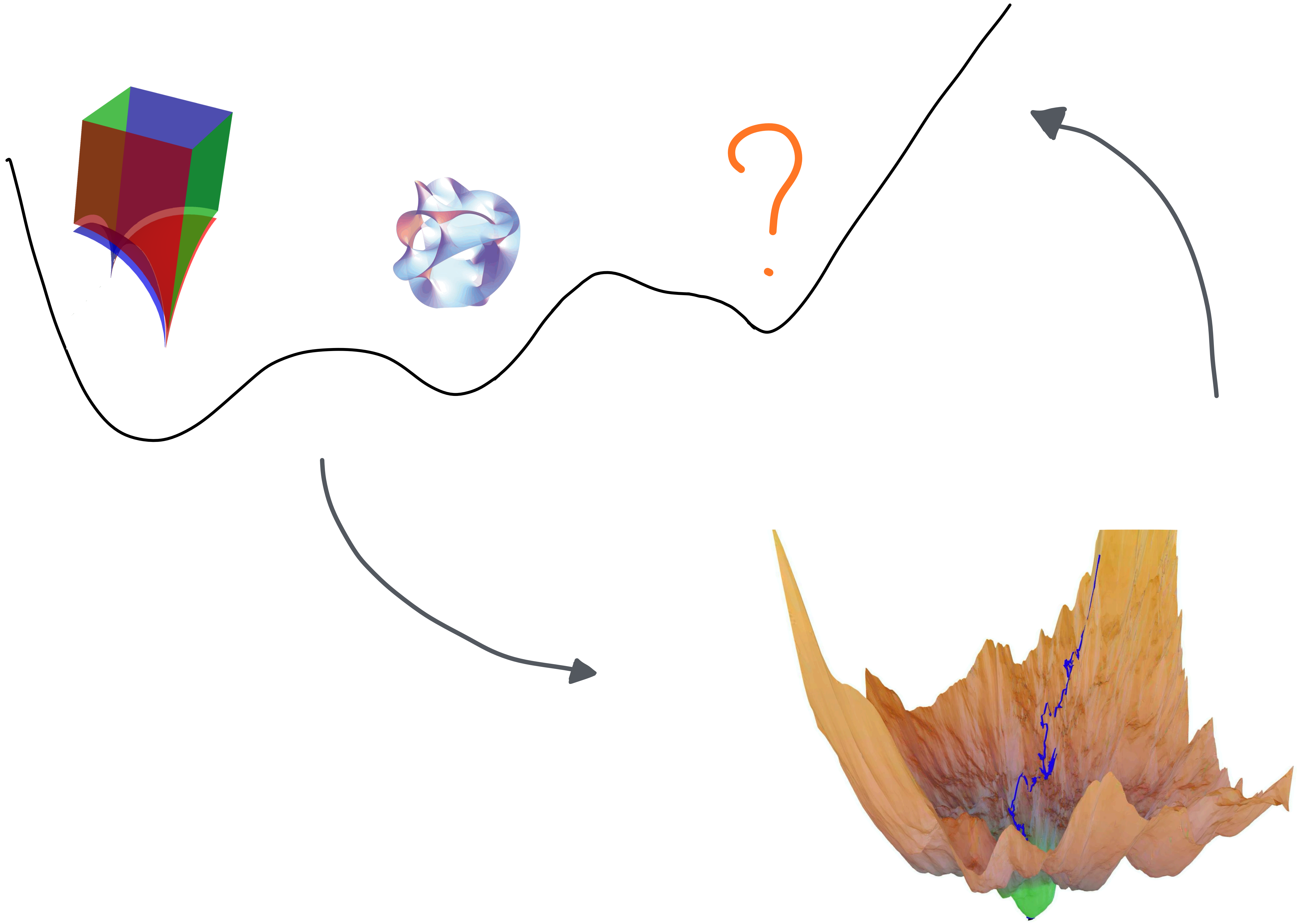
Landscapes

G. Bruno De Luca
Stanford University



Strings 2025, NYU Abu-Dhabi, Jan 08 2025

de Sitter Landscapes

- Constructing accelerating cosmologies in UV complete theories is both important and hard
- Various approaches to this problem from various parts of our community:

  statistical studies, direct constructions, conjectures about what is allowed and what is not, holographic approaches → See Gonzalo's talk tomorrow!

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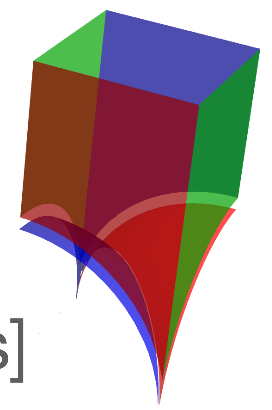
- **This talk**: various methods to explore the Landscape beyond familiar corners

1) Review of dS4 compactifications of M-theory [GBDL, Silverstein, Torroba, '21]
   - Work in progress: global solutions and explicit parametric families [GBDL, Silverstein, Torroba, in progress]



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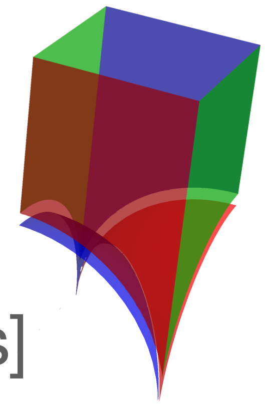
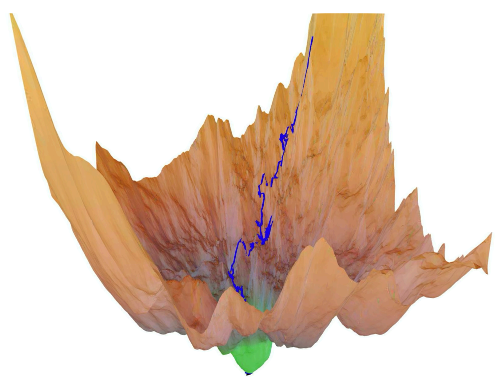
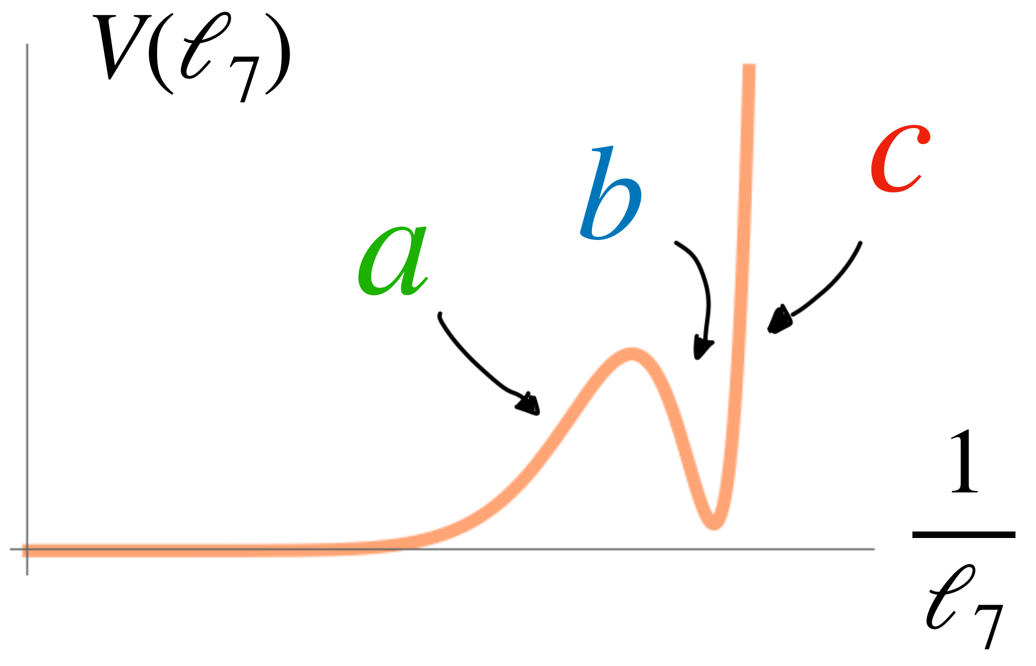


2) Can we use Machine Learning to numerically explore the Landscape, beyond Calabi-Yau's?
   - (Some) Physics methods for ML, and back [GBDL, Silverstein '21, GBDL, Gatti, Silverstein '23, GBDL, Nachman, Silverstein, Zheng, to appear]
   - Proof of concept: 3d Einstein geometries [GBDL, '25]



dS4 vacua in M-theory

- Stabilization of the volume at a minimum requires three-terms structure of the potential
  - A "negative-energy" term, needed to evade Maldacena-Nunéz no-go theorem, has to sit in the middle



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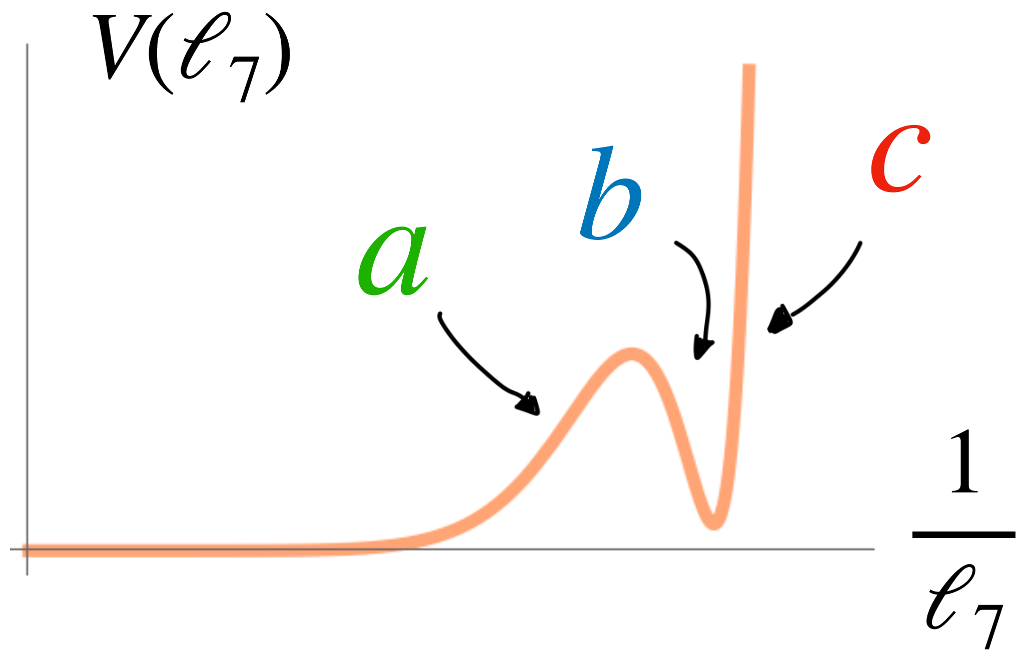

$$\langle T_{ij}\rangle \sim R_c^{-D} g_{ij} \qquad \langle T_{ab}\rangle \sim -\frac{D-k}{k} R_c^{-D} g_{ab}$$

other directions

k circle directions

small circle size ($\gg l_{11}$)

[Arkani-Hamed, Dubovsky, Nicolis, Villadoro '07]
[cf. Maldacena, Milekhin, Popov '18]

