An infrared boundary action in asymptotically flat spacetimes

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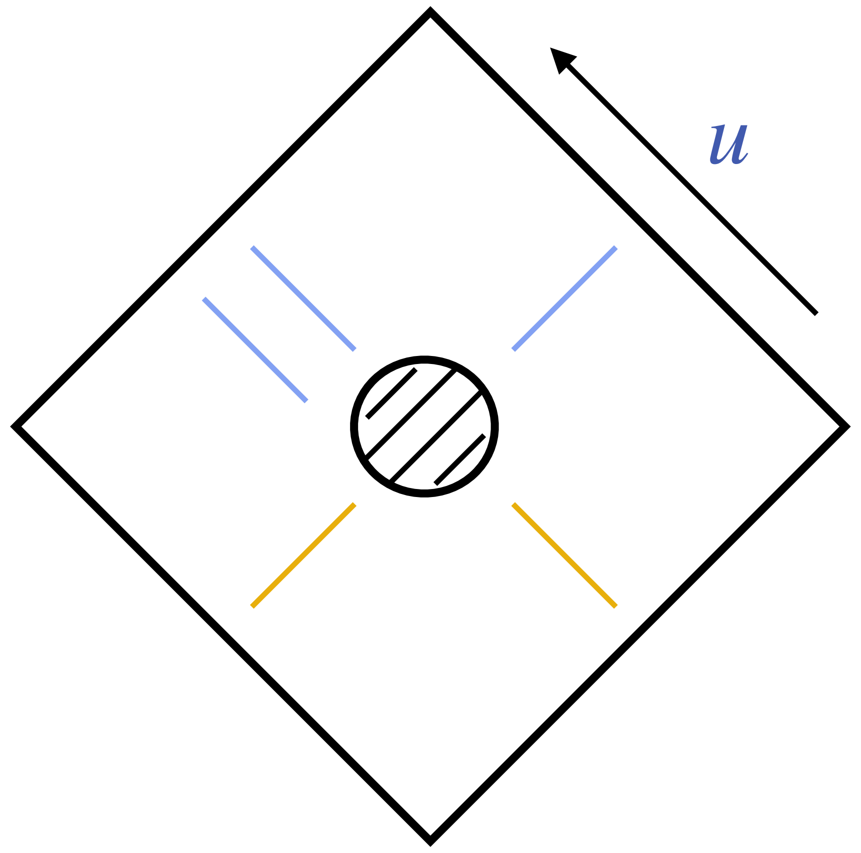
Strings 2025, NYU Abu Dhabi

based on 2408.01485 with T. He and K. Zurek [see also 2305.14411]

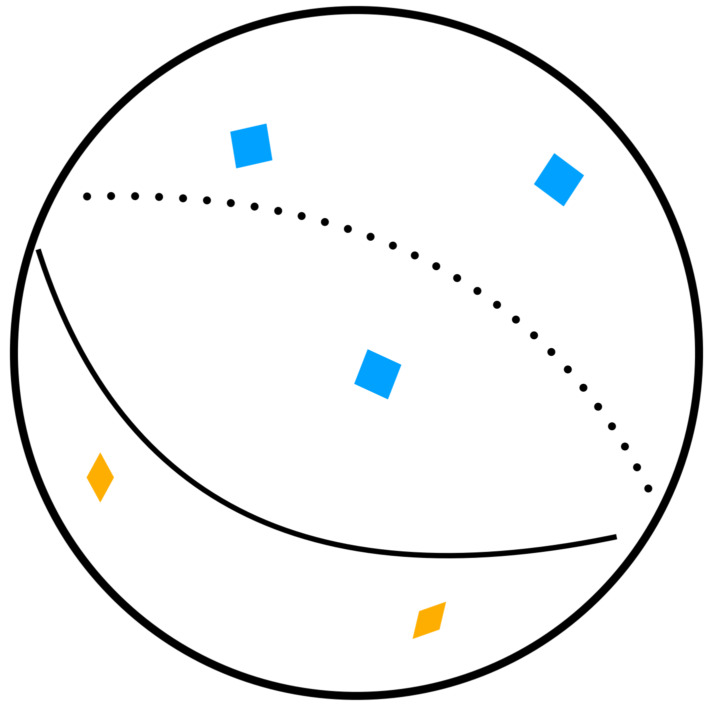
Motivation

Flat space holography from the bottom up?

gravity in
$(3+1)d$ AFS



$\longleftrightarrow$



$2d$ (C)CFT (or $3d$ Carrollian FT)

- Soft graviton modes $\longleftrightarrow$ w-algebra generators [..., Guevara, Himwich, Pate, Strominger '21]

$$\int_{-\infty}^{\infty} du u^n N_{zz}(u, z, \bar{z}) \longleftrightarrow w^n(z, \bar{z}) = \mathscr{L}_{2d}\left[\int_{-\infty}^{\infty} du u^n N_{zz}(u, z, \bar{z})\right]$$

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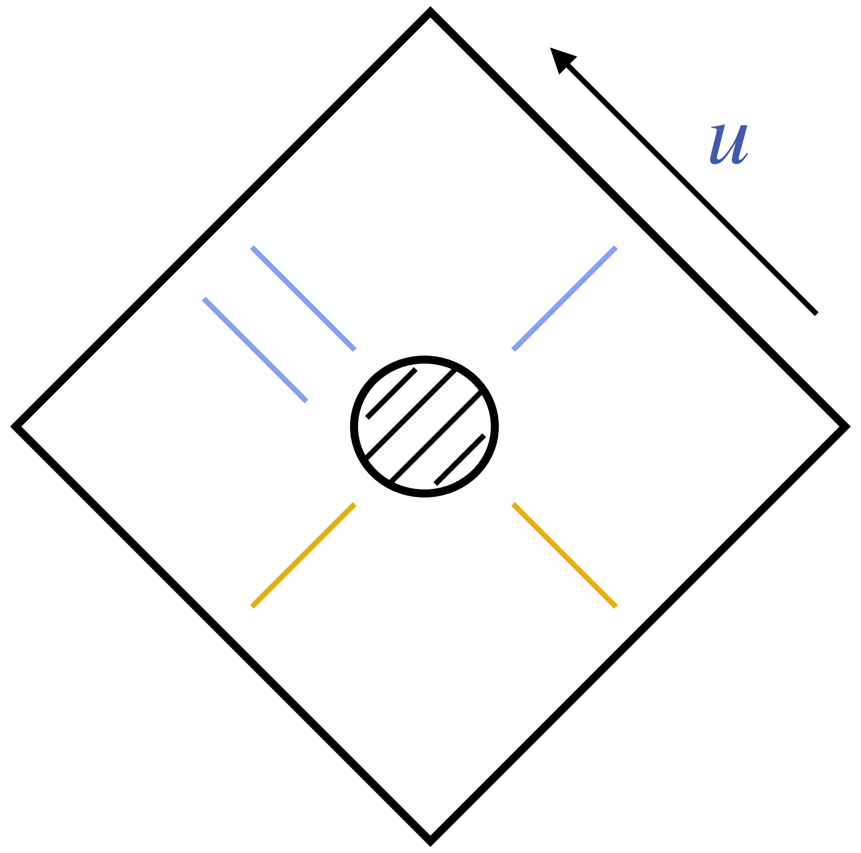

