Algebras in Field Theory and Gravity: An Overview

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Quantum mechanics textbooks usually consider an isolated system viewed from outside by an observer. As observers in the universe, we are in a different situation – we are part of the system that we are trying to observe. We cannot resolve the quantum microstate of the whole universe, because of limitations of technology but also because we cannot see beyond the cosmological horizon. So some of textbook quantum mechanics does not quite apply.

I assume, though, that enough of textbook quantum mechanics applies so that (1) observables form an algebra $\mathcal{A}$, meaning a vector space with an associative multiplication law, (2) there is a notion of hermitian conjugation ($a \to a^\dagger$, satisfying $(ab)^\dagger = b^\dagger a^\dagger$), and (3) what we measure are expectation values of observables in the state of the universe, however it should be described.

I assume, though, that enough of textbook quantum mechanics applies so that (1) observables form an algebra $\mathcal{A}$, meaning a vector space with an associative multiplication law, (2) there is a notion of hermitian conjugation ($\mathsf{a} \to \mathsf{a}^\dagger$, satisfying $(\mathsf{ab})^\dagger = \mathsf{b}^\dagger \mathsf{a}^\dagger$), and (3) what we measure are expectation values of observables in the state of the universe, however it should be described. And I assume that (4) positive observables have positive expectation values, meaning that if

$$\langle \mathsf{a} \rangle$$

denotes the expectation value of an observable in the quantum state of the universe, then

$$\langle \mathsf{a}^\dagger \mathsf{a} \rangle \geq 0,$$

for all a.

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for any two observables a, b. Because of the positivity assumption (3), this definition is consistent with all inequalities expected for inner products of Hilbert space vectors. So after dividing by null vectors (to get a pre-Hilbert space) and taking a completion (to get a Hilbert space), we arrive at a Hilbert space $\mathcal{H}$ that provides an arena for all observations we may make.

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Bousso and Penington, "Hologram in Our World," arXiv:2302.07892

but some knowledge of what is beyond the horizon is needed in order to predict an entanglement wedge that extends beyond the horizon.

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(Mathematically this is called a weak limit.) We include such limits a in the definition of $\mathcal{A}$ on the grounds that an experiment measures finitely many matrix elements with finite precision, so under the stated hypothesis, any given experiment cannot distinguish a and a_n if n is sufficiently large.

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I generally assume this in what follows. (This assumption seems well-motivated in a closed universe. In an open universe, if the observable region goes off to spatial infinity, one may need to modify it to take conserved gauge charges into account.)

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There are two kind of simplifying properties that the algebra $\mathcal{A}$ might have, in which case it is said to be of Type II or Type I rather than Type III:

II) If $\mathcal{A}$ has a trace (but no irreducible representation) it is said to be of Type II.

I) If $\mathcal{A}$ has an irreducible representation, it is said to be of Type I.

By a trace, one means a complex-valued linear function $\mathsf{a} \to \mathrm{Tr}\, \mathsf{a}$ that satisfies

$$\mathrm{Tr}\, \mathsf{ab} = \mathrm{Tr}\, \mathsf{ba}$$

and a positivity condition

$$\mathrm{Tr}\, \mathsf{a}^\dagger \mathsf{a} > 0 \quad \text{for all } \mathsf{a} \neq 0.$$

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An algebra $\mathcal{A}$ of Type I has an irreducible representation in a Hilbert space $\mathcal{H}'$ (not necessarily the same as the original Hilbert space $\mathcal{H}$ that we defined using the GNS construction). $\mathcal{A}$ is then actually the algebra $B(\mathcal{H}')$ of all bounded operators on $\mathcal{H}'$.

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If $\mathcal{H}'$ is infinite-dimensional, this trace is not defined on all elements of the algebra, since for example

$$\mathrm{Tr}_{\mathcal{H}'}1 = \infty.$$

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It is not hard to prove that such a ρ always exists and is a positive element of $\mathcal{A}$. Moreover if $\langle\Psi|\Psi\rangle = 1$, then

$$\mathrm{Tr}\,\rho = \mathrm{Tr}\,1 \cdot \rho = \langle\Psi|1|\Psi\rangle = 1,$$

so ρ has the standard normalization of a density matrix.

Once we have density matrices, we can define von Neumann entropies (or Rényi entropies, or other similar functions considered in quantum information theory):

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Here, however, there is an important difference between a Type II algebra and a Type I algebra. For a Type I algebra, the trace can be defined literally as the trace in an irreducible representation. No renormalization is required and therefore the definition of the entropy is completely natural.

