Carroll physics and flat space holography

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based on work since 2012 with Adami, Afshar, Aggarwal, Ammon, Bagchi, Basu, Bergshoeff, Chakrabortty, Ciambelli, Detournay, Ecker, Fareghbal, Fiorucci, Fredenhagen, Gary, González, Hartong, Henneaux, Merbis, Montecchio, Nandi, Parekh, Pérez, Prohazka, Radhakrishnan, Riegler, Rosseel, Ruzziconi, Sadeghian, Salgado-Rebolledo, Salzer, Schöller, Shams Nejati, Sheikh-Jabbari, Simón, Sinha, Taghiloo, Troncoso, Vassilevich, Wutte, Yavartanoo, Zwikel

Carroll symmetries

What?

Spacetime symmetries

Spacetime name	Space	Time	Energy	Speed of light c
Aristotelian	absolute	absolute	absolute	anything
Galilean	relative	absolute	relative	∞
Lorentzian	relative	relative	relative	finite

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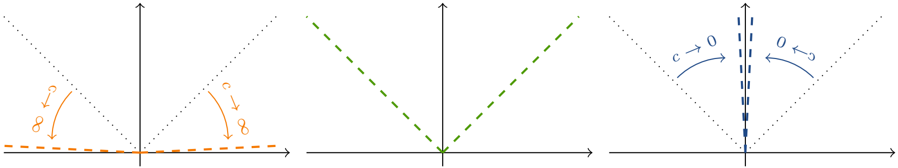

Galilean lightcone — Lorentzian lightcone — Carrollian lightcone

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Carroll spacetime, limit def.: $c \to 0$ limit of Lorentzian spacetime

Levy-Leblond '65; Sen Gupta '66

see also Bacry, Levy-Leblond '68; Figueroa O'Farrill, Prohazka '18

"Now, here, you see, it takes all the running you can do, to keep in the same place." Lewis Carroll 1871 (Through the Looking Glass)

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Intrinsic def.: metric signature $(0, +, \dots, +)$ and vector field in kernel

Henneaux '79 Teitelboim '81

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Example: flat Carroll spacetime ($c \to 0$ limit of Minkowski)

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = 0 \cdot \mathrm{d}t^2 + \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j \qquad v = v^\mu\,\partial_\mu = \partial_t$$

Carroll boosts: $t \to t + b_i\,x^i \qquad x^i \to x^i \qquad g_{\mu\nu}\,v^\nu = 0$

Carroll symmetries

Why?

- Carroll conformal Killing equations

$$\mathcal{L}_\xi \, g_{\mu\nu} = 2\lambda \, g_{\mu\nu} \qquad\qquad \mathcal{L}_\xi \, v^\mu = -\lambda \, v^\mu$$

solved by Carroll conformal Killing vectors

$$\xi = \big(T(x^j) + \frac{t}{d}\,\partial_i \, Y^i(x^j)\big)\,\partial_t + Y^i(x^j)\,\partial_i$$

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- T generates supertranslations (x^i-dependent t-translations)

special cases:

$$\begin{aligned} T &= 1: && \text{time translations} \quad H = \partial_t \\ T &= x_i: && \text{Carroll boosts} \quad B_i = x_i\partial_t \\ T &= x^i x_i: && \text{temporal SCTs} \quad K = x^i x_i\partial_t \end{aligned}$$

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- Carroll conformal algebra in D spacetime dimensions (CCA$_D$)

supertranslations	$M_T = T(x^i)\,\partial_t$
spatial translations	$P_i = \partial_i$
rotations	$J_{ij} = x_i\,\partial_j - x_j\,\partial_i$
dilatations	$D = t\,\partial_t + x^i\,\partial_i$
spatial SCTs	$K_i = x^j x_j\,\partial_i - 2x_i\big(t\,\partial_t + x^j\,\partial_j\big)$

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$$\boxed{\mathrm{CCA}_D \simeq \mathrm{BMS}_{D+1}}$$

Barnich, Troessaert '10, Bagchi '10, Duval, Gibbons, Horvathy '14

BMS_{D+1} = asymptotic symmetries of asymptotically flat spacetimes

$D=3$: Bondi, van der Burgh, Metzner '62; Sachs '62, $D=2$: Ashtekar, Bičák, Schmidt '96; Barnich, Compère '06,
$D>3$: Kapec,Lysov,Pasterski,Strominger 15; Hollands,Ishibashi,Wald '16; Aggarwal '18; Capone,Mitra,Poole,Tomova '23

e.g. metric on $\mathscr{I}^+$: $\mathrm{d}s^2 = r^2\left(\mathrm{d}\Omega^2_{S^{D-2}} - \frac{1}{r^2}\,\mathrm{d}u^2\right)\big|_{r\to\infty}$

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Flat space holography: dual is Carrollian CFT in one dimension lower

Instead of AdS$_{D+1}$/CFT$_D$ we have Flat$_{D+1}$/CCFT$_D$ CCFT = Carrollian CFT

Carroll symmetries

Why?

Additional motivations for Carroll symmetries **besides flat holography** include

- plane gravitational waves & memory effects Souriau '73; Elbistan, Zhang, Horvathy '23
- zero signature spacetimes & BKL Henneaux '79; Damour, Henneaux, Nicolai '02
- cosmology Henneaux '82; de Boer, Hartong, Obers, Sybesma, Vandoren '21
- **null hypersurfaces** Penna'15; Donnay,Marteau'19; Ciambelli,(Freidel),Leigh,Marteau,Petropoulos'19-'24
- Carroll expansion of gravity Hartong '15; Hansen, Obers, Oling, Søgaard '21
- tensionless strings Bagchi, Chakrabortty, Parekh '15; Bagchi, Banerjee, Parekh '19
- hydro de Boer, Hartong, Obers, Sybesma, Vandoren '17; Ciambelli, Marteau, Petkou, Petropoulos, Siampos '18
- black holes Penna '18; Redondo-Yuste, Lehner '22; Gray, Kubiznak, Perche, Redondo-Yuste '22
- Carroll dilaton gravity DG, Hartong, Prohazka, Salzer '20; Gomis, Hidalgo, Salgado-Rebolledo '20
- fractons Bidussi, Hartong, Have, Musaeus, Prohazka '21; Figueroa-O'Farrill, Pérez, Prohazka '23
- flat bands Bagchi, Banerjee, Basu, Islam, Mondal '22
- Hall effects Marsot, Zhang, Chernodub, Horvathy '22
- spacetime subsystem symmetries Baig, Distler, Karch, Raz, Sun '23; Kasikci, Ozkan, Pang '23
- Carroll black holes Ecker, DG, Hartong, Pérez, Prohazka, Troncoso '23; DG, Montechhio, Shams Nejati '24
- ...

