Time-like boundaries, holography and cosmology

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Goal: understand quantum gravity in cosmological spacetimes with positive cosmological constant

Several recent developments are converging:

- Multitrace deformations, $T\bar{T}$ and cutoff holography
  Aharony, Berkooz, Silverstein; Witten; Smirnov, Zamolodchikov; Cavaglia et al; Dubovsky et al; Freidel; McGough, Mezei, Velinde; Kraus, Liu, Marolf; Hartman et al; …
- Uplifts of AdS/CFT to de Sitter Dong et al; De Luca, Silverstein, Torroba; …
- Analysis of time-like boundaries in General Relativity Brown, York; Anderson; Bredberg, Strominger; Anninos et al; Andrade et al; Marolf, Rangamani; Fournodavlos, Smulevici; An, Anderson; Anninos, Galante, Maneerat; Liu, Santos, Wiseman; …
- Thermodynamics of cosmological horizons and refined entropy counts Mishayita; Draper, Farkas; Banihashemi Jacobson et al; Cardy; Sen; Hartman, Keller, Stoica; Anninos, Denef et al; Benjamin et al; …
- Quantum-mechanical models, observers, algebras. Banks et al; Anninos et al; Narovlansky, Verlinde; Susskind, Rahman; Chandrasekaran et al; Bahiru et al; Jensen et al; …

See other String's 2025 talks on these subjects!

In this talk I will describe a particular path through these developments

1. $T\bar{T}$ and relation to cutoff AdS [Zamolodchikov, Smirnov; Cavaglia et al]

2. $T\bar{T} + \Lambda_2$ and dS_3 [Gorbenko, Silvertein, GT, 2018]

3. de Sitter microstates and entropy [Coleman, Mazenc, Shyam, Silvertein, Soni, GT, Yang 2021]

4. Bulk-local dS_3 holography [Batra, De Luca, Silverstein, GT, Yang, 2024]

5. dS_4 holography [Silverstein, GT, 2024]