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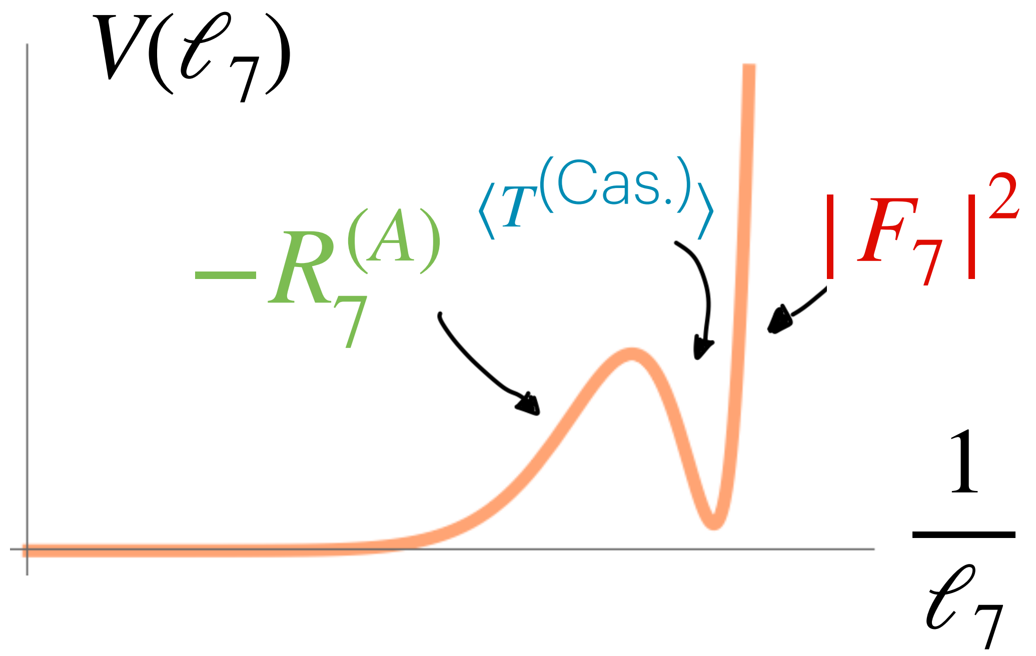

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[GBDL, Silverstein, Torroba ‘21]

$$V_{\text{eff}}[g_7, C_6; A] = \frac{1}{2\ell_{11}^9} \int_{M_7} \sqrt{g_7} e^{4A} \left(-R_7 - 12(\nabla A)^2 - \ell_{11}^9 \rho_c R_c(y)^{-11} + \frac{1}{2}|F_7|^2 \right)$$

[Douglas’ 09]

$$-R_7 - 12(\nabla A)^2 \equiv -R_7^{(A)}$$

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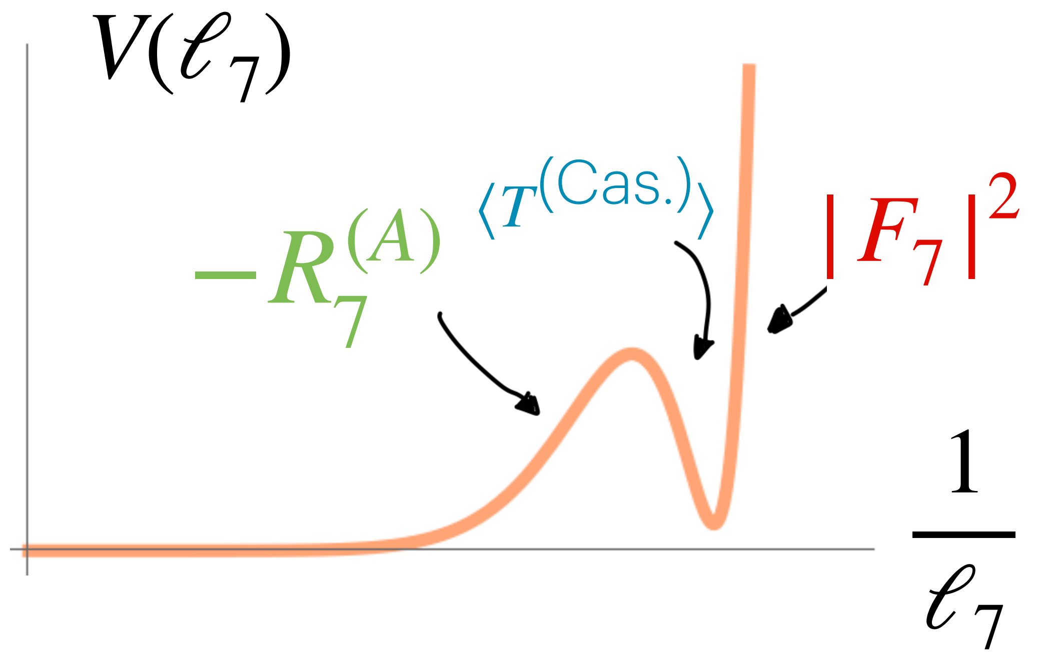

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[Douglas' 09]

1) Do negatively curved manifolds have regions with small cycles where Casimir energy can build up and compete with the classical terms?

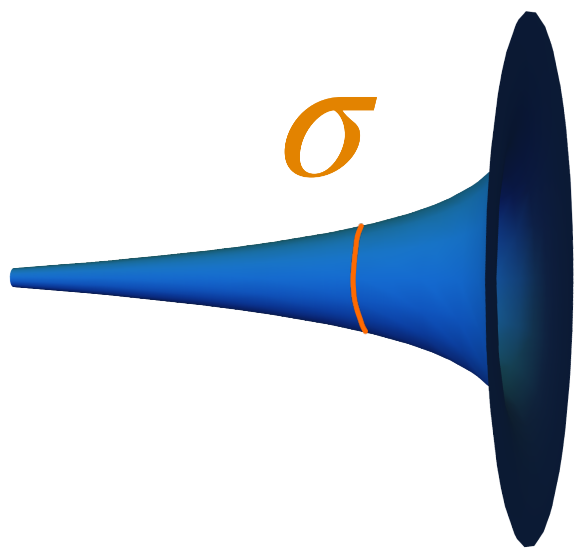
2) Can this happen with parametric control over 11D Planck-scale effects?

3) Can the semi-classical eoms be consistently solved?

$$-\frac{2}{\sqrt{-g_{11}}} \frac{\delta S_{11}^{(\text{class.})}}{\delta g_{11}^{MN}} = \langle T_{MN}^{(\text{Casimir})}\rangle$$

Hyperbolic manifolds, local solutions, and global estimates

[GBDL, Silverstein, Torroba, ‘21]



Hyperbolic manifolds have both negative curvature and regions with slowly-varying shrinking cycles, called **cusps.** Casimir builds up in the cusps, but too small sizes are problematic for control of non-perturbative effects

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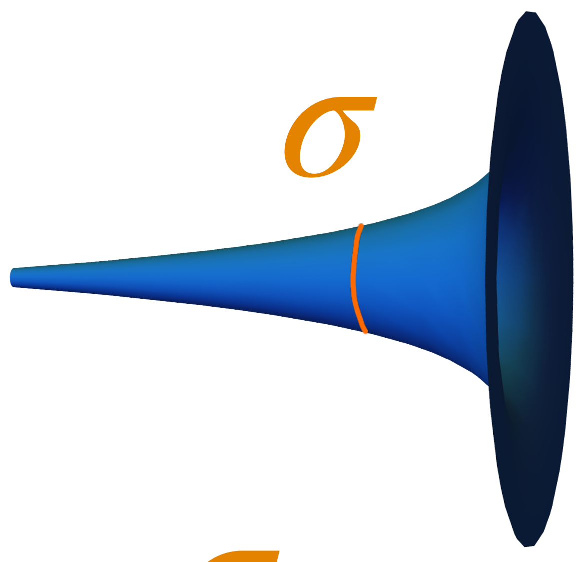

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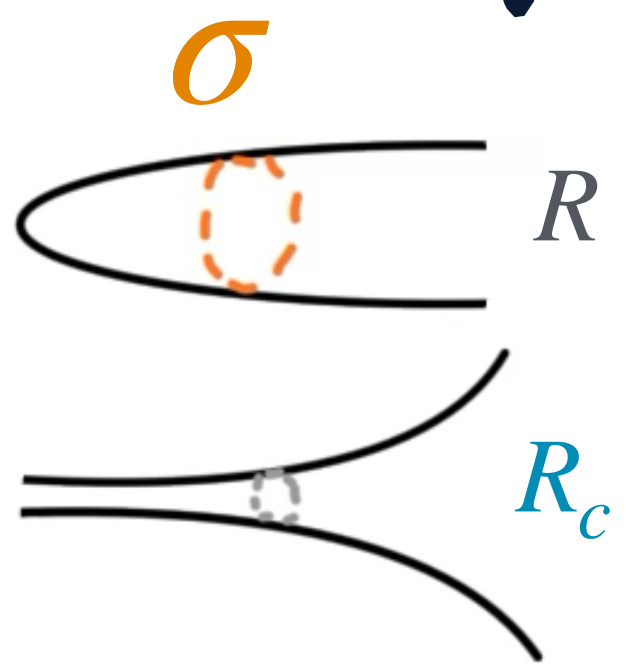
2) Anderson-Dehn filling: choose in each cusp a closed geodesics σ to kill.

The resulting manifold is Einstein, compact, and with minimal length $\sim 1/|\sigma|$

[Anderson ‘06, Bamler ‘12]



Bonus: these manifolds are rigid! ← No massless deformations, no moduli after the volume is fixed!

[e.g. Besse ‘87, Anderson ‘06]

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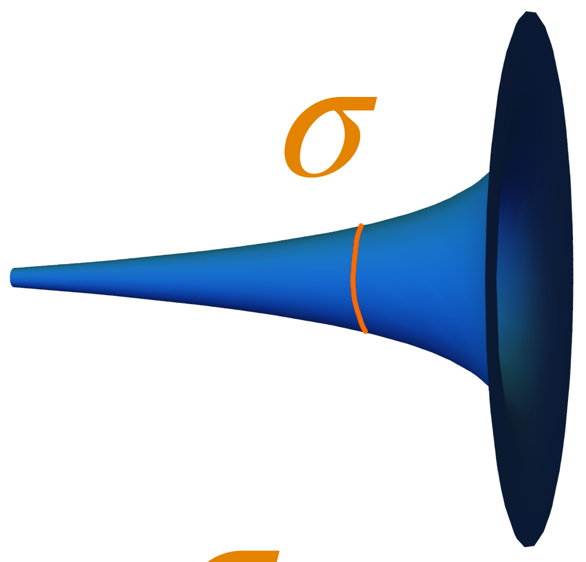

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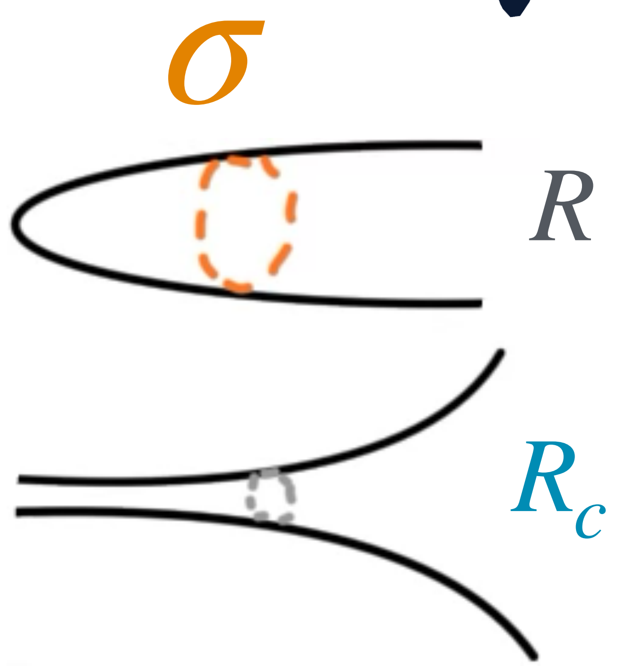