Carroll symmetries
Which?

Algebra of Carroll conformal Killing vectors in d spatial dimensions:

$$[M_T, J_{ij}] = M_{-2x_{[i}\partial_{j]}T} \qquad [M_T, P_i] = M_{-\partial_i T}$$

$$[M_T, D] = M_{T - x^i\partial_i T} \qquad [M_T, K_i] = M_{2x_i x^j\partial_j T - x^j x_j \partial_i T - 2x_i T}$$

$$[P_i \pm K_i, D] = P_i \pm K_i \qquad [P_i, K_j] = 2J_{ij} - 2\delta_{ij}D$$

$$[P_k \pm K_k, J_{ij}] = 2\delta_{k[i}(P_{j]} \pm K_{j]}) \qquad [J_{ij}, J_{kl}] = J_{[i[k}\delta_{j]l]}$$

$$[M_T, M_{T'}] = [J_{ij}, D] = 0 \qquad [P_i, P_j] = [K_i, K_j] = 0$$

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Some generalizations:

- anisotropic scale invariance with Lifshitz exponent z

$$D = zt\partial_t + x^i\partial_i$$

gravity side: DG, Pérez, Sheikh-Jabbari, Troncoso, Zwikel '19

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- include "superrotations" $Y^j(x^i)$ with fewer/no restrictions on Y^j

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- extend to Carroll-diff algebra $\xi = T(t, x^i)\partial_t + Y^j(x^i)\partial_j$

Ciambelli, Marteau, Petkou, Petropoulos, Siampos '18

see also Aganagic, Costello, McNamara, Vafa '17; Seiberg, Shao '20; Baig, Distler, Karch, Raz, Sun '23
where $T(t, x^i)$ generates fractonic time translations

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- gauge finite Carroll algebra to get Carroll gravity

Hartong '15

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- extend to Carroll-diff algebra $\xi = T(t, x^i)\partial_t + Y^j(x^i)\partial_j$
- gauge finite Carroll algebra to get Carroll gravity
- supersymmetric and/or higher spin extensions

Bagchi, Fareghbal, DG, Rosseel '13; González, Matulich, Pérez, Troncoso '13; Bagchi, DG, Nandi '22

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Subalgebra: (finite) Carroll algebra, $H = M_1$, $B_i = M_{x_i}$, P_i, J_{ij}

$$[H, B_i] = 0 \qquad [B_i, B_j] = 0 \qquad [P_i, B_j] = \delta_{ij}\, H \qquad [B_k, J_{ij}] = 2\delta_{k[i} B_{j]}$$

- Hamiltonian commutes with Carroll boosts flat bands, soft hair, tensionless strings, ...

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- H, P_i, B_i generate Heisenberg subalgebra of Carroll algebra H is central
- Carroll algebra preserves Carroll structure, $\mathcal{L}_\xi g_{\mu\nu} = 0 = \mathcal{L}_\xi v^\mu$

reminder: $g_{\mu\nu}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu = 0 \cdot \mathrm{d}t^2 + \delta_{ij}\, \mathrm{d}x^i\, \mathrm{d}x^j$ and $v^\mu \partial_\mu = \partial_t$

Carroll symmetries

Anything special in 1+1 dimensions?

- reminder: CFT_2 has infinite symmetries with central extension

$$[\mathcal{L}^\pm_n, \mathcal{L}^\pm_m] = (n-m)\,\mathcal{L}^\pm_{n+m} + \frac{c^\pm}{12}\,\big(n^3-n\big)\,\delta_{n+m,\,0}$$

holographic CFT_2: $c^\pm = \frac{3\ell}{2G}$ Brown, Henneaux '86

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- Carroll conformal algebra obtained from İnönü–Wigner contraction

$$L_n := \mathcal{L}^+_n - \mathcal{L}^-_{-n} \qquad\qquad M_n := \frac{1}{\ell}\left(\mathcal{L}^+_n + \mathcal{L}^-_{-n}\right)$$

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- AdS radius to infinity, $\ell \to \infty$: CCFT_2 algebra with central extensions

$$\boxed{\begin{aligned}[L_n, L_m] &= (n-m)L_{n+m} + \frac{c_L}{12}\left(n^3-n\right)\delta_{n+m,\,0}\\ [L_n, M_m] &= (n-m)M_{n+m} + \frac{c_M}{12}\left(n^3-n\right)\delta_{n+m,\,0}\end{aligned}}$$

$$c_L = c^+ - c^- = 0 \qquad\qquad c_M = \frac{c^+ + c^-}{\ell} = \frac{3}{G}$$

L_n: superrotations (diff S^1), M_n: supertranslations

Flat/CCFT correspondence
Main statement

Fact: $\mathrm{CCA}_D \simeq \mathrm{BMS}_{D+1}$

Duval, Gibbons, Horvathy '14

Flat/CCFT correspondence

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Conjecture: Carrollian CFT in D dimensions is field theory dual to asymptotically flat gravity in $D+1$ dimensions

precursors:
Witten (Strings 98)
Polchinski '99
Susskind '99
Giddings '99
de Boer, Solodukhin '03
Banks '03

examples in 2+1 bulk dim's: Bagchi, Detournay, DG '12; Barnich, Gomberoff, González '12

Carrollian/Celestial dictionary in 3+1 bulk dim's: Donnay, Fiorucci, Herfray, Ruzziconi '22; Bagchi, Banerjee, Basu, Dutta '22

Science history: flat space holography pioneered in 60ies

- '62: BMS symmetries
- '65: Carroll symmetries
- '65: soft graviton and photon theorems

Flat/CCFT correspondence
Main statement

Fact: $\mathrm{CCA}_D \simeq \mathrm{BMS}_{D+1}$

Conjecture: Carrollian CFT in D dimensions is field theory dual to asymptotically flat gravity in $D+1$ dimensions

Test conjecture by calculating physical observables, like

- correlation functions
- thermal entropy
- entanglement entropy

on both sides of conjectured correspondence

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Focus on three bulk dimensions

Flat$_3$/CCFT$_2$ correspondence

Stress tensor correlation functions, field theory side Bagchi, DG, Merbis '15

- Fourier modes of CCFT$_2$ stress tensor on cylinder:

$$M := \sum_n M_n e^{-in\varphi} - \frac{c_M}{24} \qquad N := \sum_n \big(L_n - inuM_n\big) e^{-in\varphi} - \frac{c_L}{24}$$

