STRINGS 2025
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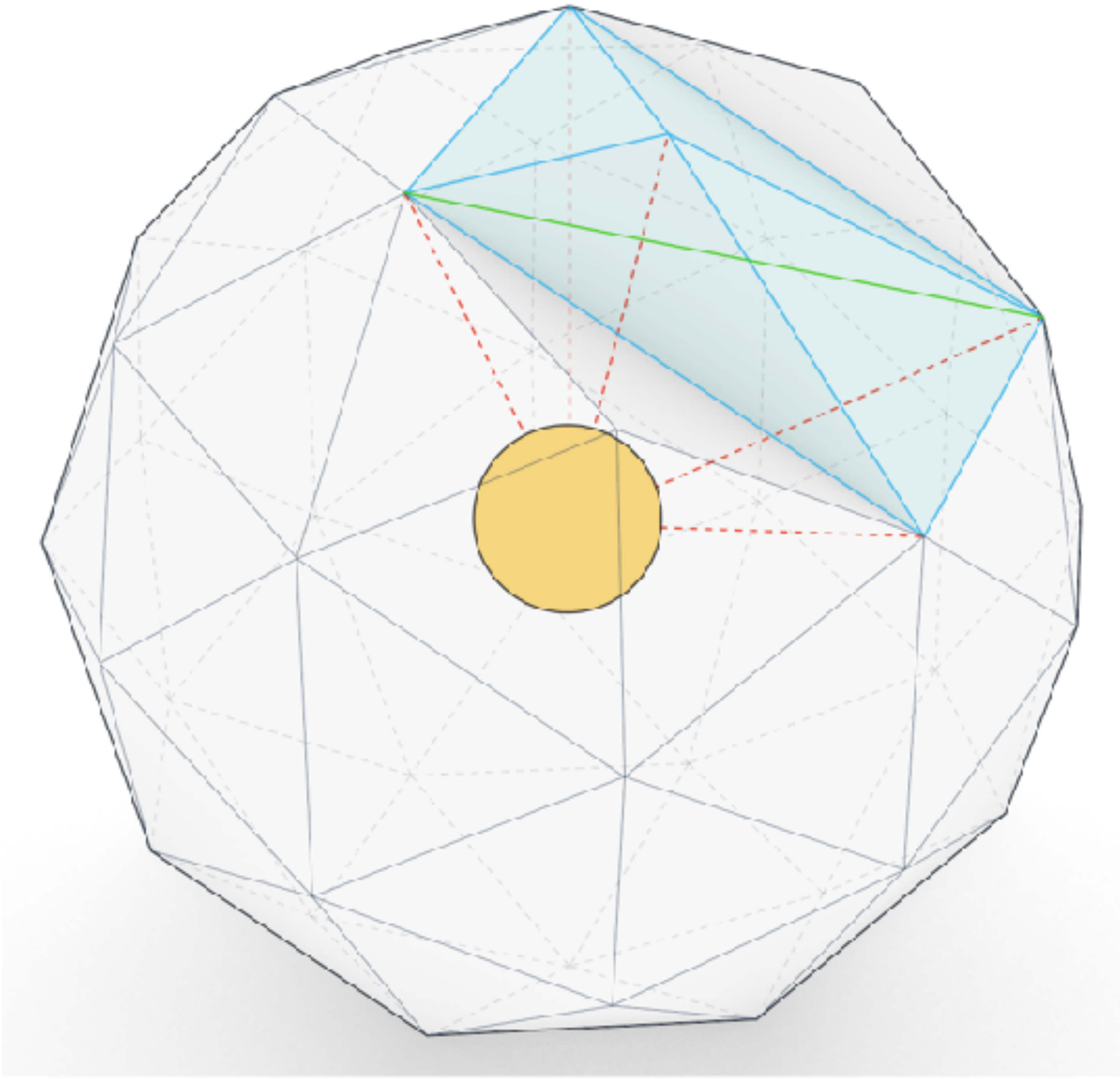
Building Up Quantum Spacetimes with BCFT Tensor Networks and Symmetric Quantum RG

LING YAN HUNG,
YMSC, TSINGHUA UNIVERSITY
7th, Jan 2025



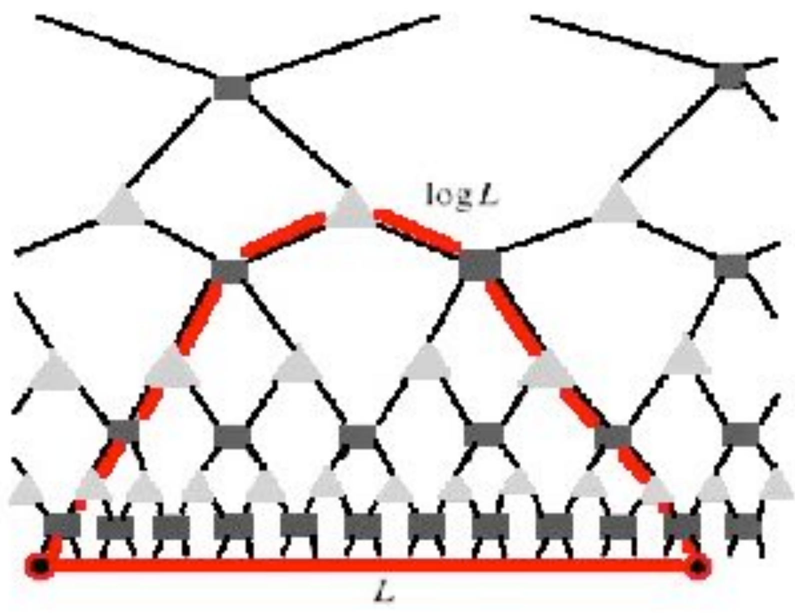
Based on works with :
Ning Bao, Yikun Jiang, Zhihan Liu , 2412.12045
Lin Chen, Yikun Jiang, Bingxin Lao
2404.00877 , 2403.03179, 2503.xxxxx
Hao Geng, Yikun Jiang, Lin Chen, 2503.xxxxx
Kaixin Ji, Liping Yang, Lin Chen, 2412.08374
Gong Cheng, Lin Chen, Zheng-Cheng Gu 2311.18005
Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang
2210.12127 Physical Review X 14 (4), 041033

清華大学 丘成桐数学科学中心
Yau Mathematical Sciences Center, Tsinghua University

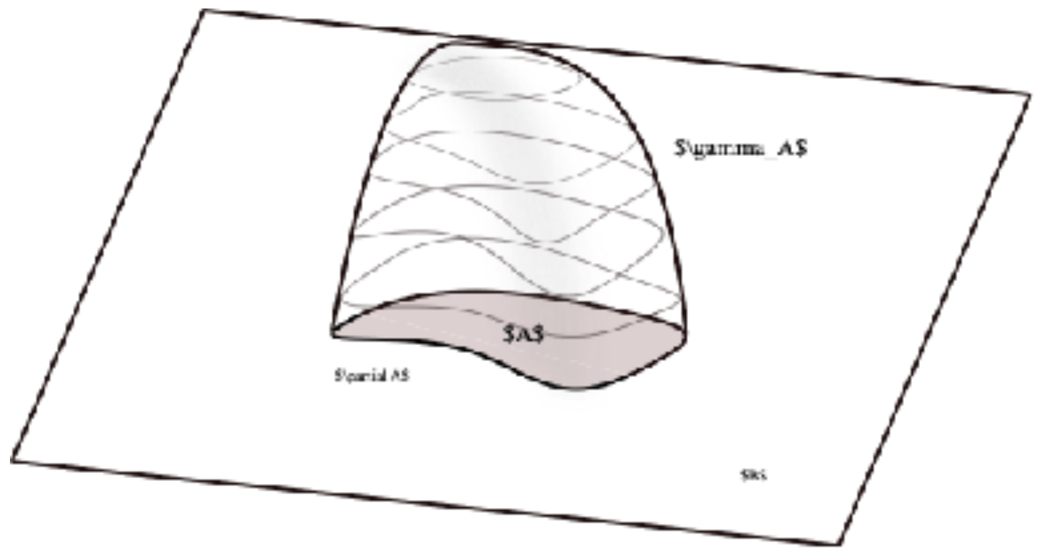
Why do we care about tensor networks for AdS/CFT?

- Realises quantum entanglement as geometry (Ryu-Takayanagi formula)
  Van Raamsdonk 2010; Swingle 2012

- Microscopic local map between CFT and AdS dofs



Picture courtesy Orus

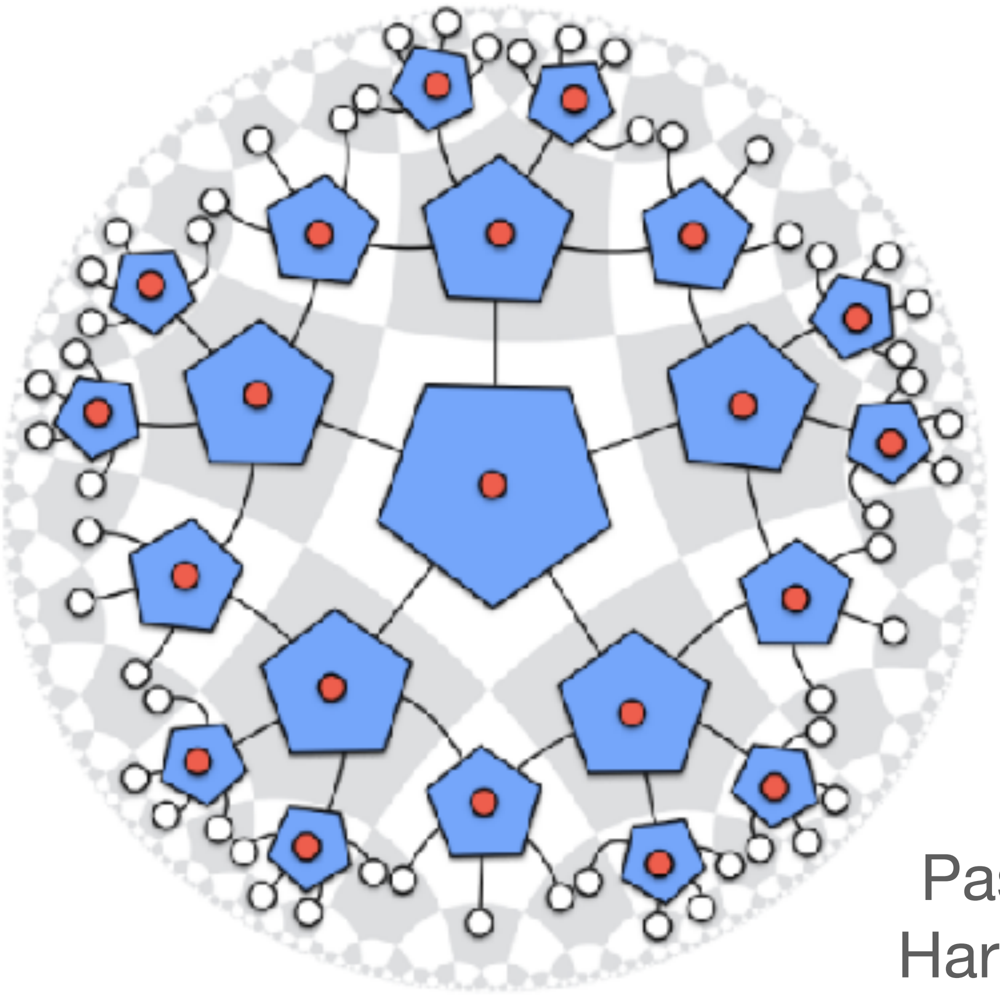


What we had for Holographic Tensor Network

Vidal 2006; Swingle 2012;
Pastawski, Yoshida, Harlow, Preskill 2015;
Hayden, Nezami, Qi, Thomas, Walter, Yang 2016 …

Amheiri, Dong, Harlow 2014; HaPPY 2015; Harlow 2016
Brown, Roberts, Susskind, Swingle, Zhao 2015; Caputa, Kundu, Miyaji, Takayanagi, Watanabe 2017; Czech 2017; Chapman, Marrochio, Myers 2017; ……

Akers, Engelhardt, Harlow, Penington, Vardhan 2022



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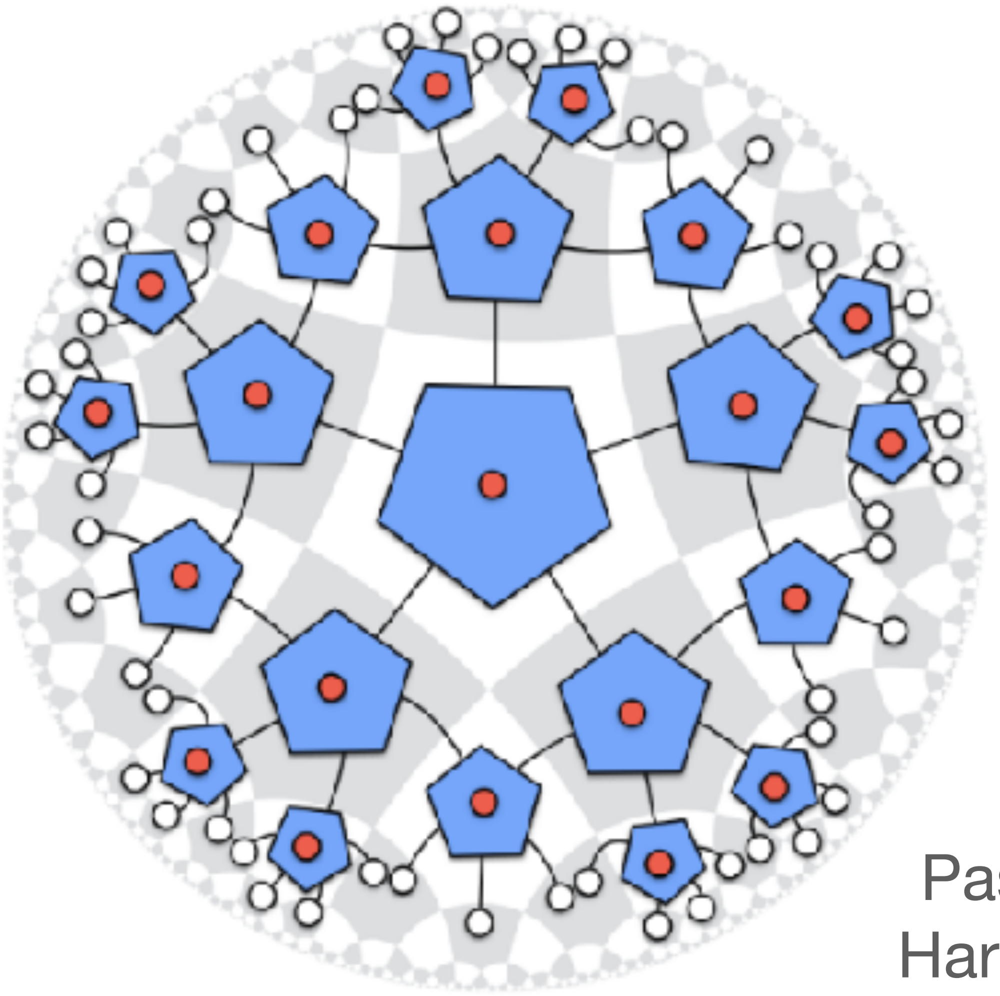
CFT data?

Einstein-Hilbert action?



Pastawski, Yoshida, Harlow, Preskill 2015;

Wishlist for Holographic Tensor Network

What we wish to have:

- Exact CFT: kinematics + *dynamics*

Some precursors, notably "BC bits", Van Raamsdonk 2018

- Graph/Background independence
- Emergent sum over geometries from CFT data

  (Einstein-Hilbert action + UV complete quantum measure)
- RG flow
- Non-perturbative boundary-to-bulk map

Brand New Strategy in AdS3/CFT2:

Deriving a **spacetime** tensor network from CFT2

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2022;
Gong Cheng, Lin Chen, Zheng-Cheng Gu LYH 2023;
Lin Chen, LYH, Yikun Jiang, Bingxin Lao 2024;
Ning Bao, LYH, Yikun Jiang, Zhihan Liu, 2024;

- **Novel** way of triangulating 2D CFT path-integral:

BCFT three-point functions + shrinkable boundaries $\longrightarrow$ ***exact*** tensor network for 2D field theories

- Read off non-perturbative 3D path-integral from BCFT OPE coefficients

$\longrightarrow$ ***exact*** holographic tensor network

- 3D bulk makes 2D global generalised symmetries manifest.

- Check large-c classical limit of the bulk preserving Virasoro topological lines

$\longrightarrow$ recover **Einstein-Hilbert action**

- Quantum symmetric RG in 2D = Radial evolution in 3D bulk

Overview

- Step 1 Cut 2D CFT path-integral into pieces
- Step 2 Emergence of 3D bulk from BCFT OPE
- Step 3 Try Liouville CFT
- Step 4 (Virasoro) SymQRG= QG $\supset T\bar{T}$

  and the Wheeler-DeWitt equation

- Conclusion and Outlook

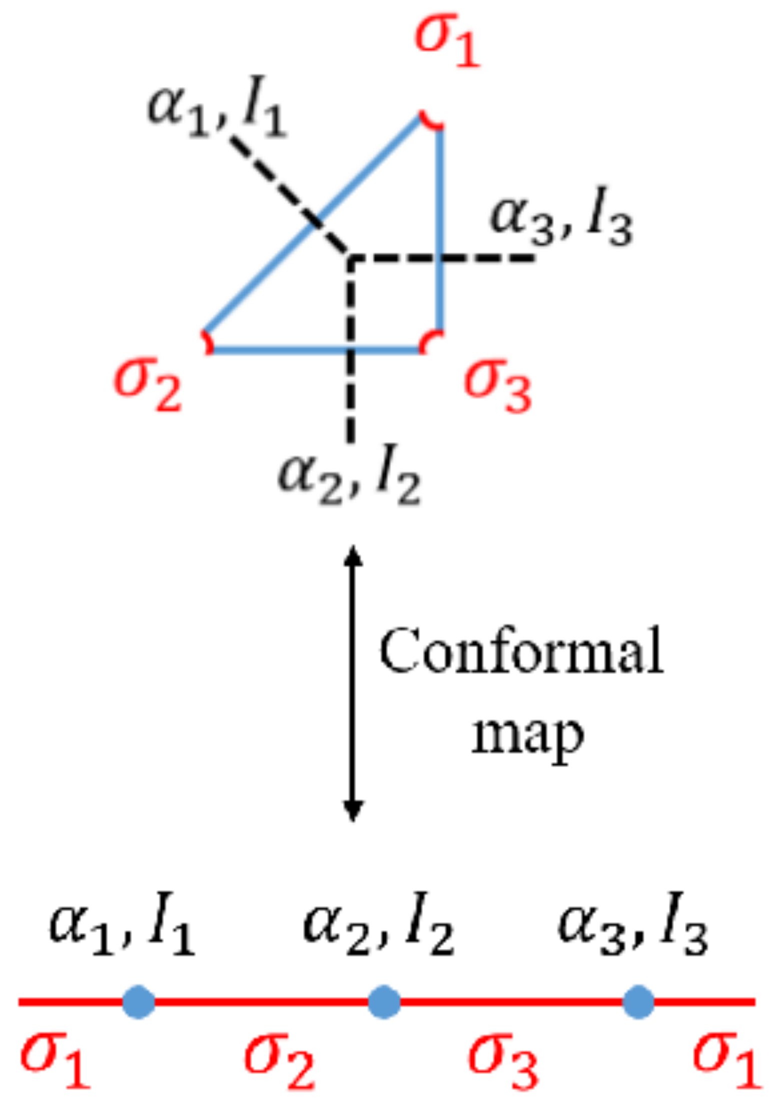
Step 1: Cut 2D CFT path-integral into pieces

Cut 2D CFT path-integral into pieces

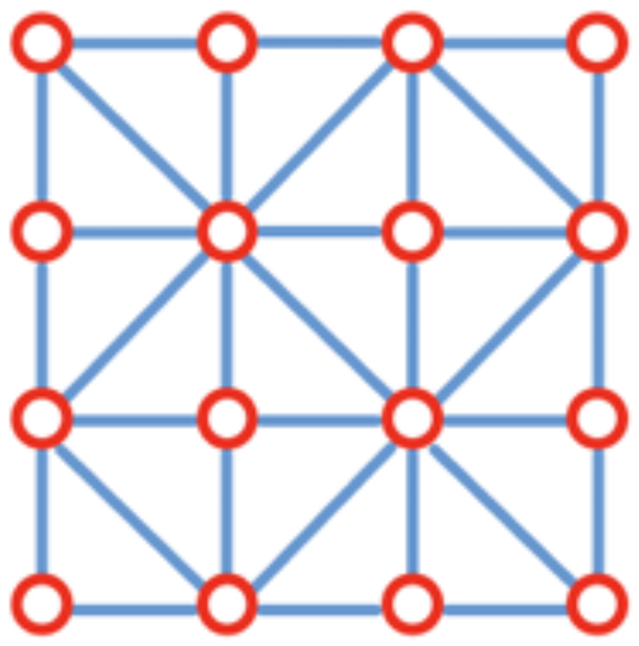
Gabriel Wong, LYH 2020
Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2022
Brehm, Runkel 2022

