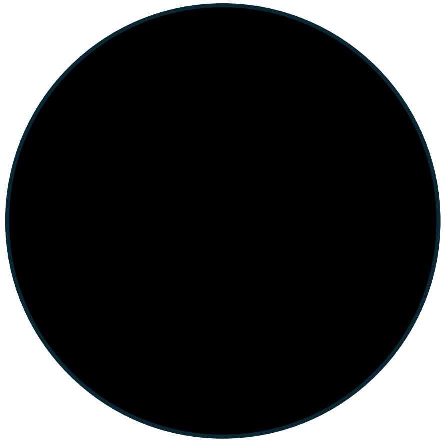
The evaporation of charged black holes

Work with Brown, Iliesiu, Usatyuk
arXiv:2411.03447

What happens to an isolated black hole?

Studying our universe (**SM coupled to gravity**) but ignoring cosmological effects (**asymptotically flat space**)



Just interested in **coarse-grained description** of evolution, not subtle effects like information being preserved

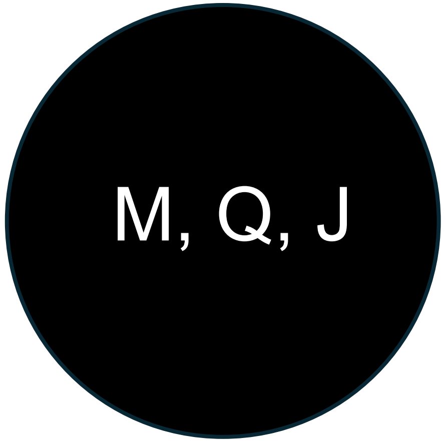
Hawking: **QFT in curved spacetime** calculation => black holes radiate and thereby evaporate

If initial black hole is **not too large**, this is a good approximation until the black hole has **Planckian mass**

Generic sufficiently large black hole: Hawking style calculation gives wrong answers for **almost the entire evaporation**

By including **quantum gravity** effects, one can give a correct analysis of the full evaporation until a Planckian mass is reached

No hair theorem

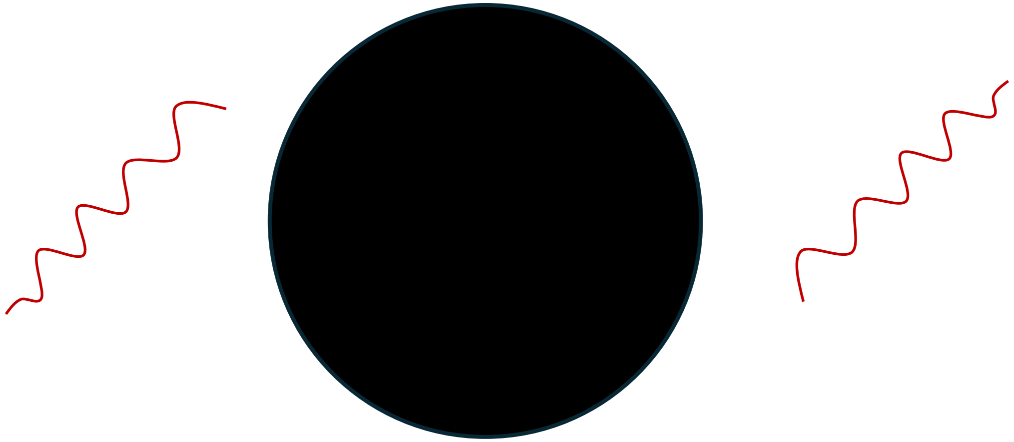


- Classical GR: black hole settles down to an equilibrium solution characterized by its **mass**, **charge**, and **angular momentum**

Generically, one finds a Kerr-Newman black hole with $Q = O(M), J = O(M^2)$

At this point, classical evolution stops - the only further changes come from Hawking radiation (**slow**)

What happens to the angular momentum?



- In a Kerr/Kerr-Newman black hole, emitted Hawking photons (or gravitons) preferentially carry angular momentum in the same direction as the black hole

Page: as a result, black hole loses angular momentum faster than it loses charge. Converges to a Schwarzschild/Reissner-Nordstrom black hole with $J \ll M^2$

What happens to the charge?

- All charged particles in our universe are massive => Hawking radiation is exponentially suppressed at low temperatures (large black holes)
- Charged particles can still be produced in strong enough electric fields due to the **Schwinger effect**
- Horizon electric field strength scales as $E \sim \frac{Q}{r_{hor}^2} \sim \frac{Q}{M^2}$ and so also becomes small large enough black holes (even if near-extremal)
- Schwinger effect is exponentially suppressed when $Q > Q_* = q/\pi m^2 \sim 10^{44}$ ($M \sim 10^6$ solar masses for an extremal black hole)

Self-tuning towards extremality

- Black holes with $Q \gg Q_{*} \sim 10^{44}$ will lose mass through radiation of photons/gravitons but don't lose charge

Eventually they will get closer and closer to extremality. As it does so, its semiclassical Hawking temperature scales as $T \sim Q^{-3/2}\sqrt{Q - M}$

When $Q - M \ll E_{brk} \sim Q^{-3}$ then the energy of a single thermal Hawking quanta becomes **larger** than the energy of the black hole above extremality

