Finite Landscape of 6d N=(1,0) Supergravity

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POSTECH & Harvard University

Based on **2411.19155** with Cumrun Vafa and Kai Xu

Strings 2025 Abu Dhabi

Introduction

- # of known SUSY string vacua (in Minkowski space) is finite
- Yau's conjecture : # of CY-manifolds in each dim. is finite.
- Finiteness conjecture [Vafa '05], [Acharya, Douglas '06]
  - The set of consistent QG vacua (for a given dim with fixed cutoff) is finite.
  - Number of massless fields has upper bound.

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  - The set of consistent QG vacua is finite.
  - Number of massless fields has upper bound.

- EFTs with 32 SUSY are (almost) uniquely determined by SUSY, and thus the finiteness of QG landscape is trivially satisfied.

32 SUSY : Toroidal compactifications of M-theory

- For 16 SUSY EFTs, using unitarity of string probes and swampland conjectures, we can show that the number of QG theories is also finite.

16 SUSY : $\mathrm{rank}(G) \leq 26 - d$ [HCK, Tarazi, Vafa '19]

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- String Lamppost Principle (or String Universality) [Kumar, Taylor '09],[Adams, De Wolfe, Taylor '10], [HCK, Tarazi, Vafa '19], [Montero, Vafa '20], ….

  All consistent quantum gravity theories arise from string theory.

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- 10d EFTs with 16 SUSY : Green-Schwarz constraints fixes groups to

$$E_8 \times E_8\,, \quad SO(32)\,, \quad \underbrace{E_8 \times U(1)^{248}\,, \quad U(1)^{496}}_{\text{No string realizations}}$$

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String Lamppost Principle (SLP) holds!

- SLP is also achieved for 16 SUSY theories in 9d, 8d (in terms of rank).

  [Montero, Vafa '20], [Dierigl, Heckman '20], [Cvetic, Dierigle, Lin, Zhang '20], [Font, Fraiman, Grana, Nunez, Parra de Freitas '20, '21], [Hamada, Vafa '21], [Bedroya, Hamada, Montero, Vafa '21], [Fraiman, Parra de Freitas '22], [Bedroya, Raman, Tarazi '23], …

This talk

- Q) Can we establish the finiteness conjecture and string universality for supersymmetric theories with 8 supercharges?

- We focus on 6d N=(1,0) supergravity theories (in Minkowski space).
  - 6d is the highest dimension with 8 supercharges.
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    $\Longrightarrow$ Finite number of 6d F-theory models [M. Gross '93]
  - However, there are infinite families of anomaly-free supergravities.

    [Schwarz '96], [Kumar, Taylor '09], [Hamada, Loges '23], [Loges '24]
  - Some of these are ruled out by unitarity of string probes.

    [HCK, Shiu, Vafa '19], [Tarazi, Vafa '21]

This talk

- Using a bottom-up argument, we can show that the number of massless fields in 6d (1,0) supergravities has upper bound, and the theories saturating the bound are realized in string theory, under two assumptions:
  - Tensionless BPS strings exist at every point on the boundary of tensor moduli space.
  - Classification of 6d SCFTs and little string theories (LSTs) given in [Morrison, Taylor '12], [Heckman et.al. '13], [Del Zotto et.al '14], [Heckman et.al. '15], [Bhardwaj et.al. '15], [Bhardwaj '15, '19], …. is complete.

Notably, all known string theory constructions are consistent with these assumptions.

Plan for the talk

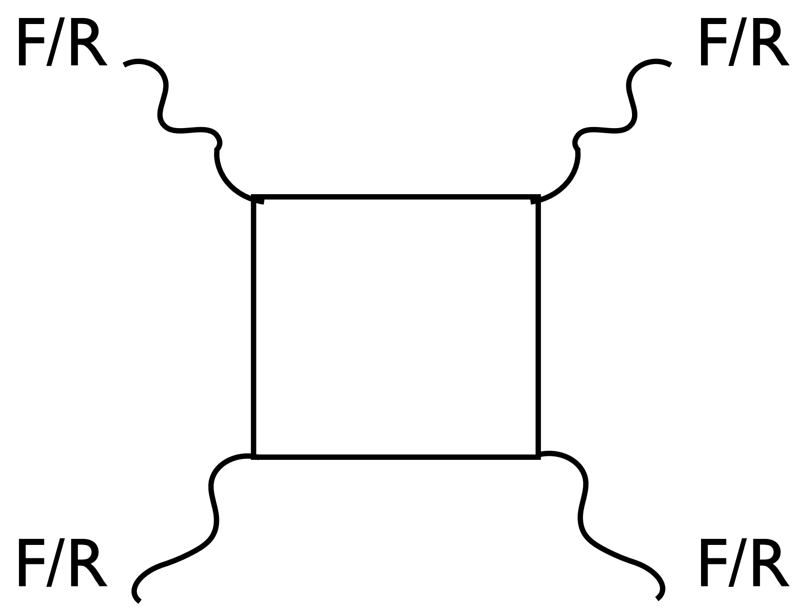
- Review on 6d N=(1,0) supergravity
- Properties of tensor moduli space and BPS strings
- Prove the finiteness of 6d N=(1,0) landscape
- Exact bounds on the number of massless fields

6d N=(1,0) Supergravity

- 4 Types of massless supermultiplets
  - Gravity multiplet $(g_{\mu\nu}, B^{+}_{\mu\nu}, \Psi_{\mu})$
  - Tensor (T) multiplet $(B^{-}_{\mu\nu}, J, \psi)$
  - Vector (V) multiplet (A_{μ}, λ)
  - Hyper (H) multiplet (q, Ψ)

Scalar vevs parametrize tensor moduli space

- Massless chiral fields, such as 2-form fields, gravitino, matter fermions, contribute to the gauge and gravitational anomalies.