Work in collaboration, 2018 — 2024

G. Batra

E. Coleman

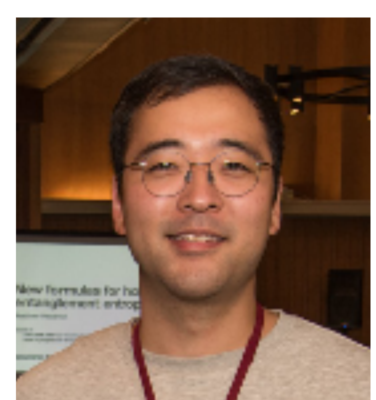
B. De Luca

V. Gorbenko

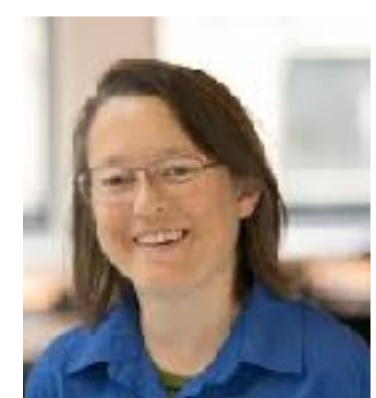
E. Mazenc

V. Shyam

R. Soni



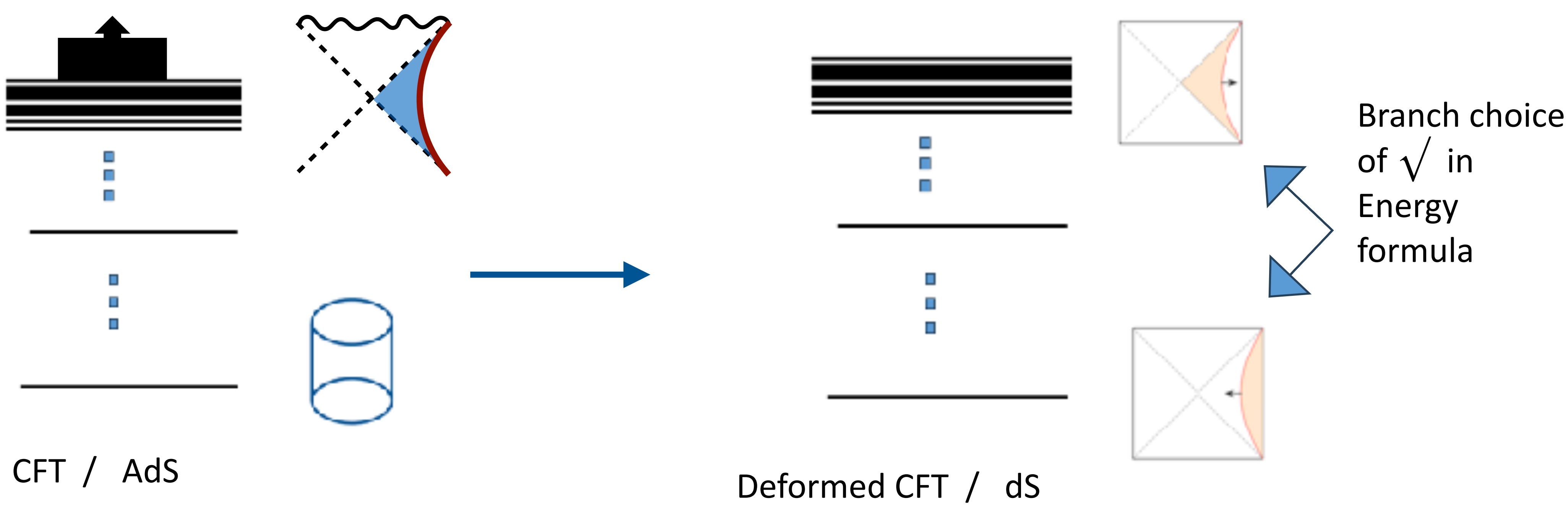
S. Yang



E. Silverstein

Key role of time-like boundaries: they introduce a well-defined boundary energy spectrum, with multiple states and associated thermodynamics.

Our main result: prescription for a finite quantum Hamiltonian formulating quantum gravity with positive cosmological constant in bulk 3d and 4d, whose spectrum produces a well-defined cosmic horizon microstate count.



I. $T\bar{T}$ and cutoff AdS

In 2004, Zamolodchikov found that the following composite operator has intriguing properties:

$$(T\bar{T})(x) = \frac{1}{8}\left(T^{\mu\nu}T_{\mu\nu} - (T^\mu_\mu)^2\right) \ , \ \Delta_{T\bar{T}} = 4$$

exact, d=2!

What happens if we perturb by TTbar? Breaks renormalizability. Surprisingly, there is a simple trajectory towards the UV that exhibits very special solvable properties:

$\lambda > 0$ for us

$$\partial_\lambda \log Z_\lambda = -2\pi \int d^2x \, \langle T\bar{T}\rangle_\lambda \quad \text{or} \quad T^\mu_\mu = -4\pi\lambda T\bar{T}$$

[Smirnov-Z]
[Cavaglia et al]

- A key result: put the CFT on spatial circle. The theory has discrete energy spectrum. Deformed energies can be computed non-perturbatively

$$E_n = \frac{L}{\pi\lambda}\left(1 - \sqrt{1 - \pi\frac{\lambda}{L}E_n^0}\right)$$

recall $E_n^0 = \frac{2\pi}{L}\left(\Delta_n - \frac{c}{12}\right)$

Natural UV cutoff for $\lambda > 0 \ : \quad E_n^0 < \dfrac{L}{\pi\lambda}$

responsible for real spectrum and finite entropy

• *Relation to holography:* take pure 3d gravity with $\Lambda = -2/\ell^2 < 0$

Consider the spacetime with a radial cutoff, where we fix the metric

$$ds^2 = dr^2 + g_{\mu\nu}(r,x)dx^\mu dx^\nu \,, \quad \text{cutoff} \;\; r < r_c \qquad K_{\mu\nu} = \frac{1}{2}\partial_r g_{\mu\nu}$$

Stress tensor of boundary theory is dual to Brown-York quasi-local tensor

[Balasubramanian, Kraus]