Clear signature that the QFT in curved spacetime approximation is **breaking down** => need to take into account backreaction of Hawking quanta + metric fluctuations

Backreaction and the long throat

Near-extremal black hole have a long throat within which the transverse sphere has approximately **constant radius**

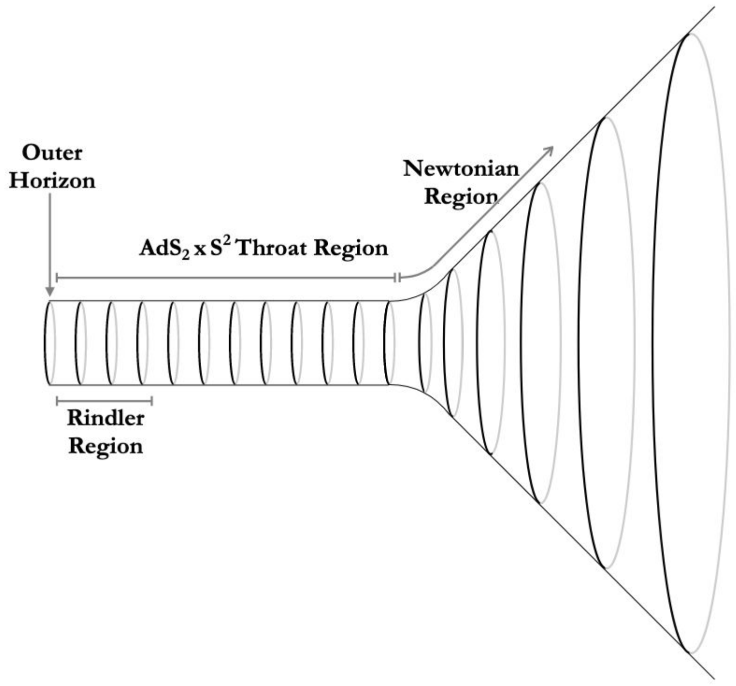
Radial and time directions look like two-dimensional **anti-de Sitter space**

As the temperature goes to zero, the length of the throat diverges

Small changes in energy lead to large changes in spacetime geometry => backreaction is important

Relatedly, **large diffeomorphisms** of AdS_2 and **rotational modes** of the transverse sphere become **almost-zero modes** of the black hole solution and will have large, strongly coupled **quantum fluctuations**

These almost-zero modes have an **effective two-dimensional description** (JT gravity + SU(2) gauge theory)

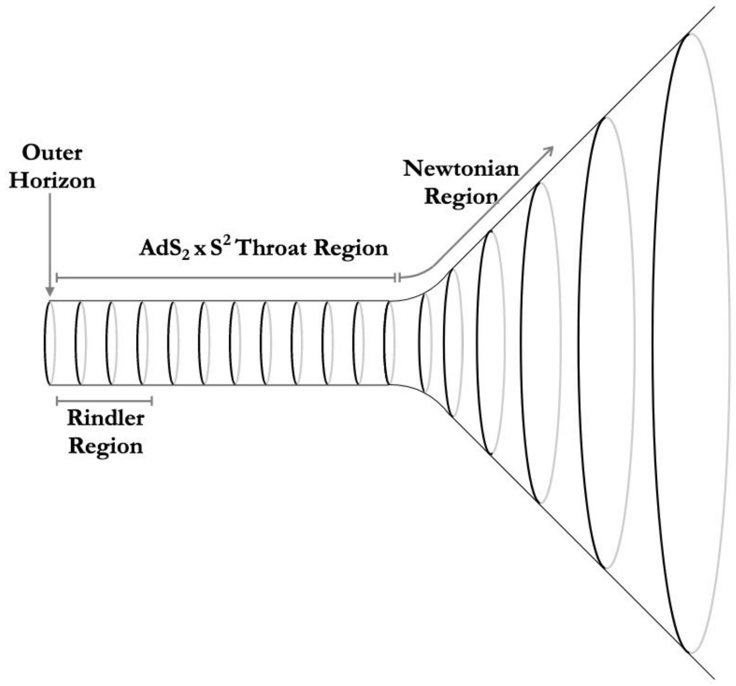


JT gravity and Schwarzian quantum mechanics

Last ten years: almost-zero modes form a solvable quantum theory: **Schwarzian quantum mechanics** coupled to a quantum **rigid rotor**

Schwarzian QM is the same theory that describes the **SYK model** in condensed matter physics at low temperatures (example of **universality**)

Strategy: nonperturbative quantum treatment of **matter fields + almost-zero modes**. Semiclassical treatment of all other spacetime fluctuations



[Iliesiu, Turiaci]