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- Carroll conservation equations: $\partial_u M = 0 \qquad \partial_u N = \partial_\varphi M$

compare with CFT_2 results:

$$\partial_{\bar z} T_{zz} = 0 = \partial_z T_{\bar z \bar z}$$

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- Carroll conservation equations: $\partial_u M = 0 \qquad \partial_u N = \partial_\varphi M$
- 2-point functions for CCFT$_2$ on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M^1 M^2 \rangle := \langle M(u_1, \varphi_1)\, M(u_2, \varphi_2) \rangle = 0$$

$$\langle M^1\, N^2 \rangle = \frac{c_M}{2 s_{12}^4}$$

$$\langle N^1\, N^2 \rangle = \frac{c_L - 2 c_M \tau_{12}}{2 s_{12}^4}$$

with $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j)\,\cot[(\varphi_i - \varphi_j)/2]$

similarly: all 3-point functions determined uniquely by CCFT$_2$ symmetries and central charges
$\langle M^1 M^2 M^3 \rangle = \langle M^1 M^2 N^3 \rangle = 0$, $\langle M^1 N^2 N^3 \rangle = c_M/(s_{12}^2 s_{13}^2 s_{23}^2)$,
$\langle N^1 N^2 N^3 \rangle = (c_L - c_M(\tau_{12} + \tau_{13} + \tau_{23}))/(s_{12}^2 s_{13}^2 s_{23}^2)$

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- 4- and higher n-point functions not universal, e.g.

$$\langle M^1\, N^2\, N^3\, N^4 \rangle = \frac{2 c_M\, g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}$$

with the conformal cross-ratio function

$$g_4(\gamma) = \frac{\gamma^2 - \gamma + 1}{\gamma} \qquad \gamma = \frac{s_{12}\, s_{34}}{s_{13}\, s_{24}}$$

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- Carroll Ward id's: $\partial_u \langle M\,\mathcal{O}\rangle = 0 \qquad \partial_u \langle N\,\mathcal{O}\rangle = \partial_\varphi \langle M\,\mathcal{O}\rangle$
- 2-point functions for CCFT$_2$ on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M^1 M^2\rangle = 0 \qquad \langle M^1\,N^2\rangle = \frac{c_M}{2s_{12}^4} \qquad \langle N^1\,N^2\rangle = \frac{c_L - 2c_M\tau_{12}}{2s_{12}^4}$$

- BPZ-type recursion relations determine all n-point functions:

$$\langle M^1\,N^2\dots N^n\rangle = \sum_{i=2}^{n}\Big(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2}\partial_{\varphi_i}\Big)\langle M^2\,N^3\dots N^n\rangle + \text{disconnected}$$

$$\langle N^1 N^2\dots N^n\rangle = \frac{c_L}{c_M}\langle M^1 N^2\dots N^n\rangle + \sum_{i=1}^{n} u_i\partial_{\varphi_i}\langle M^1 N^2\dots N^n\rangle$$

$c_{ij} := \cot[(\varphi_i - \varphi_j)/2]$

$\mathrm{Flat}_3/\mathrm{CCFT}_2$ correspondence

Stress tensor correlation functions, gravity side Bagchi, DG, Merbis '15

- Chern–Simons formulation of 3d gravity with iso$(2,1)$ connection A

  Achucarro, Townsend '86; Witten '88

$\text{Flat}_3/\text{CCFT}_2$ correspondence

Stress tensor correlation functions, gravity side Bagchi, DG, Merbis '15

- Chern–Simons formulation of 3d gravity with iso$(2,1)$ connection A
- use "holographic" gauge with fixed group element $b(r) = \exp[\frac{r}{2}\, M_-]$

$$A = b^{-1}\big(\mathrm{d} + a(u,\varphi)\big)\, b$$

and the boundary connection

$$a(u,\varphi) = \big(M_+ + \mathcal{M}(u,\varphi) M_-\big)\,\mathrm{d}u + \big(L_+ + \mathcal{M}(u,\varphi) L_- + \mathcal{N}(u,\varphi) M_-\big)\,\mathrm{d}\varphi$$

subject to the on-shell constraints

$$\partial_u \mathcal{M} = 0 \qquad\qquad \partial_u \mathcal{N} = \partial_\varphi \mathcal{M}$$

Barnich, Donnay, Matulich, Troncoso '14

non-zero iso$(2,1)$ commutators:

$[L_+, L_-] = 2L_0 \quad [L_\pm, L_0] = \pm L_\pm \quad [L_\pm, M_\mp] = \pm 2M_0 \quad [L_\pm, M_0] = \pm M_\pm \quad [L_0, M_\pm] = \mp M_\pm$

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$$A = b^{-1}\big(\,\mathrm{d}{+}a(u,\varphi)\big)\,b$$

- include chemical potentials as sources

$$a_u \to a_u - \mu_M M_+ - \mu_L L_+ + \underbrace{a_0 M_0 + a_1 L_0 + a_2 M_- + a_3 L_-}_{\text{determined on-shell}}$$

Gary, DG, Riegler, Rosseel '15

on-shell conditions on state-dependent functions (CCFT_2 Ward id's):

$$-\partial_u \mathcal{M} = \partial_\varphi \mathcal{M} \mu_L + 2\mathcal{M}\partial_\varphi \mu_L + \frac{c_M}{12}\,\partial_\varphi^3 \mu_L$$

$$-\partial_u \mathcal{N} = -\partial_\varphi \mathcal{M} + \partial_\varphi \mathcal{N} \mu_L + 2\mathcal{N}\partial_\varphi \mu_L + \partial_\varphi \mathcal{M} \mu_M + 2\mathcal{M}\partial_\varphi \mu_M + \frac{c_M}{12}\,\partial_\varphi^3 \mu_M$$

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- include chemical potentials as sources
- localize sources at insertion point(s) of n-point functions

  analogous to CFT$_2$ construction of 2-point correlators from 1-point function of deformed CFT with source $\mu(z,\bar{z})$

$$S_\mu = S_0 - \int \mathrm{d}^2 z\, \mu(z,\bar{z})\; T(z)$$

  localized at insertion point

$$\mu(z,\bar{z}) = \epsilon\, \delta^{(2)}(z-z_2,\; \bar{z}-\bar{z}_2)$$

  yielding

$$\langle\, T(z_1)\rangle_\mu = \langle\, T(z_1)\rangle + \epsilon\,\langle\, T(z_1)\, T(z_2)\rangle + \mathcal{O}(\epsilon^2)$$