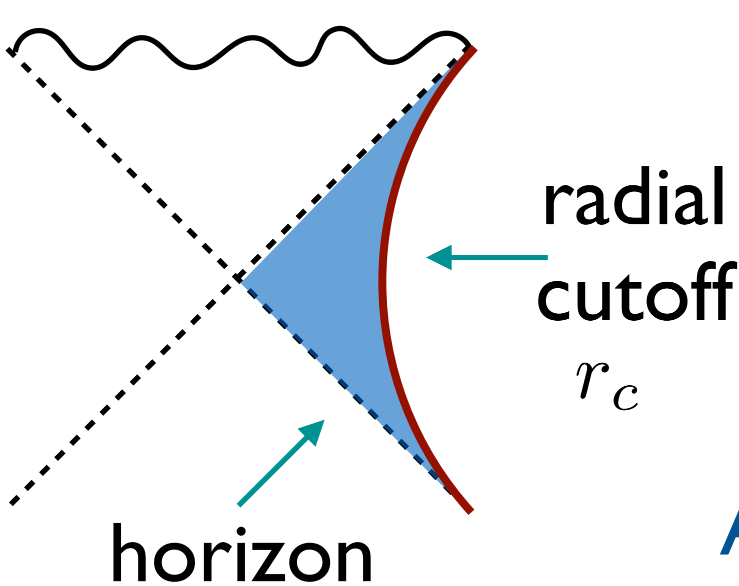
$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_{\text{on-shell}}}{\delta g^{\mu\nu}} = \frac{1}{8\pi G}\left(K_{\mu\nu} - K g_{\mu\nu} + \frac{1}{\ell}g_{\mu\nu}\right)$$

Imposing the radial Einstein equation $E^r_r = \frac{1}{2}(K^2 - K^2_{\mu\nu}) - \frac{1}{\ell^2} = 0$

and rewriting $K_{\mu\nu}$ in terms of $T_{\mu\nu}$ gives the $T\bar{T}$ flow eq., with $\frac{8G\ell}{L_c^2} = \frac{\lambda}{L^2}$

[McGough, Mezei, Verlinde] [Kraus, Liu, Marolf]

• *AdS black hole in a box:*



Quasilocal energy at cutoff surface:

$$T^t_t = \frac{1}{8\pi G\ell}\left(1 - \sqrt{1 - 8GM\frac{\ell^2}{r_c^2}}\right)$$

Agrees w/TTb. And $\sqrt{\cdots}$ captures the BH energy

2. $T\bar{T} + \Lambda_2$ and dS_3

[Gorbenko, Silvertein, GT, 2018]

Generalization: coordinated flow with $T\bar{T}$ and the identity operator

$$\partial_\lambda \log Z = \int d^2x \left(-2\pi \langle T\bar{T} \rangle + \underbrace{\frac{1-\eta}{2\pi\lambda^2}}_{\Lambda_2} \right)$$

Nontrivial nonlinear effects

Equally universal as TTb and also solvable!

- *Gravity dual:* using the relation between the stress tensor and extrinsic curvature as before, we learn that Λ_2 corresponds to changing the bulk 3d cosmological constant

$$S_{grav} = \frac{1}{16\pi G} \int_{\mathcal{M}} \left(R^{(3)} + \frac{2\eta}{\ell^2} \right) + (\text{bdry terms})\ , \ \eta = \pm 1 \quad \begin{cases} AdS_3 \\ dS_3 \ \ !!! \end{cases}$$

On cylinder of length L, the deformed spectrum is calculated as follows:

$$\lambda\partial_\lambda H = \frac{1}{2}\int dx \left\{ \frac{\pi\lambda}{2}(T^2_{\mu\nu} - (T^\mu_\mu)^2) + \frac{\eta - 1}{\pi\lambda} \right\}$$

Replacing $E = \langle E|H|E\rangle \ , \ -\frac{dE}{dL} = \langle E|T^x_x|E\rangle \ , \ \frac{J}{L^2} = \langle E|T^0_x|E\rangle$

gives a diff. eq. with general solution

$y = \lambda/L^2$

$$\mathcal{E}_n = E_n L = \frac{1}{\pi y}\left(1 \pm \sqrt{\eta + C_1 y + 4\pi^4 J^2 y^2}\right)$$

