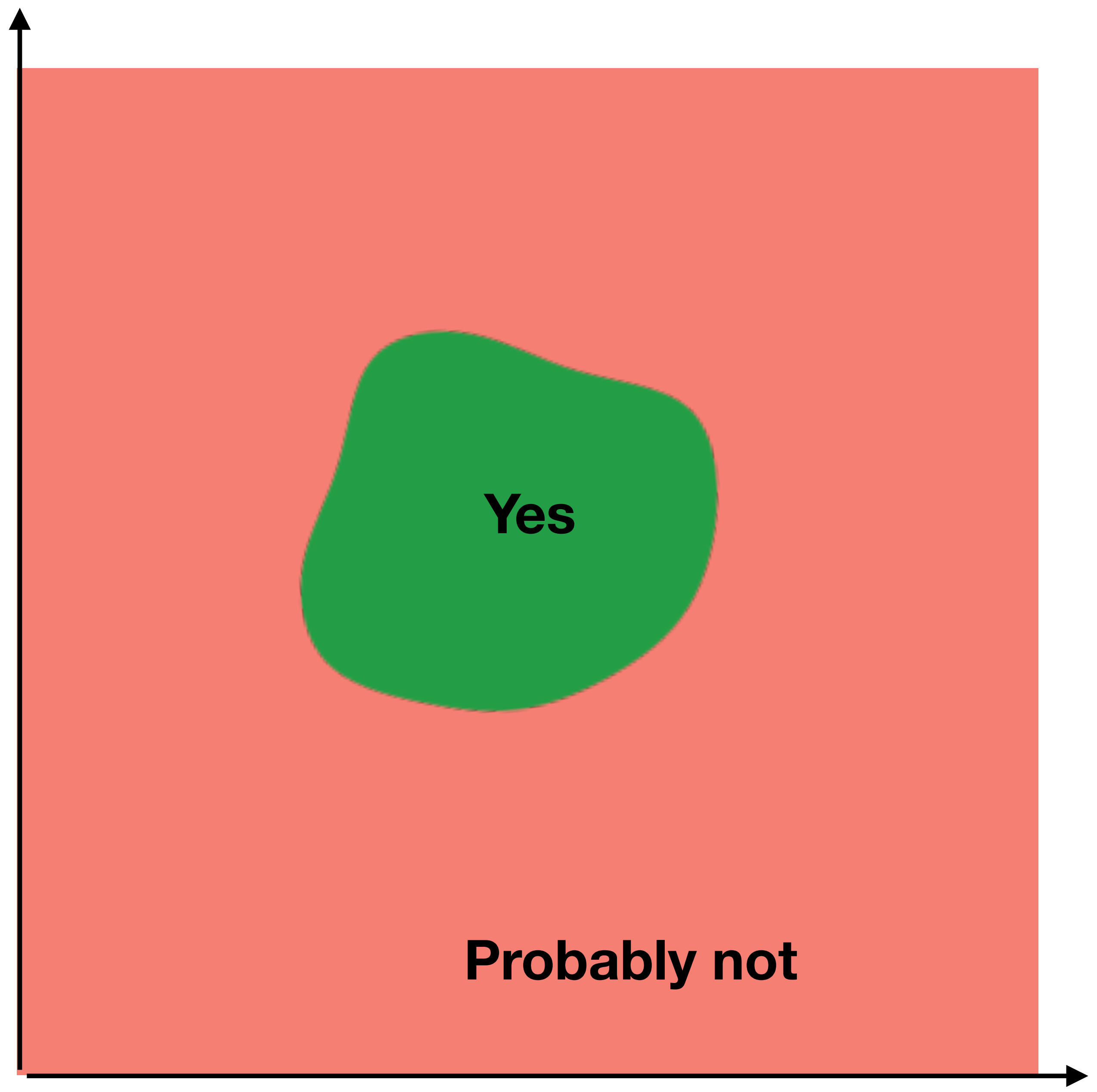
$$16\pi G_N = \ell_{\rm Pl}^{D-2} \sim \frac{g_s^2}{M_s^{D-2}}$$