Drinfeld,Sokolov '84; Polyakov '87; H.Verlinde '90; Bañados,Caro '04

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- include chemical potentials as sources
- localize sources at insertion point(s) of n-point functions
- for single localized sources recover CCFT_2 2-point functions

localized chemical potentials/sources $\mu_{L/M} = \epsilon_{L/M}\,\delta^{(2)}(u-u_2,\,\varphi-\varphi_2)$ inserted in linearized Ward id's

$$\partial_u \mathcal{M}^{(1)} = -\epsilon_L\,\frac{c_M}{12}\,\Big(\partial_\varphi^3\delta^{(2)}(u-u_2,\,\varphi-\varphi_2)+\partial_\varphi\delta^{(2)}(u-u_2,\,\varphi-\varphi_2)\Big)$$

$$\partial_u \mathcal{N}^{(1)} = \partial_\varphi\mathcal{M}^{(1)} - \epsilon_M\,\frac{c_M}{12}\,\Big(\partial_\varphi^3\delta^{(2)}(u-u_2,\,\varphi-\varphi_2)+\partial_\varphi\delta^{(2)}(u-u_2,\,\varphi-\varphi_2)\Big)$$

solved with Green function $\partial_u\partial_\varphi G(u-u_2,\,\varphi-\varphi_2) = \delta^{(2)}(u-u_2,\,\varphi-\varphi_2)$ given by

$$G(u_1-u_2,\,\varphi_1-\varphi_2) = \ln\Big((u_1-u_2)\sin\big((\varphi_1-\varphi_2)/2\big)\Big)$$

yielding

$$\mathcal{M}^{(1)} = \epsilon_L\quad\underbrace{\frac{c_M}{2s_{12}^4}}_{\langle M^1 N^2\rangle}\qquad\qquad \mathcal{N}^{(1)} = \epsilon_M\quad\underbrace{\frac{c_M}{2s_{12}^4}}_{\langle N^1 M^2\rangle}\ +\epsilon_L\,\Big(\underbrace{-\frac{c_M\,\tau_{12}}{s_{12}^4}}_{\langle N^1 N^2\rangle}\Big)$$

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- complete induction: localizing n times instead of $n-1$ times yields BPZ-type recursion relations

$$\mathcal{M}^{(n-1)} = \sum_{i=2}^{n} \epsilon_L^i \Big(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2}\,\partial_{\varphi_i}\Big)\mathcal{M}^{(n-2)}$$

$$\mathcal{N}^{(n-1)} = \sum_{i=2}^{n} \Big[\Big(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2}\,\partial_{\varphi_i}\Big)\big(\epsilon_M^i \mathcal{M}^{(n-2)} + \epsilon_L^i \mathcal{N}^{(n-2)}\big) + u_i \epsilon_L^i \Big(\frac{2c_{1i}}{s_{ij}^2} + \frac{\partial_{\varphi_i}}{s_{1i}^2}\Big)\mathcal{M}^{(n-2)}\Big] + u_1\,\partial_{\varphi_1}\mathcal{M}^{(n-1)}$$

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Consistency check that 3d flat space holography can work

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Consistency check that 3d flat space holography can work

earlier checks: Cardy-like formula Barnich '12; Bagchi, Detournay, Fareghbal, Simón '12

(holographic) entanglement entropy Bagchi, Basu, DG, Riegler '14; Basu, Riegler '15; Jiang, Song, Wen '17; Hijano, Rabideau '17; Apolo, Jiang, Song, Zhong '20

see first and second backup slides

Premise: worthwhile to explore Carroll QFTs & CFTs

- *"... the most fundamental equation of relativistic QFT"* Schwinger '63

$$\big[\mathcal{H}(x),\, \mathcal{H}(x')\big] = \delta^{ij}\big(\mathcal{H}_i(x) + \mathcal{H}_i(x')\big)\,\partial_j\delta(x-x')$$

Poisson brackets/commutators for energy density $\mathcal{H}(x)$ must be of this form due to Poincaré invariance Dirac '62 up to anomalous terms

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Poisson brackets/commutators for energy density $\mathcal{H}(x)$ must be of this form due to Poincaré invariance Dirac '62 up to anomalous terms

Carroll version Bunster, Henneaux '12

$$\big[\mathcal{H}(x),\,\mathcal{H}(x')\big] = 0$$

must hold for any Carroll QFT

Field theory side

Aspects of Carrollian field theories

- energy density obeys $[\mathcal{H}(x),\, \mathcal{H}(x')] = 0$
- electric vs. magnetic Carroll limit (free scalar field example)

Field theory side

Aspects of Carrollian field theories

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Hamiltonian Klein–Gordon action

$$\int \Big(\Pi\partial_t\Phi - \frac{c^2}{2}\Pi^2 - \frac{1}{2}\delta^{ij}\partial_i\Phi\partial_j\Phi\Big)$$

allows magnetic limit

$$\int \Big(\Pi\partial_t\Phi - \frac{1}{2}\delta^{ij}\partial_i\Phi\partial_j\Phi\Big)$$

EOM: $\partial_t\Phi = 0 = \Delta\Phi$

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Hamiltonian Klein–Gordon action ($\tilde{\Pi} = \Pi\, c$, $\tilde{\Phi} = \Phi/c$)

$$\int \Big(\Pi\partial_t\Phi - \frac{c^2}{2}\Pi^2 - \frac{1}{2}\delta^{ij}\partial_i\Phi\partial_j\Phi\Big) = \int \Big(\tilde{\Pi}\partial_t\tilde{\Phi} - \frac{1}{2}\tilde{\Pi}^2 - \frac{c^2}{2}\delta^{ij}\partial_i\tilde{\Phi}\partial_j\tilde{\Phi}\Big)$$

allows magnetic limit

$$\int \Big(\Pi\partial_t\Phi - \frac{1}{2}\delta^{ij}\partial_i\Phi\partial_j\Phi\Big)$$

or electric limit

$$\int \Big(\tilde{\Pi}\partial_t\tilde{\Phi} - \frac{1}{2}\tilde{\Pi}^2\Big) = \int \Big(\frac{1}{2}\,\partial_t\tilde{\Phi}\Big)^2$$

EOM: $\partial_t^2\tilde{\Phi} = 0$

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Neither case allows scalar field with finite propagation speed

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- energy density obeys $[\mathcal{H}(x),\, \mathcal{H}(x')] = 0$
- electric vs. magnetic Carroll limit
- swiftons: Carroll invariant interacting fields with finite propagation speed Ecker, DG, Henneaux, Salgado-Rebolledo'24

etymology: Greek prefix "tachy" = swift

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  - technical key issue: "inverse" metric $g^{\mu\nu}$ not Carroll boost invariant

    see third backup slide for details

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  - $\Rightarrow$ cannot raise spatial indices unless constraint imposed
    e.g. magnetic Carroll scalar field with constraint $\partial_t \Phi = 0$