- *New features:*

➡ cannot take $\lambda \to 0$. Need to give a boundary condition and fix C_1

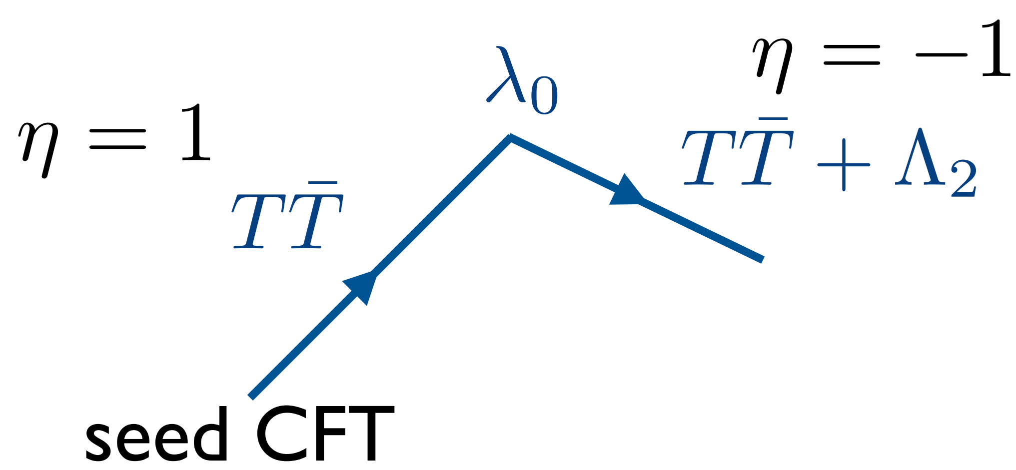
➡ $\pm\sqrt{\ }$ choice not fixed by boundary condition. In general, both contribute, corresponding to the extrinsic curvature not being fixed in gravity dual.

3. de Sitter microstates and entropy

[Coleman, Mazenc, Shyam, Silvertein, Soni, GT, Yang 2021]

Our next generalization is the following:

Start from large N CFT w/gravity dual. Deform by $T\bar{T}$
Then turn on $T\bar{T}+\Lambda_2$ at some joining value λ_0



Corresponds to

continuity of energy at $y = y_0$

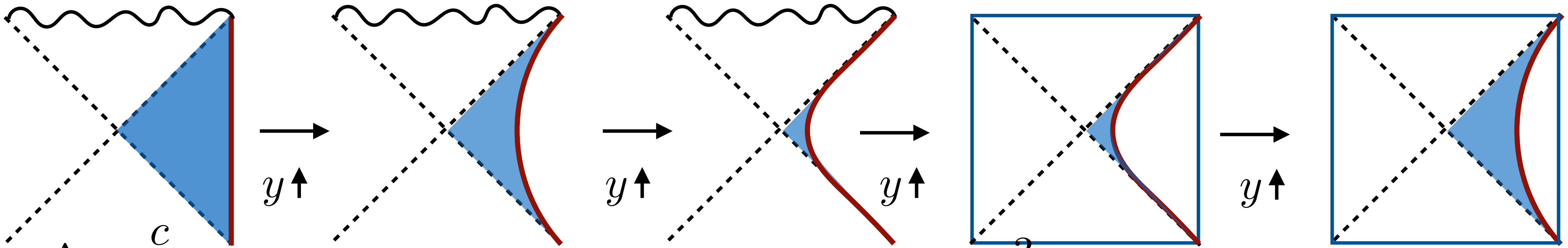
$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y(\Delta - \frac{c}{12})}\right)$$