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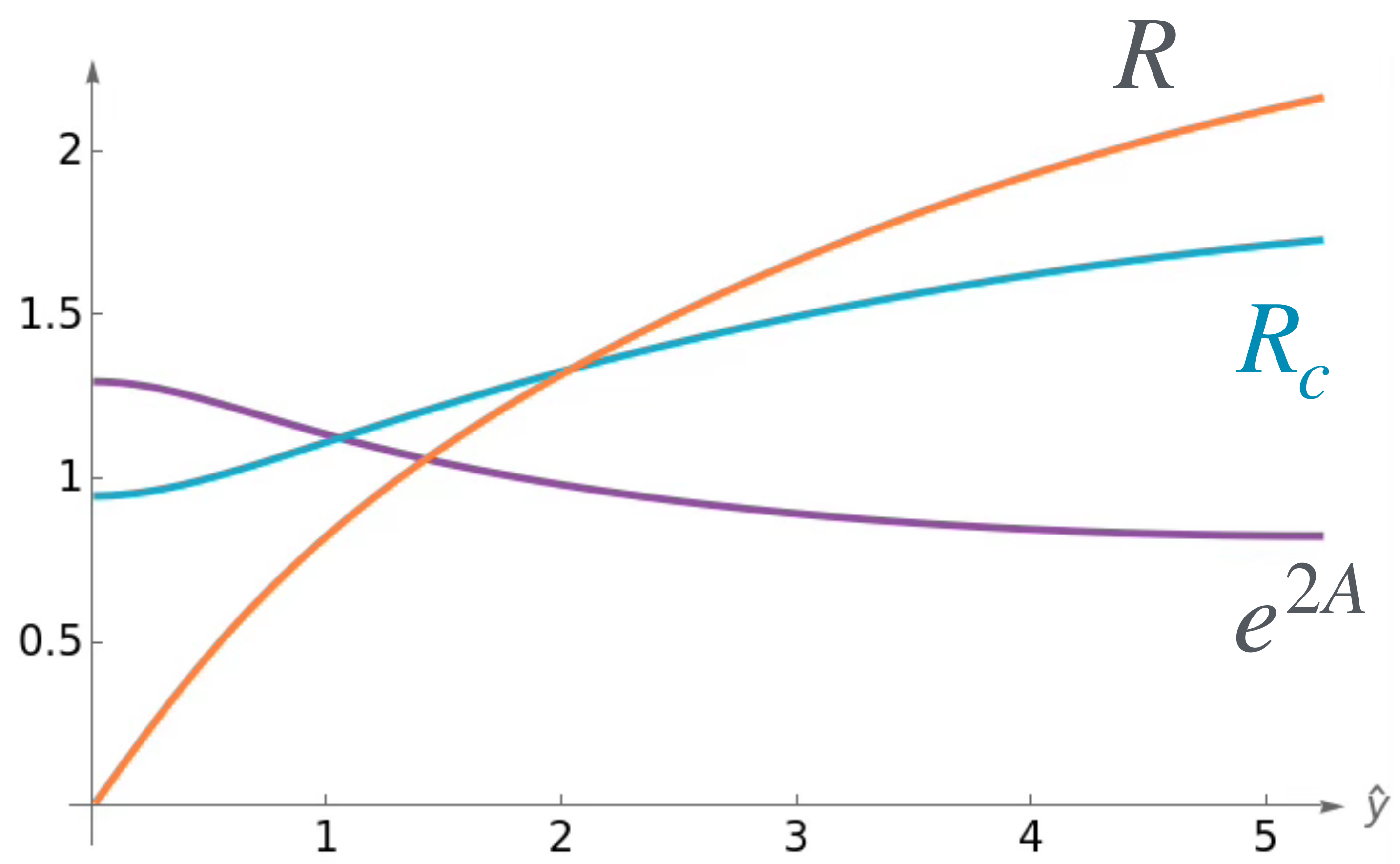
3) In the cusp: $ds^2_{11} = e^{2A(y)} ds^2_{4,\Lambda} + dy^2 + R_c^2(y) ds^2_{\mathbb{T}^5} + R^2(y) d\theta^2$

+

$$-\frac{2}{\sqrt{-g_{11}}} \frac{\delta S_{11}^{(\text{class.})}}{\delta g_{11}^{MN}} = \langle T_{MN}^{(\text{Casimir})} \rangle$$

$\Longrightarrow$

$$a \equiv \frac{\int e^{4A}(-R_7^{(A)})}{\int e^{4A} 42 \ell_7^{-2}}$$



- Gluing to the central manifold possible in regime $0 \ll a \ll 1$,
  - Tuning available via discrete choices in the manifold and flux
- No controlled dS in simple products!

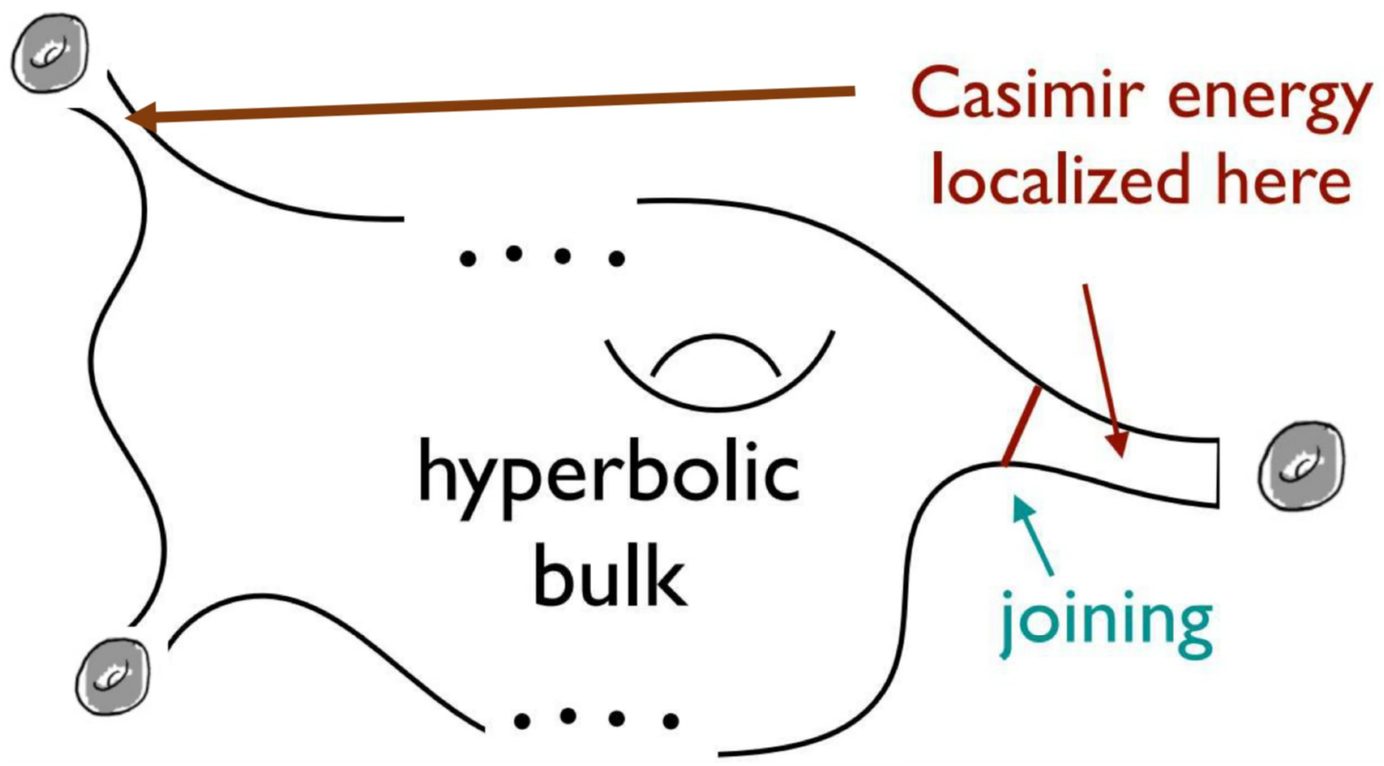
[cf. Parameswaran, Serra, '24, Montero, Bento in progress]

(functions rescaled for clarity, but $R \gg R_c \gg \ell_{11}$ parametrically)

Global solutions and parametric families

[GBDL, Silverstein, Torroba, in progress]

3') What cosmological constant values in globally consistent solutions?



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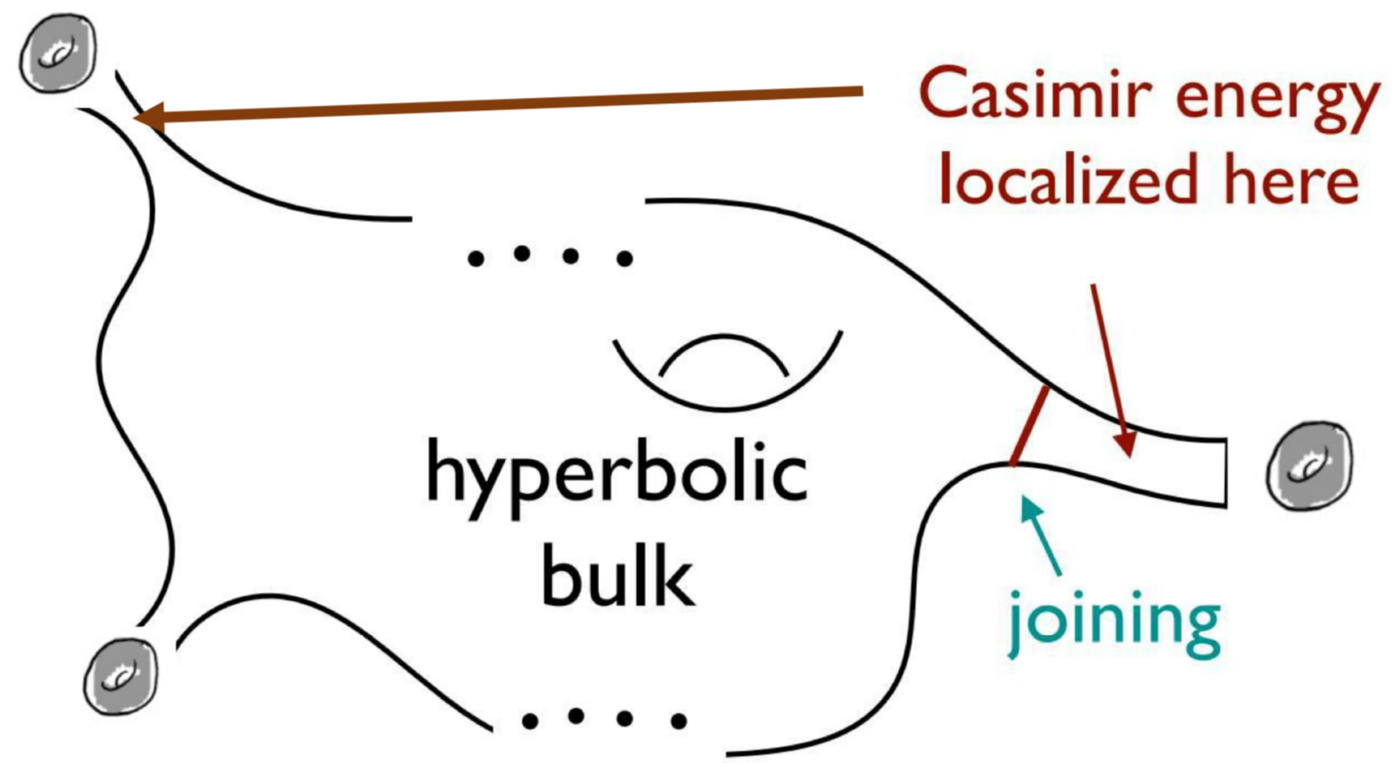
$$\frac{\delta V_{\text{eff.}}}{\delta g^i_i} = 0 \quad \Longrightarrow \quad R_7 = \frac{5}{2}|F_7|^2 - \ell_{11}^9 \langle T^{\text{Cas.}} \rangle + 12\Lambda e^{-2A} - 12(\nabla A)^2$$