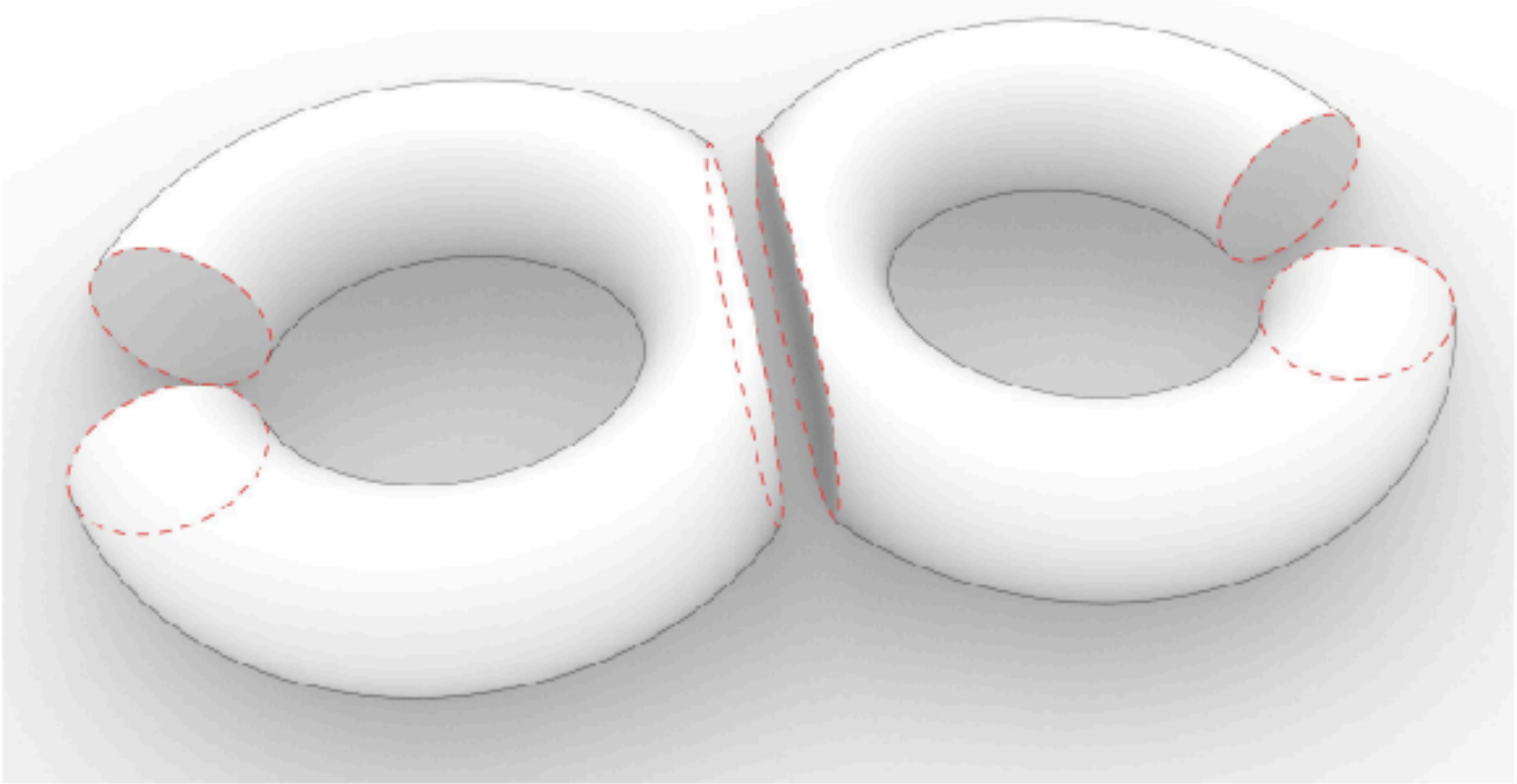
- Cut 2D manifold into triangles.
- Each triangle = three-point function of boundary-changing operators.
- Glue triangles: summing over states on the edge (primaries + descendants).
- Leaving holes (radius R) at each vertex, labeled by conformal boundary conditions.

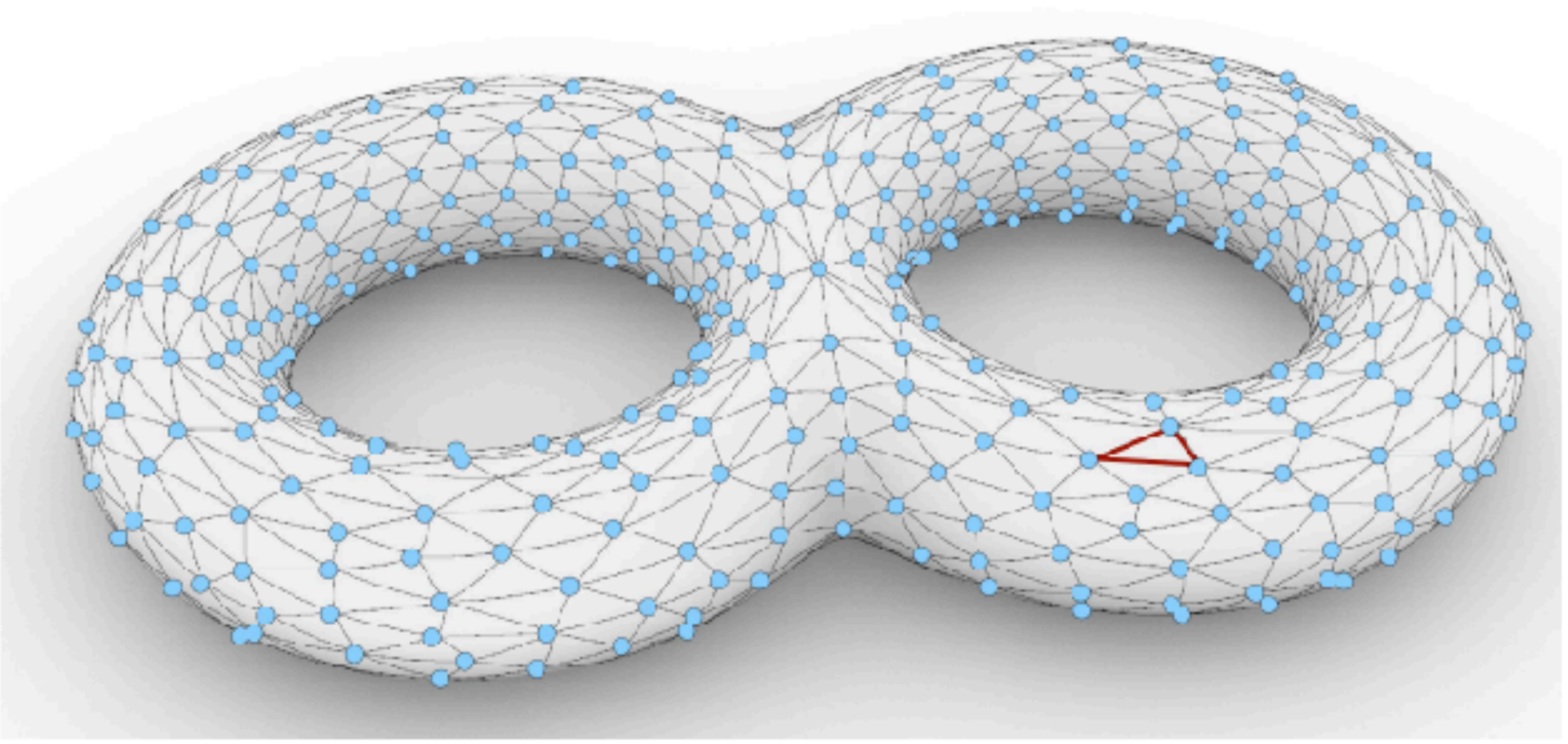
$$\mathcal{T}^{\sigma_1\sigma_2\sigma_3}_{(\alpha_1,I_1)(\alpha_2,I_2)(\alpha_3,I_3)}(\triangle, R) =$$



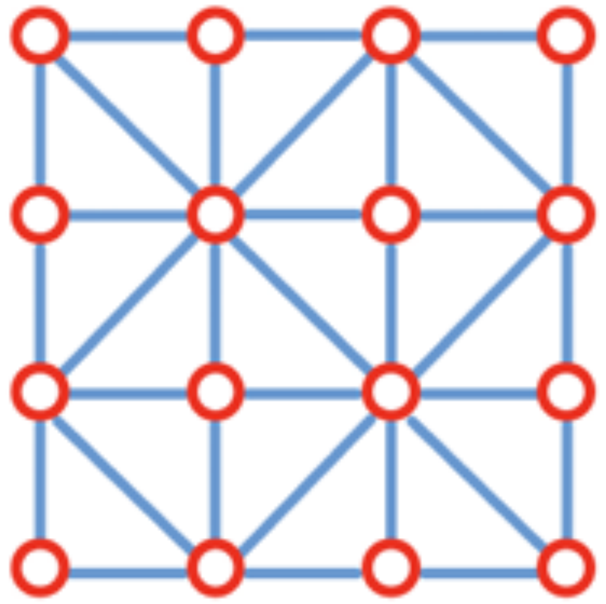
$$\mathcal{Z}(R, \sigma_a) = \sum_{\{\alpha_i\}} \sum_{\{I_i\}} \prod_{\triangle} \mathcal{T}^{\sigma_a\sigma_b\sigma_c}_{(\alpha_i,I_i)(\alpha_j,I_j)(\alpha_k,I_k)}(\triangle, R)$$







Cut a CFT into triangles



$$\mathcal{Z}(R,\sigma_a) = \sum_{\{\alpha_i\}}\sum_{\{I_i\}}\prod_{\triangle} \mathcal{T}^{\sigma_a\sigma_b\sigma_c}_{(\alpha_i,I_i)(\alpha_j,I_j)(\alpha_k,I_k)}(\triangle,R)$$

- Shrink the holes.

Shrinkable Boundary Condition =
"Entanglement Brane Boundary Condition" in RCFT
Wong, LYH 2020,
Brehm, Runkel 2022

Vacuum Ishibashi state $|e\rangle\rangle = \sum_a \omega_{\sigma_a}|\sigma_a\rangle$

"Quantum dimensions"

Shrinkable boundary designed to preserve **all** the CFT generalised symmetries.

$$\mathcal{Z}(R) = \sum_{\{\sigma_a\}}\prod_{v}\omega_{\sigma_a}\sum_{\{\alpha_i\}}\sum_{\{I_i\}}\prod_{\triangle} \mathcal{T}^{\sigma_a\sigma_b\sigma_c}_{(\alpha_i,I_i)(\alpha_j,I_j)(\alpha_k,I_k)}(\triangle,R)\,.$$

$$Z_{\rm CFT} = \lim_{R\to 0}\prod_v e^{-c/6\ln R}\mathcal{Z}(R)$$

Fixed point tensors

Gong Cheng, Lin Chen, Zheng-Cheng Gu LYH 2023

Retain few (level lower than 4) descendants, and numerically recover precise closed CFT data

Ising model

States	Numerical dim	Accurate dim
$\mathbb{1}$	0.0000	0.0000
σ	0.1250	0.1250
ϵ	0.9989	1.0000
$\partial\sigma, \bar{\partial}\sigma$	1.1253	1.1250
$\partial\epsilon, \bar{\partial}\epsilon$	2.0004	2.0000
$\mathbb{1}^{(-2)}, \bar{\mathbb{1}}^{(-2)}$	1.9986	2.0000
$\partial\bar{\partial}\sigma$	2.1099	2.1250
$\sigma^{(-2)}, \bar{\sigma}^{(-2)}$	2.1208	2.1250
$\partial\bar{\partial}\epsilon$	2.9517	3.0000
$\partial^2\epsilon, \bar{\partial}^2\epsilon$	3.0030	3.0000
$\mathbb{1}^{(-3)}, \bar{\mathbb{1}}^{(-3)}$	2.9496	3.0000
$\bar{\partial}\sigma^{(-2)}, \partial\bar{\sigma}^{(-2)}$	2.9900	3.1250
$\partial\sigma^{(-2)}, \bar{\partial}\bar{\sigma}^{(-2)}$	3.1167	3.1250
$\sigma^{(-3)}, \bar{\sigma}^{(-3)}$	3.1291	3.1250
$\mathbb{1}^{(-2,-\bar{2})}$	4.0323	4.0000

Tricritical Ising

States	Numerical dim	Accurate dim
$\mathbb{1}$	0.0000	0.000
Ω	0.075	0.075
Φ	0.200	0.200
Λ	0.873	0.875
$\partial\Omega, \bar{\partial}\Omega$	1.076	1.075
Ψ	1.196	1.200
$\partial\Phi, \bar{\partial}\Phi$	1.201	1.200
$\partial\Lambda, \bar{\partial}\Lambda$	1.871	1.875
$\mathbb{1}^{(2)}, \bar{\mathbb{1}}^{(2)}$	2.013	2.000
$\partial\bar{\partial}\Omega$	2.064	2.075
$\Omega^{(2)}, \bar{\Omega}^{(2)}$	2.084	2.075
$\Omega^{(2)\prime}, \bar{\Omega}^{(2)\prime}$	2.087	2.075
$\partial\bar{\partial}\Phi$	2.190	2.200
$\Phi^{(2)}, \bar{\Phi}^{(2)}$	2.192	2.200
$\partial\Psi, \bar{\partial}\Psi$	2.210	2.200
$\partial\bar{\partial}\Lambda$	2.877	2.875
$\Lambda^{(2)}, \bar{\Lambda}^{(2)}$	2.878	2.875
$I^{(3)}, \bar{I}^{(3)}$	2.978	3.000
Ξ	3.004	3.000

Lee-Yang

States	Numerical dim	Accurate dim
τ	-0.403	-0.400
$\mathbb{1}$	0.000	0.000
$\partial\tau, \bar{\partial}\tau$	0.598	0.600
$\tau^{(2)}, \bar{\tau}^{(2)}$	1.591	1.600
$\partial\bar{\partial}\tau$	1.600	1.600
$\mathbb{1}^{(2)}, \bar{\mathbb{1}}^{(2)}$	1.996	2.000
$\bar{\partial}\tau^{(2)}, \partial\bar{\tau}^{(2)}$	2.530	2.600
$\tau^{(3)}, \bar{\tau}^{(3)}$	2.594	2.600
[illegible]	[illegible]	[illegible]

More than a decade old question resolved!

Levin,Nave 2006;
Gu, Levin, Wen 2008;
Evenbly, Vidal 2014, 2015;
Yang, Gu, Wen 2015;
………

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Gong Cheng, Lin Chen, Zheng-Cheng Gu LYH 2023

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- Graph independence

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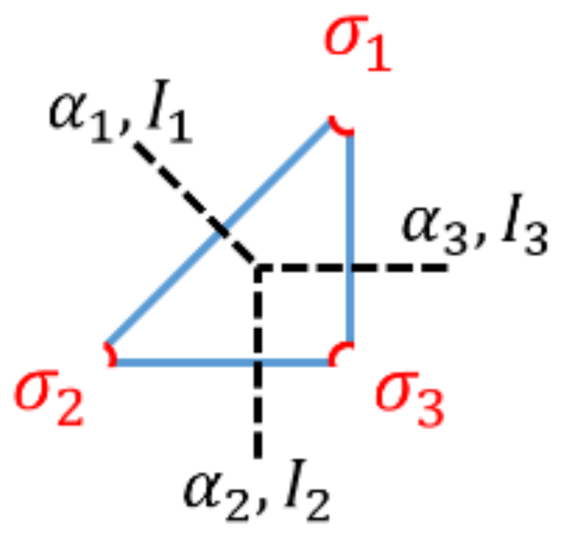
Step 2: Emergence of 3D Bulk from BCFT OPE

Emergence of 3D Bulk from BCFT OPE

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2022

$$\mathcal{Z}(R) = \sum_{\{\sigma_a\}} \prod_{v} \omega_{\sigma_a} \sum_{\{\alpha_i\}} \sum_{\{I_i\}} \prod_{\triangle} \mathcal{T}^{\sigma_a\sigma_b\sigma_c}_{(\alpha_i,I_i)(\alpha_j,I_j)(\alpha_k,I_k)}(\triangle, R)\ .$$



$$\equiv \tilde{C}^{\sigma_3\sigma_1\sigma_2}_{\alpha_1\alpha_2\alpha_3}\, \tilde{\gamma}^{\alpha_1\alpha_2\alpha_3}_{I_1I_2I_3}$$

related to “strange correlator” in lattice spin models
Bal, Williamson, Vanhove, Bultinck, Haegeman, Verstraete 2018;
Aasen, Mong, Fendley 2016; 2020;

Rewrite Z(R) as an overlap of two 3D states

$$\mathcal{Z}(R) = \langle \Omega(R) | \Psi \rangle \quad = \sum_{\{\alpha_i\}} \Omega(\{\alpha_i\})(R)\Psi(\{\alpha_i\})$$

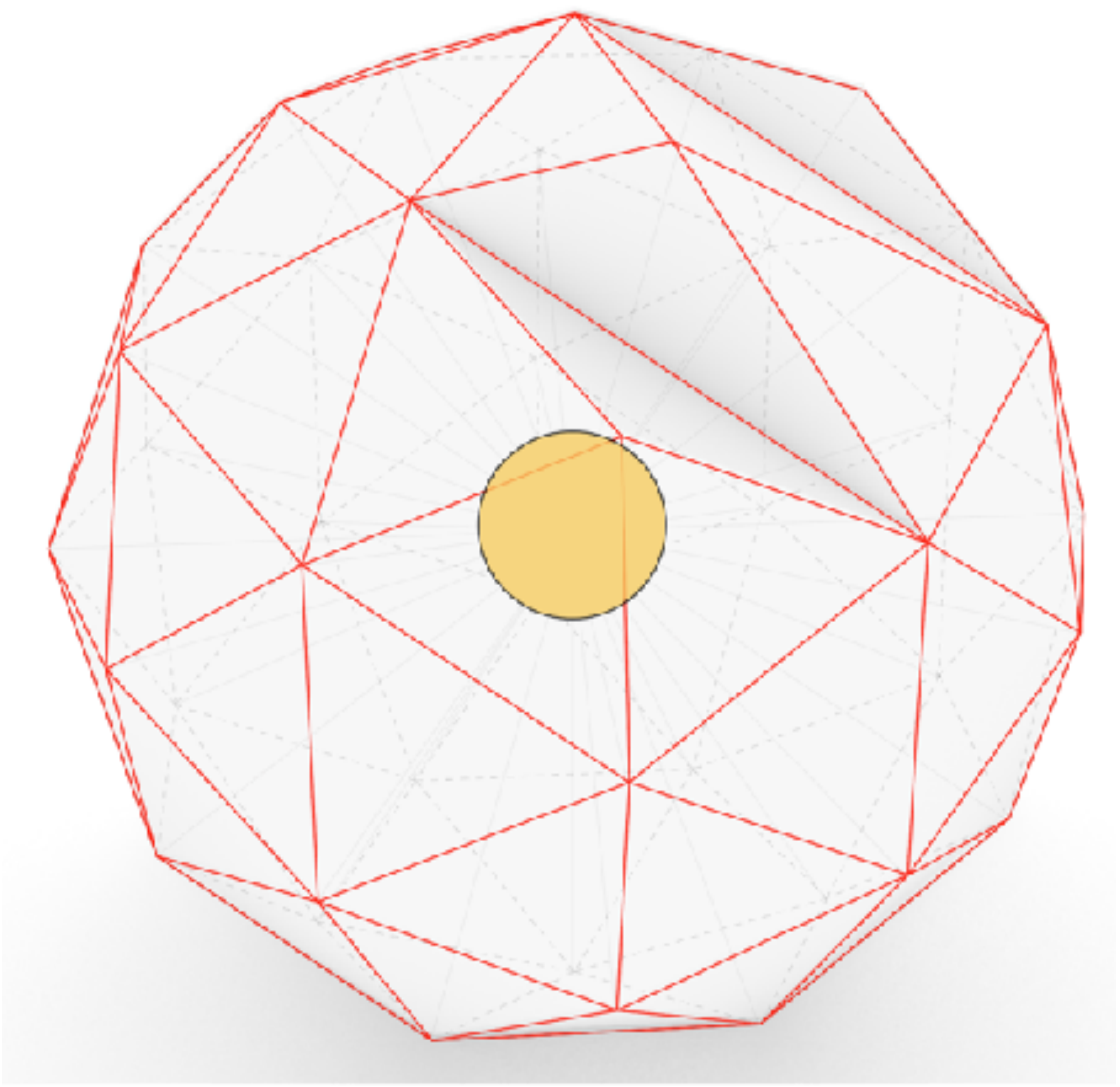
Collection of BCFT OPE (= 6j symbols= 3D tetrahedra)