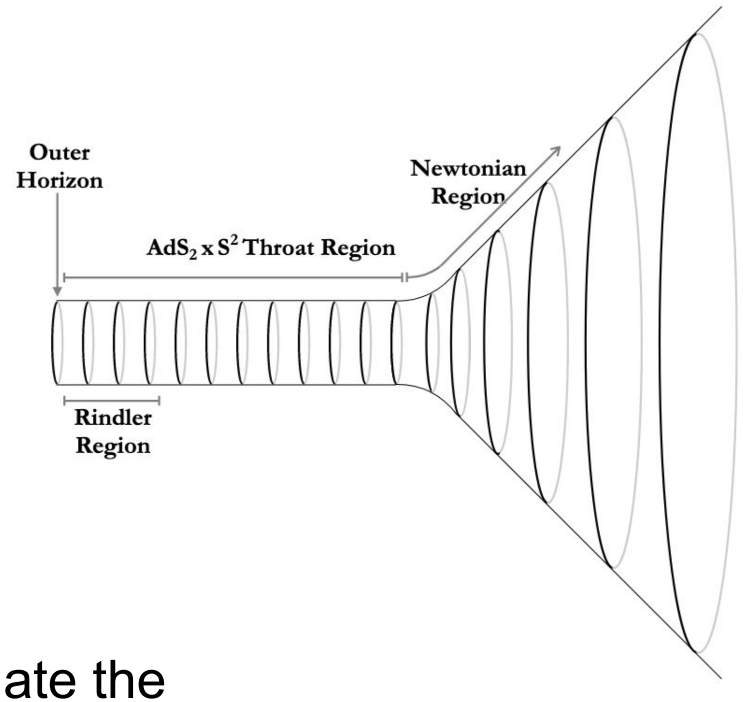
Schwarzian corrections and low temperature thermodynamics

Schwarzian effects lead to large changes in the thermodynamics of a black hole close to extremality (in the absence of light matter fields)

For $M - Q \ll E_{brk}$, we now have $M - Q = 3T/2$

Thermal photons would still lead to violations of the extremality bound

But no longer clear that the radiation is thermal

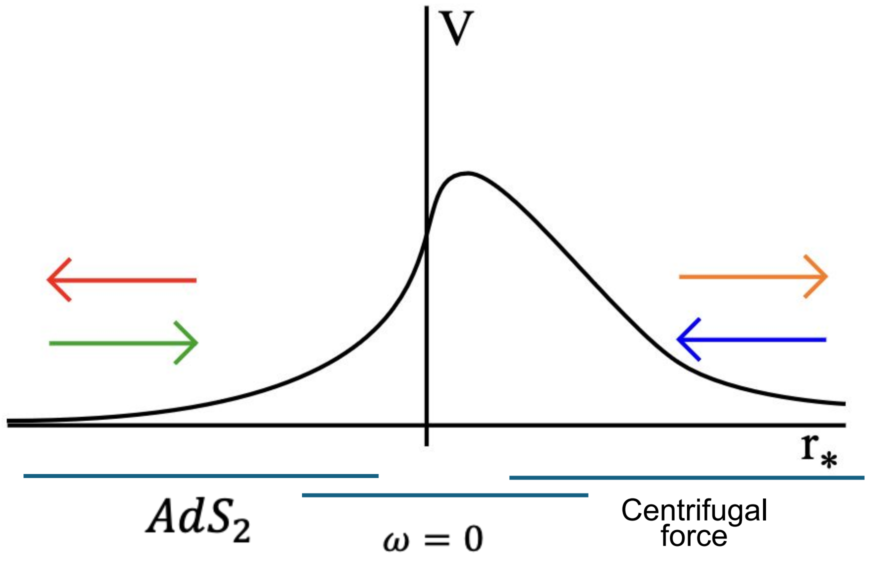


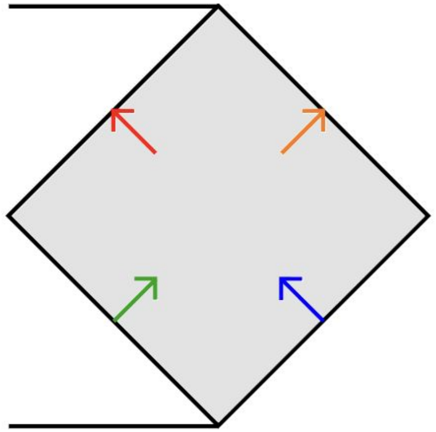
Need to actually calculate the radiation spectrum

[Iliesiu, Turiaci]

Toy example

Semiclassical massless scalar field radiation





$$\left(\frac{d^2}{dr_*^2} + \omega^2 - \underbrace{\frac{f}{r^2}\left(\ell(\ell+1) + rf'\right)}_{V_{\text{eff}}(r)}\right) ru(r) = 0.$$

Near-horizon outgoing modes are thermal. Related to outgoing modes at infinity by a classical scattering problem (graybody factors)

Analytically solvable in near-extremal limit by matching different approximate solutions in different regimes

Find that the emission probability goes to zero at low temperature $P^{\text{scalar}}_{\text{emit}}(\omega, \ell = 0) = 4(r_+\omega)^2,$

[Figure from Harlow (2014)]