Field theory side

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  - covector θ_μ transverse to Carroll vector, $v^\mu\theta_\mu = 0$:
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  - can construct derivative interaction terms $\theta_\mu \theta^\mu$ with at least 2 scalars

$$\theta_\mu = v^\nu \big(\partial_\nu \phi \partial_\mu \chi - \partial_\mu \chi \partial_\nu \phi\big)$$

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  - simplest swifton action

$$\int\Big(\underbrace{\big(v^\mu\partial_\mu\phi\big)^2 + \big(v^\mu\partial_\mu\chi\big)^2}_{\text{electric}} + \underbrace{g\,\theta_\mu\theta^\mu}_{\text{“electromagnetic”}}\Big)$$

Baig, Distler, Karch, Raz, Sun '23; Kasikci, Ozkan, Pang '23; Ecker, DG, Henneaux, Salgado-Rebolledo '24
straightforwardly generalizes to multi-scalars, vector fields, and Carroll gravity with scalars
see also Bagchi, Banerjee, Dutta, Kolekar, Sharma '22; Ciambelli '23

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  - energy density compatible with Carroll condition $[\mathcal{H}(x), \mathcal{H}(x')] = 0$ and bounded from below this is why we refrain from label "tachyon" and use "swifton" instead

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  - energy density compatible with Carroll condition $[\mathcal{H}(x), \mathcal{H}(x')] = 0$ and bounded from below
  - fluctuations in ϕ around background $\chi = t$, $\phi = 0$ propagate with finite speed, $c_{\text{eff}}^2 = -g$

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- energy density obeys $[\mathcal{H}(x),\, \mathcal{H}(x')] = 0$
- electric vs. magnetic Carroll limit
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- tantum gravity limit
  - various "nogo's" for Carroll field theories

    no propagation Levy-Leblond '65

    no (obvious) Carroll thermodynamics de Boer, Hartong, Obers, Sybesma, Vandoren '23

    no (obvious) interacting Carroll QFT Cotler, Jensen, Prohazka, Raz, Riegler, Salzer '24

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    no (obvious) interacting Carroll QFT Cotler, Jensen, Prohazka, Raz, Riegler, Salzer '24
  - example: implications of $c \to 0$ limit for BH observables ($E = Mc^2$)

$$T = \frac{\hbar\, c^5}{8\pi\, G_N\, E} \qquad S = \frac{4\pi\, G_N\, E^2}{\hbar c^5} \qquad r_h = \frac{2\, G_N E}{c^4}$$

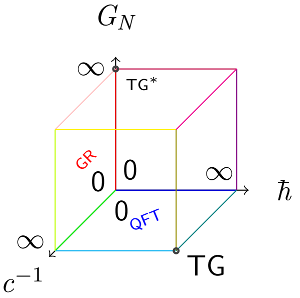
naive $c \to 0$ limit yields either 0 or ∞ for observables

Field theory side

Aspects of Carrollian field theories

- energy density obeys $[\mathcal{H}(x),\, \mathcal{H}(x')] = 0$
- electric vs. magnetic Carroll limit
- swiftons: Carroll invariant interacting fields with finite speed
- tantum gravity limit
  - various "nogo's" for Carroll field theories
  - example: implications of $c \to 0$ limit for BH observables ($E = Mc^2$)

$$T = \frac{\hbar\, c^5}{8\pi\, G_N\, E} \qquad S = \frac{4\pi\, G_N\, E^2}{\hbar c^5} \qquad r_h = \frac{2\, G_N E}{c^4}$$



finite results for $c \to 0$:

keep fixed c^4/G_N and $\hbar c$

"tantum gravity" limit

see TG corner in compactified Bronstein cube

Ecker, DG, Fiorucci '25

etymology: "tantum" = answer to question "quantum?"

Carrollian outlook

- **scattering amplitudes and Carroll/Celestial dictionary**

  - 3d: need to add massless fields
  - 4d: relation to Celestial amplitudes
  - position space amp's satisfy CCFT Ward id's
  - radiation $\simeq$ external sources in CCFT

Donnay, Fiorucci, Herfray, Ruzziconi '22
Bagchi, Banerjee, Basu, Dutta '22
Salzer '23
Bagchi, Dhivakar, Dutta '23
Saha '23
Nguyen, (West) '23
Chen, Hu '23
(Kmec,) Mason, Ruzziconi, Srikant '23
Alday, Ruzziconi, Srikant '24
(Ruzziconi,) Stieberger, Taylor, Zhu '24
Ciambelli, Pasterski, Tabor '24
Bekaert, Pekar '24
de Gioia, Raclariu '24
Adamo, Bu, Tourkine, Zhu '24
Banerjee, Basu, Bhatkar '24
Ruzziconi, Saha '24
Cotler, Jensen, Prohazka, Riegler, Salzer '24

Carrollian outlook

- scattering amplitudes and Carroll/Celestial dictionary
- **black holes in flat space holography**

  - thermal states in CCFTs
  - Cardyology and microstates
  - relation to S-matrix observables
  - information loss

?

Carrollian outlook

- scattering amplitudes and Carroll/Celestial dictionary
- black holes in flat space holography
- **string aspects**

- string constructions of CCFTs?
- Carroll limits of stringy AdS/CFT?
- Carroll strings (worldsheet vs. target space)
- boundary Carrollian CFT$_2$ and D-branes

Gibbons, Hashimoto, Yi '02
Bagchi '13
Bagchi, Chakrabortty, Parekh '15
Cardona, Gomis, Pons '16
Casali, Tourkine '16
Bagchi, Banerjee, Chakrabortty, Dutta, Parekh '20
Bagchi, DG, Nandi '22
Bagchi, DG, Sheikh-Jabbari '22
Bidussi, Harmark, Obers, Oling '23
Bagchi, Banerjee, Hartong, Have, Kolekar '23
Stieberger, Taylor, Zhu '24
Bagchi, Chakraborty, Chakrabortty, Fredenhagen, DG '24

see fourth backup slide for details

Carrollian outlook

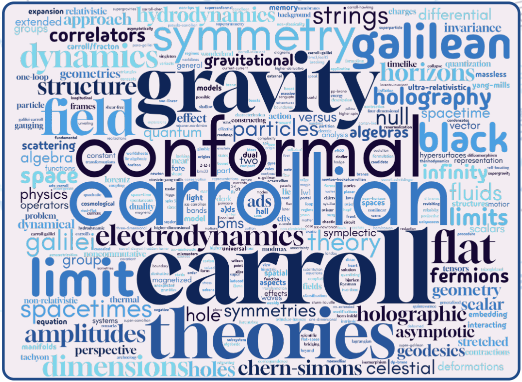
- scattering amplitudes and Carroll/Celestial dictionary
- black holes in flat space holography
- string aspects
- **Carroll CFT classification**