Fuchs, Runkel, Schweigert 2002-2006

$$\Psi(\{\alpha_i\}) = \sum_{\{\sigma_a\}} \prod_{v} \omega_{\sigma_a} \prod_{\triangle} \tilde{C}^{\sigma_a\sigma_b\sigma_c}_{\alpha_i\alpha_j\alpha_k}$$

$\xrightarrow{\text{Ground state of a } \textbf{3D} \text{ TQFT}}$

Turaev, Viro 1992;
Levin, Wen 2004



Collection of conformal blocks

$$\Omega(\{\alpha_i\})(R) = \sum_{\{I_i\}} \prod_{\triangle} \tilde{\gamma}^{\alpha_i\alpha_j\alpha_k}_{I_iI_jI_k}(R)$$

3D graph independence in the bulk triangulation

Emergence of 3D Bulk from BCFT OPE

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2022

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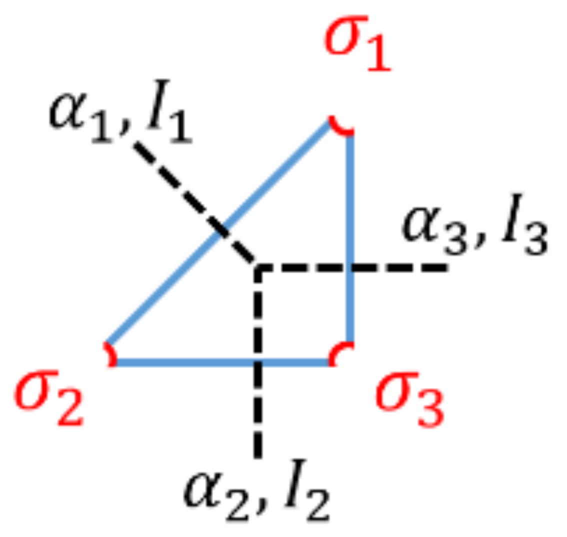

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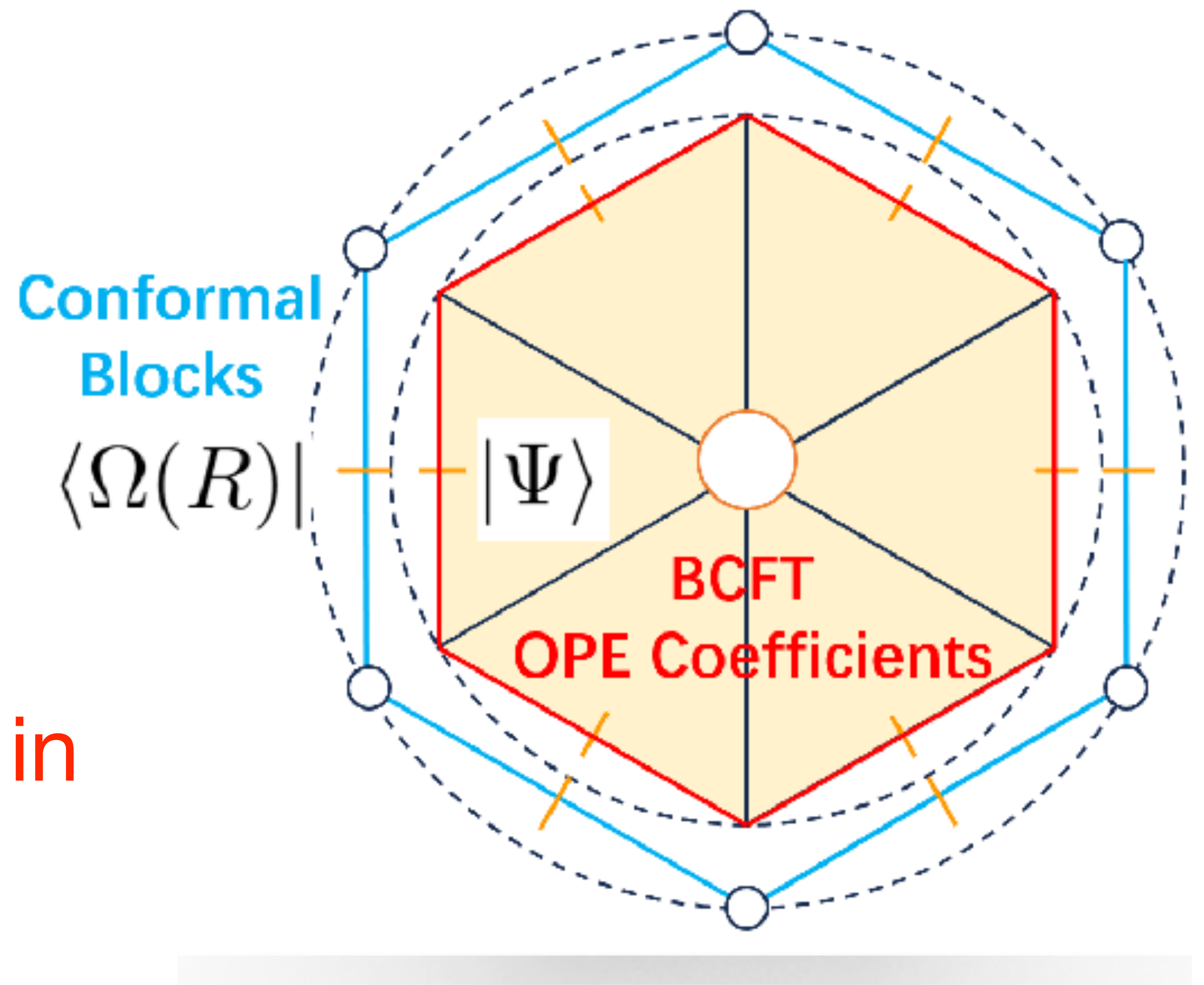

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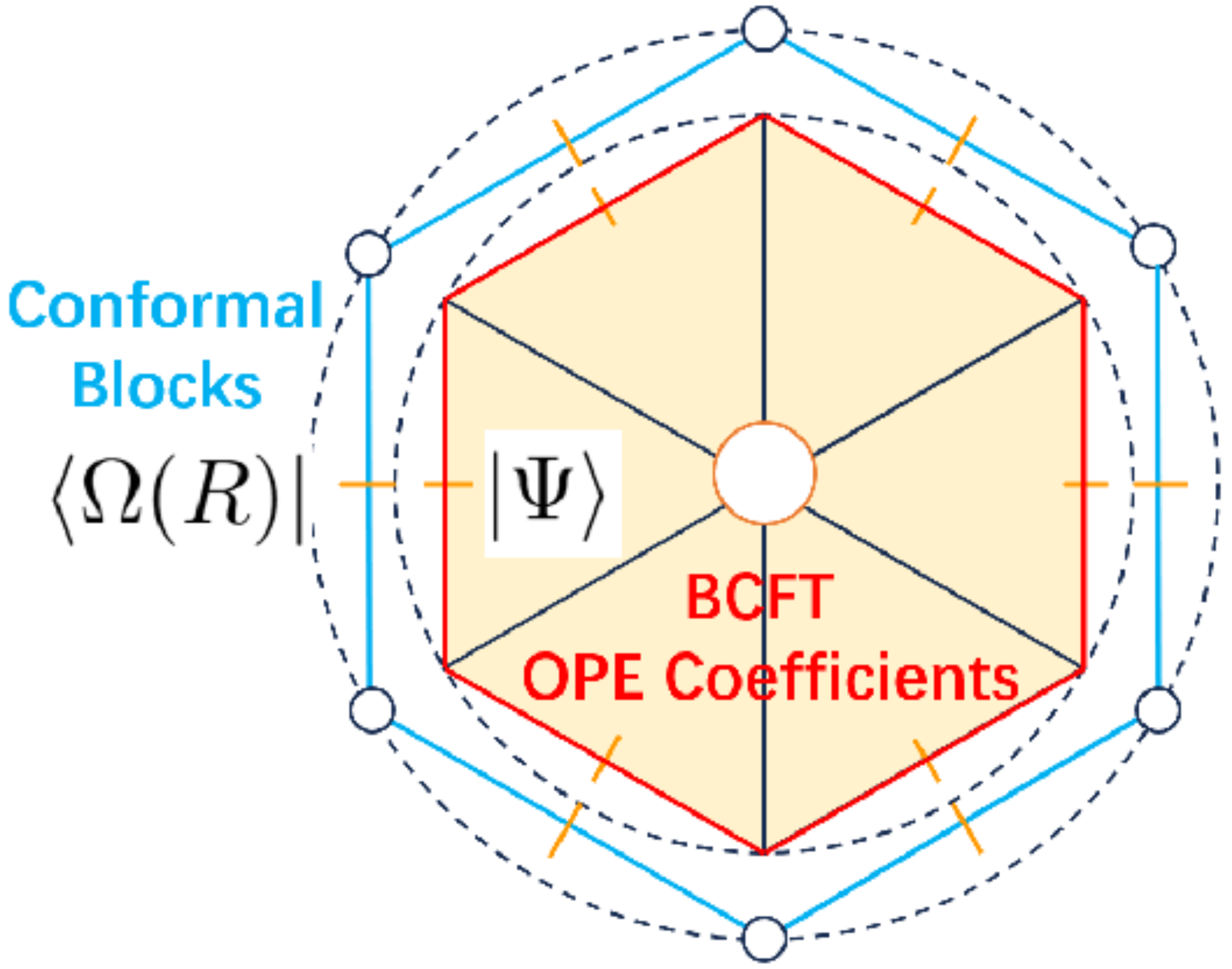
3D graph independence in the bulk triangulation

3D bulk and *Sandwich*

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2022

$$\mathcal{Z}_{2\mathrm{D}} = \langle \Omega | \Psi \rangle_{3\mathrm{D}}$$



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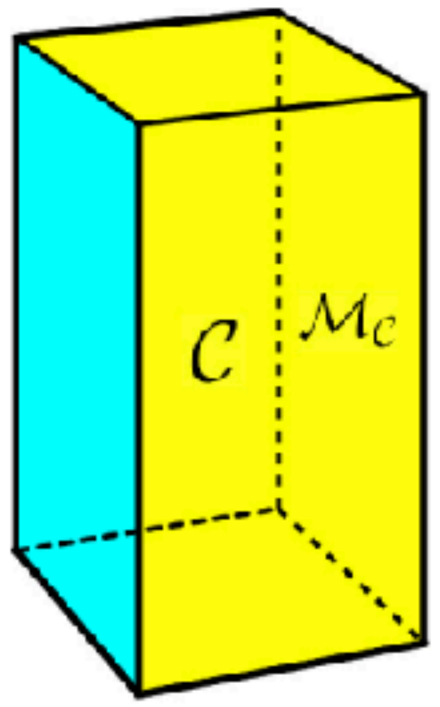

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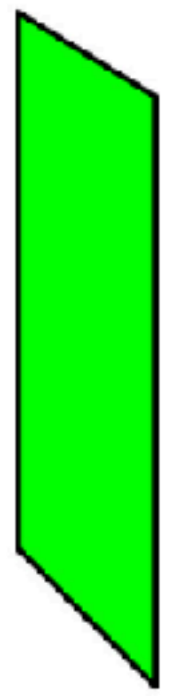

- The 3D bulk is a "gauge theory" of the CFT's generalized global symmetries (e.g: Verlinde lines).
- Each modular invariant 2D CFT $\longrightarrow$ Set of conformal boundary conditions $\longrightarrow$ 3D topological boundary condition, ***unique*** 3D bulk state.
- Explicit realisation of the "*Sandwich construction*".
- With ***maximal*** symmetry, the bulk captures CFT dynamics.

3D bulk and *Sandwich*

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2022

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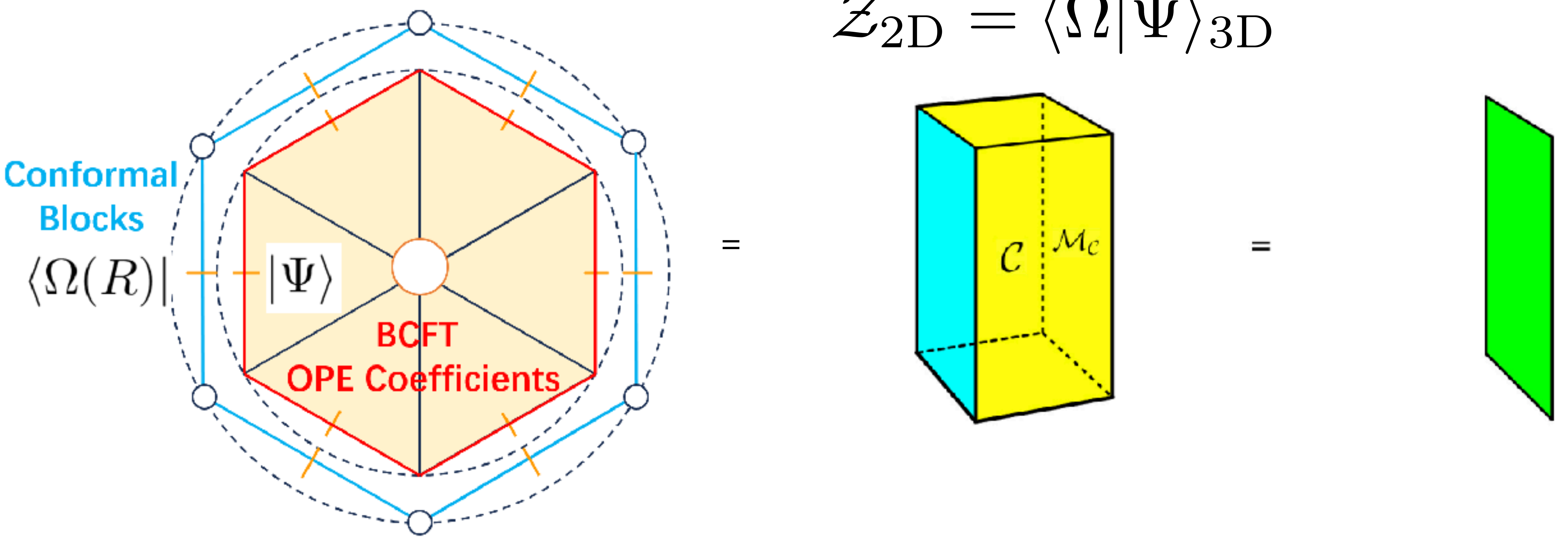

Generalized symmetries: Gaiotto, Kapustin, Seiberg, Willett 2014;
Non-invertible symmetries: Verlinde 1988; Moore, Seiberg 1988-1989; Petkova, Zuber 2000, (Frohlich), Fuchs, Runkel, Schweigert 2002-2006; Bhardwaj, Tachikawa 2017; Chang, Lin, Shao, Wang, Yin 2018; Thorngren, Wang 2019;
Sandwich and SymTFT: Kong, Zheng 2019; Ji, Wen 2019; Gaiotto, Kulp 2020; Apruzzi, Bonetti, Garcia Etxebarria, Hosseini, Schafer-Nameki 2021; Freed, Moore, Teleman 2022;
Reviews: Cordova, Dumitrescu, Intriligator, Shao 2022; Schafer-Nameki 2023; Shao 2023; McGreevy 2022

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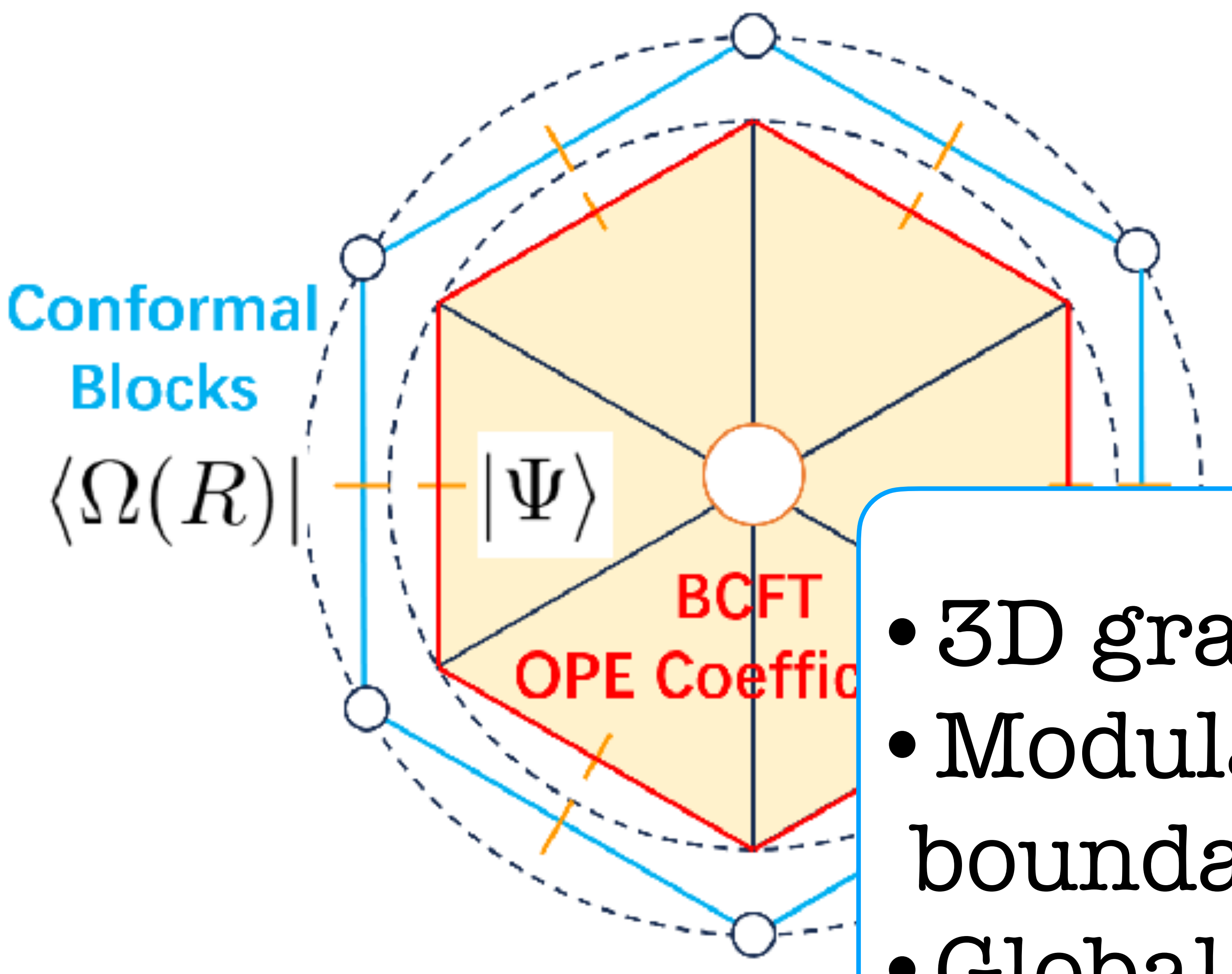

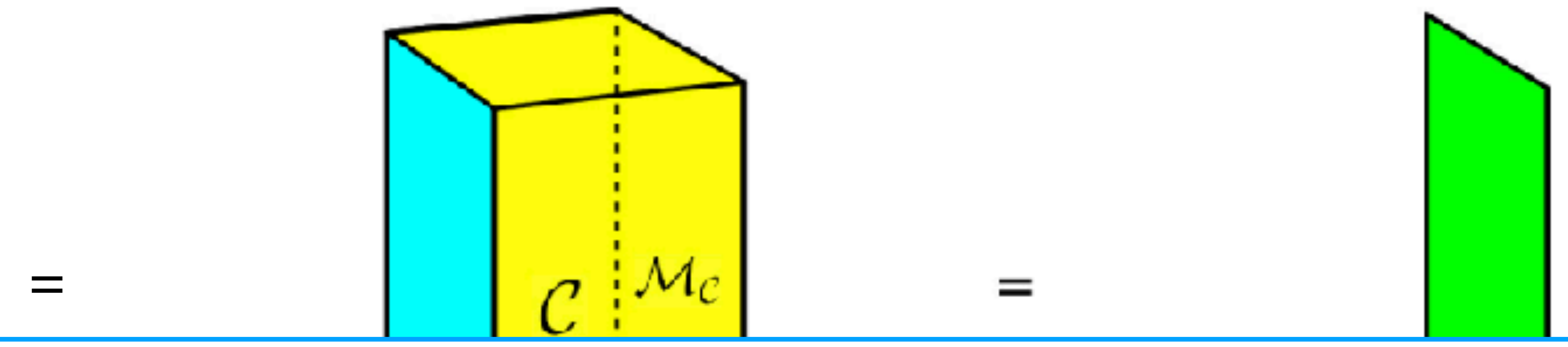

- 3D graph independence
- Modular invariance ensured by topological boundary encoded in $\Psi(\{\alpha_i\})$.
- Global symmetries of CFT showing up as gauge symmetries in the bulk

- The 3D bulk is a "gaug … Verlinde lines).
- Each modular invariant 2D CFT $\longrightarrow$ Set of conformal boundary conditions $\longrightarrow$ 3D topological boundary condition, ***unique*** 3D bulk state.
- Explicit realisation of the "*Sandwich construction*".
- With ***maximal*** symmetry, the bulk captures CFT dynamics.