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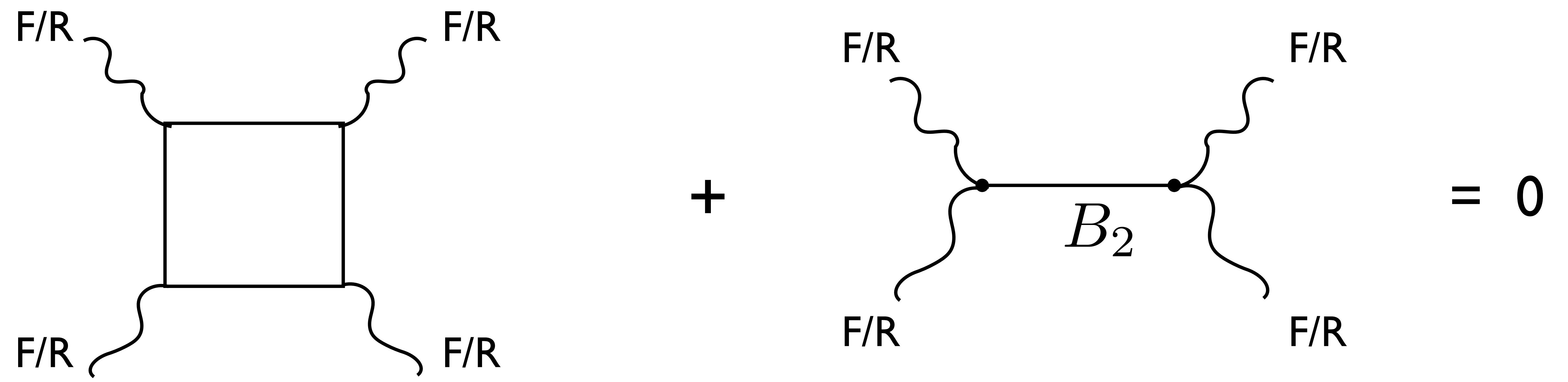
$$S_{GS} = \int \Omega_{\alpha\beta} B_2^{\alpha} \wedge X_4^{\beta}$$

[Green, Schwarz '84], [Sagnotti '92]



Green-Schwarz-Sagnotti mechamism

- Anomalies can be cancelled when the anomaly polynomial factorizes:

$$I_8^{1-\mathrm{loop}} = \frac{1}{2}\Omega_{\alpha\beta}\, X_4^\alpha X_4^\beta$$

$$X_4^\alpha = -\frac{1}{2}b_0^\alpha\,\mathrm{tr}R^2 + \frac{1}{4}\sum_i b_i^\alpha \frac{2}{\lambda_i}\mathrm{tr}F_i^2$$

$\Omega_{\alpha\beta}$: intersection form with $(1,T)$ signature

b_0, b_i : anomaly vectors $\in \mathbb{R}^{1,T}$

$\lambda_{SU(N)} = 1, \lambda_{E_8} = 60, \cdots$

- Conditions for the factorization:

$$H - V + 29T = 273\ , \quad b_0 \cdot b_0 = 9 - T\ ,$$

$$B^i_{\mathbf{adj}} = \sum_{\mathbf{r}} n^i_{\mathbf{r}} B^i_{\mathbf{r}}\ , \quad b_0 \cdot b_i = \frac{\lambda_i}{6}\left(\sum_{\mathbf{r}} n^i_{\mathbf{r}} A^i_{\mathbf{r}} - A^i_{\mathbf{adj}}\right)\ ,$$

$$b_i \cdot b_i = \frac{\lambda_i^2}{3}\left(\sum_{\mathbf{r}} n^i_{\mathbf{r}} C^i_{\mathbf{r}} - C^i_{\mathbf{adj}}\right)\ , \quad b_i \cdot b_j = 2\lambda_i\lambda_j \sum_{\mathbf{r},\mathbf{s}} n^{ij}_{\mathbf{r},\mathbf{s}} A^i_{\mathbf{r}} A^j_{\mathbf{s}} \quad i \neq j$$

$n^i_{\mathbf{r}} = \#$ of hypers in rep. $\mathbf{r}$ of G_i

$A^i_{\mathbf{r}}, B^i_{\mathbf{r}}, C^i_{\mathbf{r}}$: group theory factors for G_i

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Gravitational anomaly

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Tensor moduli space

- T-dimensional coset space of $SO(1,T)/SO(T)$ parametrized by scalars $J \in \mathbb{R}^{1,T}$ in the tensor multiplets.

- Consistency of EFT on tensor moduli space requires

$$J \cdot J > 0\,, \quad J \cdot b_i \geq 0\,, \quad J \cdot b_0 \geq 0$$

(We can fix $J^2 = 1$)

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Metric positivity along J

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Positive gauge coupling for G_i

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- Consistency of SUSY compactification on K3
- Many other arguments in [Cheung, Remmen '16], [Hamada, Noumi, Shiu '18], [Garcia Etxebarria, et.al. '20], [Ong '22], …

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- In F-theory, scalars correspond to the Kahler form $J \in H^{1,1}(B)$, and these conditions define a positive-definite Kahler cone on base B.