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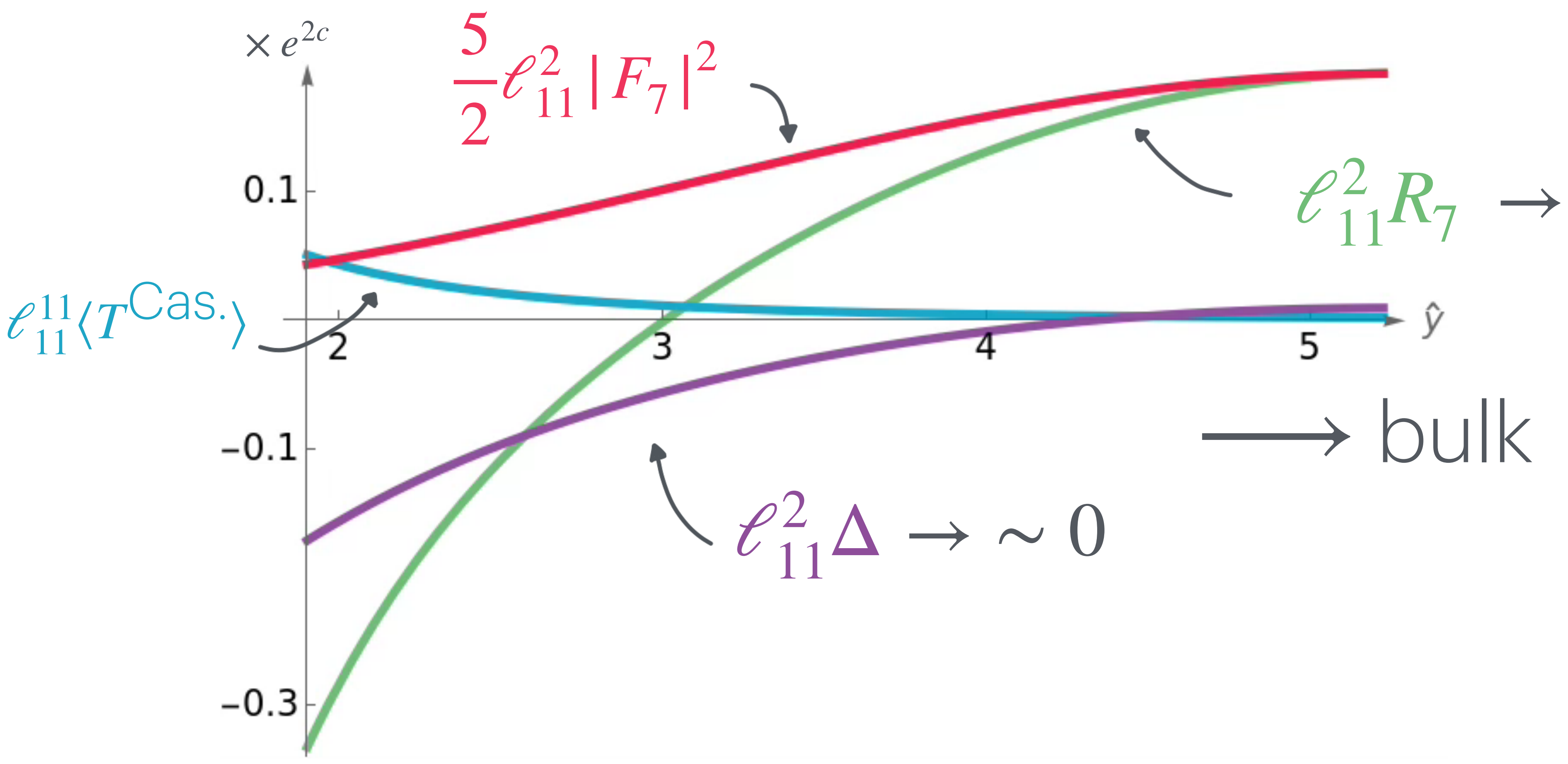

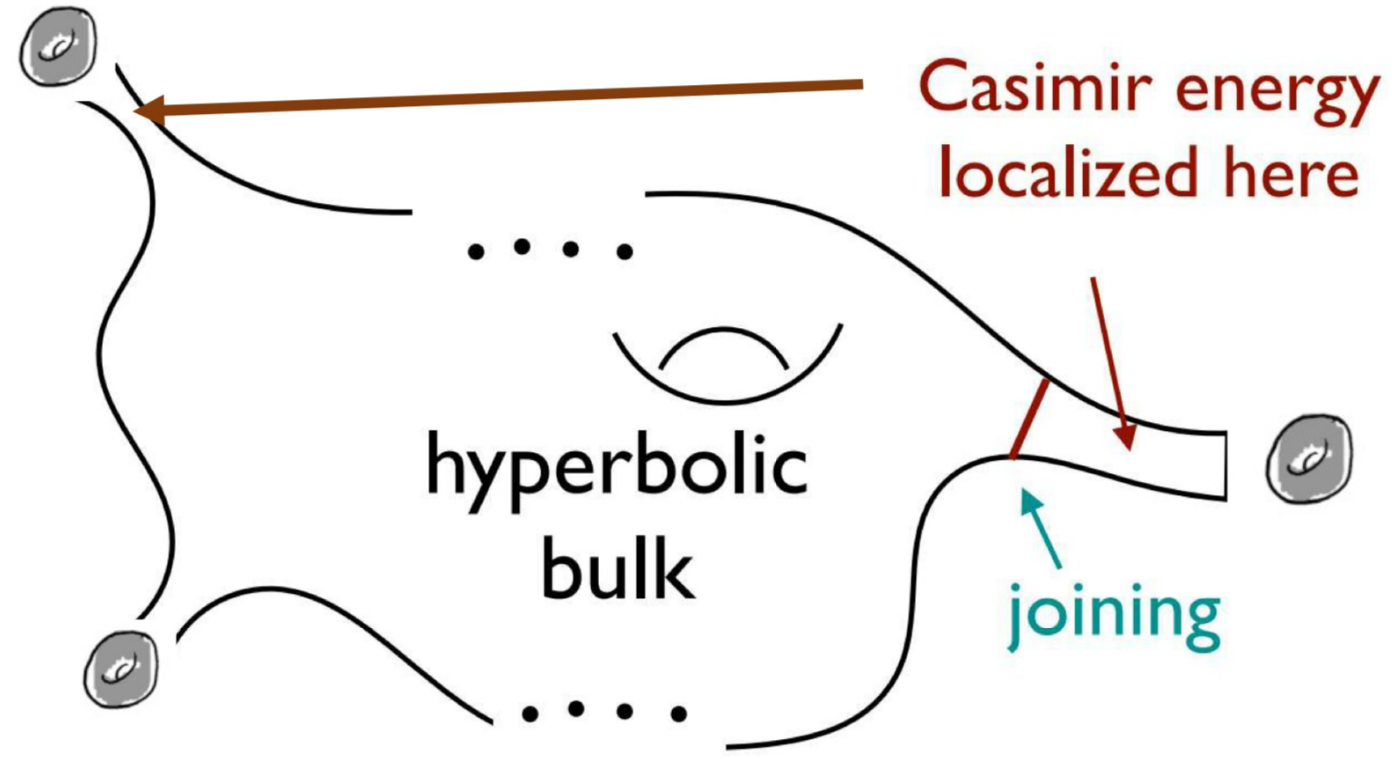

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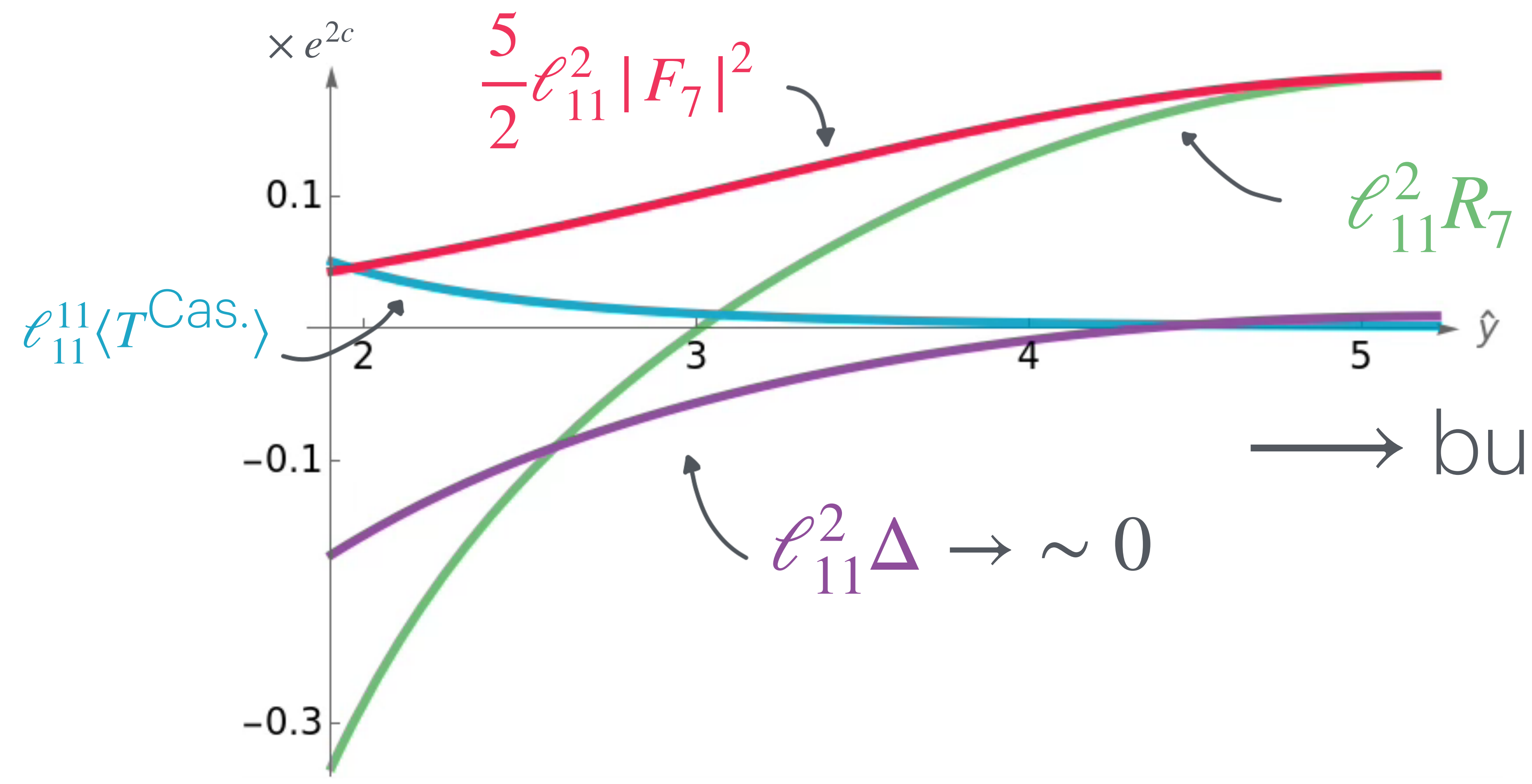
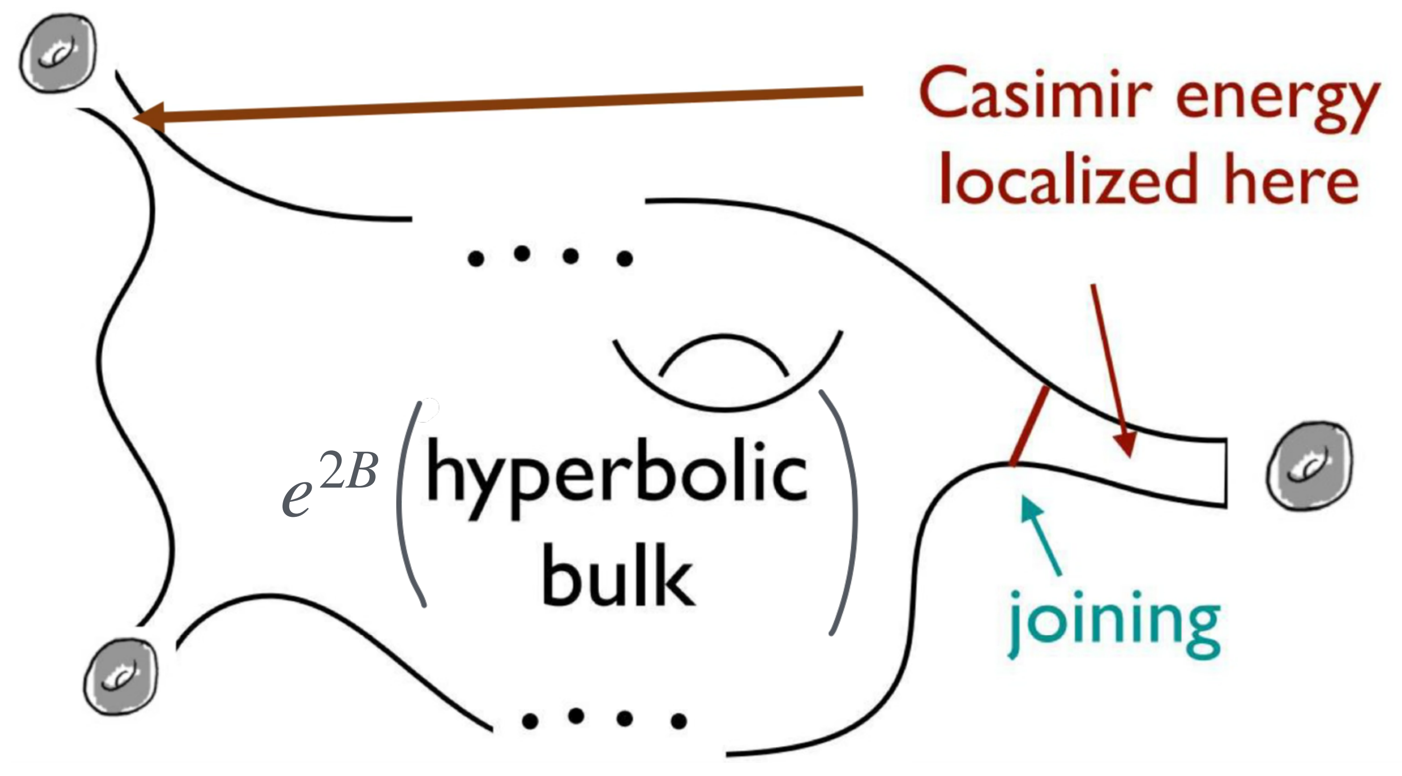