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$$\mathrm{Tr} \to \lambda \mathrm{Tr}$$

then to satisfy $\mathrm{Tr}\,\mathcal{O}\rho = \langle\Psi|\mathcal{O}|\Psi\rangle$, we need to rescale ρ by the opposite factor

$$\rho \to \lambda^{-1}\rho,$$

and then the effect on the entropy $S = -\mathrm{Tr}\,\rho\log\rho$ is

$$S \to S + \log\lambda.$$

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$$\mathrm{d}E = T\mathrm{d}S + p\mathrm{d}V,$$

but in classical physics there is no natural zero of entropy and no natural way to fix an overall additive constant.

Type II algebras have a further bifurcation into Type II_1 and Type II_∞. In a Type II_1 algebra, the identity operator has finite trace, and the trace function can be normalized to make the trace equal to 1:

$$\mathrm{Tr}\, 1 = 1$$

and more generally the trace is finite for every element of the algebra.
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and the trace is only finite for a subalgebra of "trace class" elements of the algebra. In a Type II_1 algebra, the entropy is bounded above – the density matrix of maximum entropy is $\rho = 1$ – and in a Type II_∞ algebra, the entropy is unbounded above and below.

What do we expect for the algebra $\mathcal{A}$ of observables accessible to an observer in quantum field theory – with or without gravity?

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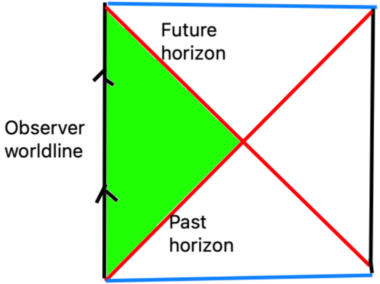
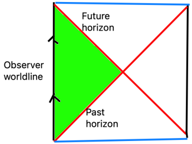
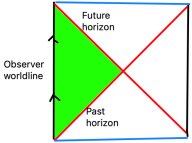




The "timelike tube theorem" of quantum field theory (which is a sort of nonperturbative version of HKLL reconstruction) says that the algebra of observables along the observer's worldline is the same as the algebra of observables in the whole "causal wedge" shown in green. (A heuristic explanation is that the observer can shine a laser beam on any object in the static patch and see what bounces back, thereby learning the full contents of the static patch.)



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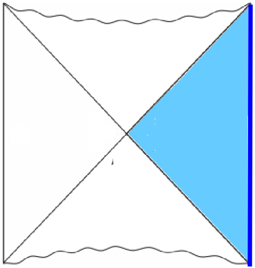

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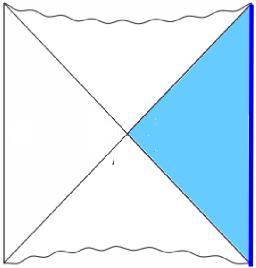

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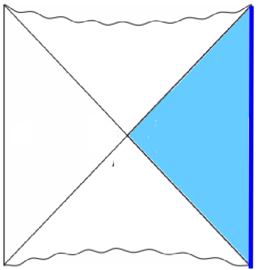

The blue wedge is the region causally accessible to the right boundary (dark blue). In the approximation of treating the bulk theory as a quantum field theory in a fixed background, the algebra of the light blue region is of Type III, so in an appropriate limit, that is also the case for the algebra of observables on the right boundary.

When the boundary dual theory is an $SU(N)$ gauge theory of some definite integer N, the thermofield double state is just a state in a tensor product $\mathcal{H}_\ell \otimes \mathcal{H}_r$ of two copies of the Hilbert space of the boundary CFT.

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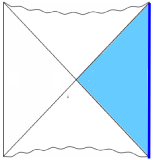

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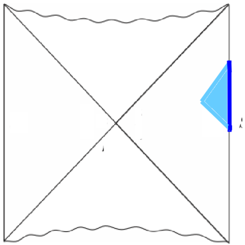

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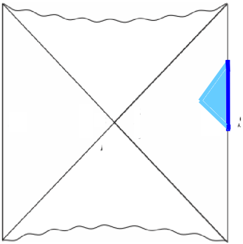

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The basic reason that one might think that Type II algebras are relevant to gravity is that in gravity we can define entropies even when we cannot define quantum mechanical microstates.

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Somewhat more specifically, in gravity we often compute, by path integrals, quantities that we give names like

$$\mathrm{Tr}\, e^{-\beta H}$$

or more generally quantities such as

$$\mathrm{Tr}\, e^{-\beta_1 H} \mathsf{a}\, e^{-\beta_2 H} \mathsf{b}\, e^{-\beta_3 H} \mathsf{c}$$

even though we do not have a Hilbert space $\mathcal{H}$ and operators $H, \mathsf{a}, \mathsf{b}, \mathsf{c}$ on $\mathcal{H}$ such that the quantities we compute really are such traces.