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- Tensions of BPS strings (with charge Q) must be non-negative.

$$T_Q \sim J \cdot Q \geq 0 \ \text{ for all BPS } Q$$

BPS cone

- BPS strings are 1/2 BPS sources for the 2-form tensor fields $B^{\pm}_{\mu\nu}$
  - Tensions of BPS strings must be non-negative:
  - String charges are sitting in a unimodular lattice $\Gamma \subset \mathbb{R}^{1,T}$ with $\Gamma = \Gamma^*$

[Seiberg, Taylor '10]

- BPS cone is a cone of BPS charges Q_i, which we refer to 'effective'.
  - Dual to tensor cone, a cone of J with $J \cdot Q \geq 0 \ \text{ for } \forall Q$.
  - Every BPS charge is a non-negative integral sum of generators $\mathcal{C}_i$

$$Q = \sum_i n_i \mathcal{C}_i \text{ with } n_i \in \mathbb{Z}_{\geq 0} \quad (\text{also, } \ J = \sum_i a_i \mathcal{C}_i \text{ with } a_i \geq 0 \text{ as } J \cdot J > 0)$$

- In F-theory, the BPS cone is the Mori cone and tensor cone is the Kahler cone on the base B.

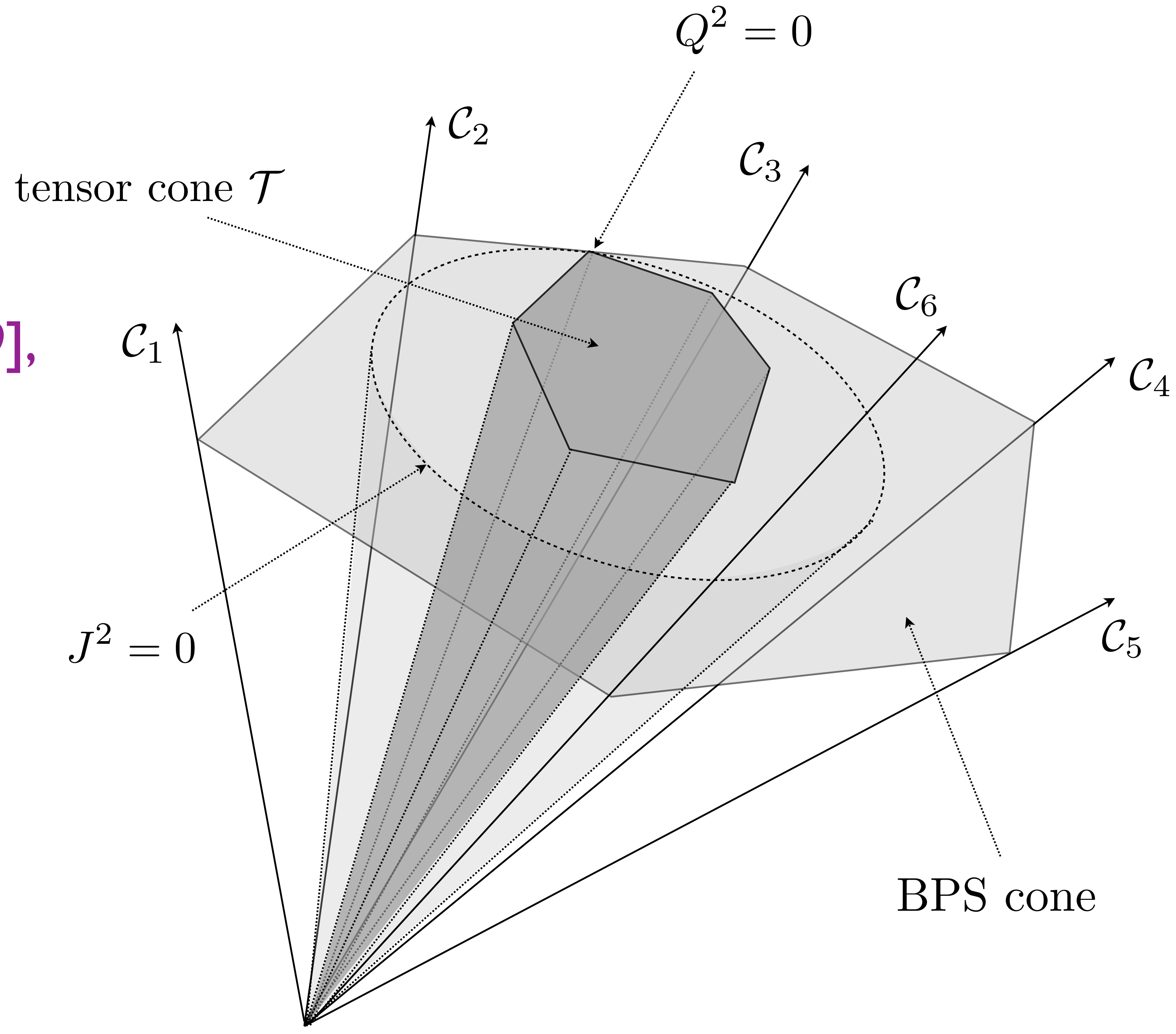
BPS cone

- Generators are 'shrinkable' providing tensionless strings where $J \cdot \mathcal{C}_i = 0$
- $\mathcal{C}_i$ shrinks either at finite distance ($\mathcal{C}_i^2 < 0$) or at infinite distance ($\mathcal{C}_i^2 = 0$)

$$\begin{cases} \mathcal{C}_i^2 < 0\,, & \mathcal{C}_i \in \mathrm{SCFT} \subset \mathrm{SUGRA} \\ \mathcal{C}_i^2 = 0\,, & \text{little string or critical string} \subset \mathrm{SUGRA} \end{cases}$$

- Generators are thus all classified!