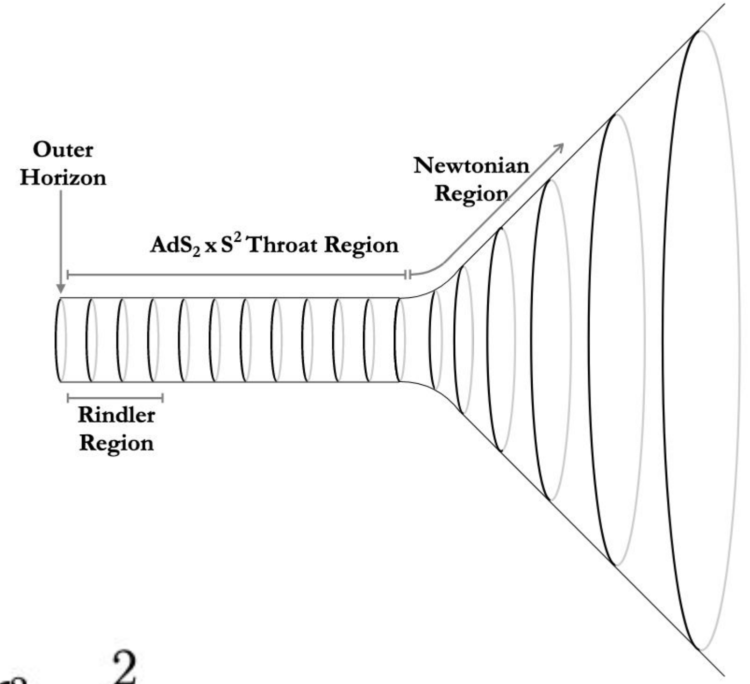
The quantum description

Near-horizon: JT gravity, 2D SU(2) gauge theory, scalar field

Far-field: scalar field in fixed black hole background

Leading order: two regions are decoupled

Perturbative coupling: far-field modes reflecting off the black hole potential barrier act as a source for the near-horizon region (and vice versa)



Interaction Hamiltonian:

$$H_I = \mathcal{N}\mathcal{O}\int_0^\infty d\omega \sqrt{r_+^2\omega}\,(a_{\omega 00} + a^\dagger_{\omega 00}), , \qquad \mathcal{N}^2 = \frac{2}{\pi}.$$

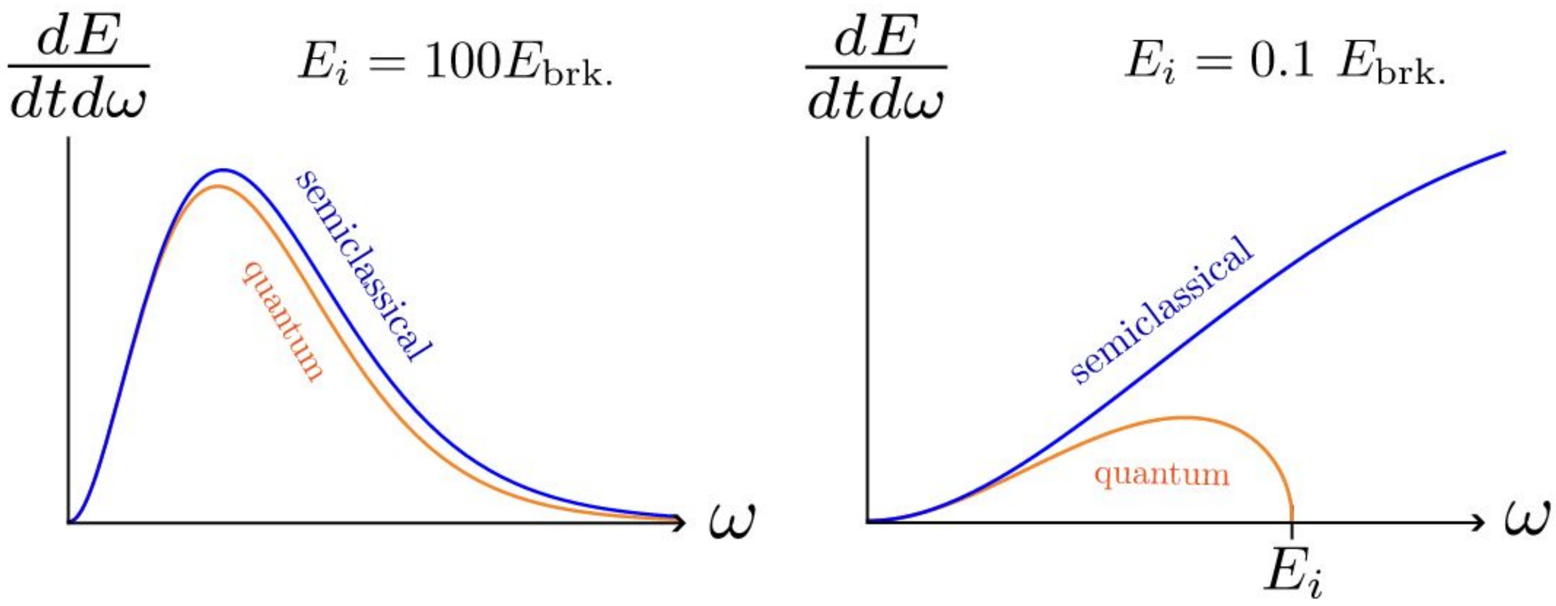
Near-horizon matter operator

Far-field creation/ annihilation operators

Quantum scalar field radiation

Emission rate follows from **Fermi's golden rule**:

$$\Gamma_{i\to f} = 2\pi|\langle E_f,\omega|H_I|E_i\rangle|^2\delta(E_f+\omega-E_i)$$
$$= 2\pi\mathcal{N}^2(r_+^2\omega)|\langle E_f|\mathcal{O}|E_i\rangle|^2\delta(E_f+\omega-E_i)\,,$$