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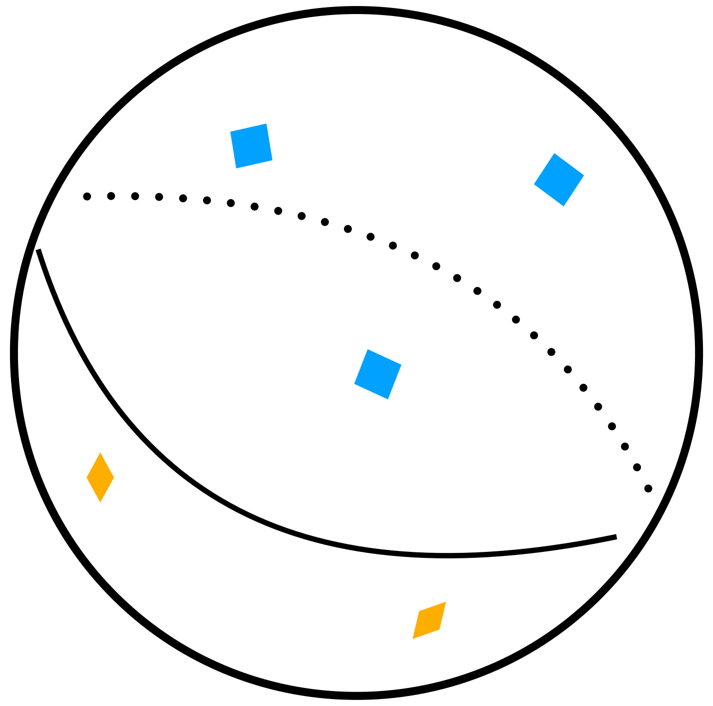

$2d$ (C)CFT (or $3d$ Carrollian FT)

What about other entries in the holographic dictionary?

Motivation

1. In AdS/CFT, the AdS partition function with specified boundary conditions acts as the generating function for correlators of primary operators in CFT:

$$Z_{\mathrm{AdS}}[\phi_0, \cdots] = \langle \exp\left(\int \phi_0 \mathcal{O} + \cdots\right)\rangle_{\mathrm{CFT}}$$

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classical (on-shell) gravity action ⟶ quantum CFT correlator: $\langle \mathcal{O}(x_1) \ \cdots \ \rangle = \frac{\delta}{\delta \phi_0(x_1)} \cdots \ Z_{\mathrm{AdS}}[\phi_0, \cdots] \big|_{\phi_0 = 0}$

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Q1: Is there an analogous relation in asymptotically flat spacetimes?

Motivation

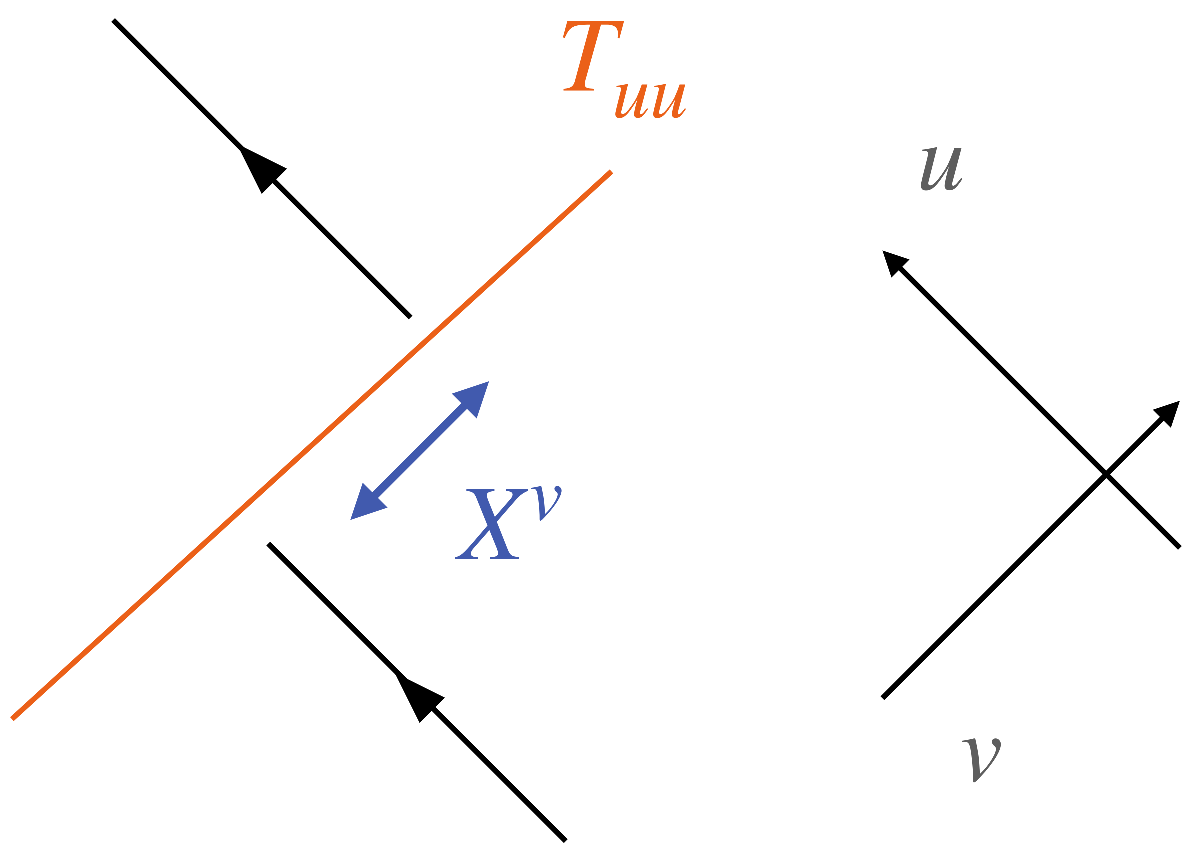
2. Proposed observability of vacuum spacetime fluctuations:

[Verlinde, Zurek '19, '22]

$$\hat{K}(z) = \int du X^u(z) T_{uu}(u,z) = X^u(z) \Box X^v(z)$$

$$\hat{K} \equiv \int_{S^2} d^2 z \hat{K}(z)$$ (bulk) area operator bilinear in X^u, X^v

[…,Jafferis, Lewkowycz, Maldacena, Suh '15]



$$ds^2 = h_{uu} du^2 + ds^2_{\text{flat}}$$

$$\Box h_{uu} \propto T_{uu}, \quad X^v \propto \int du h_{uu}$$

[Dray, 't Hooft '85]

Motivation

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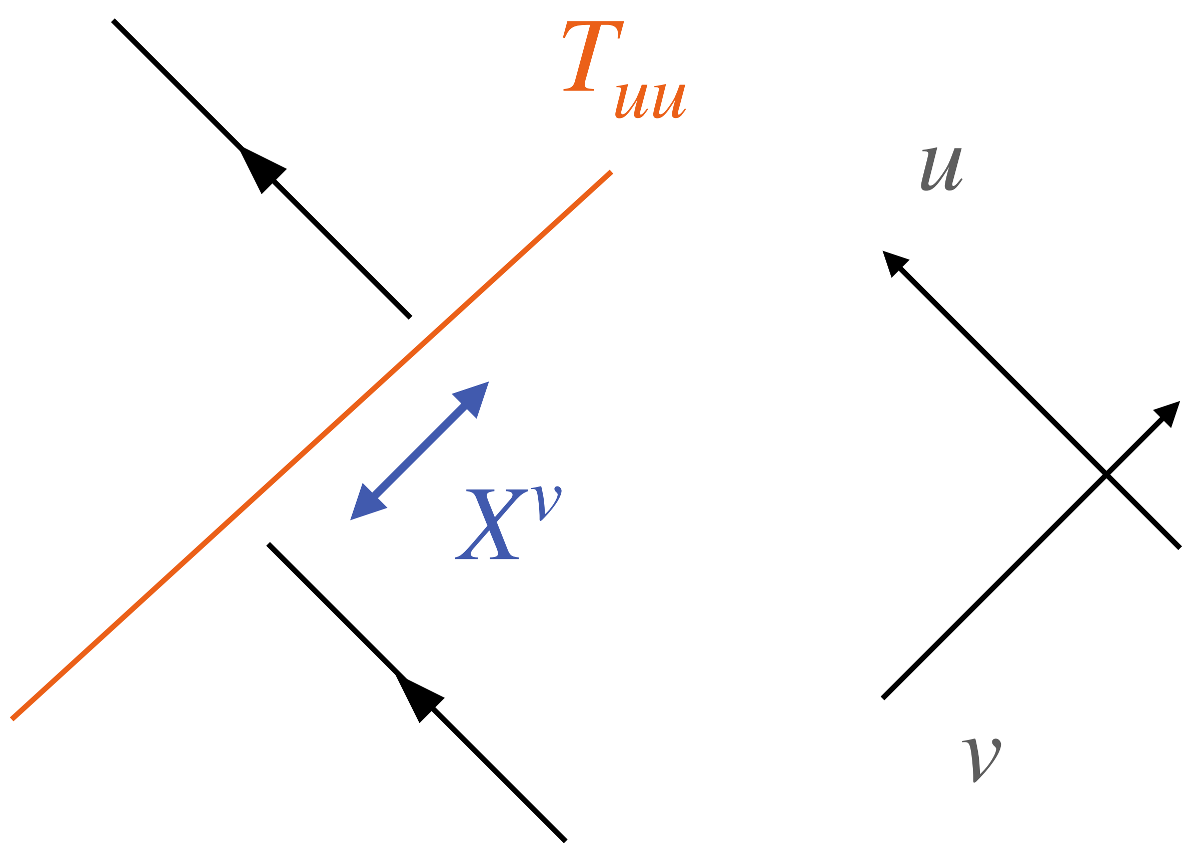

Assume that: $[X^u(z), X^v(z')] \propto G_N \delta^{(2)}(z,z') \implies$

['tHooft '96]

$$\langle \Delta \hat{K}^2 \rangle = \langle \hat{K}^2 \rangle - \langle \hat{K} \rangle^2 \sim A\delta^{(2)}(0) \rightarrow \frac{A}{\ell_p^2} \implies \frac{\Delta A^2}{\ell_P^4} \sim \frac{A}{\ell_P^2} \iff \Delta L^2 \sim L\ell_P$$