[Morrison, Taylor '12], [Heckman, Morrison, Rudelius, Vafa '15], [Bhardwaj '15, '19], [Bhardwaj, Del Zotto, Heckman, Morrison, Rudelius, Vafa '15], ….



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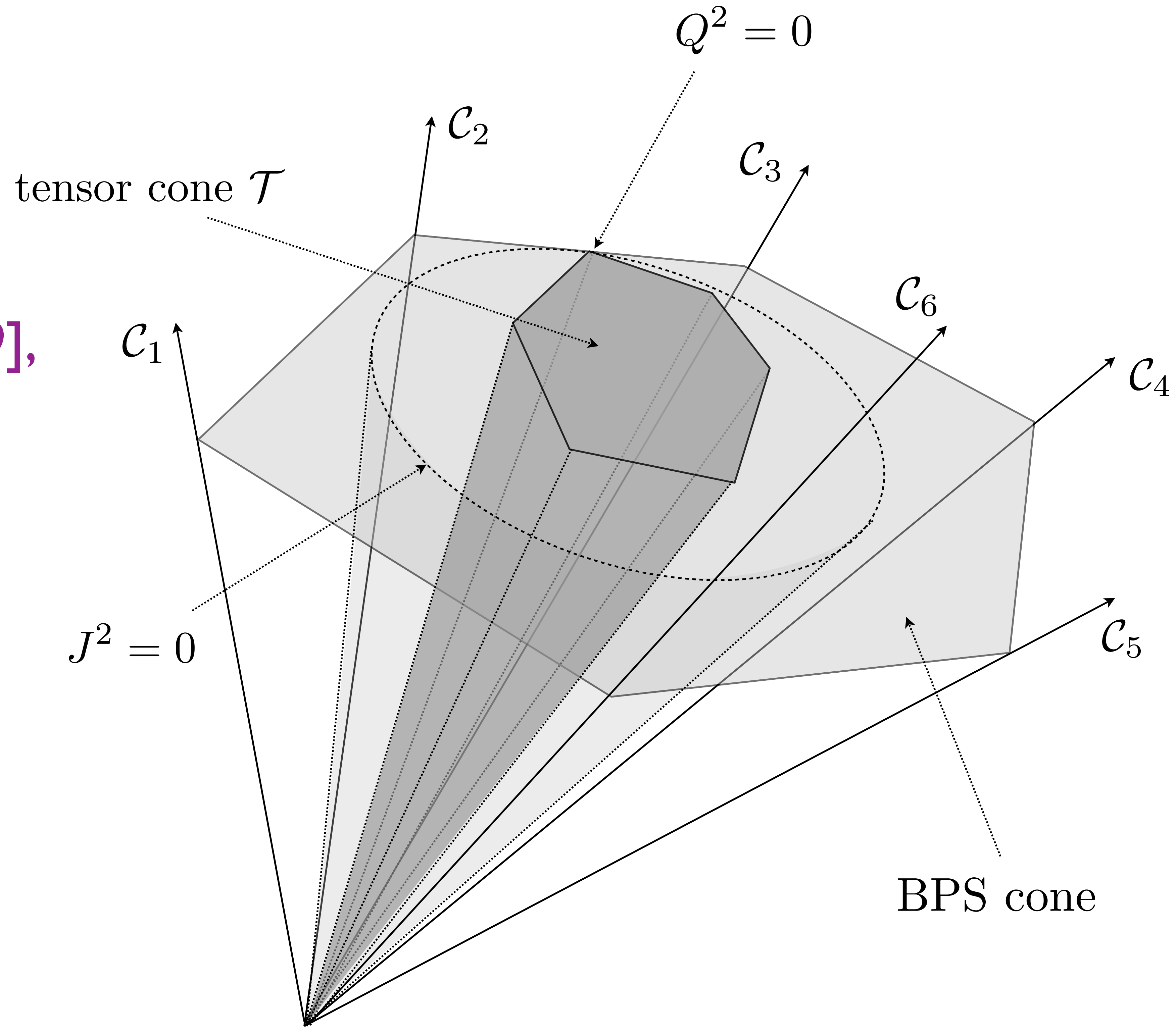
- We can prove $\mathcal{C}_i \cdot \mathcal{C}_j \geq 0$ for $i \neq j$ using
  - Mixed gauge anomaly cancellation:

$$b_i \cdot b_j = \#\text{ of charged hypers} \geq 0$$

  - Properties of E-, M-, and critical strings



H-string

- Among BPS strings, there is a distinguished class of strings, which we call 'H-string', that plays crucial role in our proof of finiteness.
- BPS charge f of an H-string satisfies

$$f^2 = 0\,,\ \ b_0 \cdot f = 2 \qquad \Longrightarrow \qquad \text{Tensionless limit at infinite distance}$$

- H-strings are critical heterotic strings and little strings with charge f .