Generalized symmetries:
Gaiotto, Kapustin, Seiberg, Willett 2014;
Non-invertible symmetries:
Verlinde 1988;
Moore, Seiberg 1988-1989;
Petkova, Zuber 2000,
(Frohlich), Fuchs, Runkel, Schweigert 2002-2006;
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Chang, Lin, Shao, Wang, Yin 2018;
Thorngren, Wang 2019;
Sandwich and SymTFT:
Kong, Zheng 2019;
Ji, Wen 2019;
Gaiotto, Kulp 2020;
Apruzzi, Bonetti, Garcia Etxebarria, Hosseini, Schafer-Nameki 2021;
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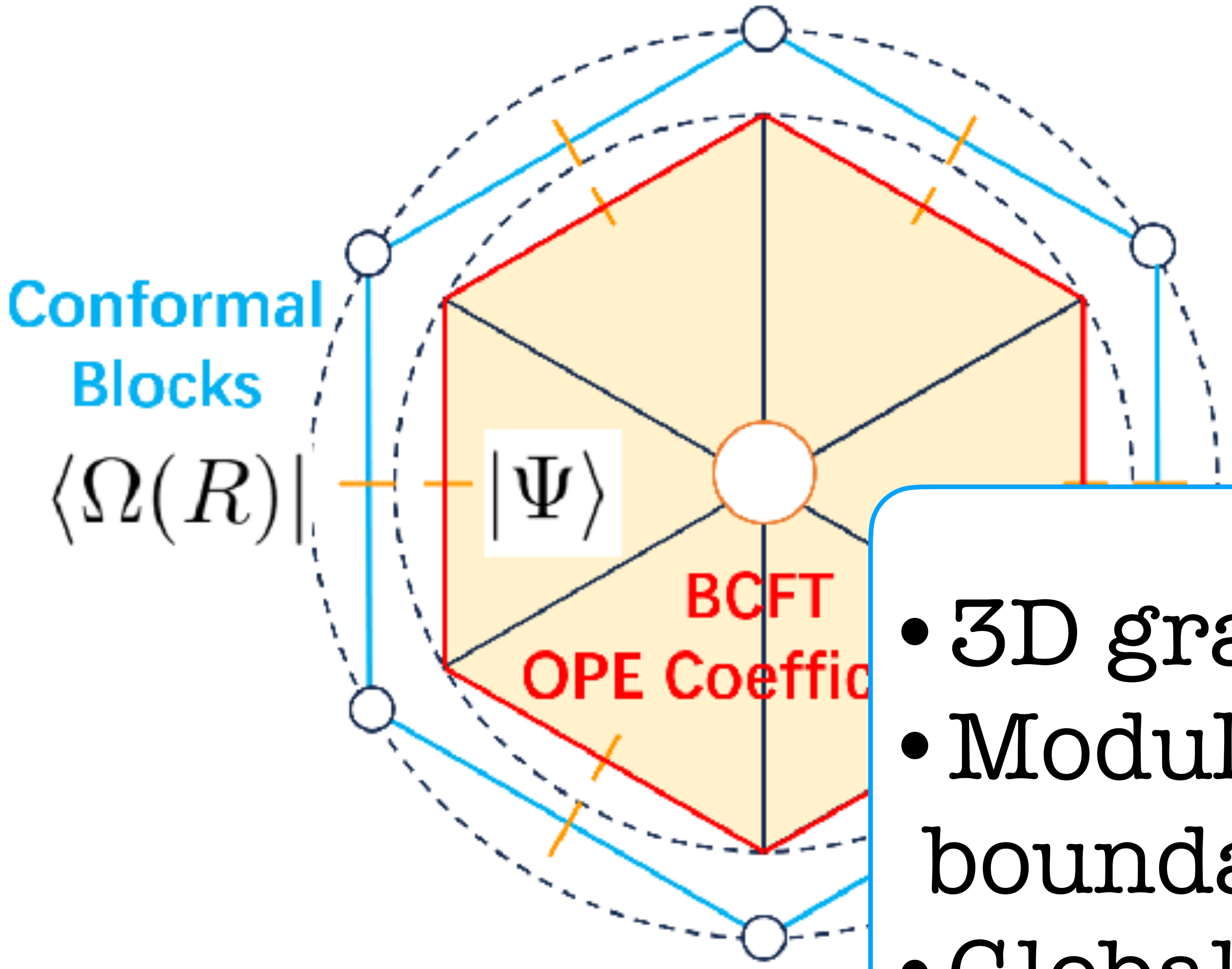
Reviews:
Cordova, Dumitrescu, Intriligator, Shao 2022; Schafer-Nameki 2023; Shao 2023; McGreevy 2022

3D bulk and *Sandwich*

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2022

$$\mathcal{Z}_{2D} = \langle \Omega | \Psi \rangle_{3D}$$



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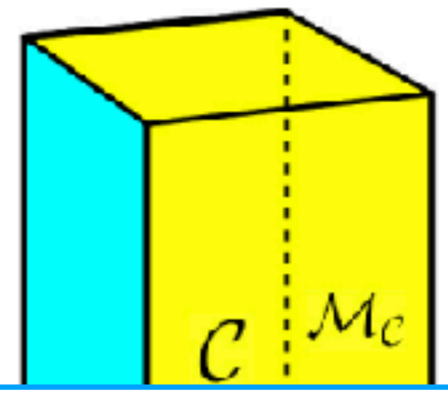

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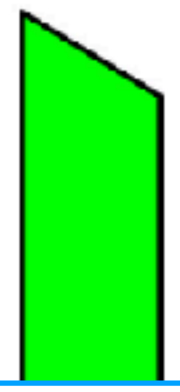

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Step 3: Liouville Theory and Quantum Gravity

Liouville CFT - Closing Holes

Lin Chen, LYH, Yikun Jiang, Bingxin Lao 2024

$$\Phi_{\alpha}^{\sigma_1\sigma_2}(x)$$

$$\sigma \equiv Q/2 + iP_\sigma, \qquad \alpha \equiv Q/2 + iP_\alpha$$

A self-consistent collection of conformal boundary conditions

Fateev, Zamolodchikov, Zamolodchikov 2000;
Teschner 2000;

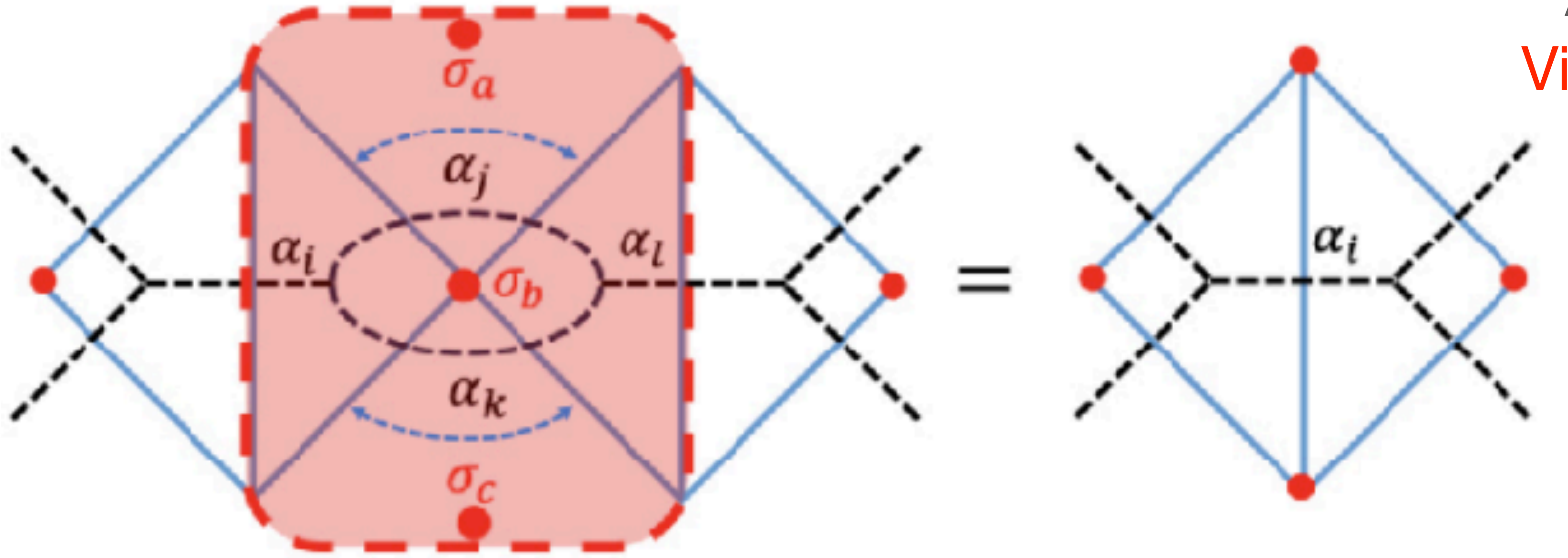
$$Q = b + b^{-1} \qquad c = 1 + 6Q^2$$

$$C^{\sigma_3,\sigma_1,\sigma_2}_{\alpha_1,\alpha_2,\alpha_3} = (\mu(P_{\alpha_1})\mu(P_{\alpha_2})\mu(P_{\alpha_3}))^{1/4} \begin{Bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \sigma_3 & \sigma_1 & \sigma_2 \end{Bmatrix}_b .$$

Ponsot, Teschner 2001;
Chen, LYH, Jiang, Lao 2024;

$$\mathcal{U}_q(SL(2,\mathbb{R})) \qquad q = e^{\pi i b^2}$$

Also Crossing Kernel of Virasoro conformal blocks



$$\mu(P) = 4\sqrt{2}\sinh(2\pi P b)\sinh\left(\frac{2\pi P}{b}\right).$$

Weight the hole with the Plancherel measure (Cardy density of states)

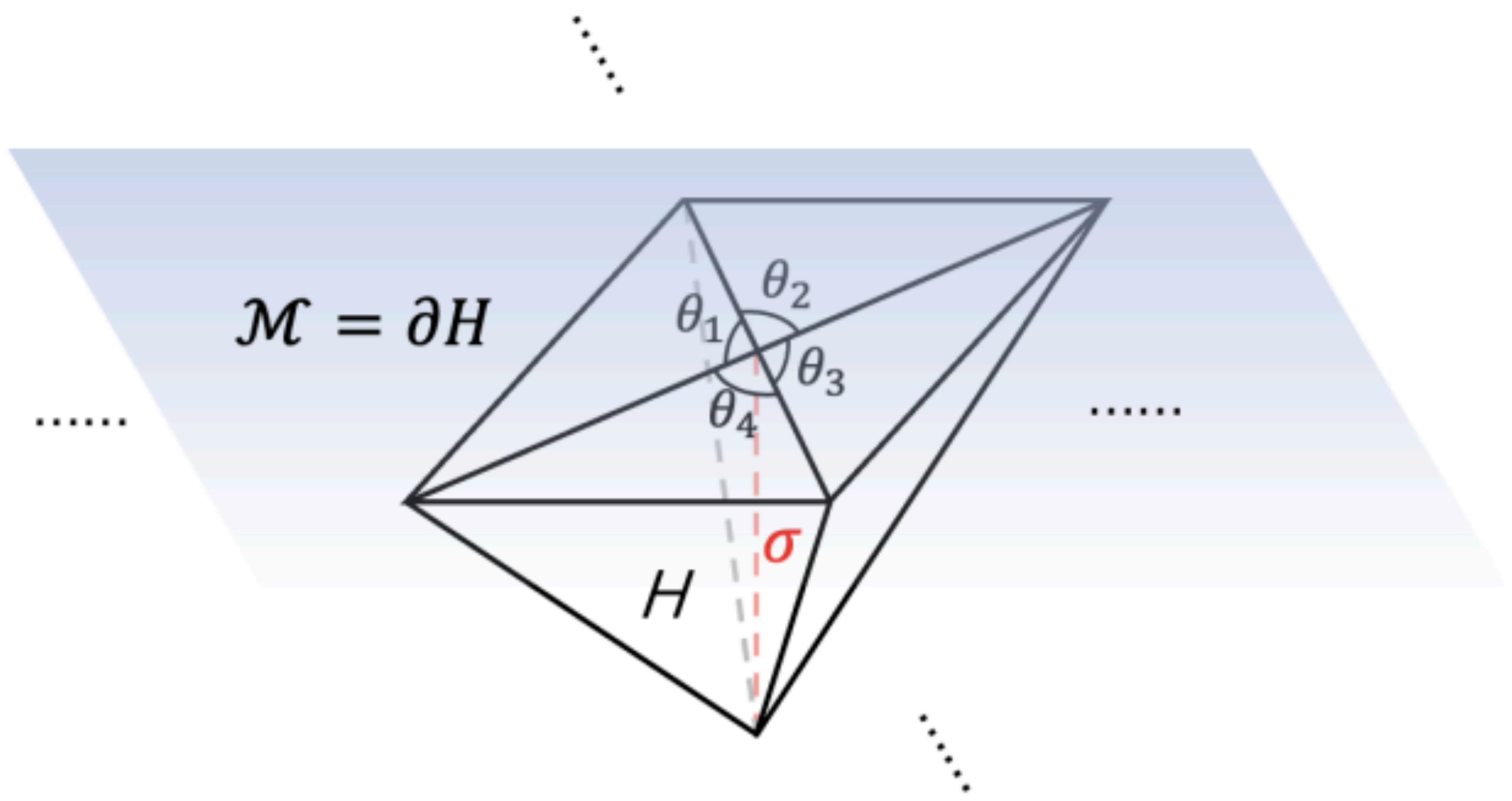
Reading off the bulk

Lin Chen, LYH, Yikun Jiang, Bingxin Lao 2024

$$Z_{\text{Liouville}}(R) = \langle \Omega(R) | \Psi \rangle$$

$$\Psi(\{\alpha_i\}) = \prod_v \int_0^\infty dP_{\sigma_a} \mu(P_{\sigma_a}) \prod_e \sqrt{\mu(P_{\alpha_i})} \prod_\triangle \begin{Bmatrix} \alpha_i & \alpha_j & \alpha_k \\ \sigma_c & \sigma_a & \sigma_b \end{Bmatrix}_b$$

$$\Omega(\{\alpha_i\})(R) = \prod_e \sum_{\{I_i\}} \prod_\triangle \left(\tilde{\gamma}^{\alpha_i \alpha_j \alpha_k}_{I_i I_j I_k}(R) \right)$$



Reading off the bulk in the large c limit

$$P = \frac{l}{2\pi b}$$

$$\begin{Bmatrix} \frac{Q}{2}+i\frac{l_4}{2\pi b} & \frac{Q}{2}+i\frac{l_5}{2\pi b} & \frac{Q}{2}+i\frac{l_6}{2\pi b} \\ \frac{Q}{2}+i\frac{l_1}{2\pi b} & \frac{Q}{2}+i\frac{l_2}{2\pi b} & \frac{Q}{2}+i\frac{l_3}{2\pi b} \end{Bmatrix}_b \xrightarrow[b\to 0]{} \exp\left(-\frac{\mathrm{Vol}(T\{l_i\})+\sum_i l_i\theta_i/2}{\pi b^2}\right),$$

Kashaev 1997;
Teschner, Vartanov 2012;
Collier, Eberhardt, Zhang 2023;
Chen, LYH, Jiang, Lao 2024;

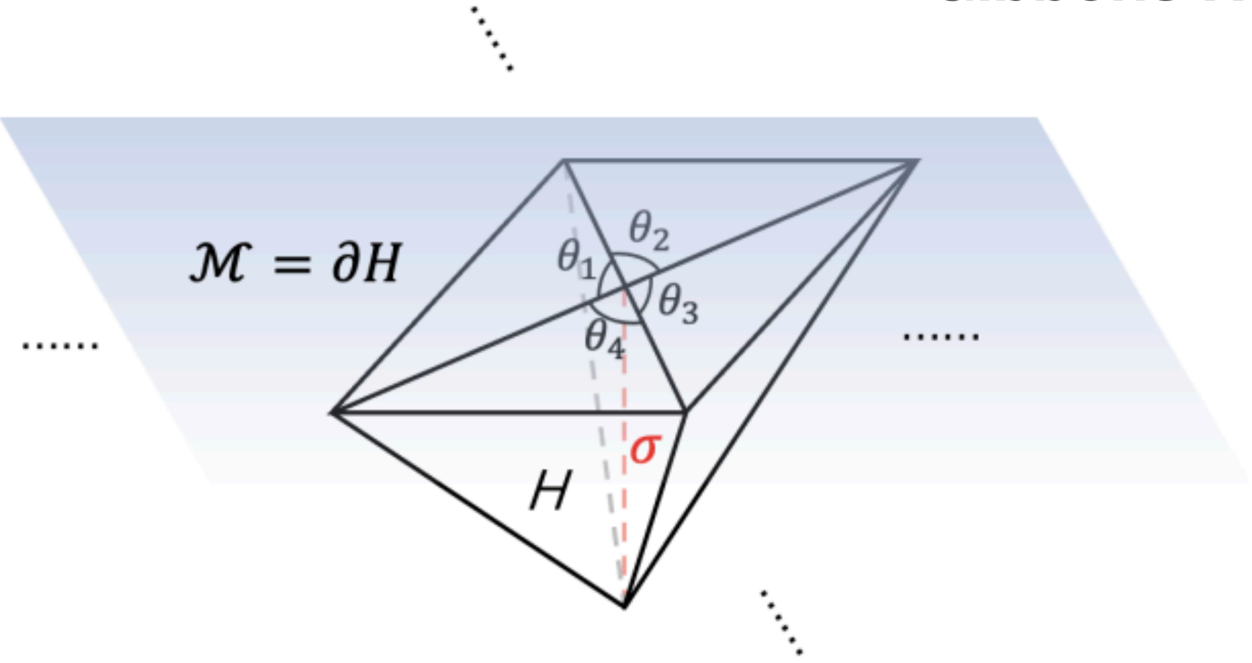
Large c limit reduces to on-shell Einstein-Hilbert action on hyperbolic tetrahedron.

$$S_{EH} = -\frac{1}{16\pi G_N}\int_H d^3x\sqrt{g}(R-2\Lambda) = \frac{V(H)}{4\pi G_N} = \frac{V(H)}{\pi b^2},$$

$$\sum_i \frac{\theta_i l_i}{2\pi b^2} = \sum_i \frac{\theta_i l_i}{8\pi G_N} = \frac{1}{8\pi G_N}\sum_i \int_{\Gamma_i} \theta_i\sqrt{h},$$