- *Use this idea to explain the dS entropy?*

We want a trajectory that includes the cosmic horizon. To achieve this, we will match AdS BH of radius $r_h = \ell$ to dS horizon of same size. Near horizon patches, indistinguishable

$$ds_3^2 = -\frac{w^2}{\ell^2}dt^2 + dw^2 + (\ell^2 + \eta w^2)d\phi^2$$

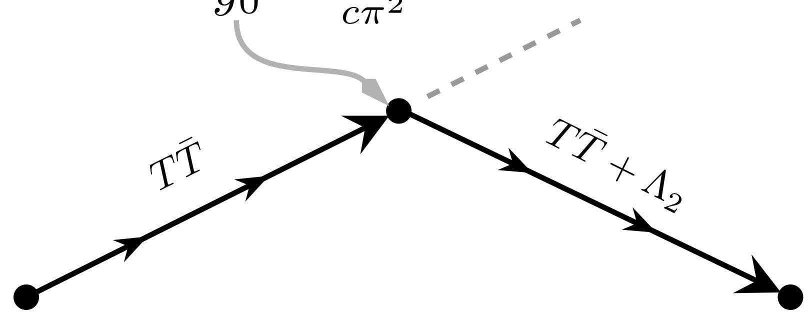
$\eta = 1 \;,\; \mathcal{E} \sim -\sqrt{\dots}$ $\qquad \sqrt{\dots} = 0$ $\qquad \eta = -1 \;,\; \mathcal{E} \sim +\sqrt{\dots}$



$\Delta \approx \frac{c}{6}$

$y = y_0 = \frac{3}{\pi^2 c}$

$-\sqrt{\dots}$ describes pole patch



CFT ($\Delta \simeq \frac{c}{6}$)

CH patch

E

$\mathcal{O}(1)$ window of states

$\Delta = \frac{c}{6} + \mathcal{O}(1)$

$\Delta = \frac{c}{6}$

E

dS ground state

$\Delta \geq \frac{c}{6} \; : \; S_{Cardy} = 2\pi\sqrt{\frac{c}{3}(\Delta - \frac{c}{12})}$

$S_{\Delta = c/6} = \frac{\pi c}{3}$ BTZ microstates

$S_{Gibbons-Hawking} = \frac{\pi c}{3}$

dS microstates

Provides quantum state count for the dS entropy!

4. Bulk-local dS3 holography

[Batra, De Luca, Silverstein, GT, Yang, 2024]

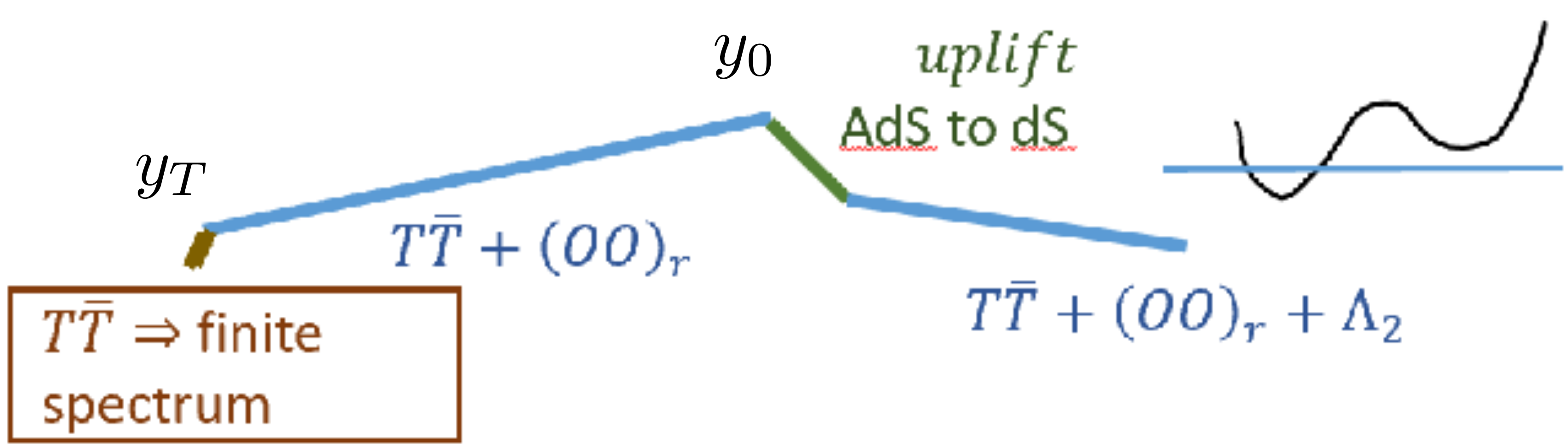
The boundary theory so far computes universal features (horizon entropy, geometry), but does not capture model-dependent details: the fine structure of energy levels, details of the uplift from AdS to dS spacetime, and local bulk matter excitations.