KK

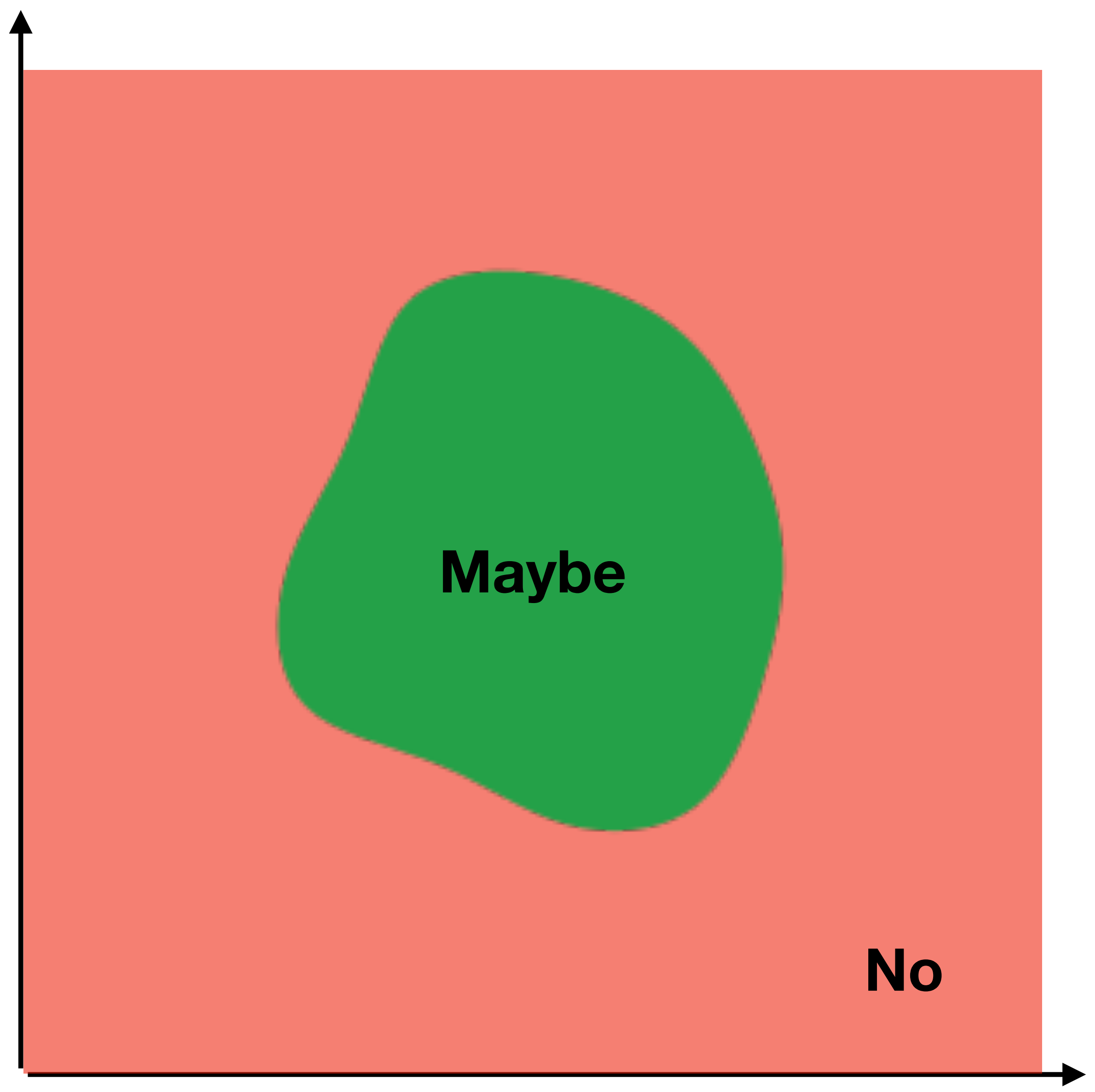
$$G_N^{(d)} = \frac{G_N^{(D)}}{\mathrm{Vol}_{KK}}$$

What is the space of theories?

Primal problem: construct amplitudes (landscape)



Dual problem: exclude theories (bootstrap bounds)



Sasha:

Can we bootstrap the M-theory S-matrix?
What are the good (experimental&theoretical) targets?
Is there a lower bound on inelasticity (eg due to BHs)?
Is any of this relevant to cosmology?

Irene:

Can we give a CFT proof for the Distance conjecture and classify the possible microscopic nature of the tower?

Can we derive from bottom-up that a QFT coupled to gravity (and light scalars) must necessarily break down at a scale that signals the existence of string states or extra dimensions?

Can we put bounds on the light spectra of a gravitational EFT with a fixed cut-off and compare with string theory/swampland expectations?

Can we give a sharp bound for the size of breaking effects of 0-form global symmetries?

Yu-tin:

Given the suggestive analysis on the G^2 contribution from the presence of black disk, can one eventually isolate production process from perturbative computation ?

Is it possible to identify bootstrap conditions that leads to tree-level completions with special spacing ?
i.e. if m^2 exists so must N m^2.

Francesco:

Can we use bootstrap methods for energy correlators (other collider observables)?

2->n scattering: prospects to incorporate it in the bootstrap. Will it actually help?