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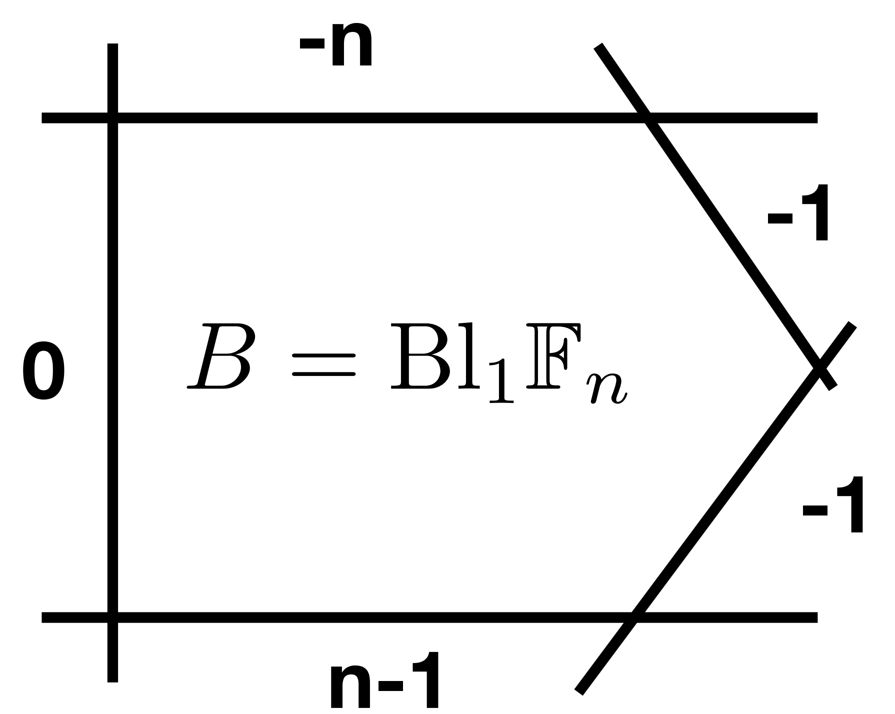
- In F-theory, D3-branes wrapping rational fiber f-curve on base B .

Example: B = Hirzebruch Fn with 1-blowup



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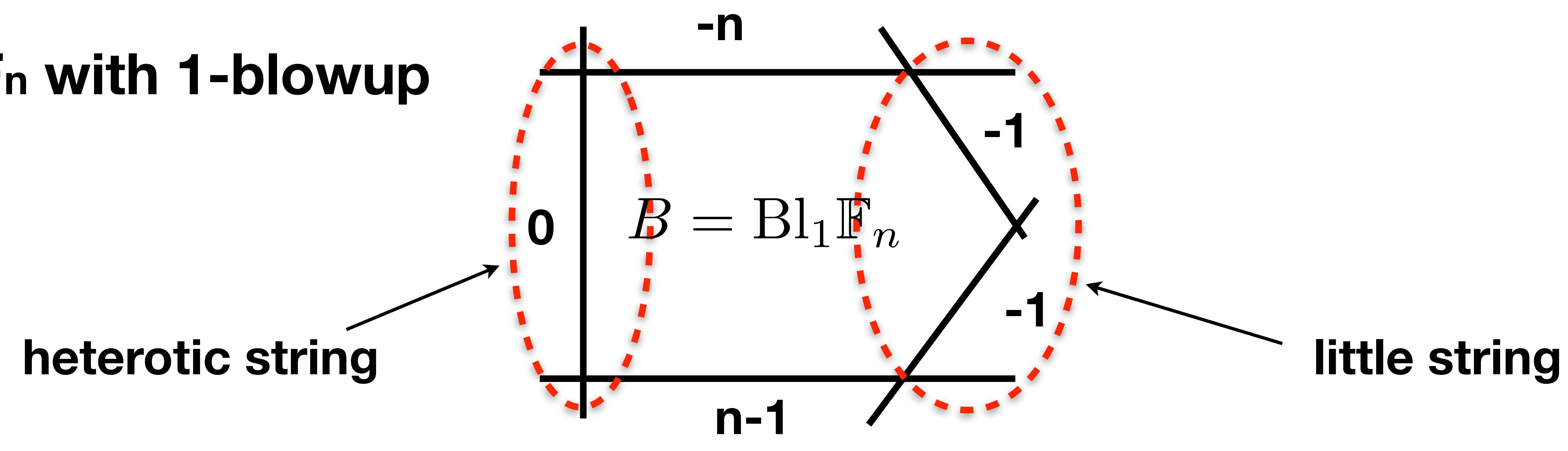


H-string

- Central charges for 2d (0,4) worldsheet CFT on an H-string can be read off from anomaly inflow:

$$c_L = 20,\ c_R = 6,\ k_i = f \cdot b_i$$

level for G_i current algebra

H-string

- Central charges for 2d (0,4) worldsheet CFT on an H-string can be read off from anomaly inflow:

$$c_L = 20, \;\; c_R = 6, \;\; k_i = f \cdot b_i$$

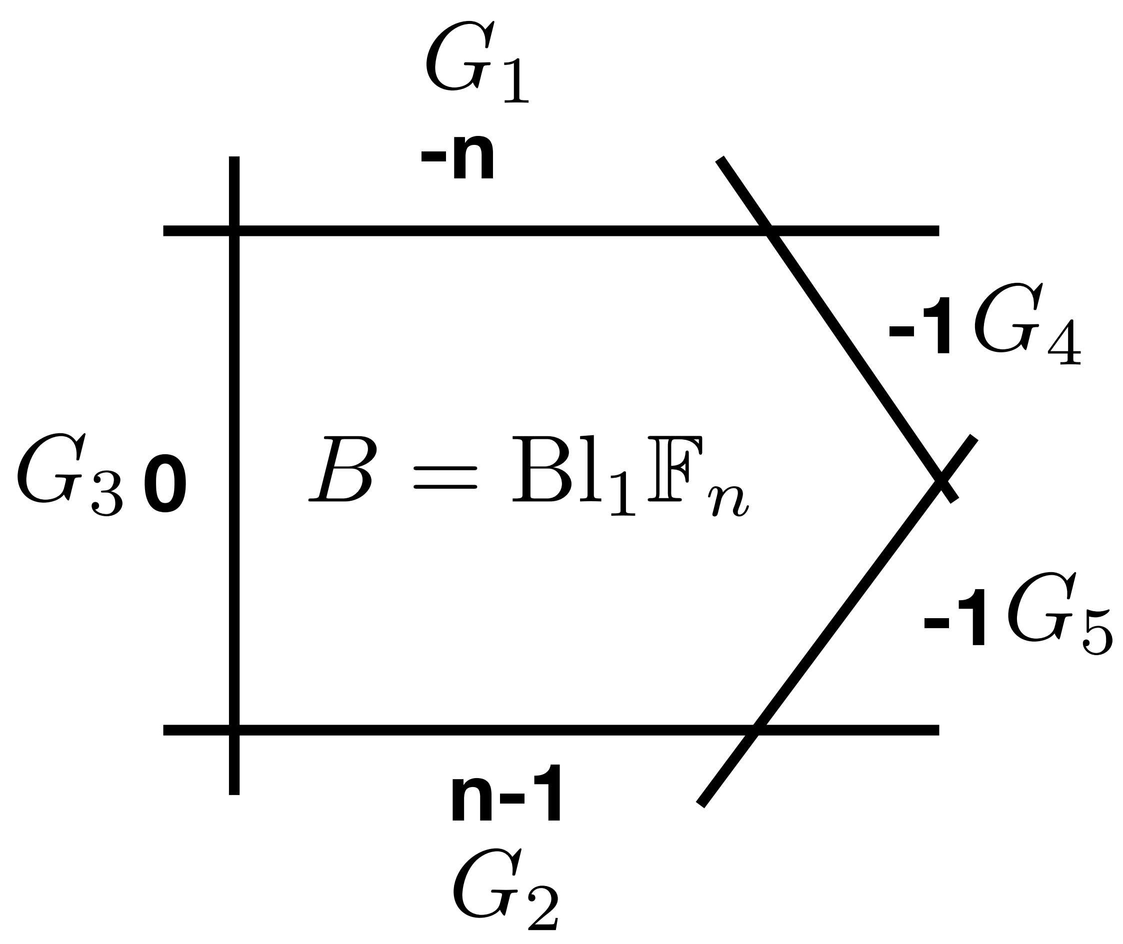
- Unitarity of the worldsheet SCFT requires:

[HCK, Shiu, Vafa '19]

$$\sum_i \frac{k_i \cdot \dim G_i}{k_i + h_i^\vee} \leq c_L \quad \Longrightarrow \quad \sum_{\text{for } k_i > 0} \mathrm{rank}(G_i) \leq 20$$

rank of 'external' gauge algebras is bounded

Example:



$\mathrm{rank}(G_1) + \mathrm{rank}(G_2) \leq 20$ as $k_{1,2} > 0\,,\;\; k_{3,4,5} = 0$

Existence of H-string

- Generators $\mathcal{C}_j$ of the BPS cone satisfy

$$b_0 \cdot \mathcal{C}_j = \begin{cases} 2 & \textbf{H-string} \\ 1 & \text{if } \ \mathcal{C}_j^2 = -1\,,\ k_L(\mathcal{C}_j) = 0 \\ 0 & \text{if } \ \mathcal{C}_j^2 = -2\,,\ k_L(\mathcal{C}_j) = 0 \ \text{ or } \ \mathcal{C}_j^2 = 0\,,\ k_L(\mathcal{C}_j) = 1 \\ \leq -1 & \text{otherwise} \end{cases}$$

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- Recall the condition on charge b_0

$$J \cdot b_0 \geq 0 \text{ with } J = \sum_i a_i \mathcal{C}_i \text{ and } a_i \geq 0$$ everywhere in tensor moduli space

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- We define a projection to subspace orthogonal to all e_i with $e_i^2 = -1$.

$$\text{Blowdown} \ : \ J \ \xrightarrow{J \cdot e_i = 0} \ J' = J|_{J \cdot e_i = 0}$$

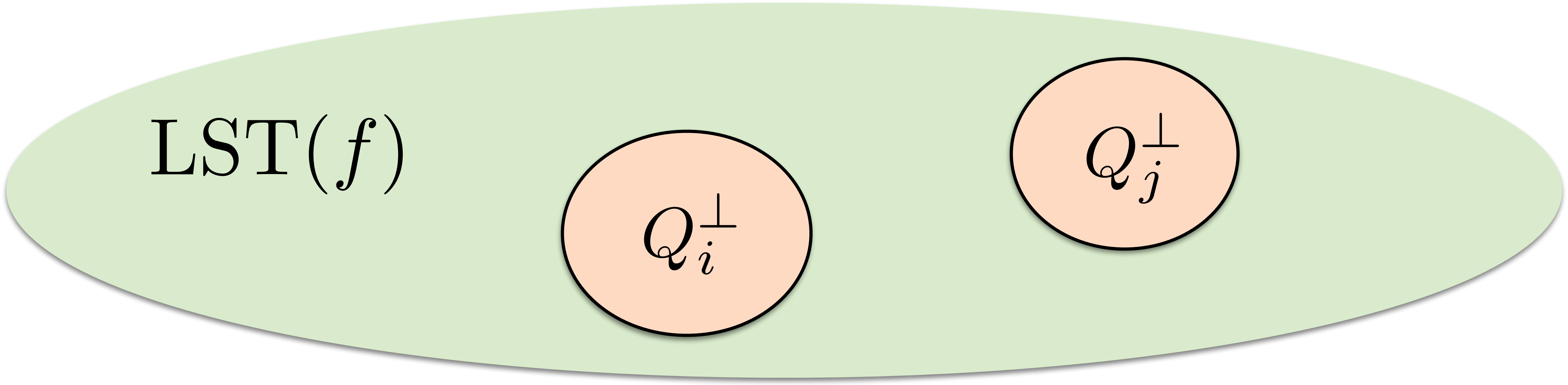
- Positivity $J' \cdot b_0' \geq 0$ requires the existence of at least an H-string!