Semiclassical photon/graviphoton radiation

There are no massless scalar fields in our universe (that we know of)

Instead: photons (spin 1) and gravitons (spin 2)

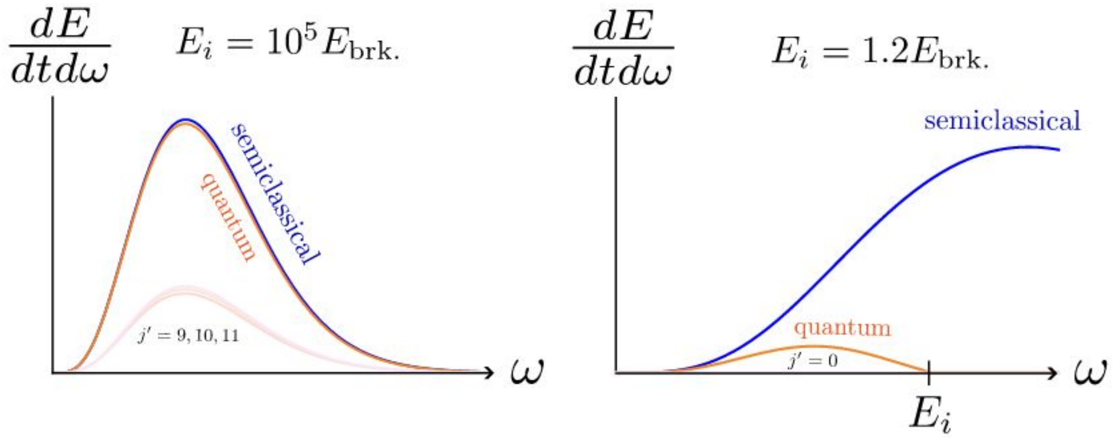
Naively, photon emission should dominate at low energies because smaller angular momentum => **smaller centrifugal barrier** in the effective potential

Not true semiclassically: black hole is charged with respect to the EM field -> leads to photon/graviton mixing

Mode with smallest scaling dimension in the near-horizon region is an $\ell = 2$ mode that is a mixture of the photon and graviton ("graviphoton")

Semiclassically, this contributes an $O(1)$ fraction of the emission (along with the $\ell = 1$ photon mode)

Can compute fully quantum emission rates for these modes, just like for a scalar field



Angular momentum and forbidden transitions

- Extremality bound says that a black hole with angular momentum j has to have mass at least

$$E_0^j = Q + \frac{j(j+1)}{2} E_{\text{brk.}}$$

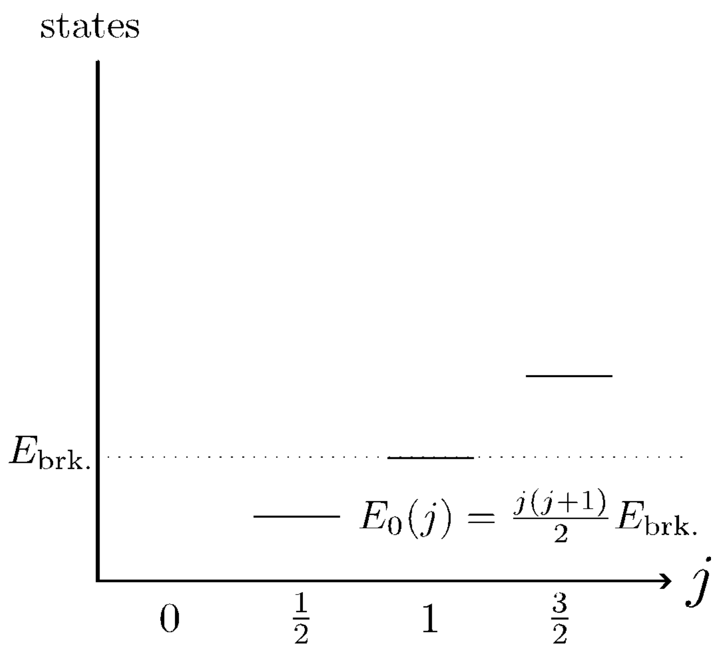
Angular momentum **selection rules** say that a $j = 0$ black hole that emits a photon must end up with $j = 1$

But if $M < Q + E_{\text{brk}}$ there are no states it can transition to with $j = 1$!

The black hole cannot emit single photons

In atomic physics, this is called a **forbidden transition** (e.g. 2s->1s transition in hydrogen)

Very slow! (2p lifetime: 1.6ns; 2s lifetime: 0.12s)



Diphoton emission

The forbidden transition occurs by emitting two photons (via an off-shell intermediate state)

Can be calculated by the **second-order perturbation theory** version of Fermi's golde

$$\Gamma_{i\to f} = 2\pi \left| \sum_I \sum_{\omega_I=\omega,\omega'} \frac{\langle E_f^{j=0}, \omega, \omega' | H_I | E_I^{j'=1}, \omega_I \rangle \langle E_I^{j'=1}, \omega_I | H_I | E_i^{j=0} \rangle}{E_I + \omega_I - E_i} \right|^2 \delta(E_i - E_f - \omega - \omega') \,,$$