$\ell_{11}^2 R_7 \to > 0$ [cf. Douglas, Kallosh '10]

$\longrightarrow$ bulk



Thm: $\forall M$ dim $\geqslant 3$, if $K \in C^\infty(M)$ is negative somewhere on M, $\exists g$ such that $R[g] = K$ and $g = e^{2B} g_{\text{hyperbolic}}$ [Kazdan, Warner '75]

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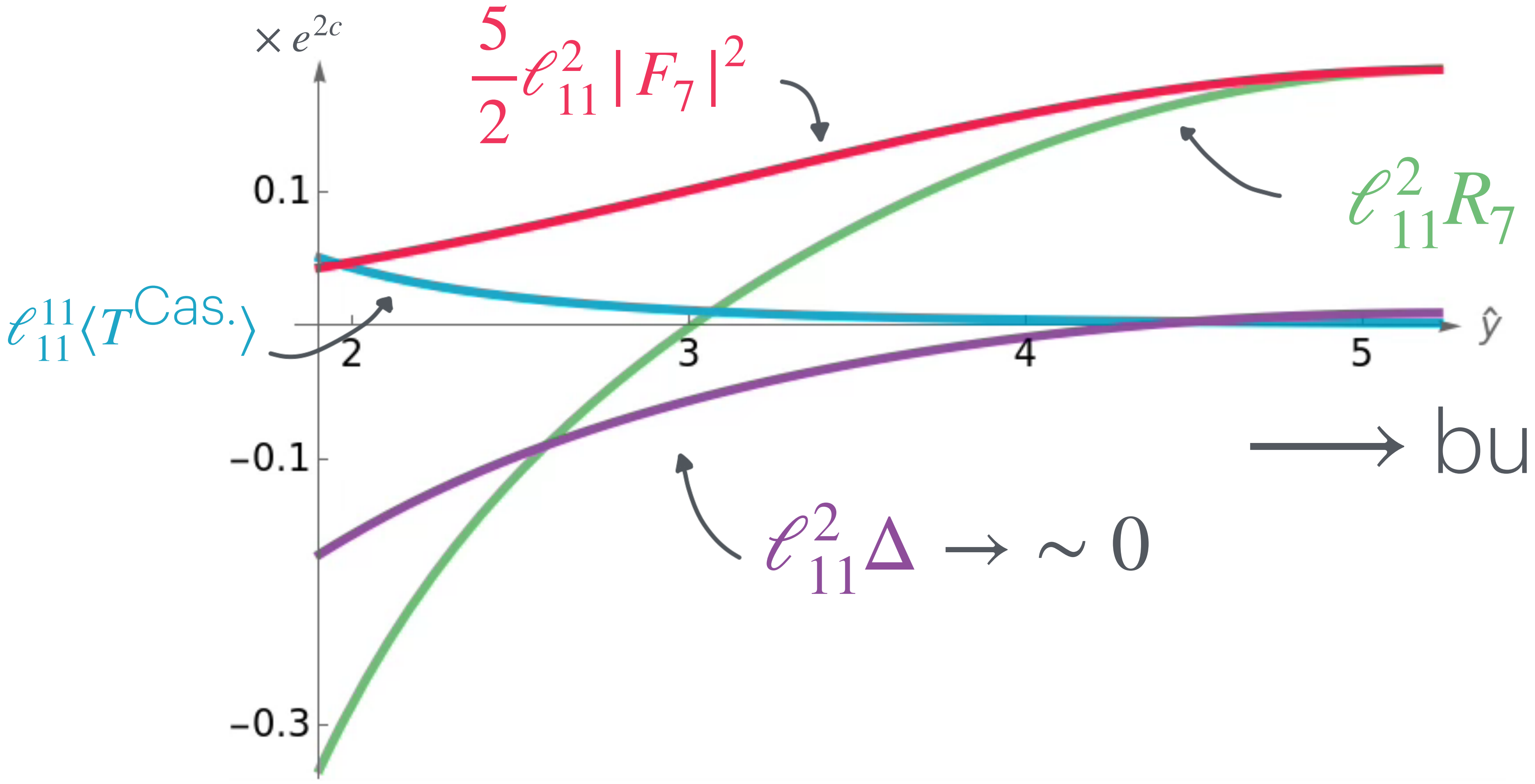

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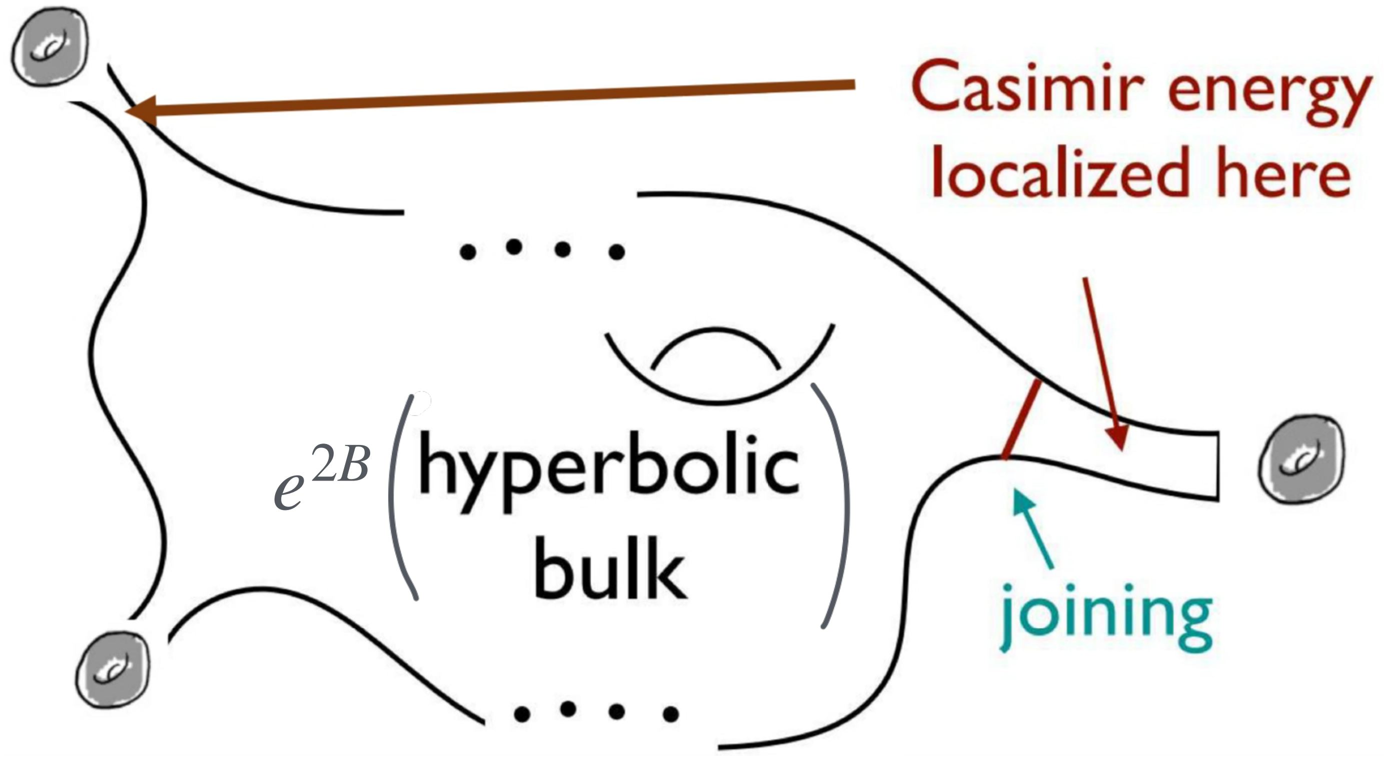

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- Similarly for the warping equation: simple Poisson equation in the bulk: only homogeneous flux source survives

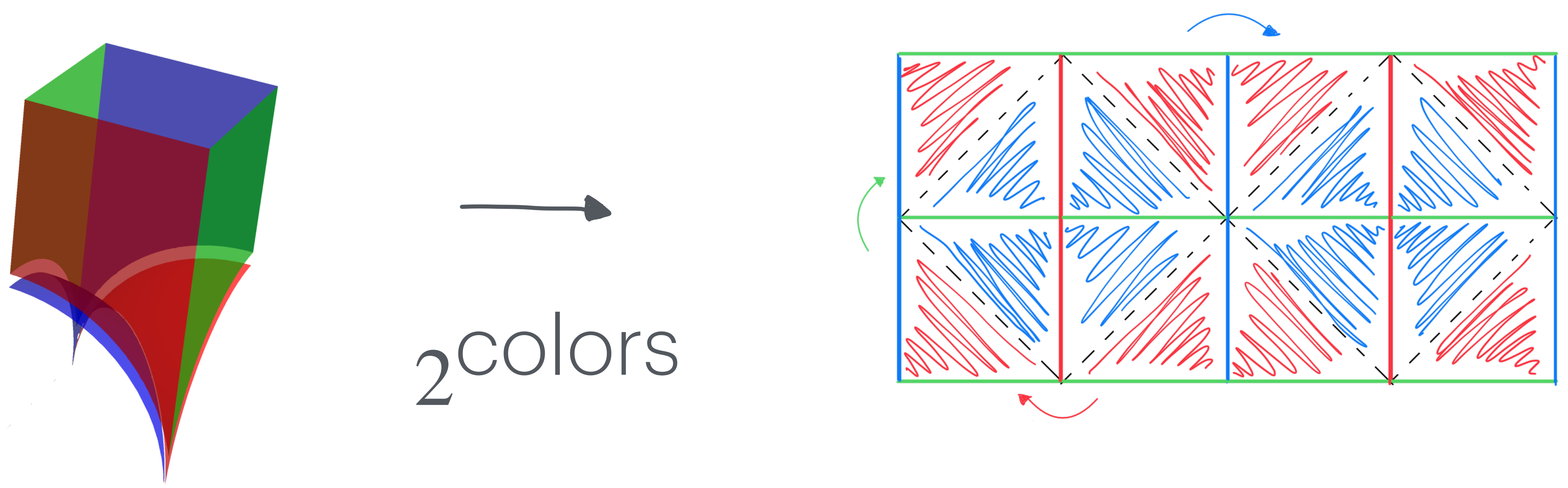
$$+\frac{\delta V_{\text{eff.}}}{\delta A} = 0 \quad \Longrightarrow \quad \nabla^2 A = \frac{1}{3}|F_7|^2 - \frac{1}{2}\ell_{11}^9 \langle T^{\text{Cas.}} \rangle + 3\Lambda e^{-2A} - 4(\nabla A)^2$$