$$\text{Two exceptions} : 1) \ T = 0\,, \quad 2) \ b_0 = 0 \ (\text{only when } T = 9)$$

SCFTs contained in H-string

- BPS strings with charge $Q_i^{\perp}$ that satisfy the following conditions are elements in a little string theory (LST) of the H-string charge class f:

$$f \cdot Q_i^{\perp} = 0 \quad \text{and} \quad \mathfrak{g}_i \neq \mathfrak{g}_{\text{small}} \quad \text{where} \quad \mathfrak{g}_{\text{small}} \in \{\varnothing, \mathfrak{su}_{2,3,4}, \mathfrak{sp}_2, \mathfrak{g}_2\}$$



$Q_i^{\perp} \in \mathrm{LST}(f)$

- We can prove this, using the properties:
  1. When H-string shrinks (at infinite distance), $Q_i^{\perp}$-string also shrinks.
  2. Intersections of $Q_i^{\perp}$ with other null charges are finite if $\mathfrak{g}_i \neq \mathfrak{g}_{\text{small}}$.

Finiteness of Tensors

- Gravitational anomaly cancellation : $\sum_i \Delta_i = 273 \text{ with } \Delta_i = H_i - V_i + 29T_i$
- Only components having $\Delta_i < 0$ are SCFT 'atoms' of charge Q_i with

$$-5 \geq Q_i^2 \geq -12 \text{ and } \mathfrak{g}_i = \mathfrak{f}_4, \mathfrak{e}_6, \mathfrak{e}_7, \mathfrak{e}_8$$

[Hamada, Loges '23]

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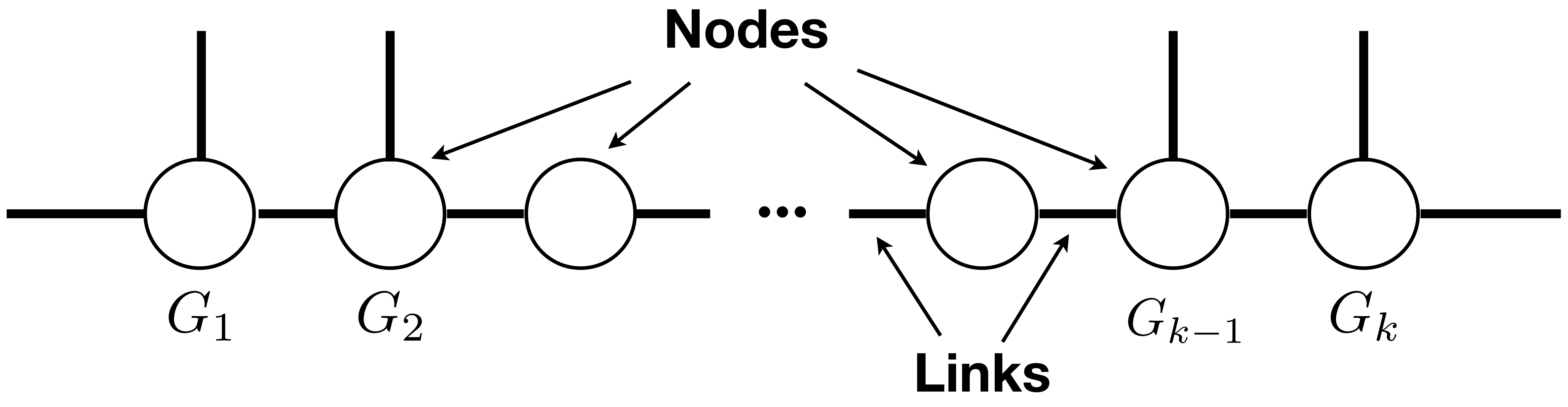
- These atoms are either $\begin{cases} 1.\ Q_i \cdot f > 0\ , & \sum_i r(\mathfrak{g}_i) \leq 20 \\ 2.\ Q_i \cdot f = 0\ , & Q_i \in \mathrm{LST}(f) \end{cases}$
- Number of tensors in the 1st case is finite due to the rank bound.
- For the 2nd case, the classification of LST tells us that these atoms are always accompanied by additional matters, so that their grav. anomaly contributions eventually become positive!

SCFT/LST classification

- SCFTs/LSTs are generalized quivers with base (tensor intersections)

[Heckman et.al. '15], [Bhardwaj et.al. '15], [Bhardwaj '15, '19], ….



(+ one more tensor for LST)

Node : Single tensor with $Q_i^2 = -4, -6, -7, \cdots, -12$

Link : Finite chain of tensors with $Q_i^2 = -1, -2, -3, -5$

- Interior links are (generalized) conformal matters: [Del Zotto et.al. '14]

E8 conf. matter : $\mathfrak{e}_8 \otimes \mathfrak{e}_8 + (-12)\,\text{tensor}: \ [\mathfrak{e}_8]1\ 2\ \overset{\mathfrak{sp}_1}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{sp}_1}{3}\ 2\ 1\ \overset{\mathfrak{e}_8}{\underline{12}} \qquad T_i = 12\,,\ \Delta_i = 30$

- Node+Link always contribute positively to grav. anomaly $\Delta_i \gtrsim 30$.