Plugging in the Schwarzian QM four-point function, one obtains

$$\begin{aligned} \frac{dE}{dt} &= \frac{2\pi \mathcal{N}^4}{9} E_{\text{brk.}}^{4\Delta} r_+^{16} \int_0^{E_i} dE_f \rho_{j=0}(E_f) \int_0^{E_i - E_f} d\omega \omega^3 (E_i - E_f - \omega)^3 (E_i - E_f) \\ &\times \mathcal{N}_{4pt} \sum_{\omega_{I,I'}=\omega,\omega'} \int_{E_{\text{brk.}}}^{\infty} \frac{dE_I dE_{I'} \rho_{j=1}(E_I) \rho_{j=1}(E_{I'}) (\Gamma_{fI}\Gamma_{fI'}\Gamma_{iI}\Gamma_{iI'})^{1/2}}{(E_I + \omega_I - E_i)(E_{I'} + \omega_{I'} - E_i)} \\ &\times \left(\frac{\delta(E_I - E_{I'})}{\rho(E_I)} + \begin{Bmatrix} \Delta & E_f & E_I - E_0^{j=1} \\ \Delta & E_i & E_{I'} - E_0^{j=1} \end{Bmatrix} \right), \end{aligned}$$

$$= 6 \times 10^{-4} \times r_+^{16} (E_{\text{brk.}} E_i)^{\frac{17}{2}}$$

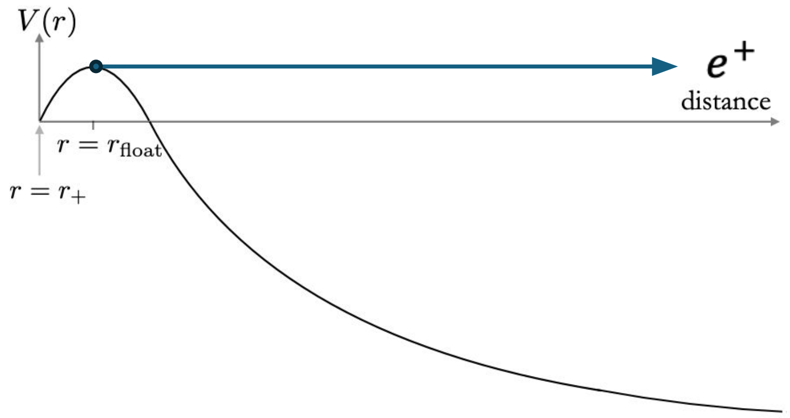
when $E_i \ll E_{brk}$

Semiclassical charged particle emission

The effective potential for an electron near a near-extremal black hole has a small barrier due to **gravitational attraction** and then goes negative due to **electrostatic repulsion**

Inside the barrier, the Hawking effect means there is a bath of thermal electrons

The Schwinger effect comes from the small probability that a thermal electron has enough energy to get over this barrier



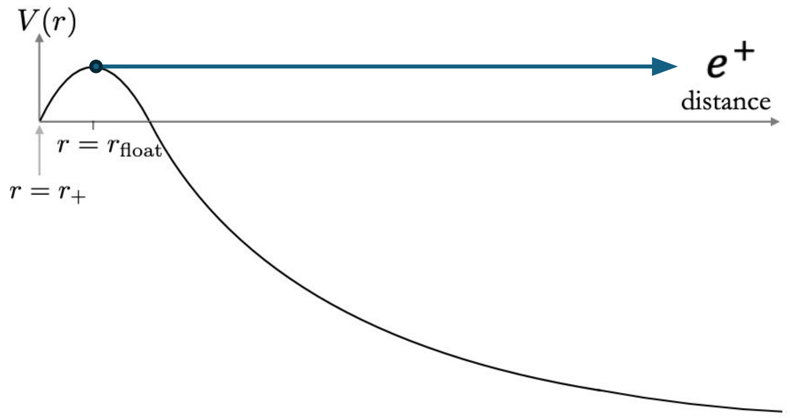
The post-emission black hole

Naively, after emitting an electron the black hole has mass and charge

$$M = M_0 - m_e \qquad Q = Q_0 - q_e$$

However once over the barrier the electron is **accelerated to ultra-relativistic speeds** and reaches nearly **Planckian energy**

Black hole gets hotter but stays near extremal



Since there were dramatic changes to neutral particle emission at low temperatures, one might expect similar corrections to the charged particle emission rates from quantum effects…

Nothing changes from the semiclassical answer at all!