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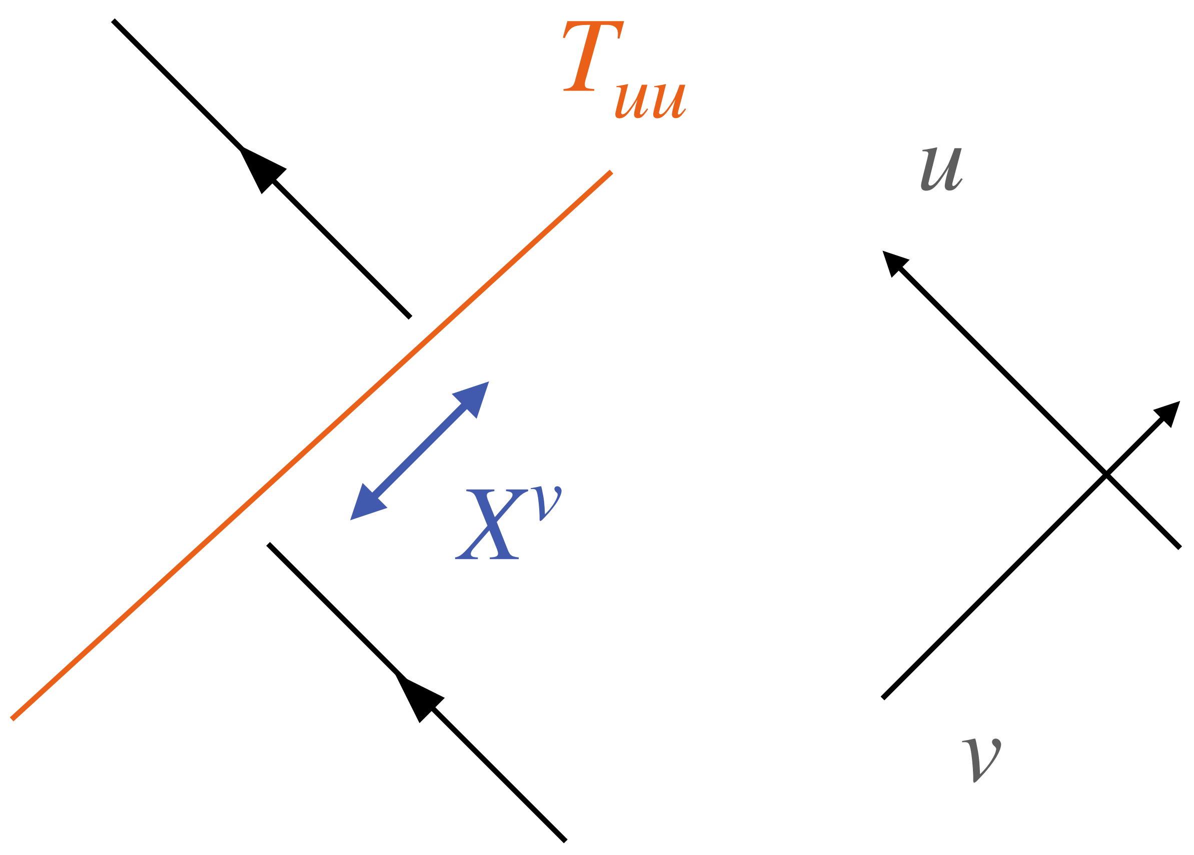

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In contrast, graviton fluctuations $\langle hh \rangle \propto \ell_P^2 \ldots$ [..., Carney, Karydas, Sivaramakrishnan'24]

Q2: Can we define $\hat{K}$ in 4d AFS and compute its fluctuations?

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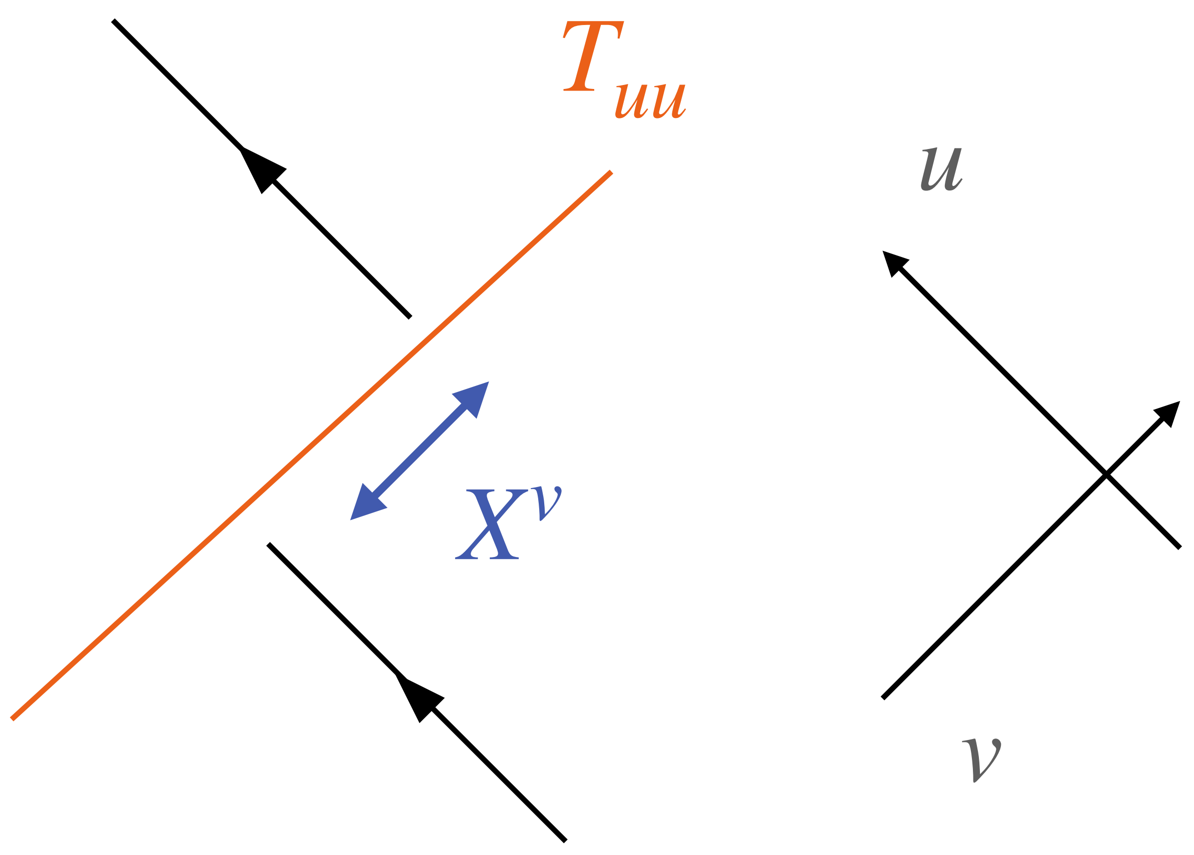

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Hint: Shockwave solutions to EE related to Minkowski + gravitational memory by gauge transformation

[Verlinde^2 '92; ...; He, A.R., Zurek '23]

Main results & outline

1. On-shell action in the IR sector of 4d AFS as generating function for CFT_2 current correlators

2. A relation between $\hat{K}$ and the soft supertranslation charge

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1. On-shell action in the IR sector of 4d AFS as generating function for CFT_2 current correlators

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- Soft modes in 4d AFS
- Boundary (on-shell) action in the (leading) soft sector of 4d AFS
- Soft charge fluctuations