Gibbons Hawking/Hayward Term

BCFT primary labels ⟺ Bulk geodesic lengths



At general c, this is Virasoro TQFT, folded

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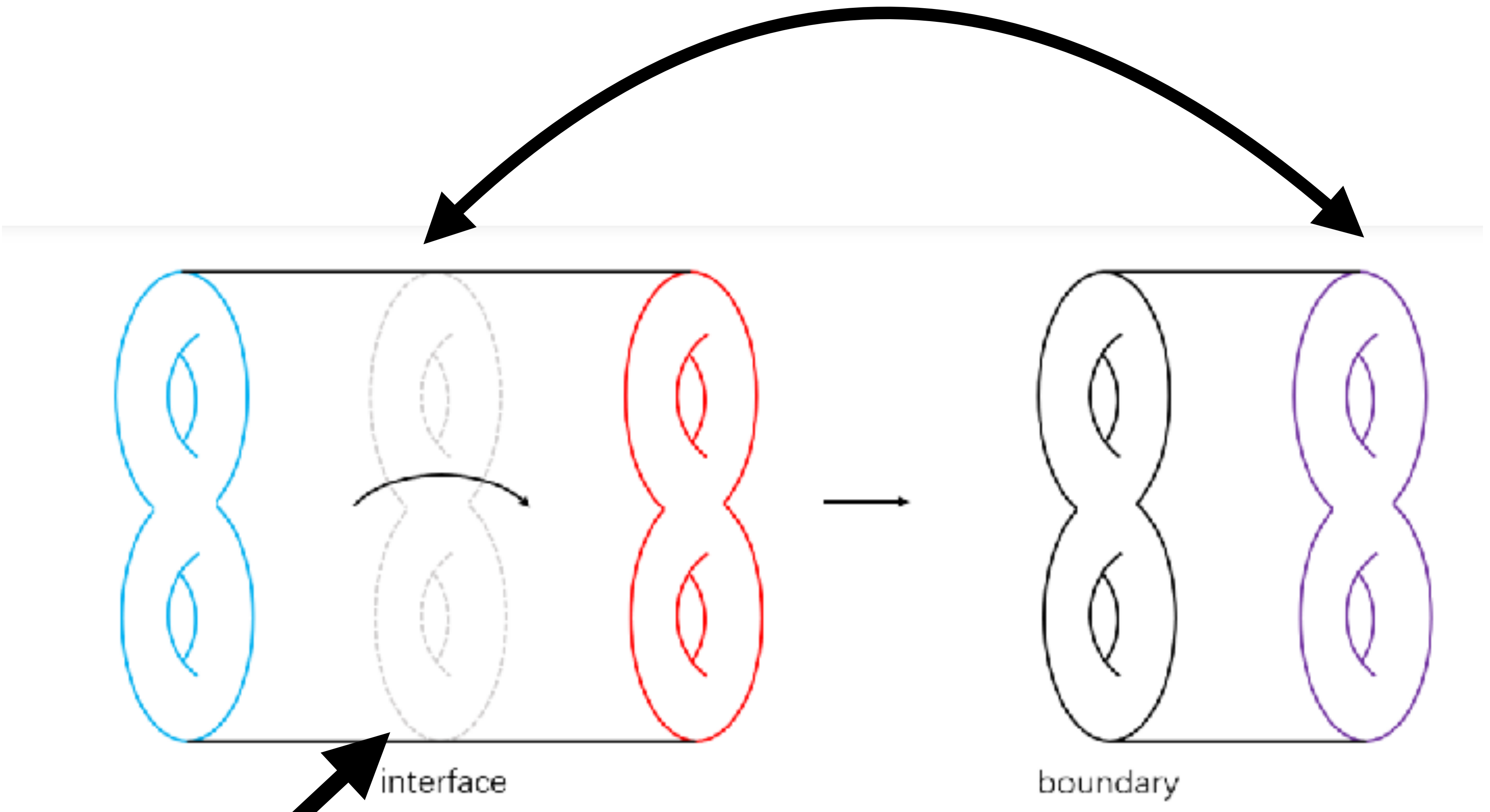
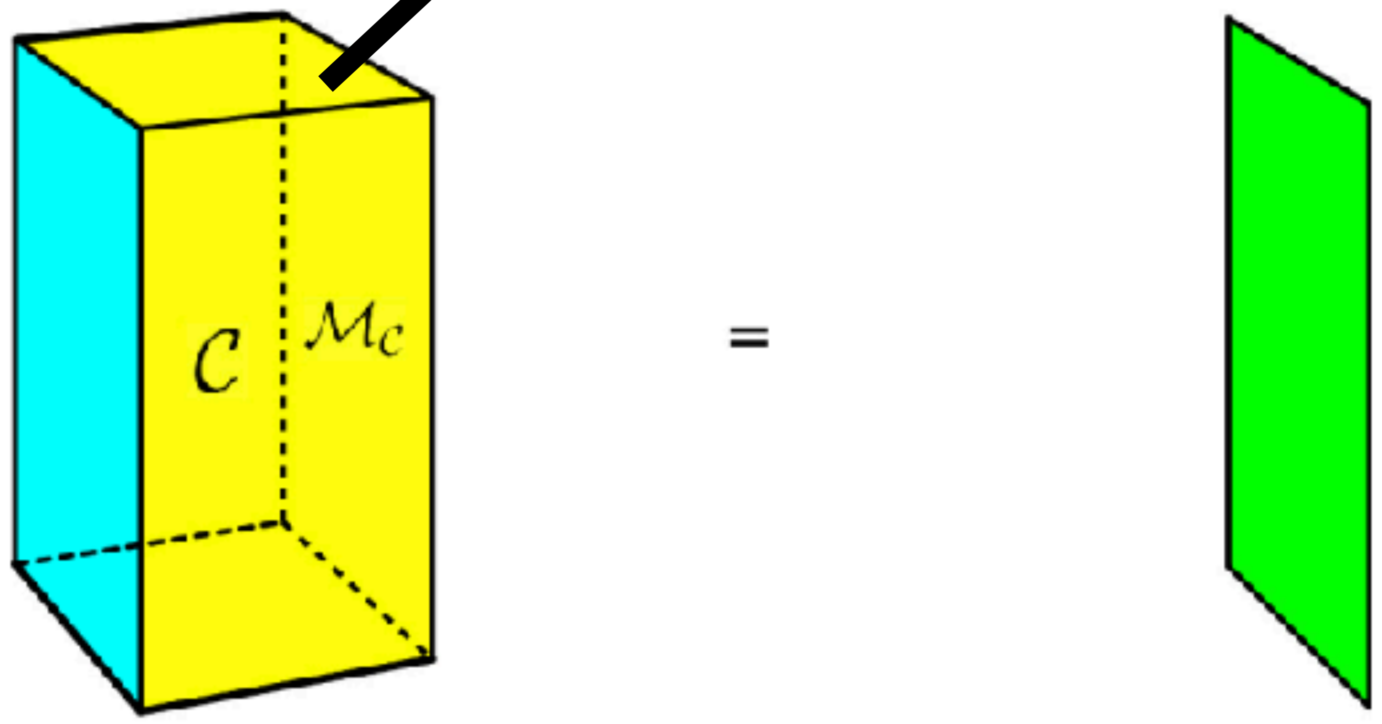


Andersen, Kashaev 2011
Collier, Eberhardt, Zhang 2023;
Chen, LYH, Jiang, Lao 2024;

Integrals over P = sum over geometries.

Liouville CFT is a non-perturbative sum over quantum geometries.

CFT spectrum/bulk quantum measure controlled by the 3D topological boundary



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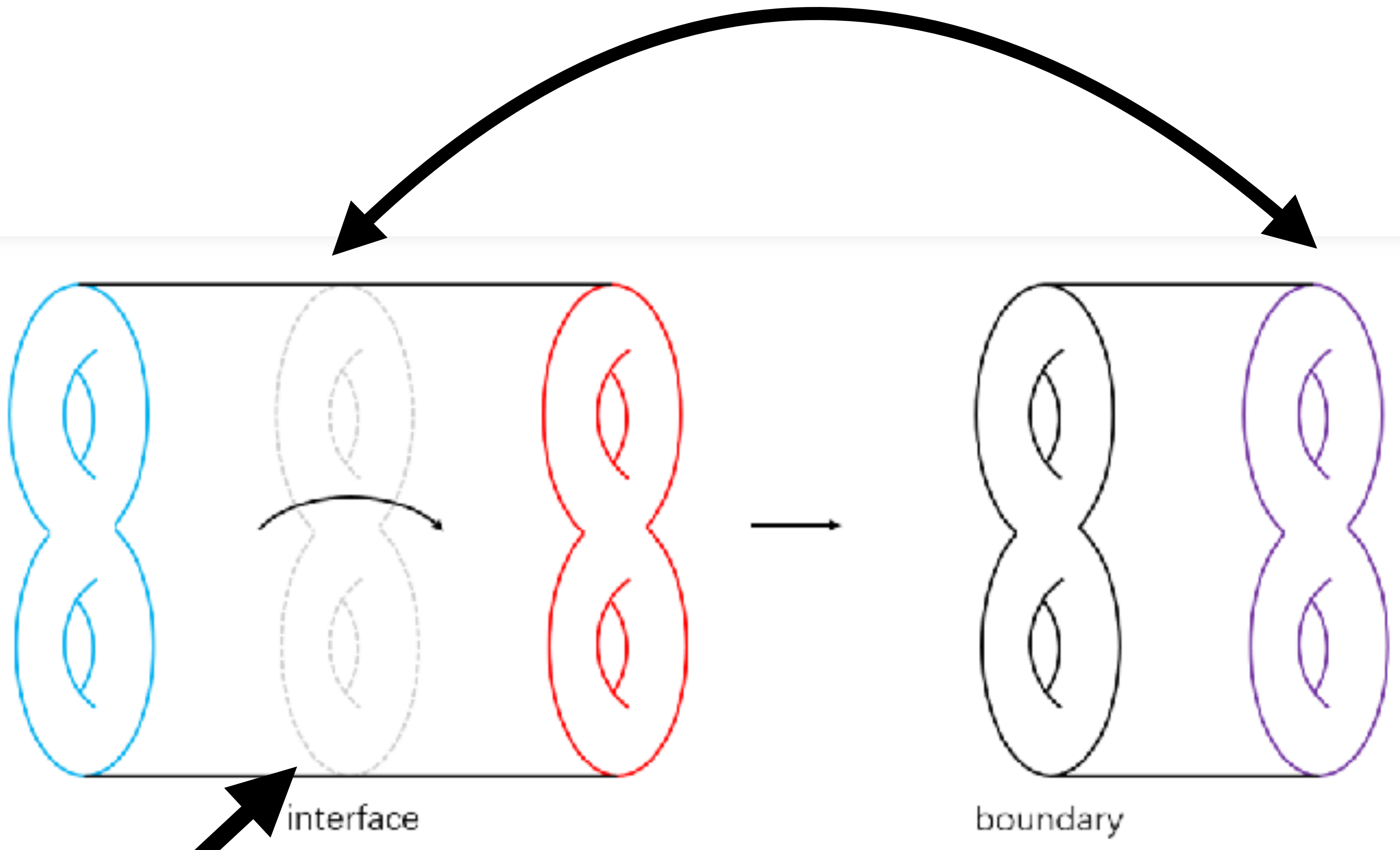

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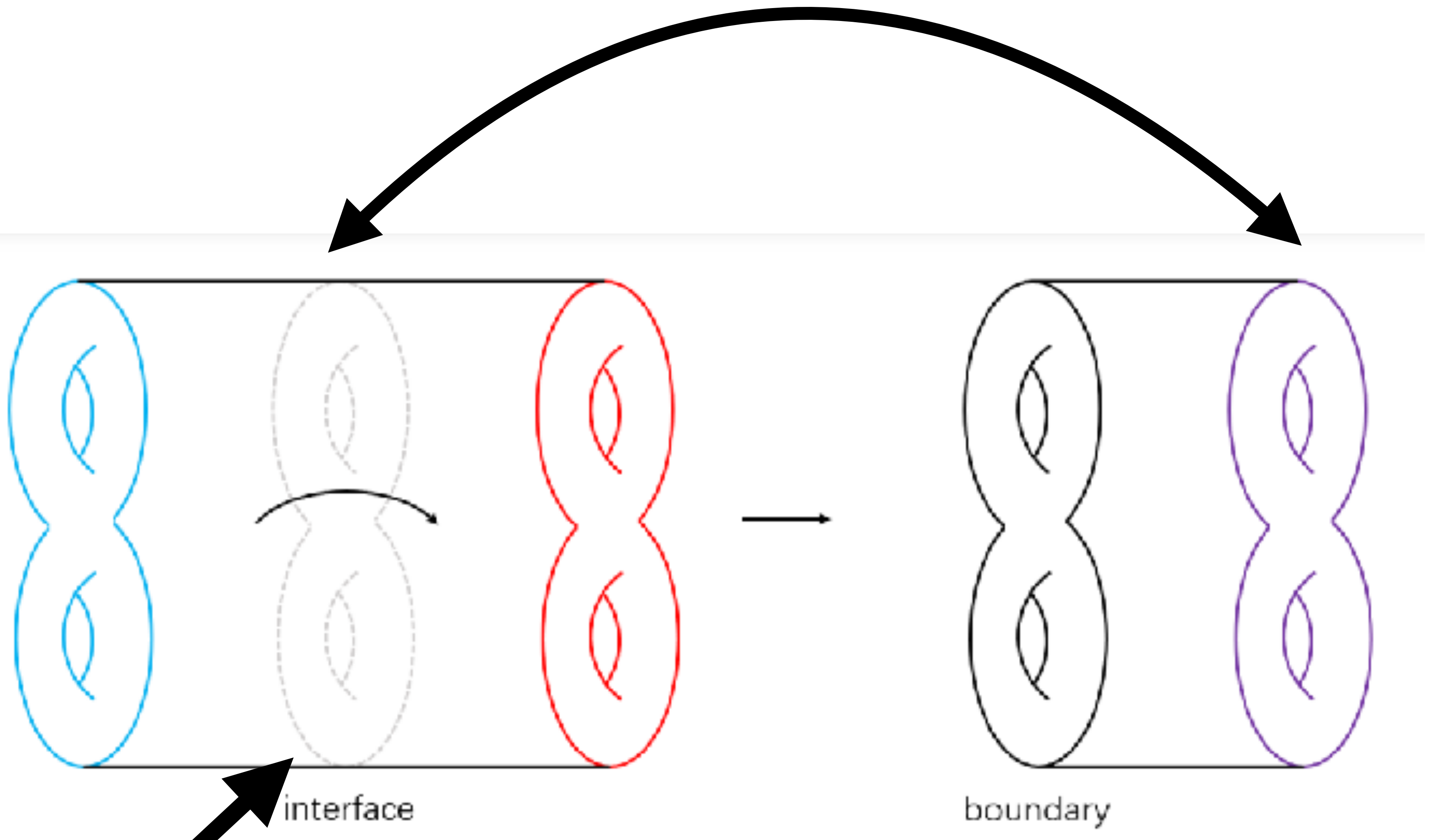

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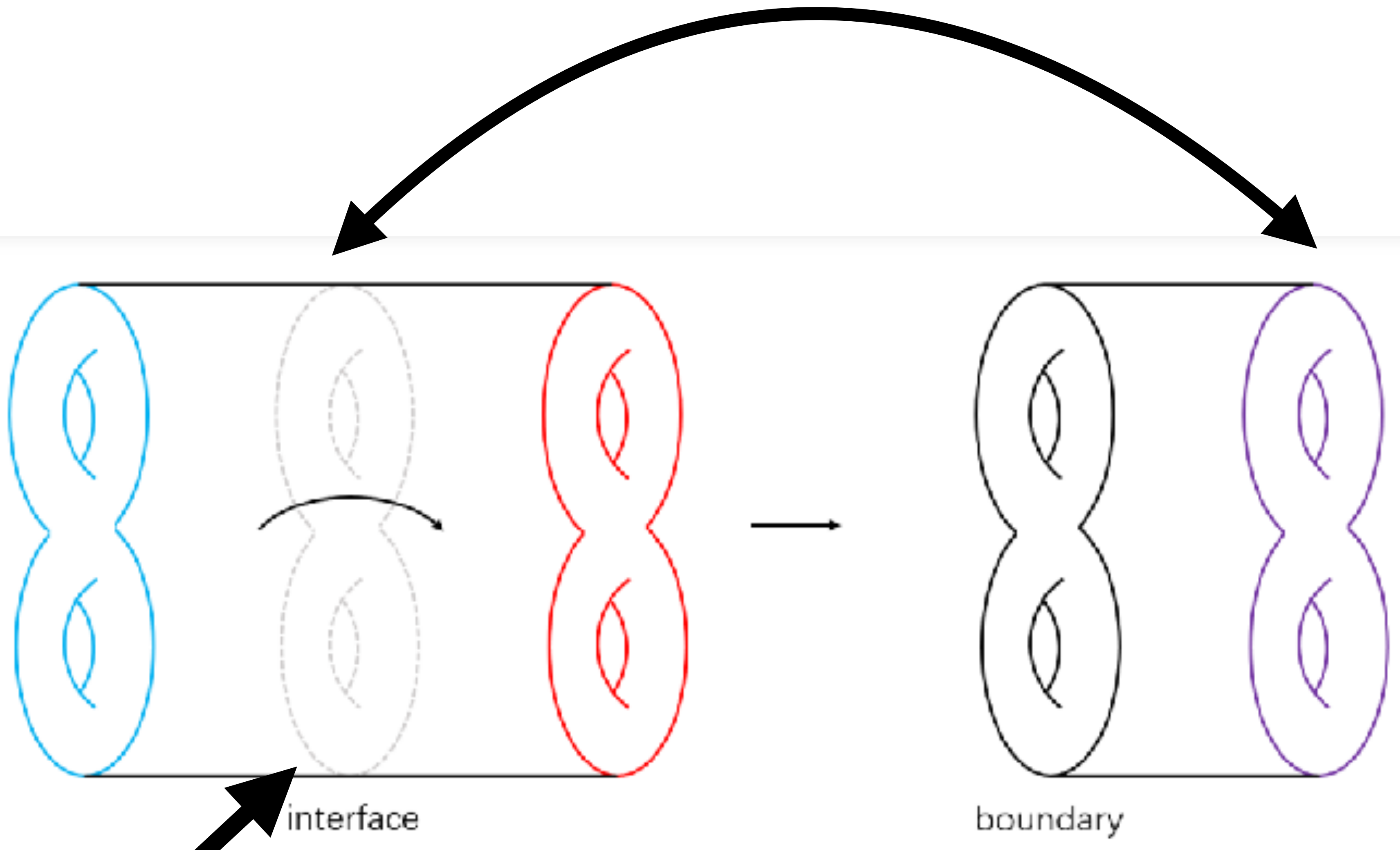

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Liouville theory is one example. We believe this framework applies quite generally.