- Global conditions from integrating it in bulk and bulk+cusp

[cf. Aubin'98]

+ equation for transverse-traceless deformations, suppressed by rigidity

1) Take a parametric family of hyperbolic manifolds. For example, using [Italiano, Martelli, Migliorini '21]:



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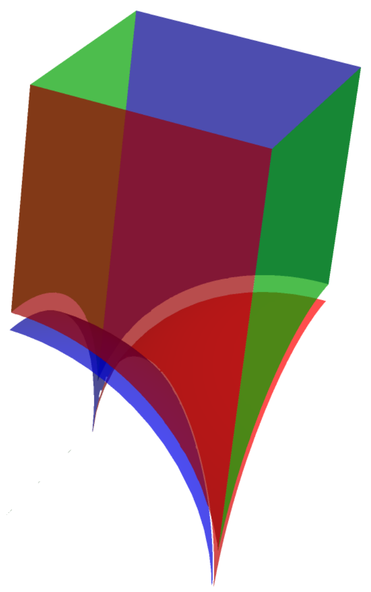
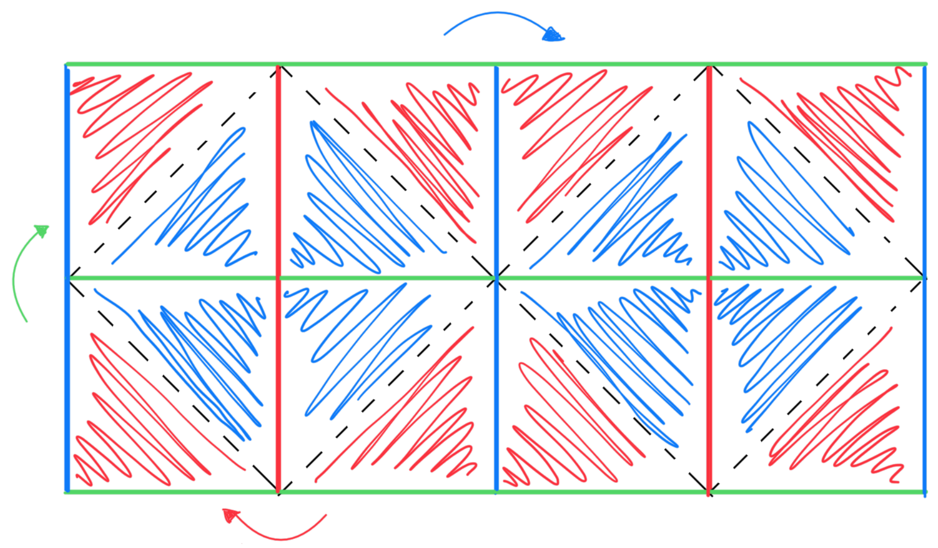


→ $_2$colors



2) Determine how to glue the local solution to the cusp(s): discretizes parameters in the local sol.

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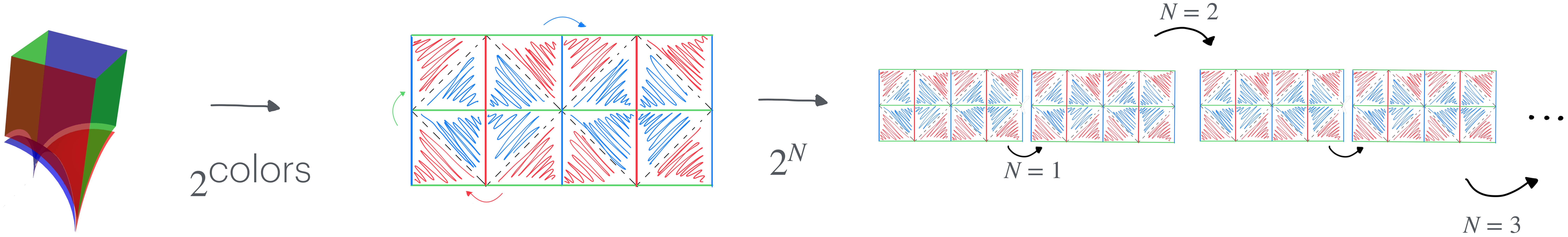

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An example: starting from M_7: $\text{Vol}_{\text{hyp.}} \sim 1.3 \times 10^5 \ell_7^7$

$\frac{\ell_{\text{dS}}}{\ell_7} \sim 0.33 \times 2^{\frac{6}{7}N}$ parametric separation of scales

$\frac{V_{\text{eff.}}}{m_4^4} \sim 13.3 \times 2^{-6N}$ tiny cc compared to 4d Planck

$R_c \sim 0.85 \times 2^{\frac{2}{9}N} \ell_{11} \gg \ell_{11}$ non pert. effects under control

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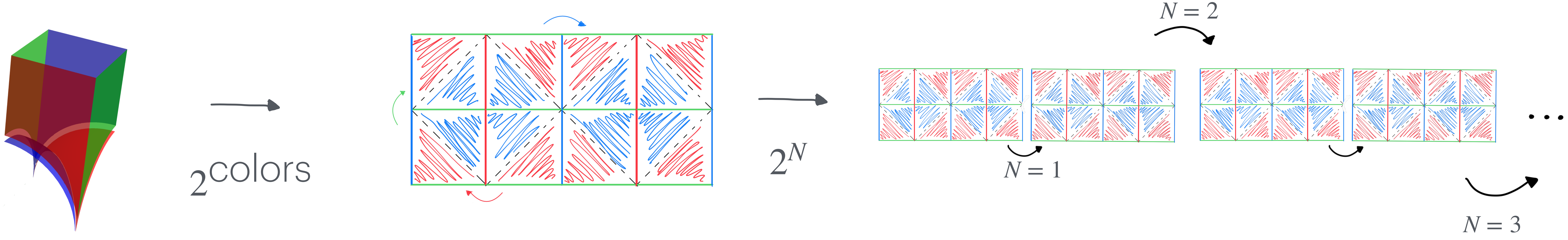

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- This is a special example: attaching a single numerical cusp and filling the others short such that Casimir does not build up in them
- Many other discrete parametric families available: different manifolds, different cutting and pasting, different (non-diagonal) independent gluings of each cusp

- A **vast landscape** to explore!

[GBDL, Silverstein, Torroba, in progress]

Direct numerical exploration of the Landscape?

- We have used different approximations in each region, and studied the existence of a global configuration.
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- Bakry-Émery-Ricci (negative-dim.) effective curvature. Studied in the context of Optimal Transport theory [Bakry-Émery '85, Villani '08, …]
- Controls the spectrum of spin 2 Kaluza-Klein modes [GBDL, Tomasiello, '21 GBDL, De Ponti, Mondino, Tomasiello, '21, '22, '23, '24]

- Coupled system of non-linear **PDEs in high dimensions** (e.g. 6 or 7)
- Standard numerical methods struggle: only few examples of direct solutions of simpler versions of the problem
    - Kaluza-Klein Black holes, Calabi-Yau metrics, Kähler-Einstein metrics, … [Headrick, Kitchen, Wiseman, '10 , Headrick, Wiseman, '05, Douglas, Karl, Lukic, Reinbacher '08, Headrick, Nassar, '13, Doran, Headrick, Herzog, Kantor, Wiseman '08, …]

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- Luckily, we now have Machine Learning! [Ashmore, He, Ovrut, '20; Jejjala, Mayorga Peña, Mishra, '20, Douglas, Lakshminarasimhan, Qi, '20 Anderson, Gerdes, Gray, Krippendorf, Raghuram, Ruehle, '21; Larfors, Lukas, Ruehle, Schneider, '21, '22; Douglas, '22; Gerdes, Krippendorf, '22, …]
    - Already successfully used for finding Calabi-Yau metrics, including α' corrections [Fraser-Taliente, Harvey, Kim, '24]
- Can we use it to directly solve all the eoms, including backreaction of matter fields? [GBDL, '25]

Landscapes in the Landscape

1) Parametrize the problem using Neural Networks

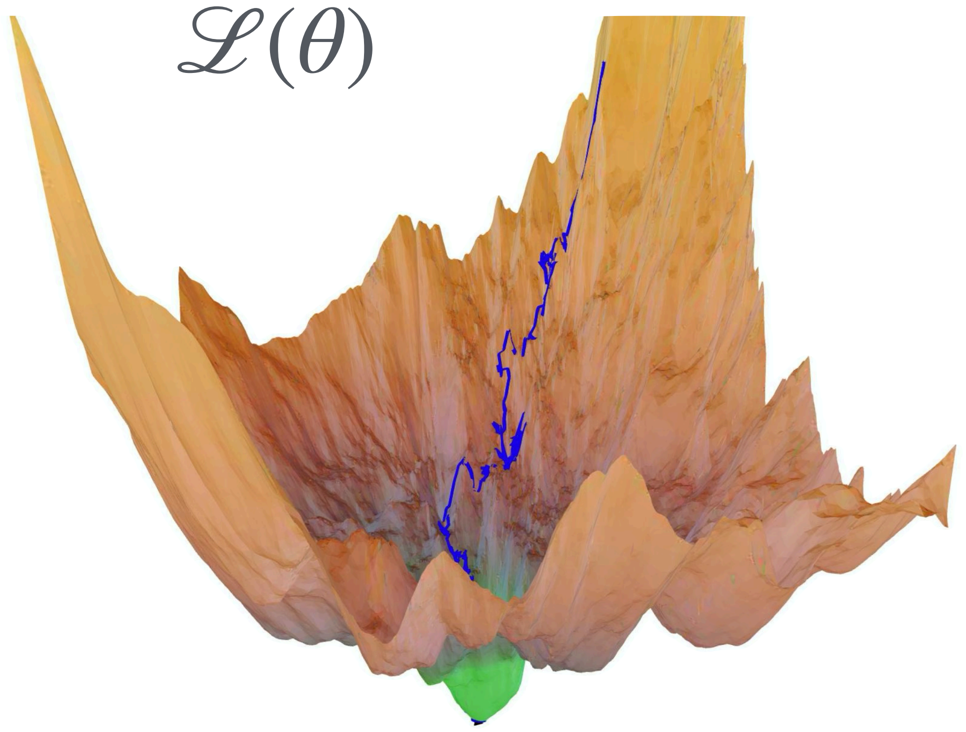
$$g_{mn}(y) \equiv \mathcal{N}_{mn}(y;\theta)$$
$$f(y) \equiv \mathcal{N}(y;\theta)$$

[Lagaris et al., '98; Weinan, Yu, '17; Sirignano, Spiliopoulos, '18; Raissi et al. '19; Bar-Sinai et al. '19, Piscopo et al., '19; Lu et al, '22; Wang et al, '22, ...]