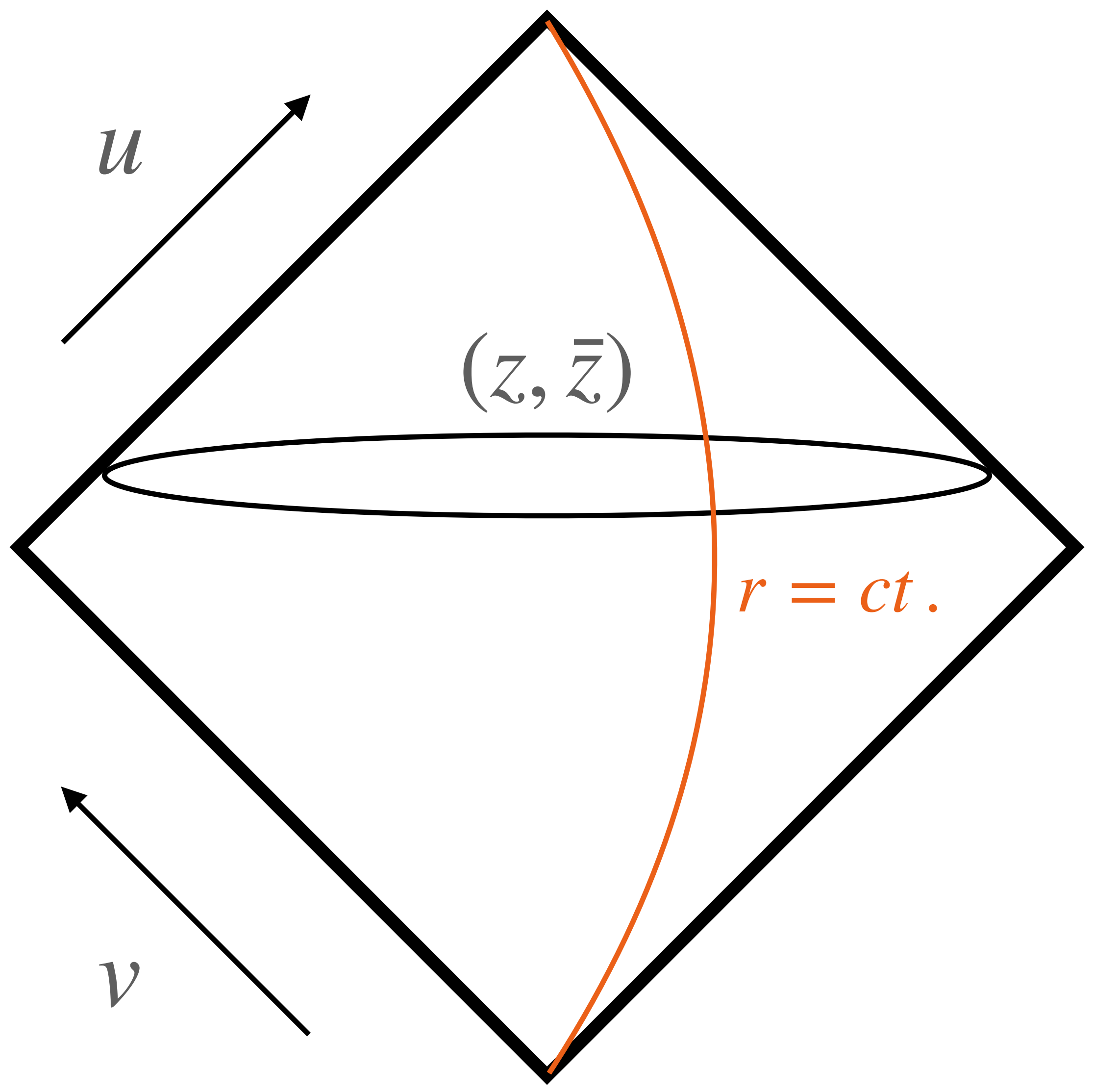
Asymptotically Flat Spacetimes (AFS)

Solutions to the Einstein eq. at large r:

$$ds^2 = ds^2_{\text{Mink}} + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + \frac{2m_B}{r}du^2 + \cdots$$

$C_{zz}(u, z, \bar{z})$: shear, free data

$N_{zz}(u, z, \bar{z}) = \partial_u C_{zz}(u, z, \bar{z})$: news (gravitational radiation)



Equations of motion determine the subleading components in terms of C_{zz}, eg. :

$$G_{uu} = 0 \implies \partial_u m_B = \frac{1}{4}\left(D_z^2 N_{\bar{z}\bar{z}} + D_{\bar{z}}^2 N_{zz}\right) - \frac{1}{4} N_{zz} N_{\bar{z}\bar{z}}$$