Quantum charged particle emission

- Schwinger effect is described by a **semiclassical instanton**

Gravitational backreaction of the instanton can be approximated by gluing a smaller black hole inside the electron worldline to a larger black hole outside

Smooth solution inside worldline => Euclidean exterior black hole solution has an **effective conical defect**

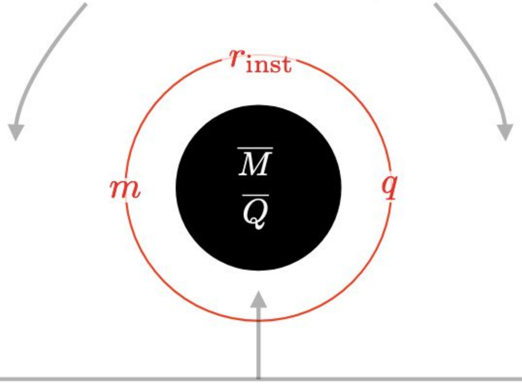
Take into account metric fluctuations in (multi-)instanton solutions by integrating over the moduli space of **hyperbolic geometries with defects**

This can be evaluated via the technology of the **string equation**: one finds

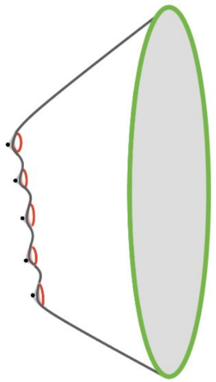
$$\frac{Z_{\text{all inst}}}{Z_{\text{BH}}} = e^{i\beta\Gamma}|_{\beta\to\infty} \, ,$$

where Γ is exactly the semiclassical Schwinger emission rate plus an exponentially small correction

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2 + \frac{dr^2}{1-\frac{2M}{r}+\frac{Q^2}{r^2}} + r^2 d\Omega^2$$



$$ds^2 = -A^2\left(1-\frac{2\overline{M}}{r}+\frac{\overline{Q}^2}{r^2}\right)dt^2 + \frac{dr^2}{1-\frac{2\overline{M}}{r}+\frac{\overline{Q}^2}{r^2}} + r^2 d\Omega^2$$



Back to neutral radiation?

$$\frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2}$$

- The black hole remains near extremal but with $M - Q \gg E_{brk}$ and again begins to emit semiclassical photon and graviphoton radiation

Something has changed: electron is a fermion and carries half-integer spin

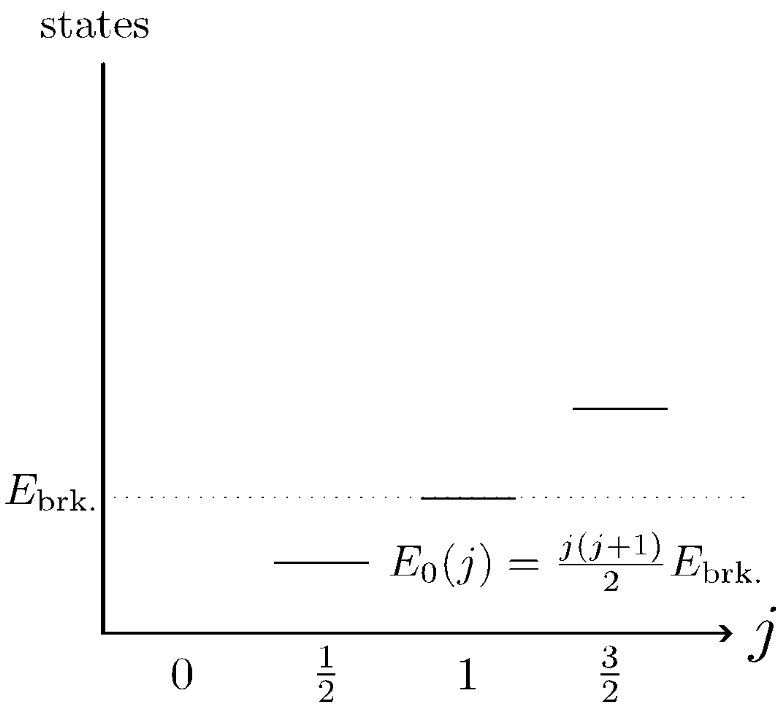
So the black hole is now also **fermionic** and has half-integer spin

Since the photons and graviphotons carry integer spin, the black hole remains fermionic through the neutral radiation phase

It therefore can never reach $j = 0$ and instead settles down at $j = 1/2$

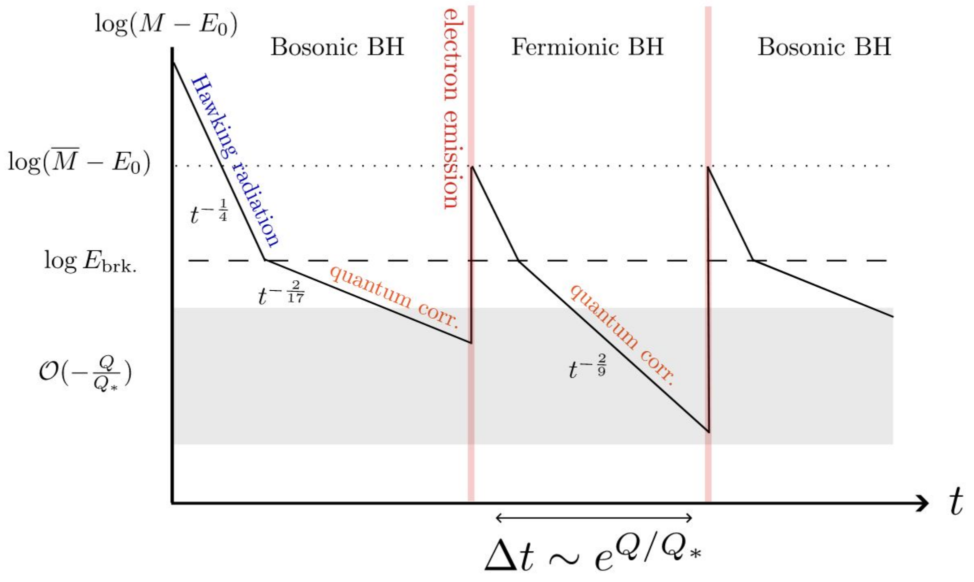
Selection rules allow $j = \frac{1}{2} \to \frac{1}{2}$ transitions via a single photon (not allowed via a graviphoton)!