The boundary dual that incorporates Dirichlet b.c. for bulk matter is

[Hartman, Kruthoff, Shaghoulian, Tajdini]

$$\partial_\lambda \log Z \sim \int (T\bar{T} + \Lambda_2 + \mathcal{O}\mathcal{O}) \quad \text{where } \mathcal{O} \leftrightarrow \phi \text{ in the bulk}$$

OK at large c, but is there a complete finite c theory? Our proposal:



[Silverstein]

I. $\partial_\lambda \log Z = -2\pi \int T\bar{T}$, $0 < y < y_T \ll 1$

Gives finite real spectrum, with well-defined composite operators,

$$\langle n|\mathcal{O}^2|m\rangle = \sum_p \langle n|\mathcal{O}|p\rangle\langle p|\mathcal{O}|m\rangle - \text{subtractions}$$

only affects very high UV modes

II. $y_T < y < y_0 = \dfrac{3}{\pi^2 c}$: $\lambda\partial_\lambda \log Z \sim \int (\lambda T\bar{T} + (\lambda c)^{\Delta-1}\mathcal{O}^2_{ren} + \ldots)$

III. Uplift $T\bar{T} + \mathcal{O}_r^2 \to T\bar{T} + \mathcal{O}_r^2 + \Lambda_2$ in terms of appropriate

bulk fields Φ_u with potential. Examples of dS3 and dS4.

See B. De Luca's String talk.

[Dong, Horn, Silverstein, GT]
[De Luca, Silverstein, GT]

IV. $y_0 < y < \infty$ Deformed CFT with $T\bar{T} + \mathcal{O}_r^2 + \Lambda_2$

- Finite quantum mechanics, type I algebra
- Describes states in GR+EFT+…, as well as thermodynamics
- Explanation of dS entropy $S_{gen} = \dfrac{A}{4G_N} - c_1 \log \dfrac{A}{G_N} + S_{mat}$

5. Generalization to dS4 holography

[Silverstein, GT, 2024]

We want to extend previous construction to the realistic 4d case, starting from CFT on cylinder $\mathbb{R} \times S^2$, and deforming it to describe dS4. Novel aspects: special 2d properties of TTbar absent in d>2; related: bulk dynamical gravitons.

- *Dirichlet b.c. for gravity:* $T\bar{T} \to T^2 \equiv T^2_{\mu\nu} - \frac{1}{d-1}(T^\mu_\mu)^2$ (d=3 for us)

  [Hartman et al; Taylor]

  Not solvable, unlike 2d.

- *New solvable deformation:*

$$\lambda \partial_\lambda H = \int [\lambda (T^2)_s + \Lambda_3 + \frac{C_3^{2/3}}{\lambda^{1/3}} R^{(3)}]$$