2) Design a *loss function* minimized on the solution

$$\mathcal{L}(\theta) \equiv \sum_{y\in M} \left| \mathrm{Ric}^f_{mn}(x,\theta) - \Lambda g_{mn}(y,\theta) - \tilde{T}_{mn}(y,\theta) \right|^2 + \sum_{y\in M} \left| \left(-\nabla^2 e^f\right)(y;\theta) + (D-2)\left(\Lambda - \frac{1}{d}\hat{T}^{(d)}(y;\theta)e^{f(y;\theta)}\right) \right|^2$$

$$+ \left|\text{matter eqs.}\right|^2 + \sum_{y\in\partial M} \left|\mathrm{BC}(y;\theta)\right|^2 + \left|\text{constraints}\right|^2$$



Landscapes in the Landscape

1) Parametrize the problem using Neural Networks

$$g_{mn}(y) \equiv \mathcal{N}_{mn}(y;\theta)$$
$$f(y) \equiv \mathcal{N}(y;\theta)$$

[Lagaris et al., '98; Weinan, Yu, '17; Sirignano, Spiliopoulos, '18; Raissi et al. '19; Bar-Sinai et al. '19, Piscopo et al., '19; Lu et al, '22; Wang et al, '22, …]

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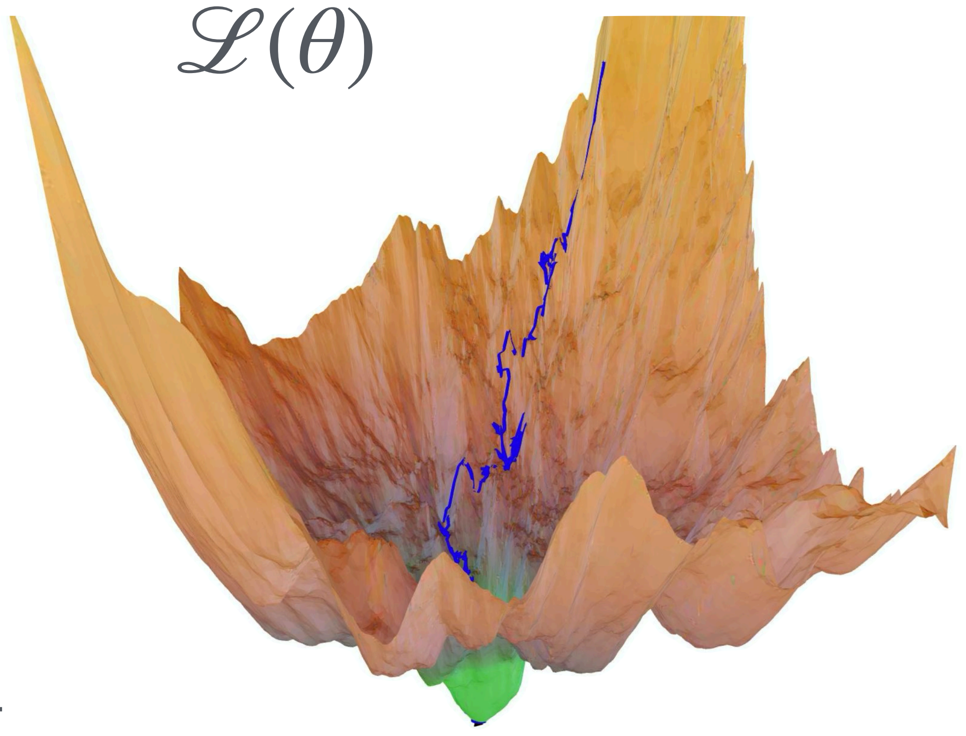
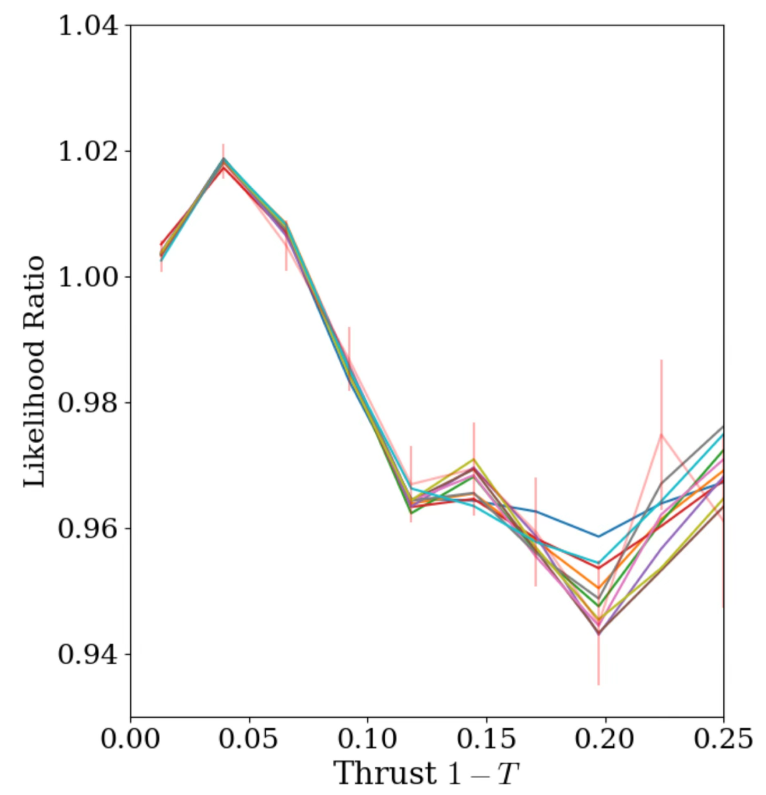


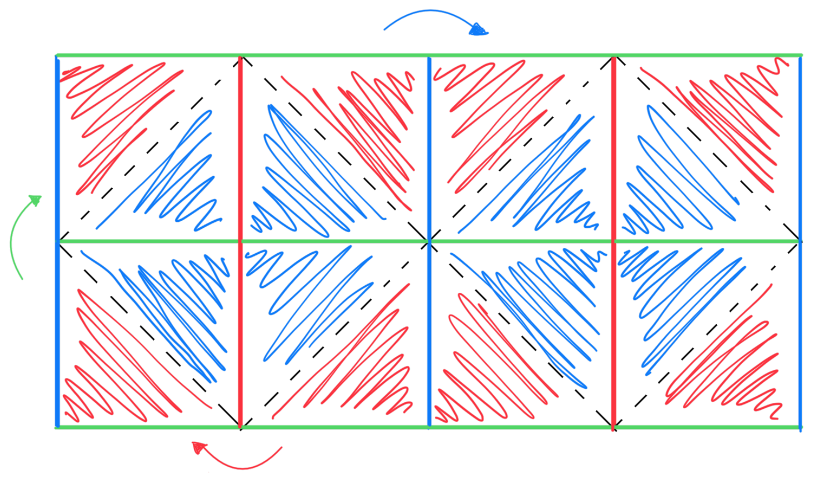
3) It's now a physics problem! Prescribe a dynamics on this new landscape and follow it

- Standard approach: (Stochastic) Gradient Descent (and relatives) ⟶ friction dominated motion!
- Can we use other physical dynamics to solve this problem?
  - Yes: Energy Conserving Descent: Hamiltonian evolution in chaotic regime, concentrate Liouville measure [GBDL, Silverstein '21, GBDL, Gatti, Silverstein' '23]
    - Convergence due to kinetic effects (e.g. relativistic) rather than friction
    - Measure concentration useful for precise optimization for simulation based-inference in particle physics [GBDL, Nachman, Silverstein, Zheng, to appear]
  - Yes: [Your idea here]



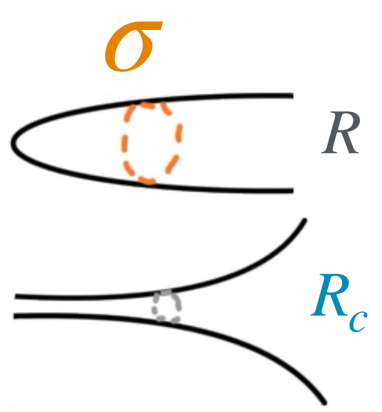
Proof of concept: filled metrics in 3d

[GBDL, '25]

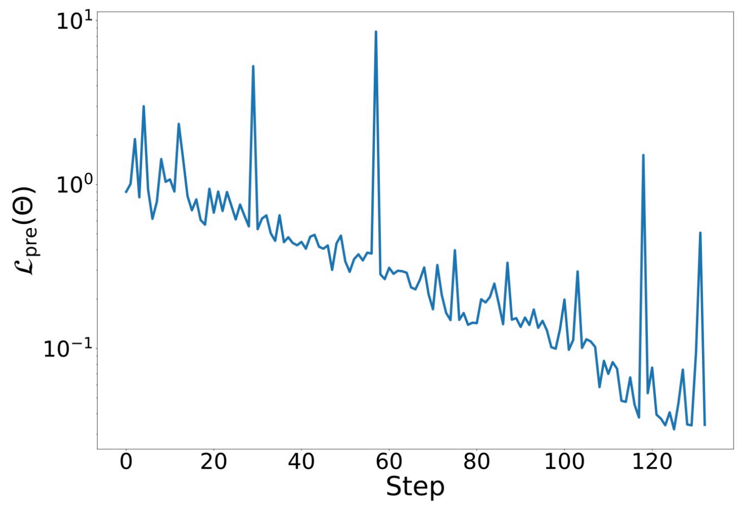


- Can we directly solve the Einstein equations to find the filled geometries, starting from 3d?

  1) Parametrize the problem using Neural Networks



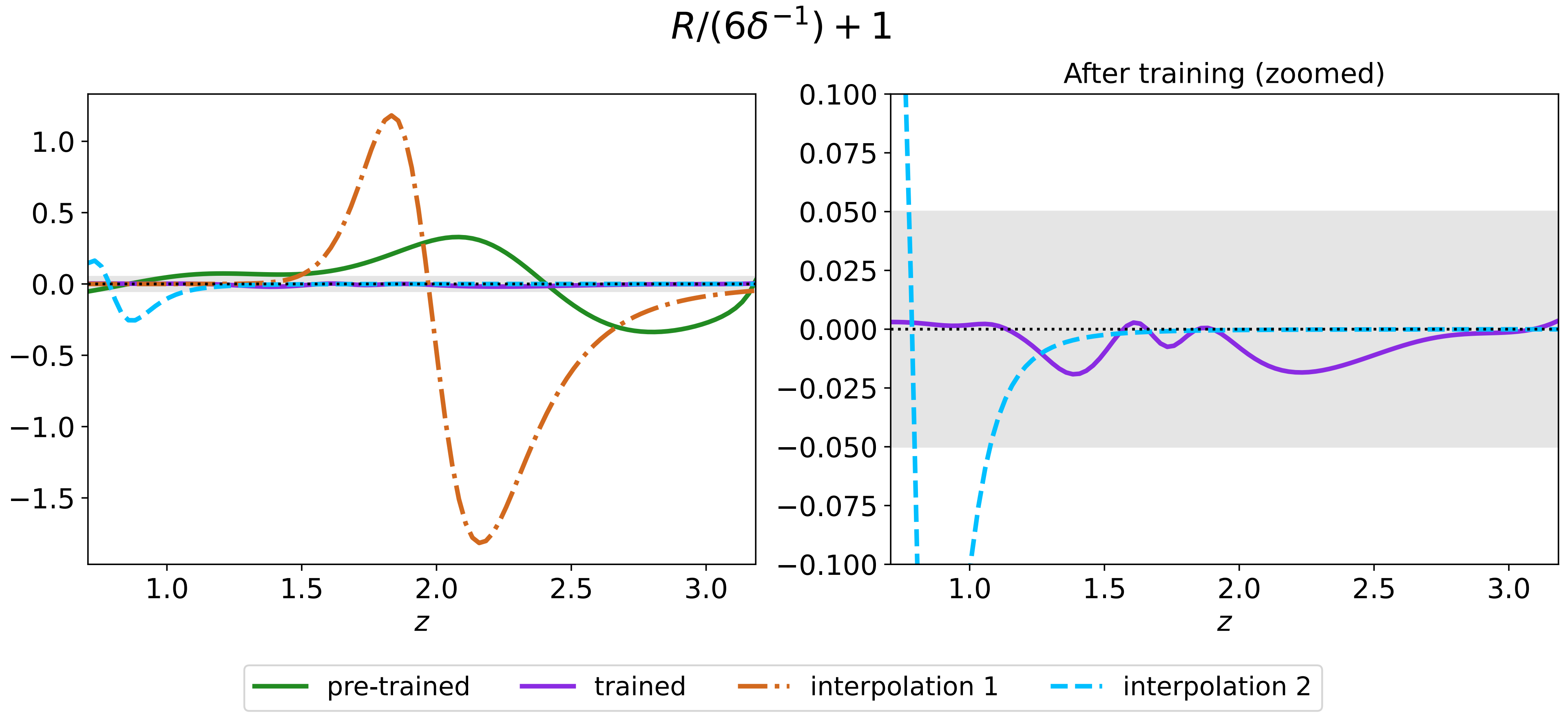
  2a) Pre-train to fit Anderson's approximate metric