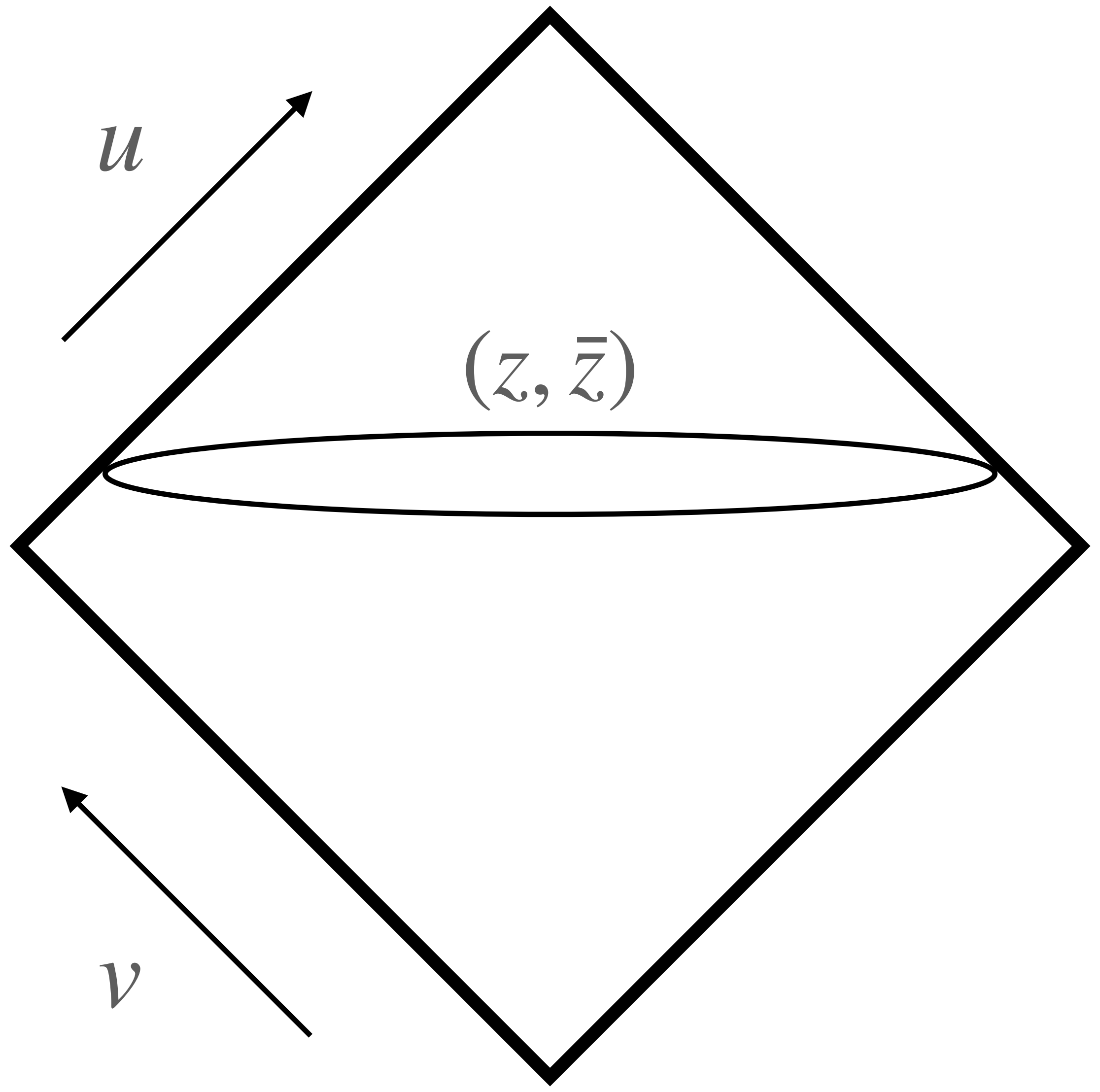
Leading soft sector in AFS

Restrict to shear profiles of the form:

$$C_{zz} = -2D_z^2 C(z,\bar{z}) + D_z^2 N(z,\bar{z})\Theta(u-u_0)$$

- $C(z,\bar{z})$ labels the vacuum degeneracy
- $N(z,\bar{z})$ is related to a soft graviton:

$$\int du N_{zz} = D_z^2 N = \Delta C_{zz}, \quad N_{zz} \equiv \partial_u C_{zz}$$



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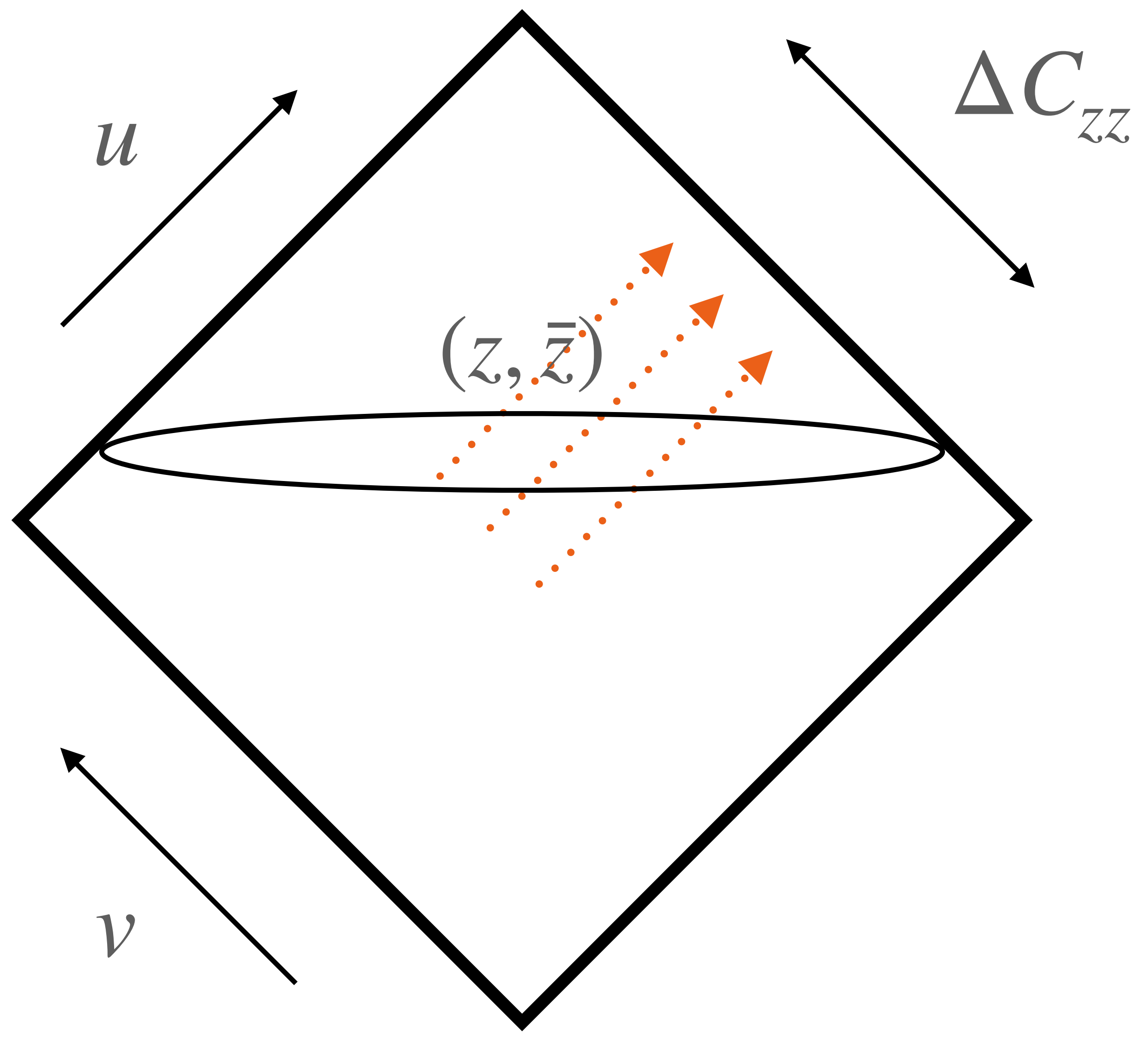