Step 4 (Virasoro) SymQRG= QG $\supset T\bar{T}$ and the Wheeler-DeWitt equation

Wheeler-DeWitt equation and SymQRG

Ning Bao, LYH, Yikun Jiang, Zhihan Liu, 2024

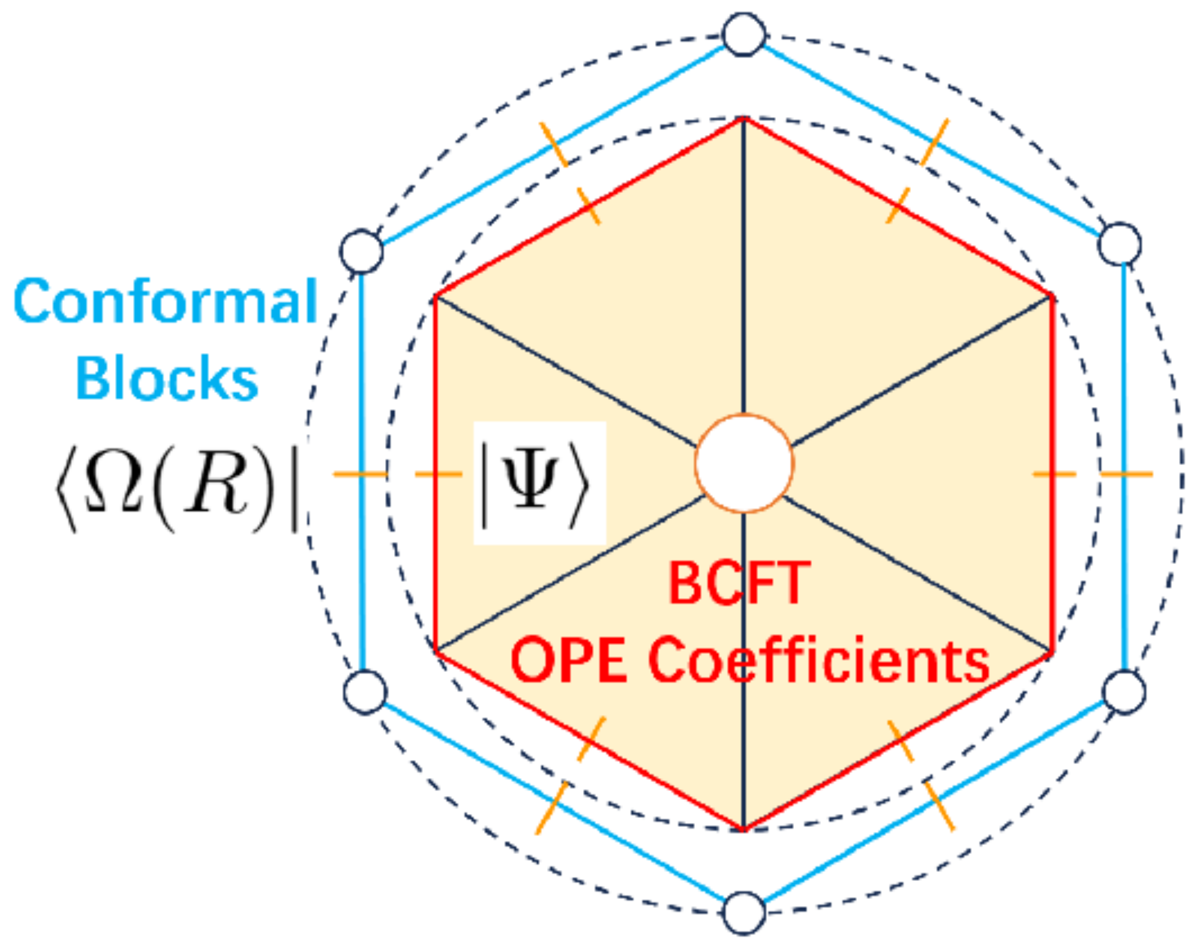
- AdS/CFT: CFT partition function= QG Wheeler-DeWitt wavefunction

de Boer, Verlinde, Verlinde 1999; Freidel 2008; Lee 2013; McGough, Mezei, Verlinde 2016;

$$\mathcal{Z} =_\Lambda \langle \Omega | \Psi \rangle_\Lambda$$

triangulation lattice scale provides natural RG scale

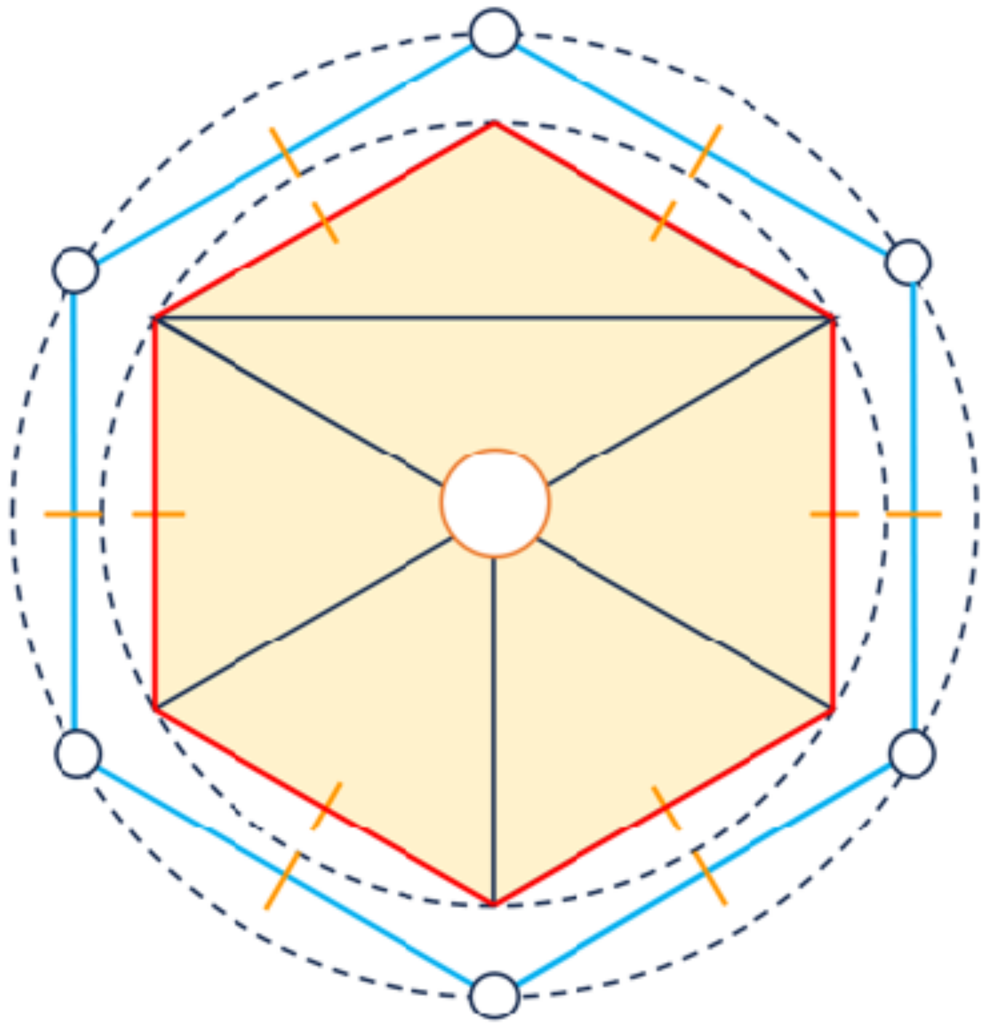
- Wheeler-DeWitt equation: no-flux constraint $\hat{\mathcal{H}} |\Psi\rangle_\Lambda = 0$



$$|\Psi\rangle_\Lambda = U^{\Lambda\Lambda'}_{\mathrm{SymQRG}} |\Psi\rangle_{\Lambda'}$$

Re-triangulation

CFT crossing



- RG operator $U^{\Lambda,\Lambda'}_{\mathrm{SymQRG}}$ preserves ***all*** generalised symmetries, explicitly expressed as quantum 6j symbols (tetrahedra) — 3D path-integral between two radial slices.

Wheeler-DeWitt equation and SymQRG

Ning Bao, LYH, Yikun Jiang, Zhihan Liu, 2024

For more details, please see Yikun Jiang's poster.

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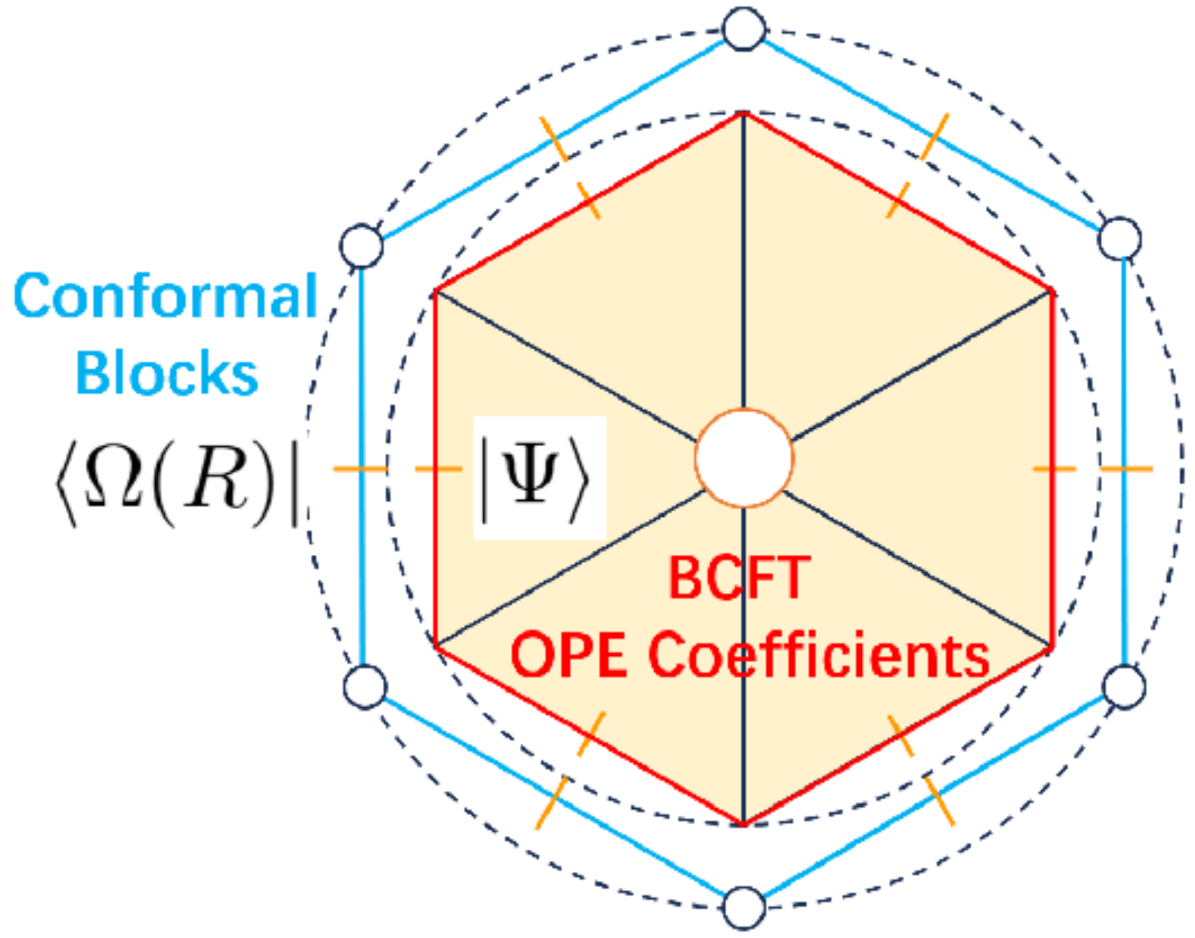

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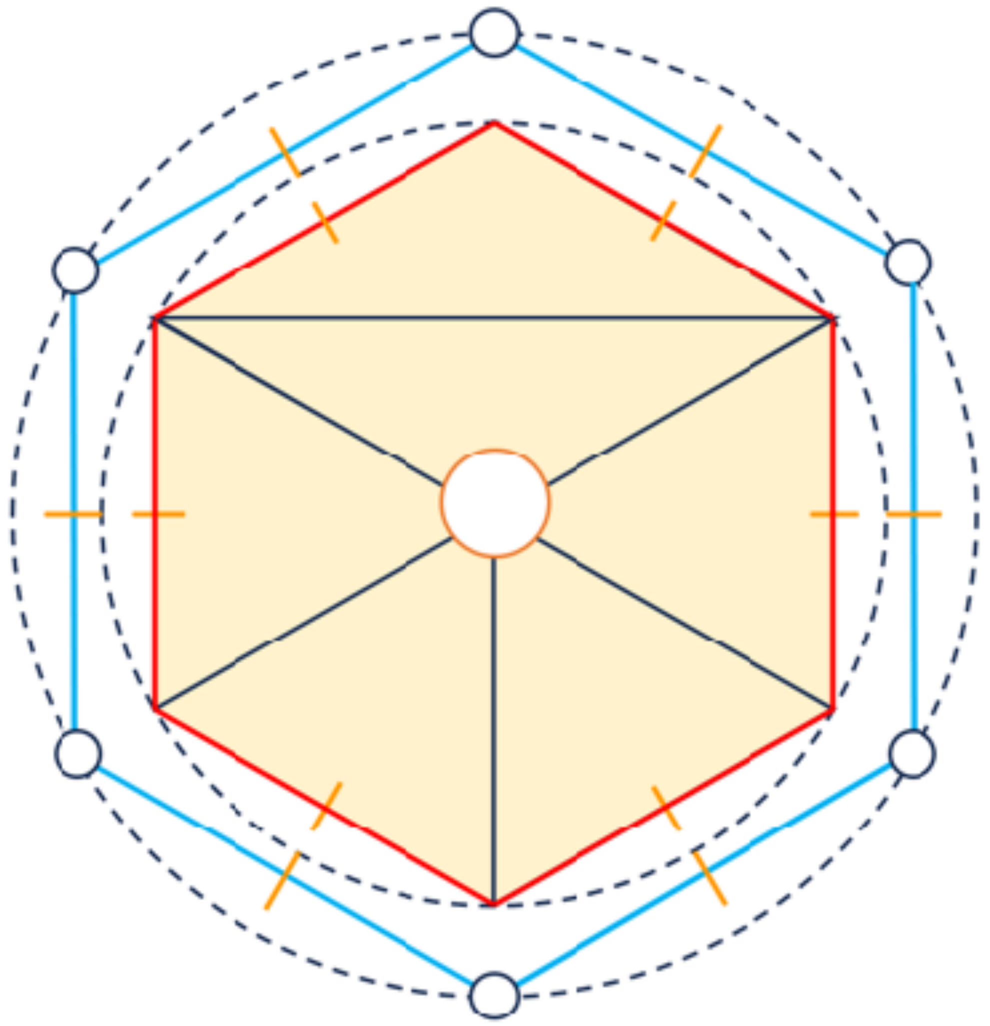

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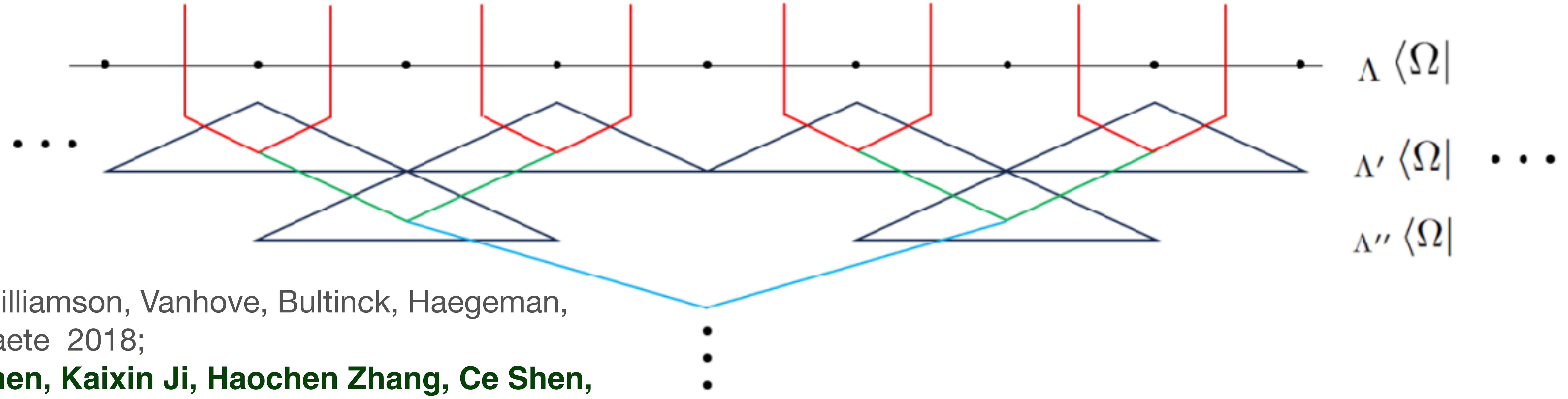
$U^{\Lambda,\Lambda'}_{\mathrm{SymQRG}}$ is a holographic tensor network

Coarse-graining

UV/IR correspondence

Susskind, Witten 1998

Deeper in the bulk



Bal, Williamson, Vanhove, Bultinck, Haegeman, Verstraete 2018;
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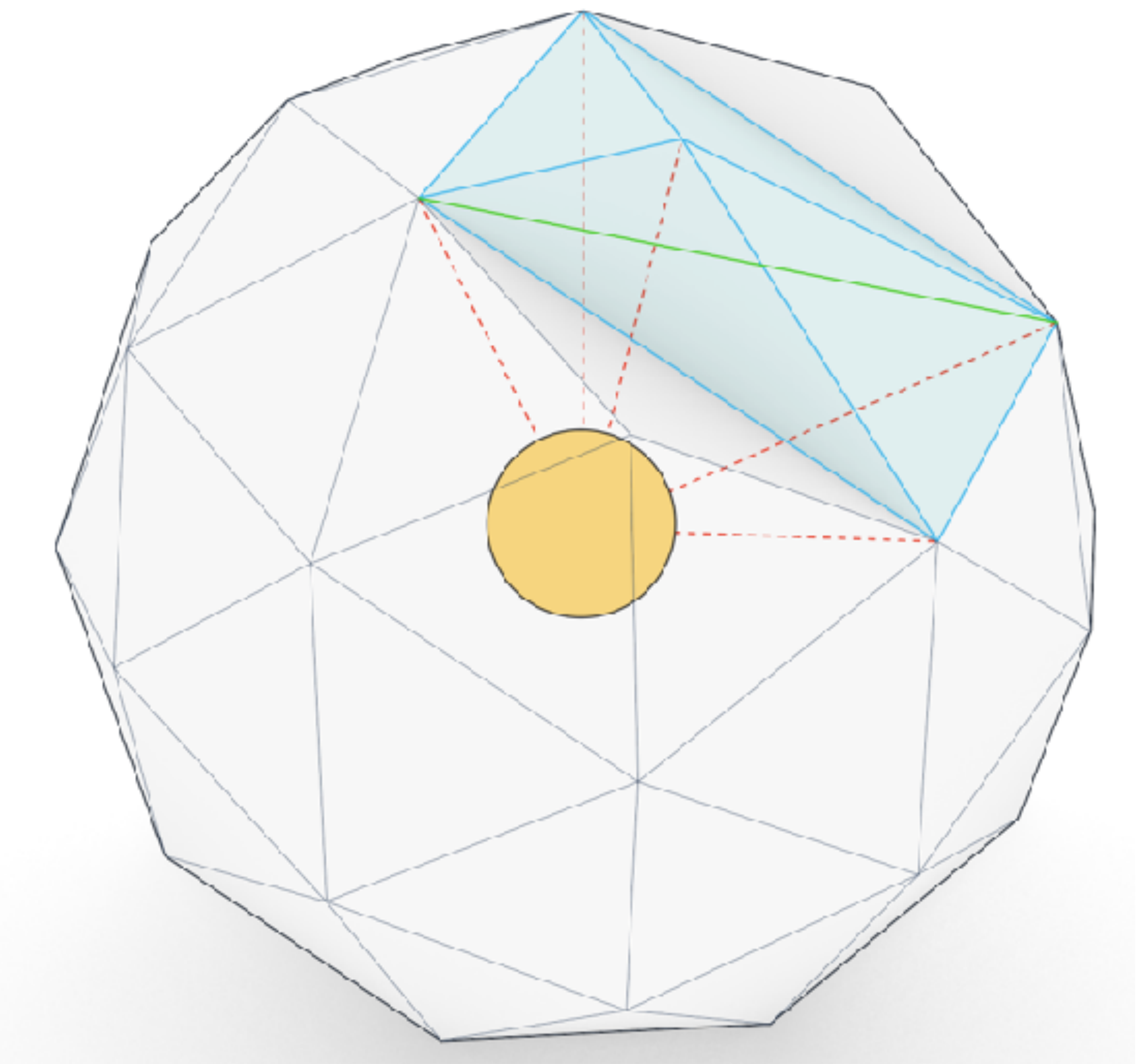