Black hole cools at a much higher rate!



The evaporation of very large black holes

1. A black hole initially evaporates semiclassically by photon and graviphoton emission
2. As it approaches $M - Q \sim E_{brk}$ its angular momentum is driven towards $J = 0$
3. When $M - Q \ll E_{brk}$ it evaporates by diphoton emission
4. Eventually after an exponential time it emits a positron/electron and jumps to $\mathrm{E_{brk}} \ll M - Q \ll Q$
5. Because it now has half-integer rather than integer spin it cannot reach $J = 0$ through photon emission and instead settles at $J = \frac{1}{2}$
6. Now at low temperatures single photon emission dominates
7. After emitting another particle it returns to step 1

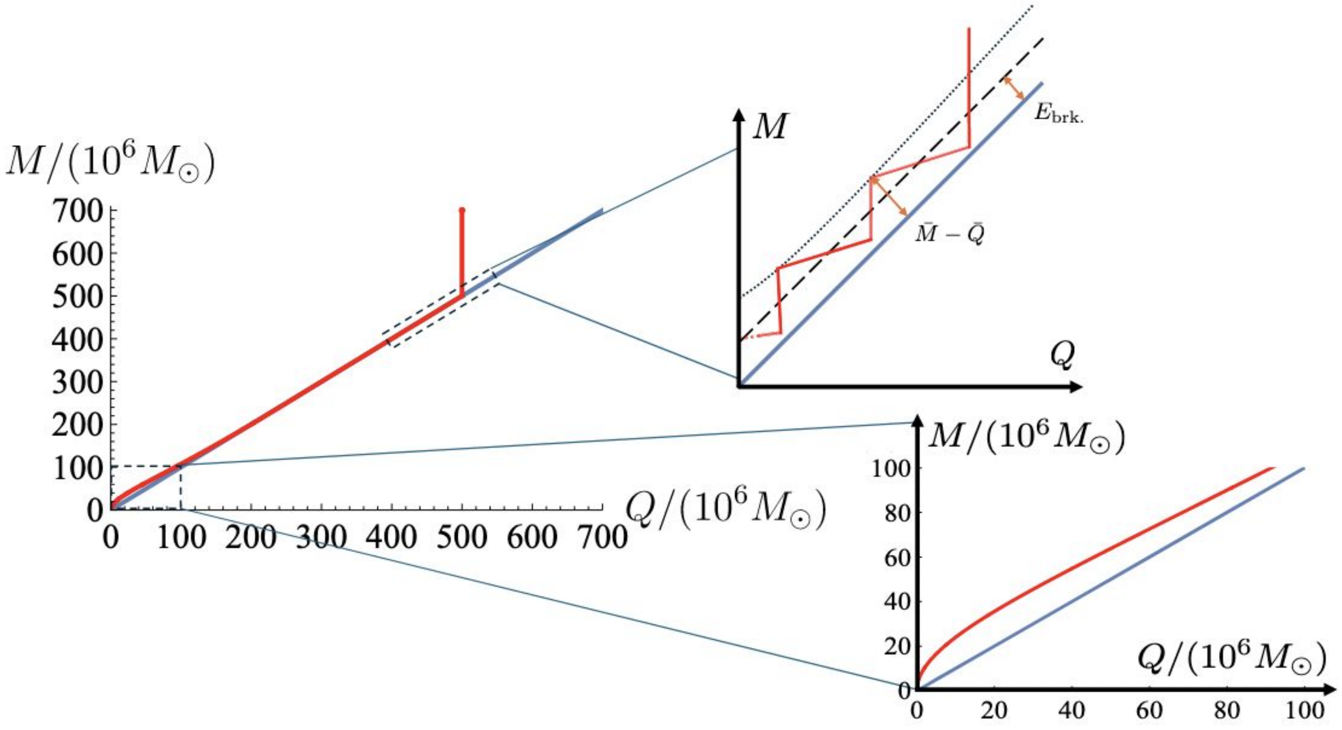


The evaporation of very large charged black holes

8. After many cycles, the charge becomes small enough that Schwinger emission stops being suppressed

9. From this point the charge scales as $Q \sim M^2/Q_*$ so that the electric field at the horizon remains close to the critical electric field from the Schwinger effect

10. Eventually the mass becomes Planckian and we lose control



What have we learned?

Even the simplest of problems can have an awful lot of physics in it?

Sometimes you do a tricky calculation and find stuff that's really fun and interesting … and sometimes you don't?

Just because we don't understand quantum gravity (in our universe) doesn't mean we can't do genuine quantum gravity calculations (in our universe) if we're careful and we get a bit lucky?

If you want to be a physicist, it's always worth paying attention in undergraduate quantum mechanics classes?

Thanks!