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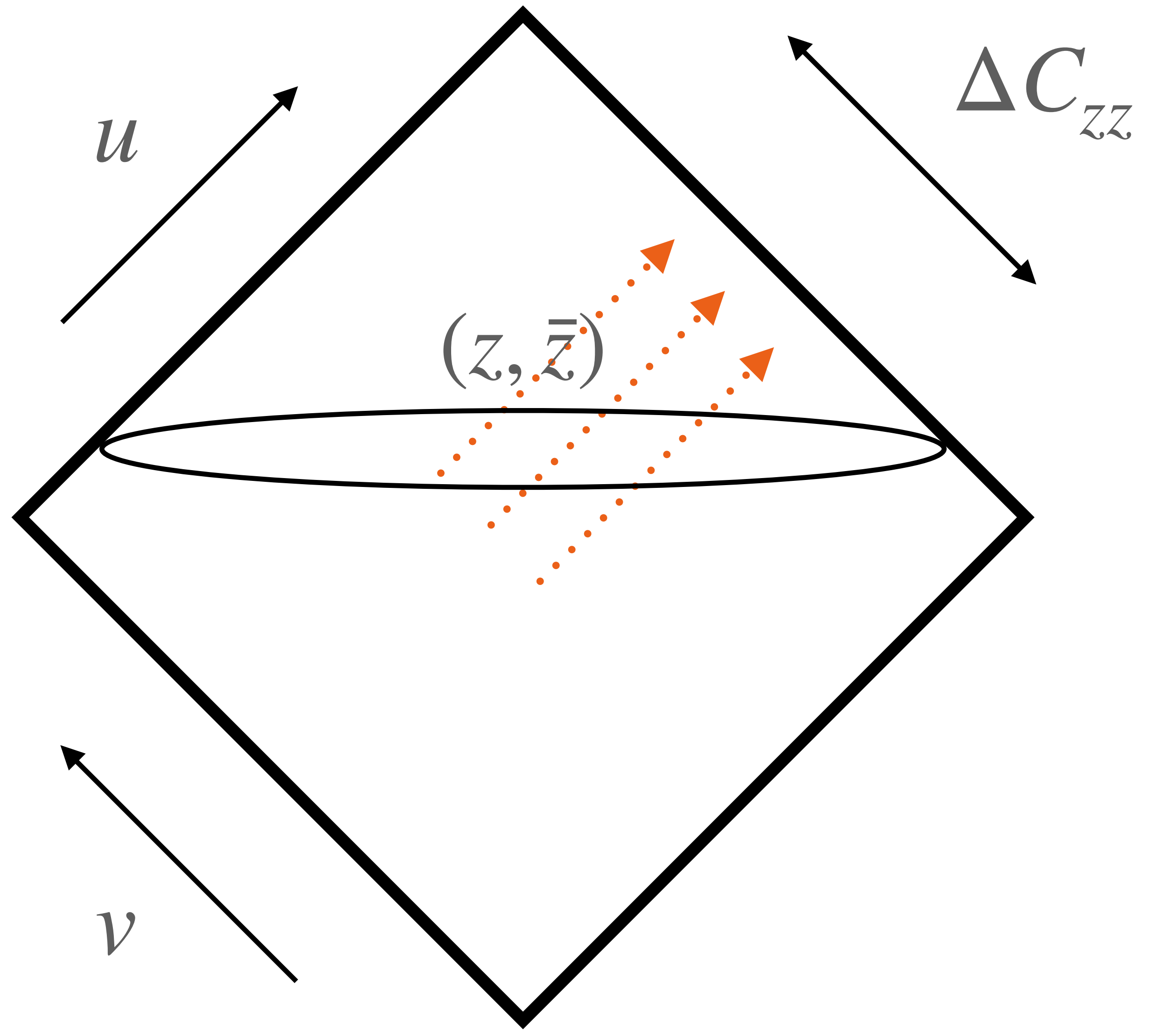

C and N are canonically paired variables of the leading IR sector of 4d AFS

Leading soft sector in AFS

Variation of Lagrangian form:

$$\delta \mathrm{L} = \mathrm{EOM} \cdot \delta g + d\theta(g, \delta g)$$

[Iyer, Wald '94]

Einstein-Hilbert action evaluated on asymptotically flat solution labeled by the soft shear:

$$\Theta(g, \delta g) = \frac{1}{32\pi G} \int du \int d^2 z \left[4\delta m_B + \frac{1}{4}\delta \left(\partial_z^2 N \partial_{\bar{z}}^2 N \right) \delta(u - u_0) + \frac{1}{2}\delta \left(\partial_z^2 C \partial_{\bar{z}}^2 N + h.c. \right) \delta(u - u_0) \right]$$
$$- \frac{1}{16\pi G} \int du \int d^2 z \left(\partial_z^2 \delta C \partial_{\bar{z}}^2 N + h.c. \right) \delta(u - u_0)$$

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$$-\frac{1}{16\pi G}\int du \int d^2z \left(\partial_z^2 \delta C \partial_{\bar{z}}^2 N + h.c.\right)\delta(u-u_0) \implies \delta\Theta(g,\delta g) = -\frac{1}{16\pi G}\int d^2z \left(\partial_z^2 \delta C \wedge \partial_{\bar{z}}^2 \delta N + h.c.\right)$$

symplectic form

Leading soft sector in AFS

$\implies$ C and N are canonically paired with bracket:

$$[N(z,\bar{z}),C(w,\bar{w})]=4iG|z-w|^2\log|z-w|^2$$

[He, Lysov, Mitra, Strominger '14]

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Boundary action in AFS

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In order for variational principle to be well defined:

1. Add boundary action to cancel the first line

$$S^{\text{IR}} = S_{\text{EH}} + S_{\text{bdry.}}, \quad S_{\text{bdry.}} = -\frac{1}{32\pi G}\int d^2z \left[4\int du\, m_B + \frac{1}{4}\partial_z^2 N \partial_{\bar{z}}^2 N + \frac{1}{2}\left(\partial_z^2 C \partial_{\bar{z}}^2 N + h.c.\right)\right]$$

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Boundary action in AFS

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since $N = 0$ would otherwise set the gravitational memory to 0

Boundary action in AFS

$$S^{\mathrm{IR}}_{\mathrm{on-shell}} = -\frac{1}{32\pi G}\int d^2z \left[4\int du m_B + \frac{1}{4}\partial_z^2 N \partial_{\bar{z}}^2 N + \frac{1}{2}\left(\partial_z^2 C \partial_{\bar{z}}^2 N + h.c.\right)\right]$$

- m_B dependence can be traded for $\mathcal{O}(N^2)$ term by constraint, coefficient is IR divergent (related to Weinberg soft S-matrix?)