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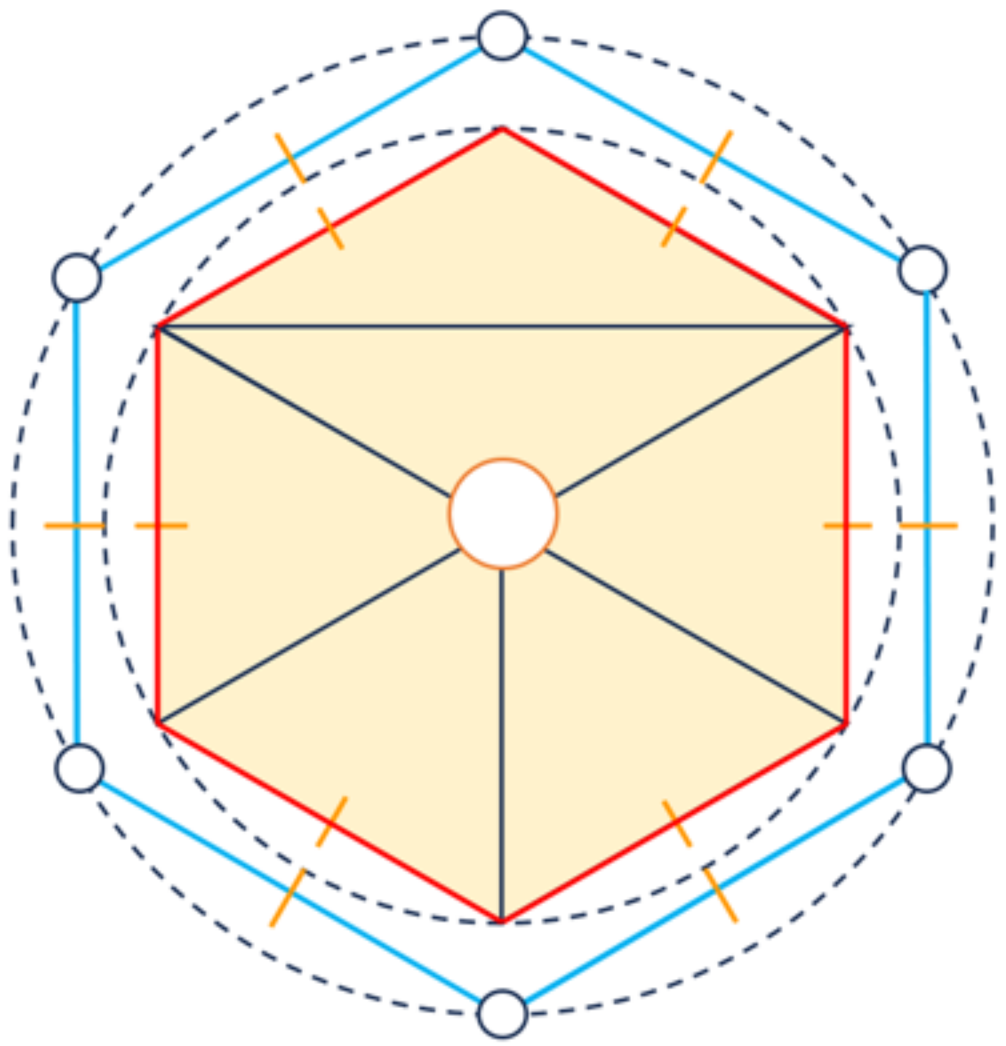

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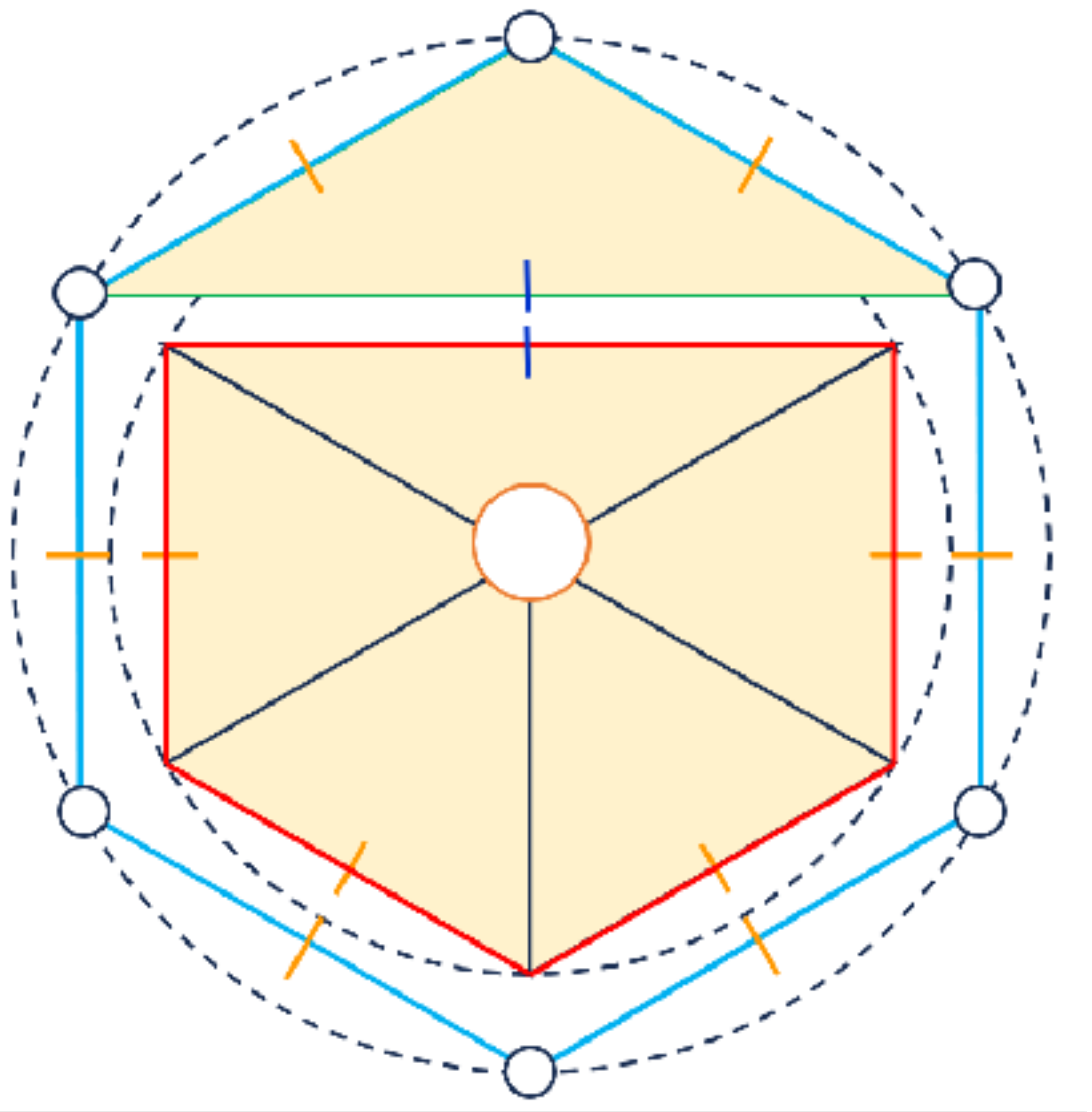
Symmetry-preserving real space RG transform



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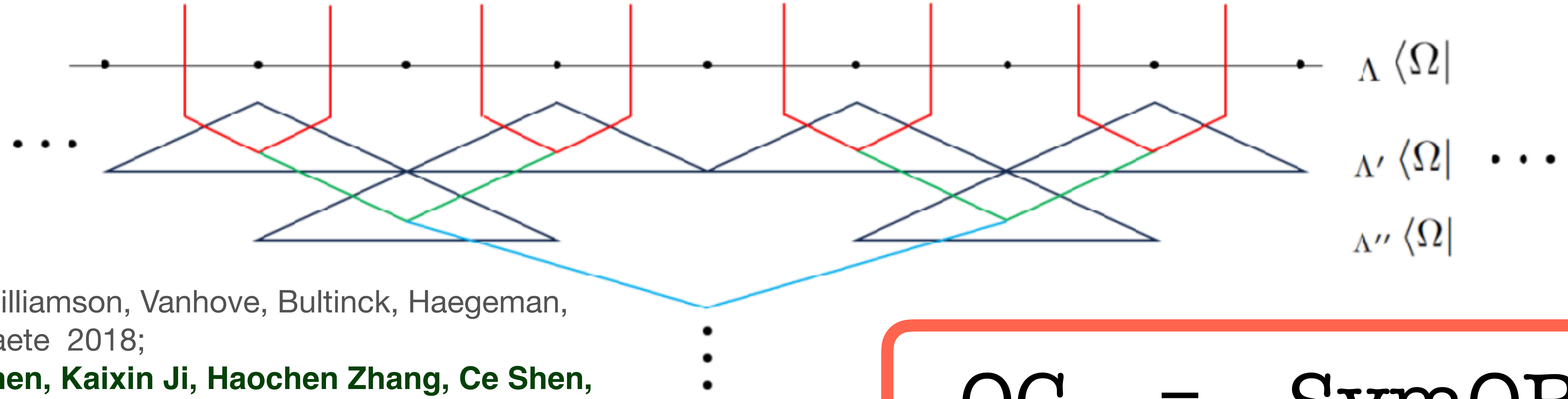

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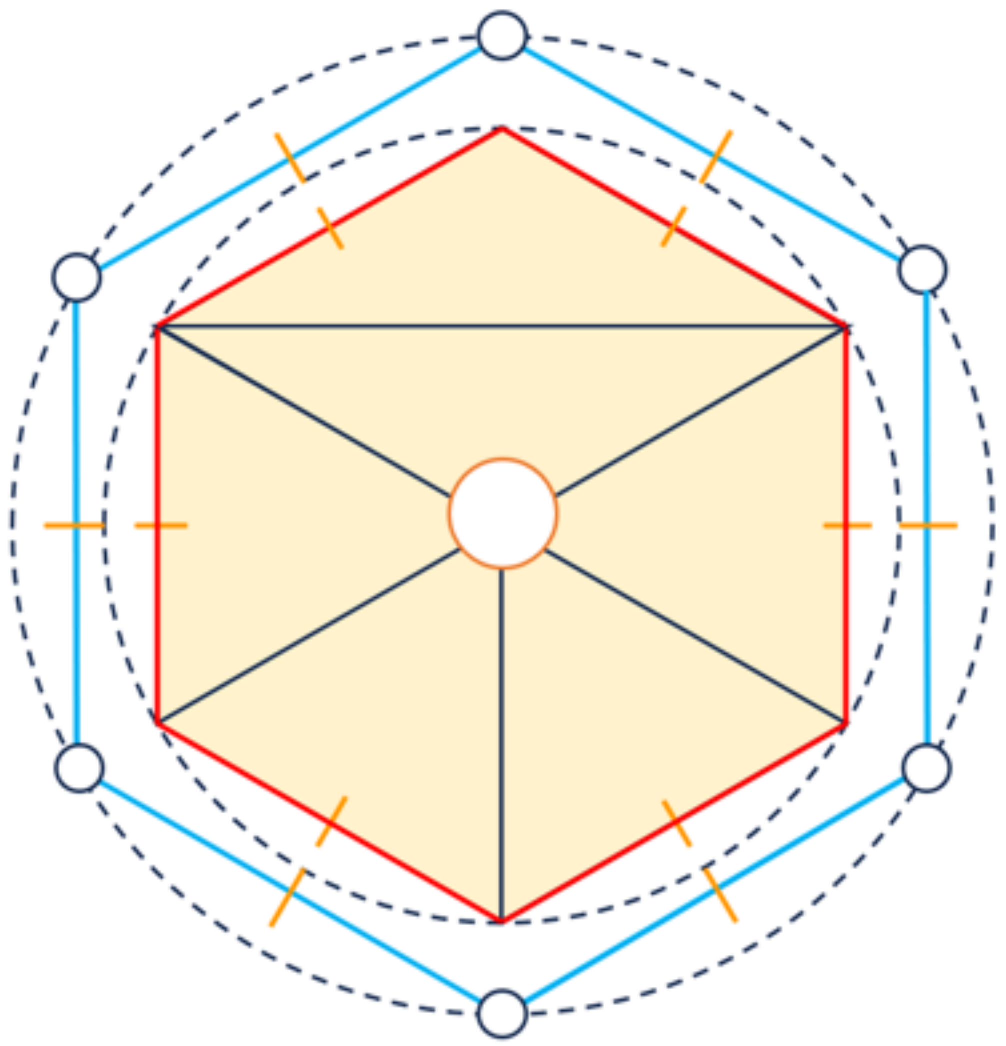

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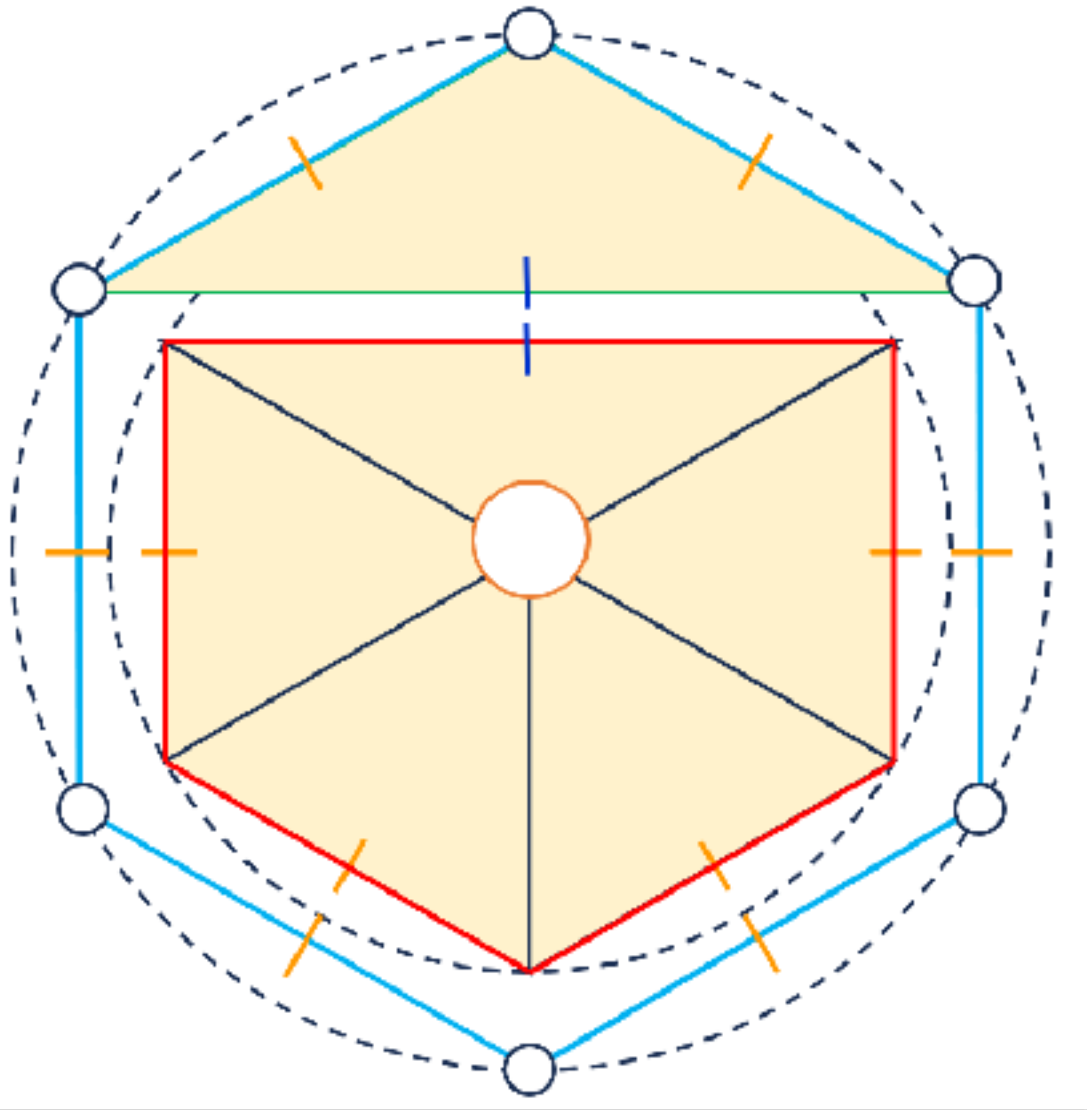
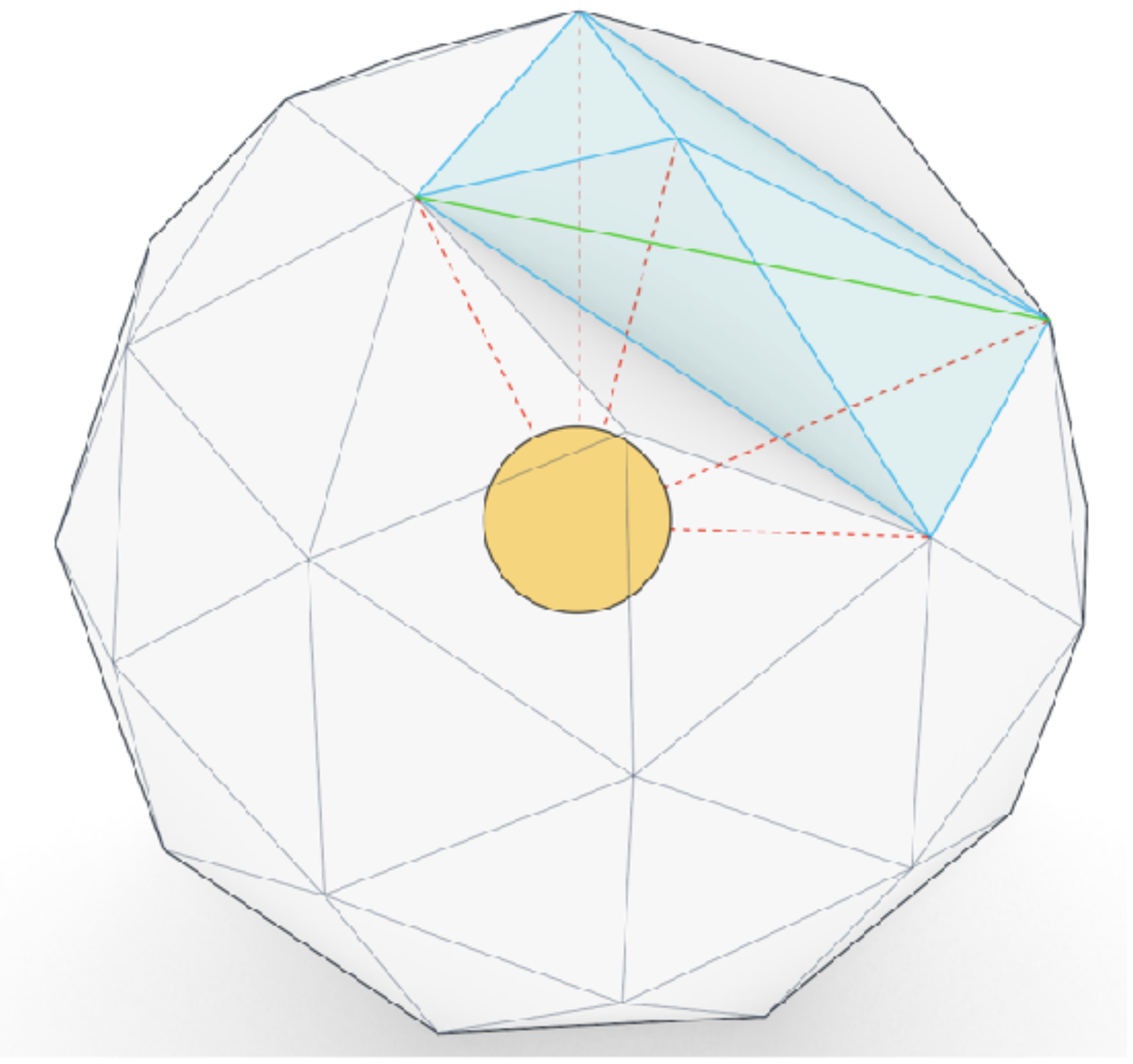




TTbar deformation and Callan-Symanzik equation

B. Zamolodchikov 2004; Brehm, Runkel 2022, 2024

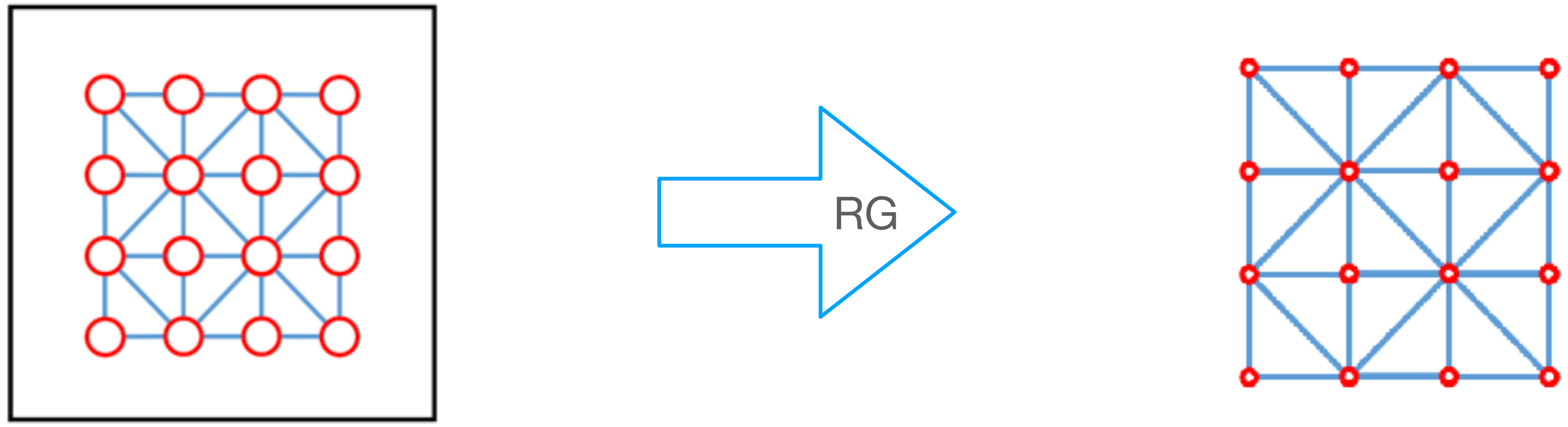
Ning Bao, LYH, Yikun Jiang, Zhihan Liu, 2024

TTbar deformation: changing size R of the holes near fixed point.