- electric vs. magnetic vs. intrinsic
- tame zoo of Carroll conformal symmetries
- construct examples of various zoo members
- find candidates for holographic CCFTs

Barnich, Gomberoff, González '12
Bagchi, Mehra, Nandi '19
Banerjee, Basu, Mehra, Mohan, Sharma '20
Chen, Liu, Zheng '21
Bagchi, DG, Nandi '22
Baiguera, Oling, Sybesma, Søgaard '22
Bekaert, Campoleoni, Pekar '22
Bhattacharyya, Nandi '23
de Boer, Hartong, Obers, Sybesma, Vandoren '23
Afshar, Bekaert, Najafizadeh '24

Carrollian outlook

- scattering amplitudes and Carroll/Celestial dictionary
- black holes in flat space holography
- string aspects
- Carroll CFT classification
- **Carroll applications besides holography**



Once you have seen Carroll symmetries you cannot unsee them...

$[H, B_i] = 0$ signature $(0, +, \ldots, +)$ $[\mathcal{H}(x), \mathcal{H}(x')] = 0$

Once you have seen Carroll symmetries you cannot unsee them...

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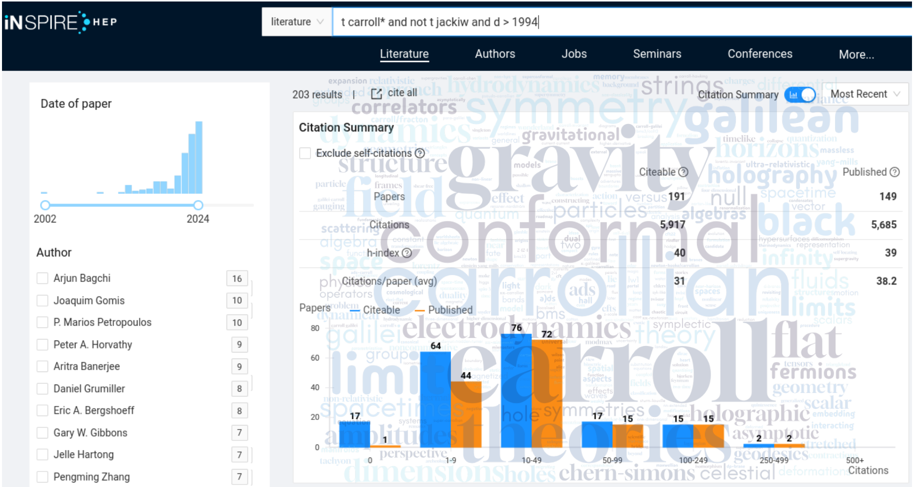


... which may explain the recent increase in Carroll papers on INSPIRE

شكرًا لكم

Thanks for your attention!

created with NightCafe using the prompt “Carroll goes down the rabbit hole in Abu Dhabi”

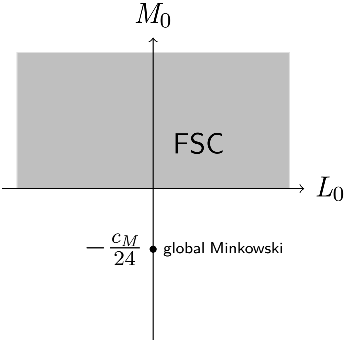
$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide I

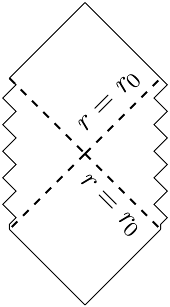
Cardyology Barnich '12; Bagchi, Detournay, Fareghbal, Simoón '12; Aggarwal, Bagchi, Detournay, DG, Riegler, Simón '25

- essence of Cardy-formula: S-duality

  high-T partition function (dominated by black holes) equivalent to low-T partition function (dominated by ground state)

  key assumptions: gap in spectrum, modular invariance of partition fct





FSC Penrose diagram Cornalba, Costa '02

$$\mathrm{d}s^2_{\text{FSC}} = r_+^2\,(1 - \tfrac{r_0^2}{r^2})\,\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{r_+^2\,(1 - \frac{r_0^2}{r^2})} + r^2\,(\mathrm{d}\varphi - \tfrac{r_+ r_0}{r^2}\,\mathrm{d}t)^2$$

r_0: horizon radius

$$M_0 = \frac{c_M}{24}\, r_+^2$$

$$L_0 = \frac{c_M}{12}\, r_+ r_0$$

$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide I

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- essence of Cardy-formula: S-duality
- Carroll modular trafos $(ad - bc = 1,\ a, b, c, d \in \mathbb{Z})$

$$\sigma \to \frac{a\sigma + b}{c\sigma + d} \qquad\qquad \rho \to \frac{\rho}{(c\sigma + d)^2}$$

$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide I

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$$\sigma \to \frac{a\sigma + b}{c\sigma + d} \qquad\qquad \rho \to \frac{\rho}{(c\sigma + d)^2}$$

- S- and T-trafos:

$$S:\ \sigma \to -\frac{1}{\sigma} \qquad \rho \to \frac{\rho}{\sigma^2} \qquad\qquad T:\ \sigma \to \sigma + 1 \qquad \rho \to \rho$$

obey usual braiding relations $S^2 = \mathbb{1} = (ST)^3$

Flat$_3$/CCFT$_2$ correspondence — backup slide I

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- modular invariance of partition function yields desired duality

$$Z_{\text{CCFT}}(\sigma, \rho) = Z_{\text{CCFT}}(-1/\sigma, \rho/\sigma^2)$$

Flat$_3$/CCFT$_2$ correspondence — backup slide I

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$$Z_{\text{CCFT}}(\sigma, \rho) = Z_{\text{CCFT}}(-1/\sigma, \rho/\sigma^2)$$

- CCFT$_2$ Cardy-like entropy formula

$$S_{\text{CCFT}} = (1 - \rho\partial_\rho - \sigma\partial\sigma)\, Z_{\text{CCFT}}(\sigma, \rho) = 2\pi L_0 \sqrt{\frac{c_M}{24 M_0}} = \frac{2\pi r_0}{4G} = S_{\text{BH}}$$