[Weinberg '65]

- Linearize in N (keeping the C background fixed)

- $S^{\mathrm{IR}}_{\mathrm{on-shell}}$ becomes gauge invariant after adding a corner term $\propto \int d^2z \partial_{\bar{z}}^2 N \partial_z C$ [Donnelly, Freidel '16]

- Also including the contribution from $\mathscr{I}^-$: $S^{\mathrm{IR}} = -\dfrac{1}{8\pi G}\displaystyle\int d^2z \partial_{\bar{z}}^2 N \partial_z^2 C$

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a) generating function for Kac-Moody current correlators

b) soft supertranslation charge

a) Current correlators in 2d CFT

Associate with $S^{\rm IR}_{\rm on-shell}[N, C]$ the partition function:

$$Z[C] \equiv \int [dN] e^{-S^{\rm IR}_{\rm on-shell}[N,C]}$$

- $Z[C]$ is the generating function for 2d correlators of Kac-Moody (supertranslation) currents (these are dual to soft gravitons in 4d) [He, Lysov, Mitra, Strominger '14]

- The on-shell action agrees with the action derived from a 2d effective action of Goldstone modes after integrating in the memory/soft mode [Himwich, Narayanan, Pate, Paul, Strominger '20; Kapec, Mitra '21]

- Potentially related to partition function for edge modes [Donnelly, Wall '14, …Chen, A.R., Myers '24]

b) Soft supertranslation charge

Supertranslation charge in 4d AFS:

$$
\begin{aligned}
Q_{f=C} &= \frac{1}{4\pi G}\int d^2z C(z,\bar{z}) m_B(z,\bar{z}) \\
&= \underbrace{-\frac{1}{16\pi G}\int_{\mathcal{I}^+} du d^2z C(z,\bar{z})\left(\partial_z^2 N_{\bar{z}\bar{z}} + h.c.\right)}_{Q_S} + Q_H
\end{aligned}
$$

$$
N_{zz} = \partial_z^2 N \delta(u - u_0) \implies Q_S = -\frac{1}{8\pi G}\int d^2z \partial_z^2 C \partial_{\bar{z}}^2 N = S^{\text{IR}} + \mathcal{O}(N^2)
$$

- C is a function

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$$N_{zz} = \partial_z^2 N \delta(u-u_0) \quad \Longrightarrow \quad Q_S = -\frac{1}{8\pi G}\int d^2z \partial_z^2 C \partial_{\bar{z}}^2 N = S^{\rm IR} + \mathcal{O}(N^2)$$

- C is a function
- Promote C to an operator (recall $\{C, N\} \neq 0$):

$$\hat{Q}_S = -\frac{1}{8\pi G}\int d^2z \partial_z^2 \hat{C} \partial_{\bar{z}}^2 \hat{N}$$

Soft charge fluctuations?

When $m_B = 0$ (but in the presence of matter) the supertranslation charge vanishes and $Q_S + Q_H = 0$

This is condition is equivalent to the $G_{uu} = T^M_{uu}$ constraint:

$$\delta(u)\,\Box\, h(z,\bar{z}) = T^M_{uu}(u,z,\bar{z}) \implies \underbrace{\int d\Omega f(\Omega)\,\Box\, h}_{-Q_S} = \underbrace{\int du \int d\Omega f(\Omega) T^M_{uu}}_{Q_H = K}$$

- In vacua with $m_B = 0$, $\hat{Q}_S \propto \hat{K} \implies \langle \Delta \hat{Q}_S^2 \rangle \propto \langle \Delta \hat{K}^2 \rangle \sim \dfrac{A}{\ell_p^2}$ [provided that $f \to \hat{C}$]

Summary

- The on-shell action for the (leading) soft sector of gravity in (3+1)d AFS is non-trivial
- Fixing the zero mode of the shear (Goldstone), we recover the generating function for 2d correlators of Kac-Moody currents dual to 4d soft gravitons
- The boundary action is related to the soft component of the supertranslation charge
- Allowing for the Goldstone to fluctuate leads to a new soft charge operator whose fluctuations are enhanced by an IR scale (related to the area of the celestial sphere)

Outlook

- Why should C be promoted to an operator? Finite size causal diamonds?
- Relation to edge modes and entanglement in gravitational theories in AFS?
- How could vacuum spacetime fluctuations be experimentally probed? Can we measure $\hat{Q}_S$?

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Thank you!

Extra slides

Current correlators in 2d CFT

Weinberg soft S-matrix $S = e^{-\Gamma}$, $\Gamma \propto \log \frac{\Lambda_{UV}}{\Lambda_{IR}} \int d^2z J_{ab} J^{ab}$ reproduced from 2d effective action of Goldstones:

[Kapec, Mitra '21]

$$e^{-\Gamma} = \int [dC] e^{-S_{\text{soft}}[C] - S_{\text{int}}[C,j]} \implies \quad S_{\text{soft}}[C] \propto \int d^2z\, \widetilde{C}_{ab}(z,\bar{z}) \widetilde{C}^{ab}(z,\bar{z}), \quad C_{zz} = -2\partial_z^2 C \quad \text{pure gauge}$$

$$S_{\text{int}} \propto \int d^2z J_{ab} \widetilde{C}^{ab}$$

- In the presence of external soft gravitons, integrate in soft graviton N and further demand that $\int [dN] e^{-S'_{\text{soft}}[N,C]} = e^{-S_{\text{soft}}[C]} \equiv Z[C]$
- $S'_{\text{soft}}[N,C]$ agrees (at least to linear order in N) with the on-shell action we found!

[He, Lysov, Mitra, Strominger '14]