Type change and quantum reference frames

CJ Fewster

Department of Mathematics and YCQT, University of York

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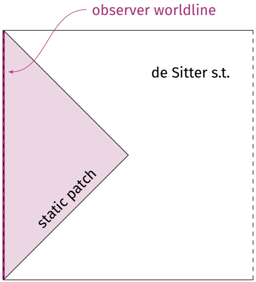
CMP **406**:19 (2025), `arXiv:2403.11973` with D.W. Janssen, L.D. Loveridge, K. Rejzner, J. Waldron

The CLPW model Chandrasekaran, Longo, Penington, Witten

Consider QFT on de Sitter in representation induced from Bunch–Davies.



- QFT observables $\mathcal{M}_S$ in static patch have type III
- Invoke a specific, simple 'observer system' and, in the presence of gravity, regard H_{tot} as a constraint.

  Only invariant elements of the joint algebra are observable.
- Important observation: Resulting algebra has type II_1.

Questions How fine tuned is the observer system? How essential is gravity?

The role of gravity

CLPW's use of H_{tot} as a constraint arises from linearisation (in)stability in GR Moncrief suggesting that a gravity plays a decisive role.

However, it is natural to restrict to isometry-invariant observables in any QFT on CST, as the fixed-background remnant of the hole argument for generally covariant theories.

If GR is pointless, QFT in CST only has points modulo isometries.
(Normally this is ignored!)

The role of the observer

The CLPW observer isn't coupled to the system – what (if anything) is it observing?

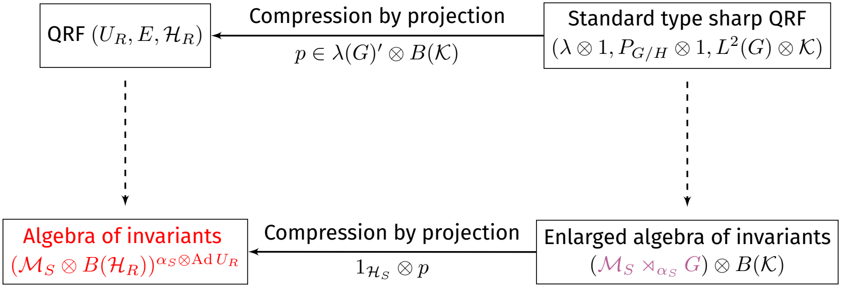
Our approach: start from a description of measurement theory in QFT CJF & Verch, CMP 2020

- The QFT is measured by locally coupling to a probe theory
- Isometrically related couplings should be indistinguishable
- Ambiguity is resolved by introducing a quantum reference frame
  CLPW's observer can be given a QRF structure.
- Readings of the QRF have no intrinsic physical significance...
- ...but invariant elements of the joint QFT-QRF algebra are physical observables.
- Investigate in generality what properties are required for change of algebra type.

Main results (1): QRFs and the algebra of invariants

Let G be the isometry group of spacetime M. Suppose that for every compact $K \neq \emptyset$, the stabliser subgroup $\{g \in G : g.K = K\}$ is compact [as holds for dS static patch].

Any QRF that resolves isometry-related measurements has a particular form that can be classified using Mackey theory, and which allows the algebra of invariants to be computed by crossed product techniques.



Main results (2): the type of the algebra of invariants

Special case: $G = \mathbb{R} \times H$ with compact H and $\alpha_S(\cdot, \mathrm{id}_H) =$ modular action for $\mathcal{M}_S$ w.r.t. a β-KMS state (includes CLPW).

Theorem The algebra of QRF-QFT invariants $\mathcal{A}_{\mathrm{inv}}$ is always semifinite (not type III).

Furthermore, $\mathcal{A}_{\mathrm{inv}}$ is finite if the QRF admits a particular well-behaved β-KMS weight (effectively constrains the growth of $\#$ QRF d.o.f. with energy).

Summary: CLPW is generalised within a broader, operationally motivated, setting, elucidating the roles of good thermodynamic behaviour for the QRF and gravity.

Quantum reference frames (QRFs)

The idea of studying QRFs goes back to Aharonov and beyond that to Eddington. There are several recent formalisations

- operational [our choice] Busch, Loveridge, Miyadera...
- perspective neutral Höhn, Giacomini, Bruckner...
- information-based Bartlett, Spekkens...

Given a quantum system $\mathcal{H}_S$ with symmetry group G represented unitarily by U_S, a QRF is another quantum system $\mathcal{H}_R$, with G represented unitarily by U_R, and a G-covariant observable E, technically, a POVM on value space Σ on which G acts.

- $\langle E(\Delta)\rangle_\psi$ = probability that the QRF 'reads' a value in $\Delta \subset \Sigma$ in state $\psi \in \mathcal{H}_R$.
- G-covariance means $U_R(g)E(\Delta)U_R(g)^{-1} = E(g.\Delta)$

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While $B(\mathcal{H}_S)$ might not have many G-invariant observables, the joint system $B(\mathcal{H}_S \otimes \mathcal{H}_R)$ often does, e.g., relative position of two quantum particles.
A large subclass can be constructed and parameterised using the observable E.

The CLPW clock as an unsharp QRF for time translations

For $G = \mathbb{R}$, let $\mathcal{H}_R = L^2(\mathbb{R}^+, dq)$, $H_R = q$, $U_R(t) = e^{iH_R t}$. Value space $\Sigma = \mathbb{R}$.

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Define the covariant POVM E as follows: for $\Delta \subset \mathbb{R}$,

$$E(\Delta) = W^* \chi_\Delta W, \qquad (W\psi)(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}^+} dq\, e^{-iqt} \psi(q),$$

and one has $U_R(t)E(\Delta)U_R(t)^* = E(\Delta + t)$, i.e., E is a time observable.

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W is a nonunitary isometry $\implies$ $E(\cdot)$ is a POVM and not a PVM, an unsharp QRF.

However, it is a compression of a sharp QRF on $L^2(\mathbb{R}, dq)$.
(Pauli: a sharp time observable requires $\sigma(H) = \mathbb{R}$).