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even though we do not have a Hilbert space $\mathcal{H}$ and operators $H, \mathsf{a}, \mathsf{b}, \mathsf{c}$ on $\mathcal{H}$ such that the quantities we compute really are such traces. We call these functions traces because they do satisfy $\mathrm{Tr}\, \mathcal{O}\mathcal{O}' = \mathrm{Tr} \mathcal{O}'\mathcal{O}$, $\mathrm{Tr}\, \mathcal{O}\mathcal{O}^\dagger > 0$, for any $\mathcal{O}$, $\mathcal{O}'$ for which these quantities are finite, such as

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with for example a local operator $\mathcal{V}$. Moreover, typically we believe we are computing large N limits (or asymptotics) of functions that really would be traces for positive integer N.

What do these formulas mean?

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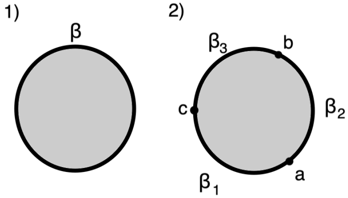

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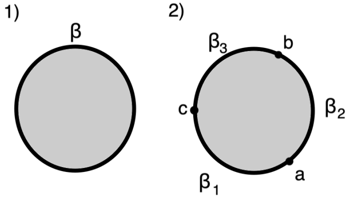

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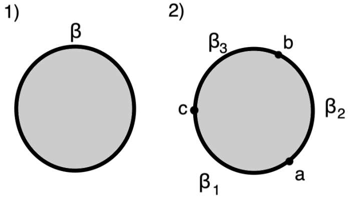

as traces even though there is no "one-sided" Hilbert space in which they are traces. An interpretation (Penington and EW, 2301.07527, Kolchmeyer 2303.04701) is that objects such as $e^{-\beta H}$, etc., as elements of an algebra of Type II_∞ in which the trace function is computed by a path integral on the disc.

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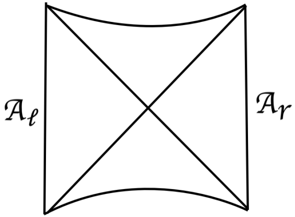

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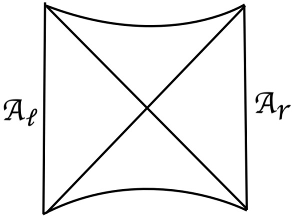

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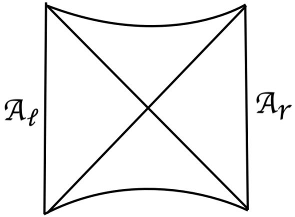

In this situation, one can define Type II_∞ algebras $\mathcal{A}_r$ and $\mathcal{A}_\ell$ for the right and left boundaries, respectively. One can show that they are each "factors" (trivial center) and are each other's commutants.

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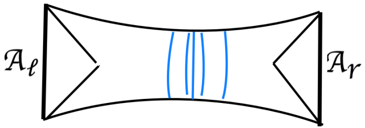

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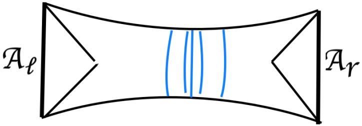

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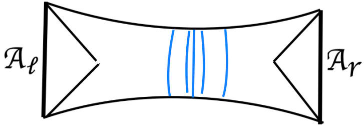

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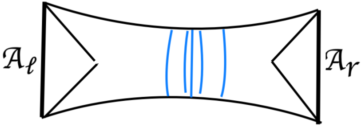

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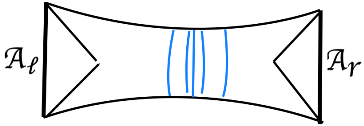

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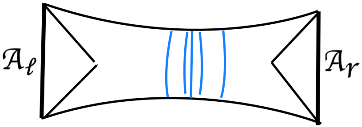

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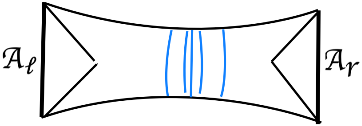

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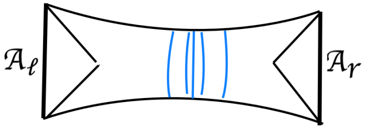

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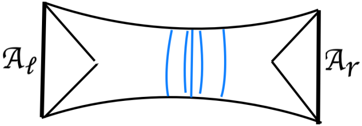

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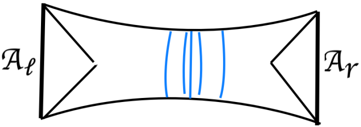