$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide II

Entanglement entropy Bagchi, Basu, DG, Riegler '14

- CCFT_2: follow CFT_2 Holzhey, Wilczek, Larsen '94; Cardy, Calabrese '04

$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide II

Entanglement entropy Bagchi, Basu, DG, Riegler '14

- CCFT_2: follow CFT_2 Holzhey, Wilczek, Larsen '94; Cardy, Calabrese '04
- EE for CCFT_2 on plane with intervals Δu and Δx

$$S_{\text{EE}} = \underbrace{\frac{c_L}{6}\,\ln\frac{\Delta x}{\epsilon_x}}_{S_L} + \underbrace{\frac{c_M}{6}\Big(\frac{\Delta u}{\Delta x} - \frac{\epsilon_u}{\epsilon_x}\Big)}_{S_M}$$

$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide II

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$c_M = 0$: recover chiral CFT_2 result for EE on plane, $S_{\text{EE}} = S_L$

Δx: size of entangling region

ϵ_x: UV cutoff

$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide II

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- EE for states dual to FSC or other orbifolds from uniformization map

Flat$_3$/CCFT$_2$ correspondence — backup slide II

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- EE for states dual to FSC or other orbifolds from uniformization map

  DG, Parekh, Riegler '19

  conceptually the same as uniformization maps in AdS$_3$/CFT$_2$ using solutions of Hill's equation $\Rightarrow$ constructed flat Hill's equation
  example: EE for global Minkowski/CCFT on cylinder ($\varphi \sim \varphi + 2\pi$)

$$S_{\text{EE}} = \frac{c_L}{6}\,\ln\frac{2\sin\frac{\Delta\varphi}{2}}{\epsilon_\varphi} + \frac{c_M}{6}\Big(\frac{\Delta u}{2}\,\cot\frac{\Delta\varphi}{2} - \frac{\epsilon_u}{\epsilon_\varphi}\Big)$$

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- EE for states dual to FSC or other orbifolds from uniformization map

relatedly, infinitesimal diffeos

$$x \to \xi(\varphi) = \varphi + \underbrace{\sigma(\varphi)}_{\text{superrotation}} \qquad\qquad u \to \zeta(u,\varphi) = \underbrace{\eta(\varphi)}_{\text{supertranslation}} + u\,\xi'(\varphi)$$

transform CCFT_2 EE as

$$\delta S_L = \sigma S_L' - \frac{c_L}{12}\,\sigma' \qquad\qquad \delta S_M = \sigma S_M' + \zeta \dot{S}_M - \frac{c_M}{12}\,\zeta'$$

$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide II

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- EE for states dual to FSC or other orbifolds from uniformization map
- Is there an (H)RT-like prescription of holographic EE?

$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide II

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- Is there an (H)RT-like prescription of holographic EE?
- yes, using Wilson lines in CS-formulation Basu, Riegler '15

analogous to holographic EE for gravity/higher spin theories in AdS_3

Ammon, Castro, Iqbal '13; de Boer, Jottar '13

$\text{Flat}_3/\text{CCFT}_2$ correspondence — backup slide II

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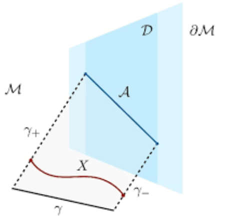
- EE for states dual to FSC or other orbifolds from uniformization map
- Is there an (H)RT-like prescription of holographic EE?
- yes, using Wilson lines in CS-formulation Basu, Riegler '15
- yes, using swing surfaces Jiang, Song, Wen '17; Hijano, Rabideau '17; Apolo, Jiang, Song, Zhong '20



$\partial\mathcal{M}$: asymptotic boundary
$\mathcal{A}$: entangling region in CCFT_2
$\gamma_\pm$: null geodesics ("ropes")
X, γ: spacelike surfaces ("bench")
$\gamma_+ \cup \gamma \cup \gamma_-$: extremal surface ("swing")
S_{EE}: area of swing surface; reproduces EE above

Carroll geometry and Carroll boosts — backup slide III

Essence for swifton interactions Ecker, DG, Henneaux, Salgado-Rebolledo '24

- vielbein formulation: temporal einbein τ, spatial vielbein e^a

notation alert: temporal einbein τ a.k.a. "Ehresmann connection"

Carroll geometry and Carroll boosts — backup slide III

Essence for swifton interactions Ecker, DG, Henneaux, Salgado-Rebolledo '24

- vielbein formulation: temporal einbein τ, spatial vielbein e^a
- dual vector fields: Carroll vector field v and spatial vielbein e_a

$$\tau_\mu v^\mu = -1 \qquad e^a_\mu e^\mu_b = \delta^a_b \qquad \tau_\mu e^\mu_a = 0 = v^\mu e^a_\mu$$

$a = 1, 2, \ldots, d$ where d is the spatial dimension

Carroll geometry and Carroll boosts — backup slide III

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- action of Carroll boosts on vielbein variables:

$$\delta_\lambda \tau_\mu = -\lambda_a e^a_\mu \qquad \delta_\lambda e^\mu_a = -\lambda_a v^\mu \qquad \delta_\lambda v^\mu = 0 = \delta_\lambda e^a_\mu$$

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- implication I: Carroll metric variables

$$g_{\mu\nu} = e^a_\mu e^b_\nu \delta_{ab} \qquad v^\mu$$

are Carroll boost invariant, $\delta_\lambda g_{\mu\nu} = 0 = \delta_\lambda v^\mu$

Carroll geometry and Carroll boosts — backup slide III

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- implication I: Carroll metric variables Carroll boost inv.
- implication II: "inverse" Carroll metric variables

$$g^{\mu\nu} = e^\mu_a e^\nu_b \delta^{ab} \qquad\qquad \tau_\mu$$

  are not Carroll boost invariant

$$\delta_\lambda g^{\mu\nu} = -\delta^{ab}\lambda_a \big(v^\mu e^\nu_b + v^\nu e^\mu_b\big) \qquad\qquad \delta_\lambda \tau_\mu = -\lambda_a e^a_\mu$$

Carroll geometry and Carroll boosts — backup slide III

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- implication I: Carroll metric variables Carroll boost inv.
- implication II: "inverse" Carroll metric variables not Carroll boost inv.

$$\delta_\lambda g^{\mu\nu} = -\delta^{ab}\lambda_a\big(v^\mu e^\nu_b + v^\nu e^\mu_b\big) \qquad\qquad \delta_\lambda \tau_\mu = -\lambda_a e^a_\mu$$