Finiteness of Tensors

- Gravitational anomaly cancellation : $\sum_i \Delta_i = 273 \;\; \text{with} \;\; \Delta_i = H_i - V_i + 29T_i$
- Only components having $\Delta_i < 0$ are SCFT 'atoms' of charge Q_i with

$$-5 \geq Q_i^2 \geq -12 \;\; \text{and} \;\; \mathfrak{g}_i = \mathfrak{f}_4, \mathfrak{e}_6, \mathfrak{e}_7, \mathfrak{e}_8$$ [Hamada, Loges '23]

- These atoms are either $\begin{cases} 1.\; Q_i \cdot f > 0 \;, & \sum_i r(\mathfrak{g}_i) \leq 20 \\ 2.\; Q_i \cdot f = 0 \;, & Q_i \in \mathrm{LST}(f) \end{cases}$
- Number of tensors in the 1st case is finite due to the rank bound.
- For the 2nd case, the classification of LST tells us that these atoms are always accompanied by additional matters, so that their grav. anomaly contributions eventually become positive!

$$\Longrightarrow \quad T \text{ is finite}$$

Tensor and Vector Bound

- We first identify the composition of ‘atoms’ for $\mathrm{LST}(f)$ that can host large number of tensors and high rank of gauge algebras.
- Adding ‘external’ matters for $Q_i \cdot f > 0$ properly, supergravity theories with maximum tensors and rank of gauge algebras can be constructed.

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- $T_{\max}$ theory : $\overset{\mathfrak{e}_8}{(12)} \otimes \underbrace{\overset{1}{\mathfrak{e}_8} \otimes \mathfrak{e}_8 \otimes \cdots \otimes \mathfrak{e}_8 \otimes \overset{1}{\mathfrak{e}_8}}_{15} \otimes \overset{\mathfrak{e}_8}{(12)}$ with $T = 193$, $G = \mathfrak{e}_8^{17} \times \mathfrak{f}_4^{16} \times \mathfrak{g}_2^{32} \times \mathfrak{sp}_1^{32}$

  $Q^2 = -12,\ Q \cdot f = 1$ $\qquad$ $Q^2 = -12,\ Q \cdot f = 1$

- $r_{\max}(G)$ theory : $\overset{\mathfrak{so}_{64}}{4}\ \overset{\mathfrak{sp}_{56}}{1}\ \overset{\substack{\mathfrak{sp}_{40} \\ 1}}{\overset{\mathfrak{so}_{176}}{4}}\ \overset{\mathfrak{sp}_{72}}{1}\ \overset{\mathfrak{so}_{128}}{4}\ \overset{\mathfrak{sp}_{48}}{1}\ \overset{\mathfrak{so}_{80}}{4}\ \overset{\mathfrak{sp}_{24}}{1}\ \overset{\mathfrak{so}_{32}}{(4)}$ with $T = 9\,,\ r(G) = 480$

  $Q^2 = -4,\ Q \cdot f = 1$

- Therefore, we find the bounds $T \leq 193\,,\ r(G) \leq 480$!

Maximal SUGRAs from string theory

- T_{max} theory : $\overset{\mathfrak{e}_8}{(12)} \otimes \underbrace{\overset{1}{\mathfrak{e}_8} \otimes \mathfrak{e}_8 \otimes \cdots \otimes \mathfrak{e}_8 \otimes \overset{1}{\mathfrak{e}_8}}_{15} \otimes \overset{\mathfrak{e}_8}{(12)}$ **with** $T = 193$, $G = \mathfrak{e}_8^{17} \times \mathfrak{f}_4^{16} \times \mathfrak{g}_2^{32} \times \mathfrak{sp}_1^{32}$

T-dual $\updownarrow$

- $r_{\text{max}}(G)$ theory : $\begin{array}{ccccccccc} & & \overset{\mathfrak{sp}_{40}}{1} & & & & & & \\ \overset{\mathfrak{so}_{64}}{4} & \overset{\mathfrak{sp}_{56}}{1} & \overset{\mathfrak{so}_{176}}{4} & \overset{\mathfrak{sp}_{72}}{1} & \overset{\mathfrak{so}_{128}}{4} & \overset{\mathfrak{sp}_{48}}{1} & \overset{\mathfrak{so}_{80}}{4} & \overset{\mathfrak{sp}_{24}}{1} & \overset{\mathfrak{so}_{32}}{(4)} \end{array}$ **with** $T = 9\,,\ r(G) = 480$

- These theories are realized by 24 instantons in $E_8 \times E_8$ and $SO(32)$ heterotic strings, respectively, on K3 with an E_8 singularity.

[Aspinwal, Morrison '97], [Candelas, Perevalov, Rajesh '97], [Morrison, Taylor '12]

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- This result, 6d SUGRAs saturating the bounds arise from string theory, provides non-trivial evidence for string lamppost principle.

Conclusion

- We have demonstrated that, in 6d (1,0) supergravity, the number of tensors and the rank of the gauge algebra are bounded above, based on the consistency of tensor branch physics, the unitarity of BPS strings, and the classification of 6d SCFTs and LSTs: $T \leq 193$, $r(G) \leq 480$

- Our results predict sharp bounds $h^{1,1}(\mathrm{CY}) \leq 491,\ h^{1,1}(\mathrm{Base}) \leq 194$ for elliptic Calabi-Yau 3-folds, and the 3-fold saturating the bounds is explicitly constructed via 192 blowups of Hirzebruch base $\mathbb{F}_{12}$ with specific fiber choices. This provides further evidence for SLP!

- # of allowed 6d (1,0) SUGRAs is finite?
- Lower dimensional and lower supersymmetric cases?