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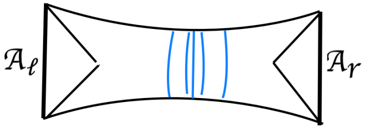

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$$\mathrm{Tr}\, e^{-\beta H} \quad \text{or} \quad \mathrm{Tr}\, e^{-\beta_1 H} \mathsf{a}\, e^{-\beta_2 H} \mathsf{b}\, e^{-\beta_3 H} \mathsf{c}$$

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Another way to say the same thing is that to have a continuous spectrum (which a black hole has in any approximation that we actually understand) but a finite partition function $\mathrm{Tr}\, e^{-\beta H}$ (such as we also compute) is not possible if "Tr" is literally the trace of an operator $e^{-\beta H}$ in Hilbert space.

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A more explicit construction of a Type II algebra is available in the case of a two-sided Schwarzschlild black hole, and also has a close analog in quantum cosmology. (EW 2112.12828, V. Chandrasekharan, G. Penington and EW (CPW) 2209.10454, CPW and R. Longo (CPLW) 2206.10780).

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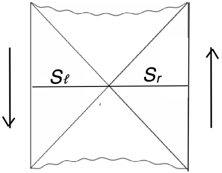

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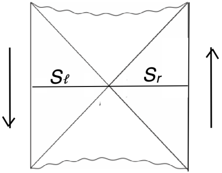

It can be written as an integral over an initial value surface S, which we take to be at $t = 0$ where t is Schwarzschild time:

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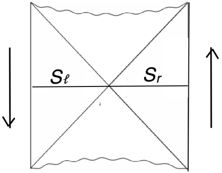

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$$H = \int_S \mathrm{d}^d x \, V^\mu T_{\mu 0},$$

where T is the stress tensor and $V \sim \frac{\partial}{\partial t}$. In heuristic arguments, it is convenient to decompose S as a union $S = S_r \cup S_\ell$ of portions to the right and left of the bifurcation surface:

$$H = H_r - H_\ell,$$

with

$$H_r = \int_{S_r} \mathrm{d}^d x \, V^\mu T_{\mu 0}, \qquad H_\ell = -\int_{S_\ell} \mathrm{d}^d x \, V^\mu T_{\mu 0}.$$

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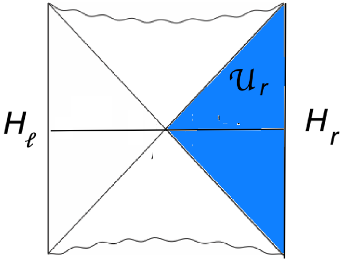
is not really correct. Trying to make this decomposition leads to ultraviolet divergences at the "bifurcation surface" at which S_r and S_ℓ meet. In gravity, there is no problem with the decomposition $H = H_r - H_\ell$. Indeed, this is just the ADM expression for the total conserved charge H in terms of ADM energies H_r and H_ℓ measured on the right and left boundaries.

Now let us try to describe an algebra of observables accessible to an observer who lives in the asymptotically AdS region $\mathcal{U}_r$ on the right of the bifurcation surface:



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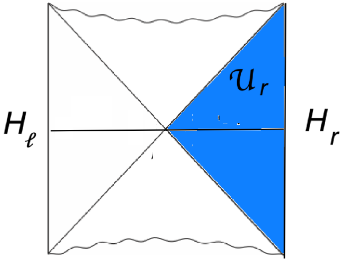
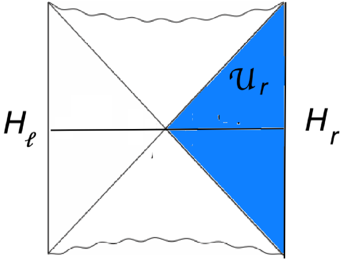


In ordinary quantum field theory (and with linearized gravity treated as just one more quantum field), the algebra of observables in region $\mathcal{U}_r$ is an algebra $\mathcal{A}_0$ of Type III. However, in the presence of gravity, the full algebra $\mathcal{A}$ of observables has one more generator H_r, so $\mathcal{A} = \{\mathcal{A}_0, H_r\}$ is the von Neumann algebra generated by $\mathcal{A}_0$ together with (bounded functions of) H_r.