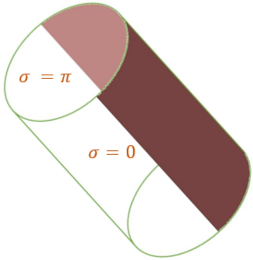
- transverse covector, $v^\mu \theta_\mu = 0$: Carroll boost invariant norm

$$\delta_\lambda(\theta_\mu\theta_\nu g^{\mu\nu}) = -\delta^{ab}\lambda_a\big(v^\mu e^\nu_b + v^\nu e^\mu_b\big)\theta_\mu\theta_\nu = 0$$

assuming Carroll boost invariance of covector, $\delta_\lambda \theta_\mu = 0$

Boundary Carrollian CFT$_2$ & open null strings — backup slide IV

More on BCCFT$_2$: see Bagchi, Chakraborty, Chakrabortty, Fredenhagen, DG, Pandit '24



- CCFT$_2$ generators on cylinder

$$L_n = e^{in\sigma}\left(\partial_\sigma + in\tau\partial_\tau\right) \qquad M_n = e^{in\sigma}\,\partial_\tau$$

not suitable for BCCFT$_2$ with boundaries at $\sigma = 0, \pi$

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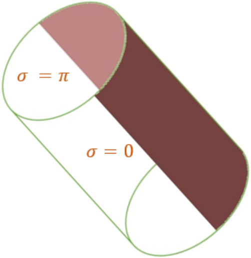

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- use instead BCCFT$_2$ generators $\mathcal{O}_n$, P_n defined by

$$\mathcal{O}_n := L_n - L_{-n} = 2i\sin(n\sigma)\,\partial_\sigma + 2in\tau\cos(n\sigma)\,\partial_\tau \qquad P_n := M_n + M_{-n} = 2\cos(n\sigma)\,\partial_\tau$$

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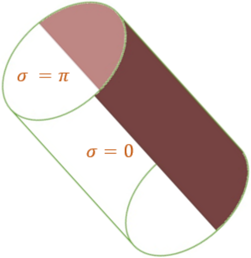

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$$\mathcal{O}_n := L_n - L_{-n} = 2i\sin(n\sigma)\,\partial_\sigma + 2in\tau\cos(n\sigma)\,\partial_\tau \qquad P_n := M_n + M_{-n} = 2\cos(n\sigma)\,\partial_\tau$$

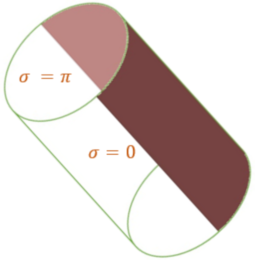
- BCCFT$_2$ algebra with central extension: $[P_n,\, P_m] = 0$ and

$$[\mathcal{O}_n,\, \mathcal{O}_m] = (n-m)\,\mathcal{O}_{n+m} - (n+m)\,\mathcal{O}_{n-m}$$

$$[\mathcal{O}_n,\, P_m] = (n-m)\,P_{n+m} + (n+m)\,P_{n-m} + \frac{c_M}{12}\left(n^3 - n\right)\left(\delta_{n,-m} + \delta_{n,m}\right)$$

Boundary Carrollian CFT$_2$ & open null strings — backup slide IV

More on BCCFT$_2$: see Bagchi, Chakraborty, Chakrabortty, Fredenhagen, DG, Pandit '24



- CCFT$_2$ generators on cylinder

$$L_n = e^{in\sigma}\left(\partial_\sigma + in\tau\partial_\tau\right) \qquad M_n = e^{in\sigma}\,\partial_\tau$$

  not suitable for BCCFT$_2$ with boundaries at $\sigma = 0, \pi$

- use instead BCCFT$_2$ generators $\mathcal{O}_n$, P_n defined by

$$\mathcal{O}_n := L_n - L_{-n} = 2i\sin(n\sigma)\,\partial_\sigma + 2in\tau\cos(n\sigma)\,\partial_\tau \qquad P_n := M_n + M_{-n} = 2\cos(n\sigma)\,\partial_\tau$$

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- obtained also from contraction ($\epsilon \to 0$) of single Virasoro $\mathcal{L}_n$ (BCFT$_2$)

$$\mathcal{O}_n = \mathcal{L}_n - \mathcal{L}_{-n} \qquad\qquad P_n = \epsilon\left(\mathcal{L}_n + \mathcal{L}_{-n}\right)$$

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- global subalgebra: iso$(1,1)$ (Poincaré$_2$) generated by $P_0, P_1, \mathcal{O}_1$

$$[\mathcal{O}_1,\, P_0] = 2P_1 \qquad [\mathcal{O}_1,\, P_1] = 2P_0 \qquad [P_0,\, P_1] = 0$$

Boundary Carrollian CFT$_2$ & open null strings — backup slide IV

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- e.g.: tensionless null string with action Isberg, Lindstrom, Sundborg, Theodoridis '93

$$S_{\text{ILST}} = \frac{1}{4\pi\, c'} \int_{\mathcal{M}} \mathrm{d}^2x\; V^\alpha\, V^\beta (\partial_\alpha X^\mu)(\partial_\beta X^\nu)\eta_{\mu\nu}$$

allows three types of bc's: c': tantum gravity-like coupling const.

$\delta X^\mu\big\vert_{\sigma=0,\pi} = 0$	Dirichlet	X^μ	: target space coord's
$V^\beta \partial_\beta X^\nu\big\vert_{\sigma=0,\pi} = 0$	Neumann	V^α	: Carroll vector density
$n_\alpha\, V^\alpha\big\vert_{\sigma=0,\pi} = 0$	Null	n_α	: normal to b'dry

Boundary Carrollian CFT$_2$ & open null strings — backup slide IV

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- for Dirichlet: null open string from tensile open string gauge fixing: $V^\alpha = (1,0)$

$$X^\mu(\tau,\sigma) = x_0^\mu + \sqrt{2\alpha'}\alpha_0^\mu\sigma + i\sqrt{\tfrac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^\mu e^{-in(\tau+\sigma)} + \alpha_{-n}^\mu e^{in(\tau-\sigma)}\right)$$

with Carroll/tensionless limit $\tau\to\epsilon\tau$, $\sigma\to\sigma$, $\alpha'\to c'/\epsilon$, $\epsilon\to 0$ and

$$\mathcal{O}_n = \tfrac{1}{2}\sum_p \left(\alpha_p\cdot\alpha_{n-p} - \alpha_{-p}\cdot\alpha_{p-n}\right) \qquad P_n = \tfrac{\epsilon}{2}\sum_p \left(\alpha_p\cdot\alpha_{n-p} + \alpha_{-p}\cdot\alpha_{p-n}\right)$$

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- BCCFT analog of classical Virasoro constraints: $\mathcal{O}_n = 0 = P_n$