  2b) Continue training to solve the Einstein conditions, plus continuity and differentiability

$$\mathscr{L}(\theta) \equiv \sum_{y\in M} \left|\mathrm{Ric}_{mn}(x,\theta) - \Lambda g_{mn}(y,\theta)\right|^2 + \sum_{y\in\partial M} \left|\mathrm{BC}(y;\theta)\right|^2$$

  - Average percentage error: $\sim 3\times 10^{-3}$



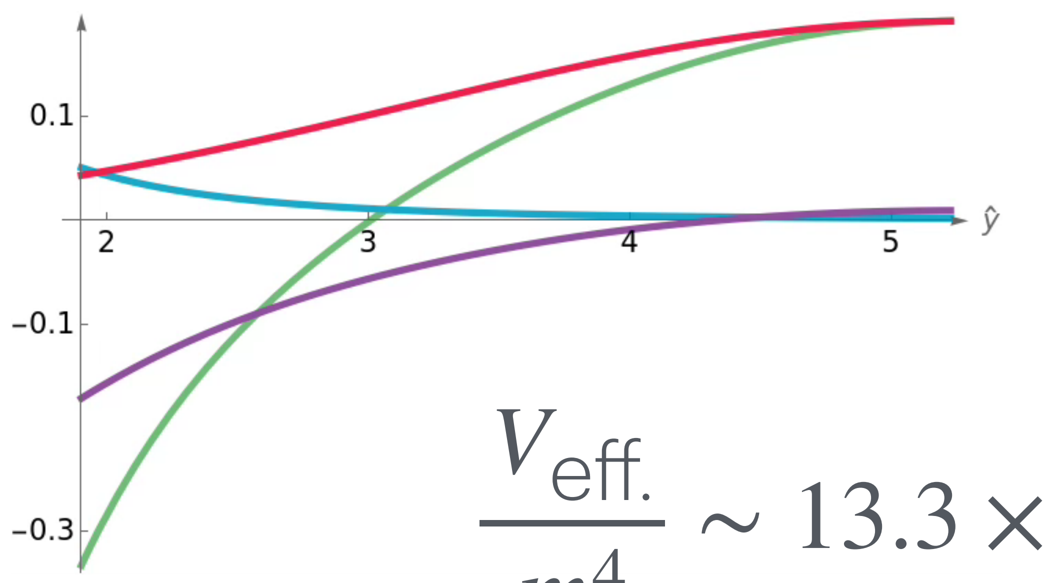
- **Next steps**: add matter fields and scale up to 7d!
  - Intermediate step: explicit numerical filling in 4d is an open problem in hyperbolic geometry [Martelli, '15]

[GBDL, Law, in progress]

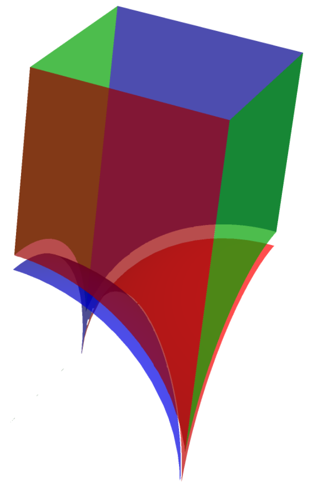
- Other approach: direct minimization of effective potential or slow-roll parameters? [cf. GBDL, Silverstein, Torroba, '21]

Recap

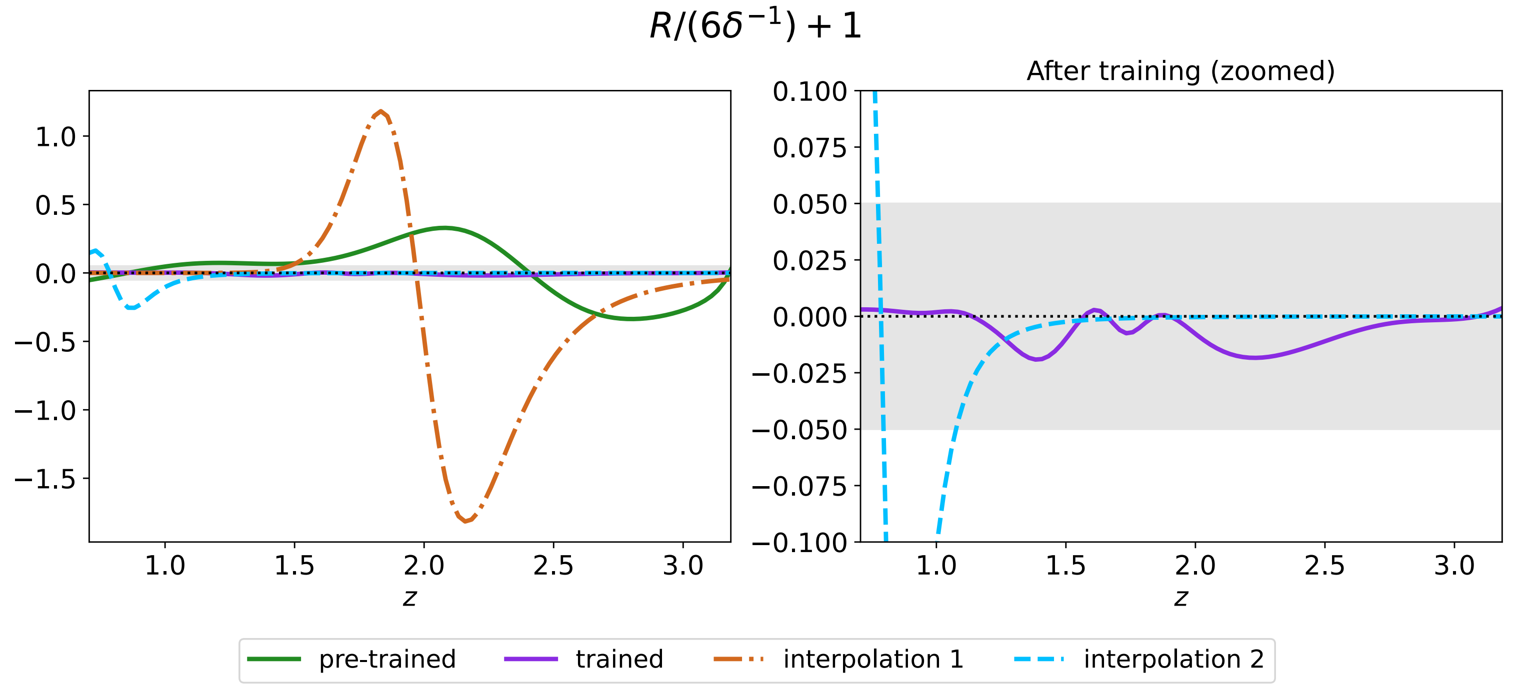
Thank you!

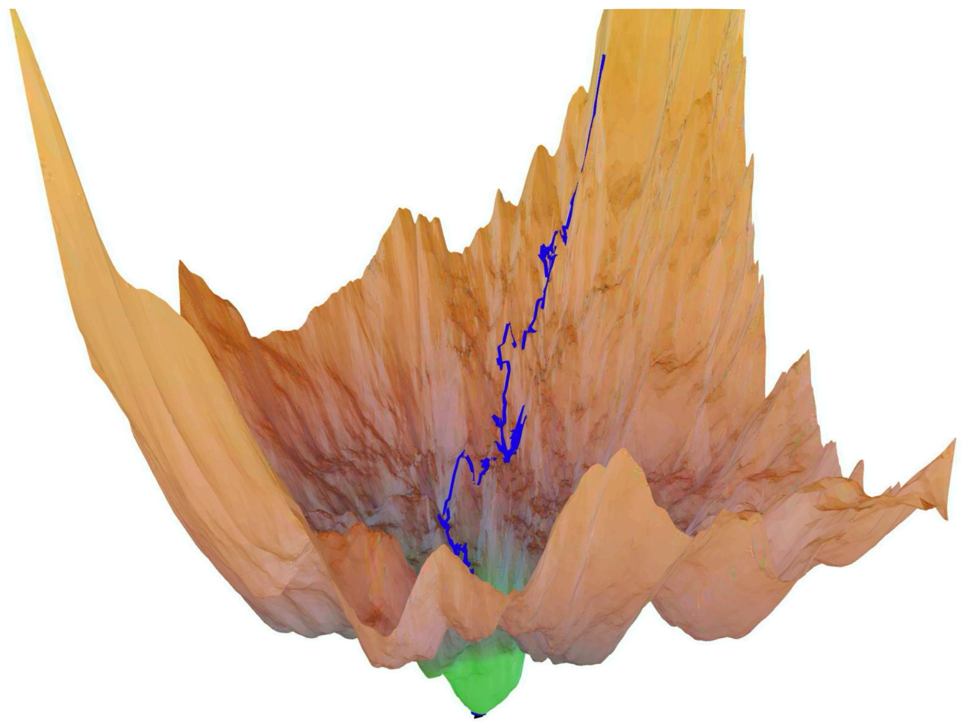


$$\frac{V_{\text{eff.}}}{m_4^4} \sim 13.3 \times 2^{-6N}$$

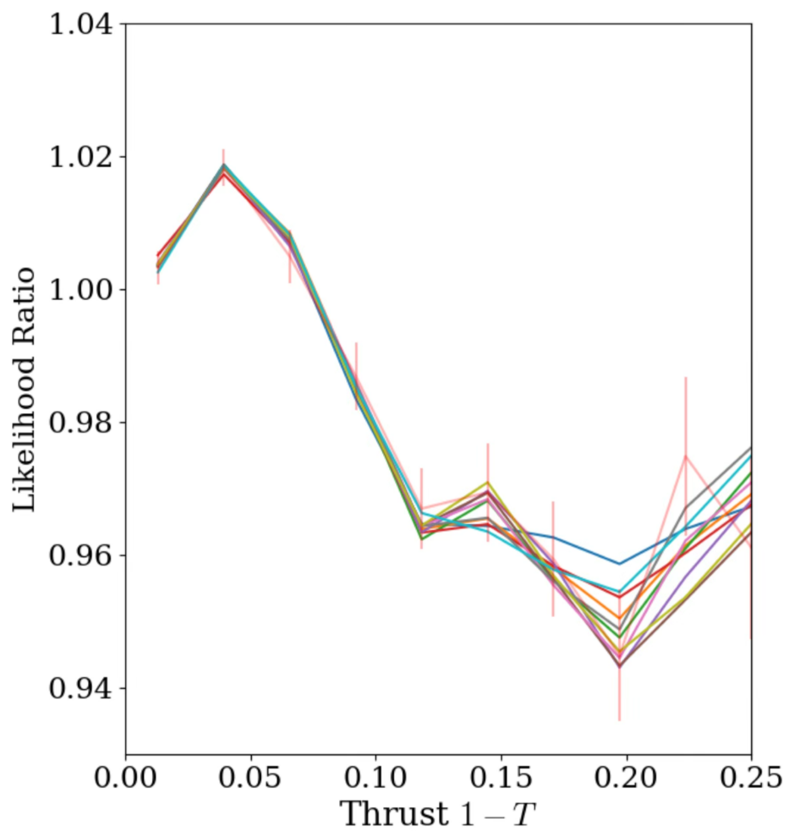


de Sitter Landscape





Neural Networks Landscapes



Many Physics applications!