$\sim N$

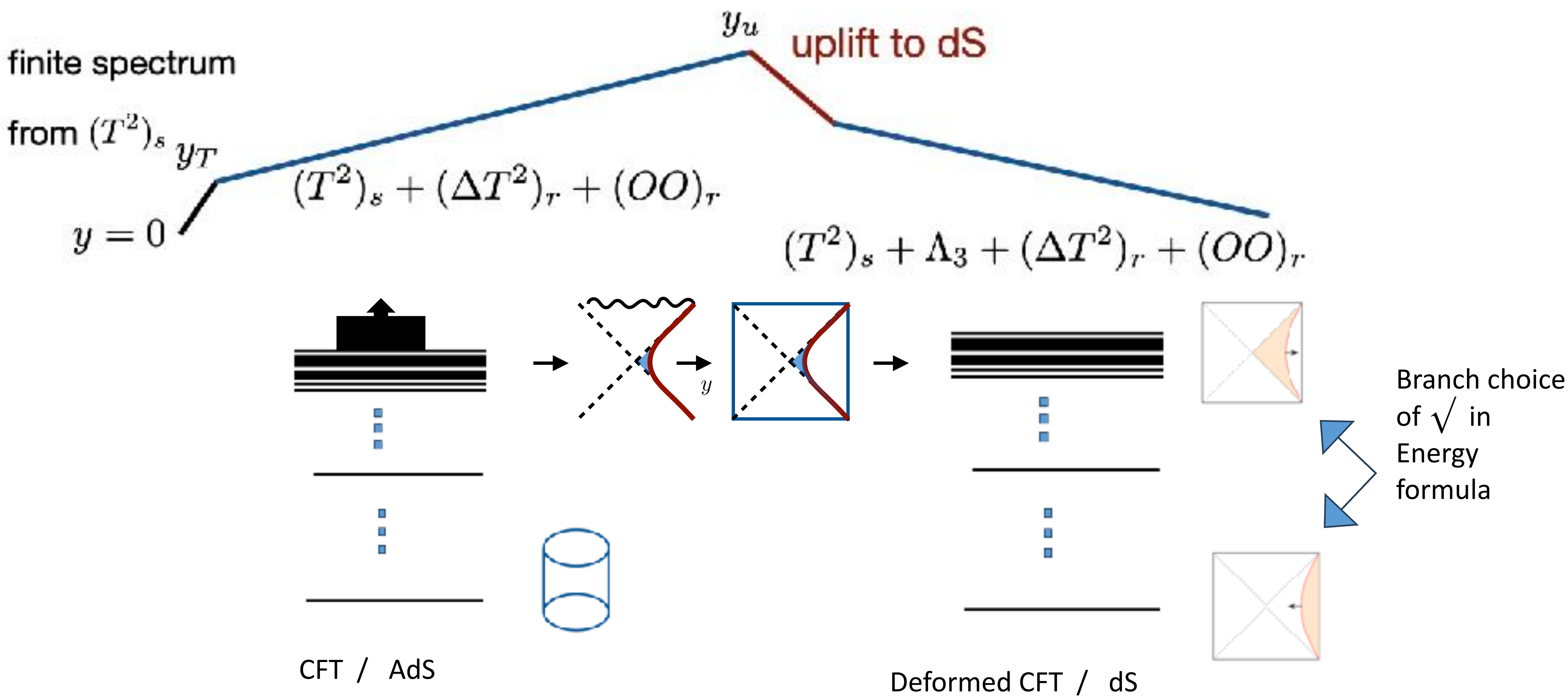
with $\langle E | T^\mu_{\nu,s} | E \rangle = \mathrm{diag}\left(-E/V, -dE/dV, -dE/dV\right)$, $\Lambda_3 \sim \frac{1-\eta}{\lambda}$

Captures entropically dominating effects. Solvable deformed spectrum:

$$V^{1/2} E_n(y) = \frac{3}{\pi y}\left(1 + (C_3 y)^{2/3} \pm \sqrt{\eta + (C_3 y)^{2/3} + c_1 y}\right) \qquad y = \frac{\lambda}{V^{3/2}}$$

integr. const.

Given this, the deformed CFT is defined by the following steps:



$$(T^2)_{solvable} + (\Delta T^2)_{renormalized} + (OO)_{renormalized}$$

captures most entropic energy bands

they capture local dynamics of gravitons and bulk matter. Requires fine-tuning

Fine-tuning is done by matching to GR+EFT+… below Planck scale

$$\langle n|_{def-CFT}(T^2)_r|m\rangle_{def-CFT} \equiv \langle n|_{GR+EFT+\ldots}(T^2)_r|m\rangle_{GR+EFT+\ldots}$$

taking into account the limited resolution of GR and the huge $e^{A/4G_N}$ available Hilbert space.

In summary, we have defined a quantum mechanics system that

➡ describes emergent dS radial geometry
➡ captures local bulk dynamics using its vast Hilbert space
➡ reproduces the generalized dS horizon entropy and thermodynamics

Thanks!!

Future directions

- Continue developing UV completions of time-like boundaries and their quantum properties.
- Consider models with different boundary conditions (e.g. conformal and umbilic b.c.)
- Observational signatures of time-like boundaries in cosmology
- Connect with other quantum mechanics model for dS, as well as explicit constructions in string/M-theory.