While H_r is observable for an observer in the right region $\mathcal{U}_r$, the corresponding left energy H_ℓ



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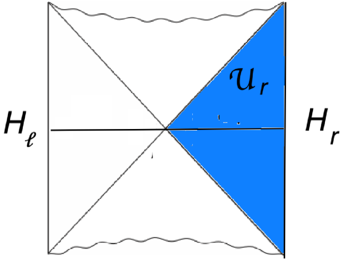

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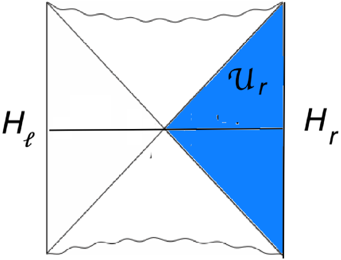

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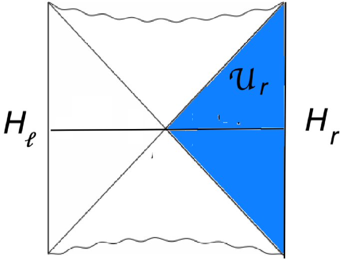

commutes with all observables in that region. Taking the perspective of an observer in region $\mathcal{U}_r$ who knows nothing about H_ℓ, let us just call that operator X. We can trivially write the formula $H = H_r - H_\ell = H_r - X$ in the form

$$H_r = H + X.$$

So, taking gravity into account, we can describe the observable algebra in region $\mathcal{U}_r$:

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and we see that this is what we have just found. This construction can be defined for any automorphism of a von Neumann algebra, but if the symmetry is the modular automorphism group (for some state), which is the case for the black hole with symmetry generator H (Sewell 1982), and the initial algebra is generic Type III, also true here, then the crossed product algebra is Type II_∞.

If $\mathcal{A}_0$ acts on a Hilbert space $\mathcal{H}_0$, and H is an operator on $\mathcal{H}_0$, then the crossed product algebra $\mathcal{A} = \{\mathcal{A}_0, H + X\}$ acts on a Hilbert space $\mathcal{H} = \mathcal{H}_0 \otimes L^2(X)$, where $L^2(X)$ is the space of square-integrable functions of X.

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for any $\widehat{\mathsf{a}} \in \mathcal{A}$. The algebra is Type II_∞, not Type II_1, since $\mathrm{Tr}\, 1 = \langle \widehat{\Psi} | 1 | \widehat{\Psi} \rangle = \langle \widehat{\Psi} | \widehat{\Psi} \rangle = \int_{-\infty}^{\infty} \mathrm{d}X\, e^X = \infty$.

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In this construction, it is important that the two-sided AdS Schwarzschild (or asymptotically flat Schwarzschild) black hole has an equilibrium state – the thermofield double state. In important examples such as a Kerr black hole or a Schwarzschild-de Sittter spacetime, there is no equilibrium state and the discussion as presented does not apply. However, by starting instead with the Unruh state, and operators on the past horizon, it is possible to make a somewhat similar construction of a Type II_∞ algebra for those spacetimes (J. Kudler-Flam, S. Leutheusser, and G. Satishchandran (KFLS) 2309.15897). There are also extensions in cosmology that I will not have time to describe (KFLS 2406.01669, C.-H. Chen and G. Penington 2406.02116).

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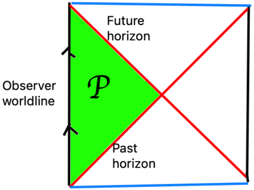

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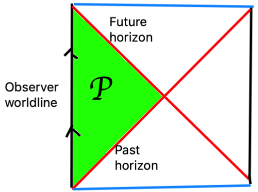

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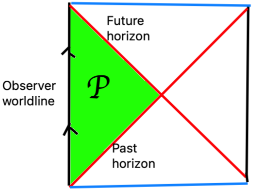

and the green region $\mathcal{P}$ (called a static patch) is the region causally accessible to the observer. In ordinary quantum field theory, we associate to $\mathcal{P}$ an algebra of observables $\mathcal{A}_0$ which is of Type III.

In gravity, we observe first that the region $\mathcal{P}$ is invariant under a one-parameter group of "time translations" whose generator I will call H.

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$$\mathcal{A} = \Pi \mathcal{A}_1^H \Pi.$$

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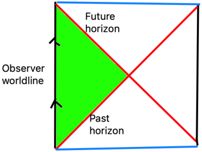

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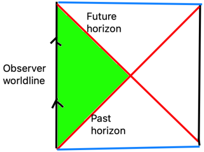

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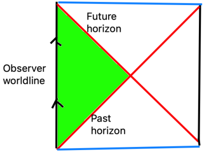

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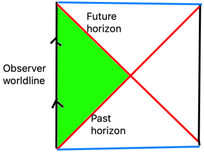

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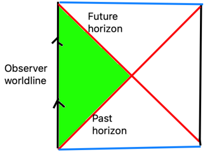

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