- States on the holes at finite radius

$$|e;R\rangle\rangle \;=\; R^{-\frac{c}{6}}\Big(|0\rangle + R^4\,\tfrac{2}{c}\,L_{-2}\overline{L}_{-2}|0\rangle + (\text{higher powers of } R)\Big)\;,$$

R plays the role of TTbar coupling



Ratio R/Λ flows under this coarse graining

- Callan-Symanzik equation from this flow agrees with field theoretic analysis.

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Conclusion and Outlook

Wishlist for Holographic Tensor Network

What we wish to have:

- Exact CFT: kinematics + *dynamics*
- Graph/Background independence
- Emergence of geometries from CFT data
- Einstein-Hilbert action + UV completion
- Sum over geometries (precise quantum measure)
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- Exact CFT: kinematics + *dynamics*
- Graph/Background independence
- Emergence of geometries from CFT data
- Einstein-Hilbert action + UV completion
- Sum over geometries (precise quantum measure)
- RG flow : Callan-Symanzik + Wheeler-DeWitt equations
- Non-perturbative boundary-to-bulk map
- Conformal bootstrap (crossing and modular invariance) and bulk emergence
- *Generalised global symmetries* realised as *bulk gauge symmetries*

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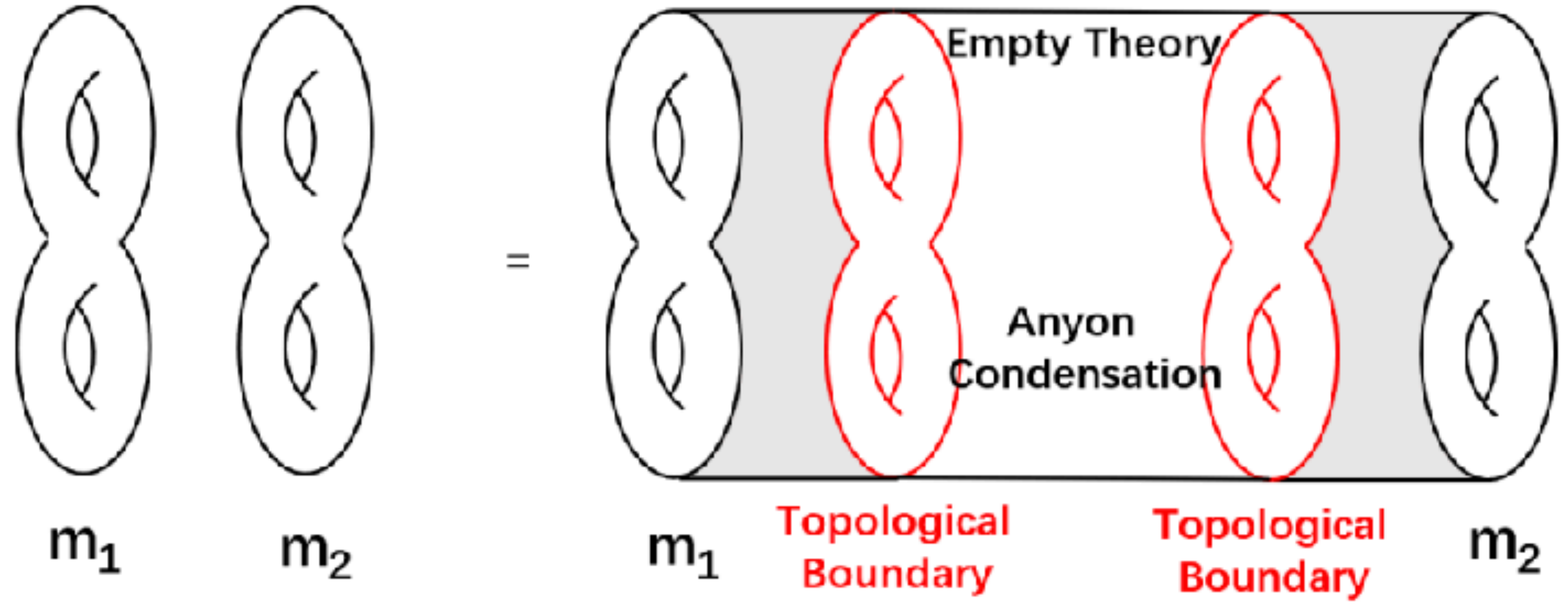
Conclusion and Outlook

- Exact triangulation of the CFT allows one to construct explicit tensor networks.

- ***Maximal*** set of generalized symmetries constrain the dynamics sufficiently that it allows one to reconstruct the bulk precisely. We believe this is a general feature of AdS/CFT.

- Does **NOT** require sum over **all** possible geometries — the sum is tailored for each CFT for an exact correspondence. Quantum measures can vary, corresponding to different individual CFTs or ensemble averages.

Implication: Factorization puzzle.
Topological boundary provides a precise bulk quantum measure that allows two unrelated CFTs to factorize.



Maldacena, Maoz;
Marolf, Maxfield;
Saad, Shenker, Stanford, Yao;
Benini, Copetti, Di Pietro;
Blommaert, Iliesiu, Kruthoff; …..

Conclusion and Outlook

- AdS3 Black Holes? — ZZ boundary states ? — Chua, Jiang; Mertens; Mertens, Turiaci, Verlinde
- AdS/BCFT — Takayanagi...
- Other theories: e.g. symmetric orbifolds — Eberhardt, Gaberdiel, Gopakumar...
- Ensemble Averages — Belin, de Boer; Stanford; Belin, de Boer, Jafferis, Nayak, Sonner; Chandra, Collier, Hartman, Maloney; Jafferis, Rozenberg, Wong;

open structure coefficients - sum over loops emerges.
Heavy operators become generalised free fields. — 2503.XXXXX with Lin Chen, Yikun Jiang, Bingxin Lao

- Lower Dimensions : 2D lattice gauge theory and JT gravity — Blommaert, Mertens, Verschelde; 2503.XXXXX with Hao Geng, Yikun Jiang, Lin Chen
- DSSYK and de Sitter? — Berkooz, Isachenkov, Narovlansky, Torrents; Lin, Stanford; Verlinde; Susskind;….
- Higher Dimensions? — Reviews: Cordova, Dumitrescu, Intriligator, Shao; Schafer-Nameki; Shao; McGreevy

3+1 D toric code with different boundary conditions can produce 3d phase transitions

Chen, Ji, Zhang, Shen, Wang, HLY 2023 ; Kaixin Ji, Lin Chen, Liping Yang, HLY 2024

- Lorentzian evolution and HRT formula — Hartman, Maldacena; Hubeny, Rangamani, Takayanagi;
- More general non-conformal theories

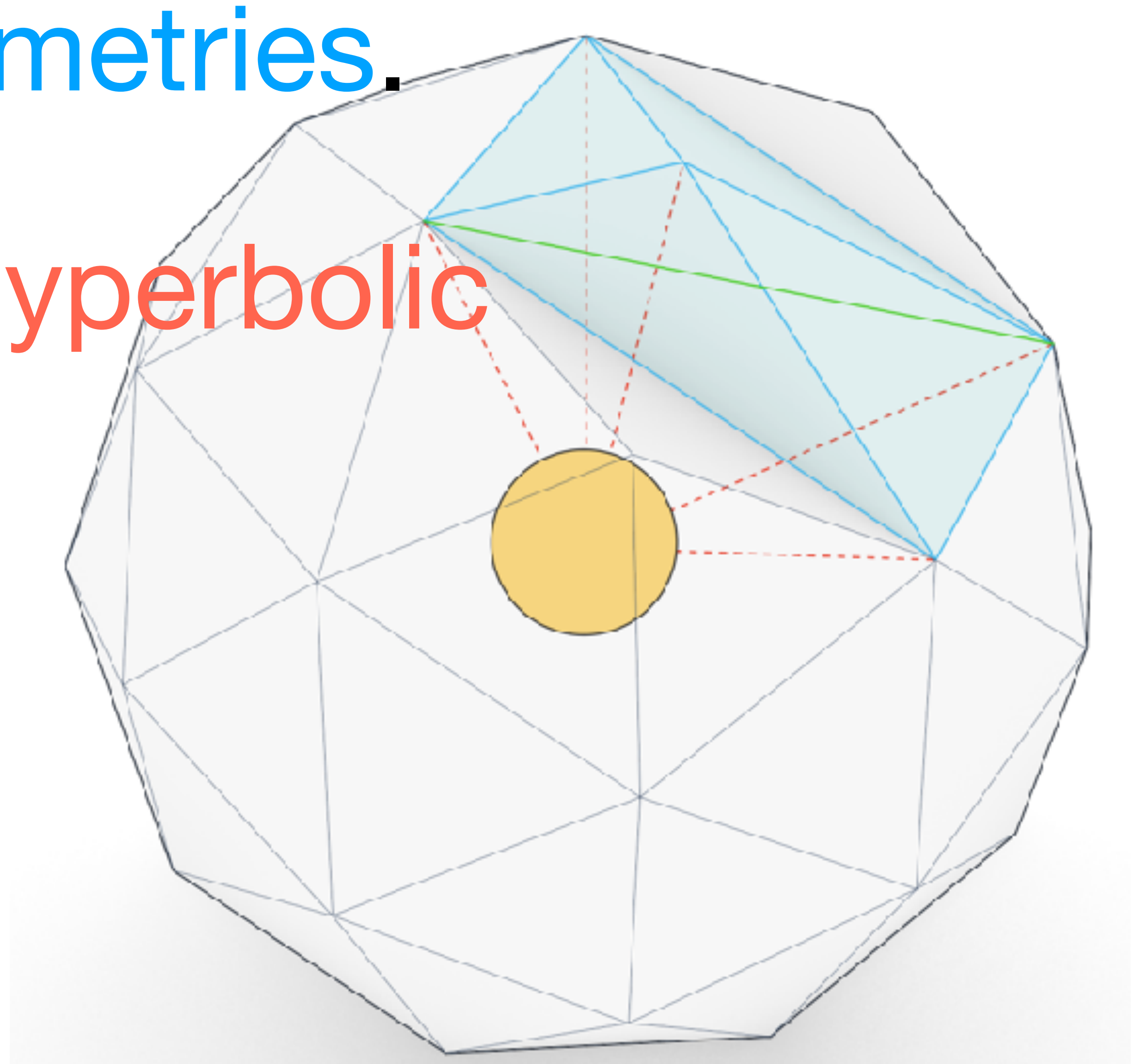
Summary

$$\mathcal{Z}_{\rm 2D} = \langle \Omega | \Psi \rangle_{\rm 3D}$$

- Triangulate 2D CFT path-integral:

BCFT three-point functions + shrinkable boundaries $\longrightarrow$ ***exact*** tensor network for 2D field theories.

- Emergent 3D path-integral from BCFT OPE coefficients.
- ***Deriving non-perturbative*** 3D/2D ***equivalence***.
- 3D gauge theory explicitly encodes 2D global generalised symmetries.
- Virasoro topological lines $\longrightarrow$ 3D bulk sums over quantum hyperbolic geometries (Example: Liouville CFT).
- Quantum symmetric RG in 2D = radial evolution in 3D bulk.



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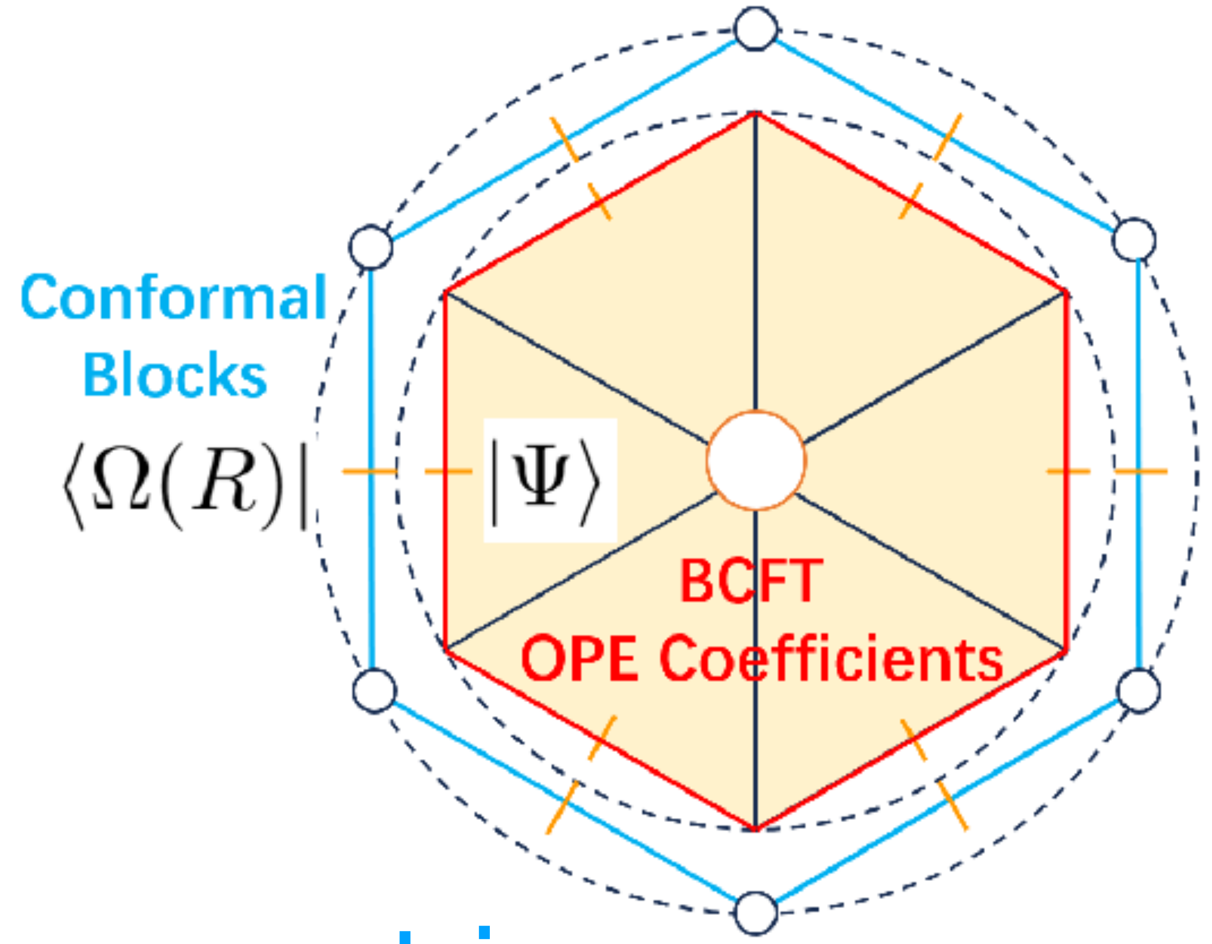

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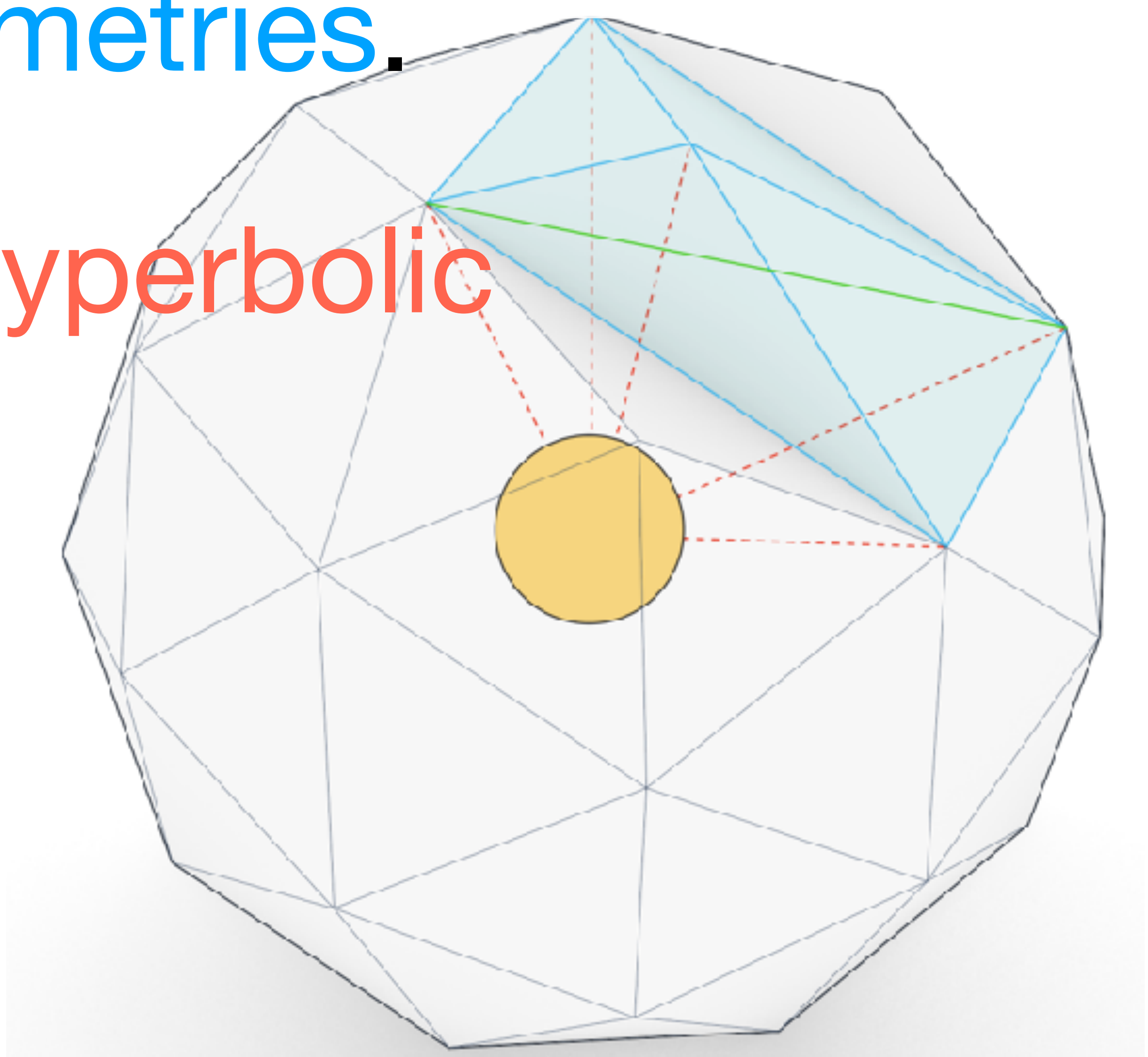
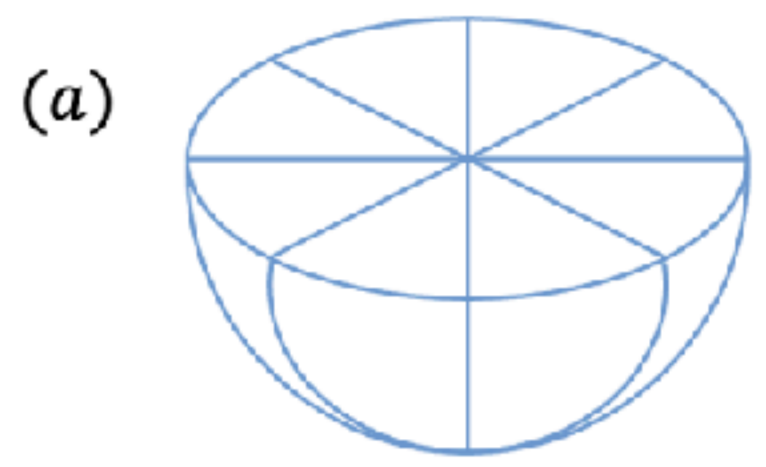
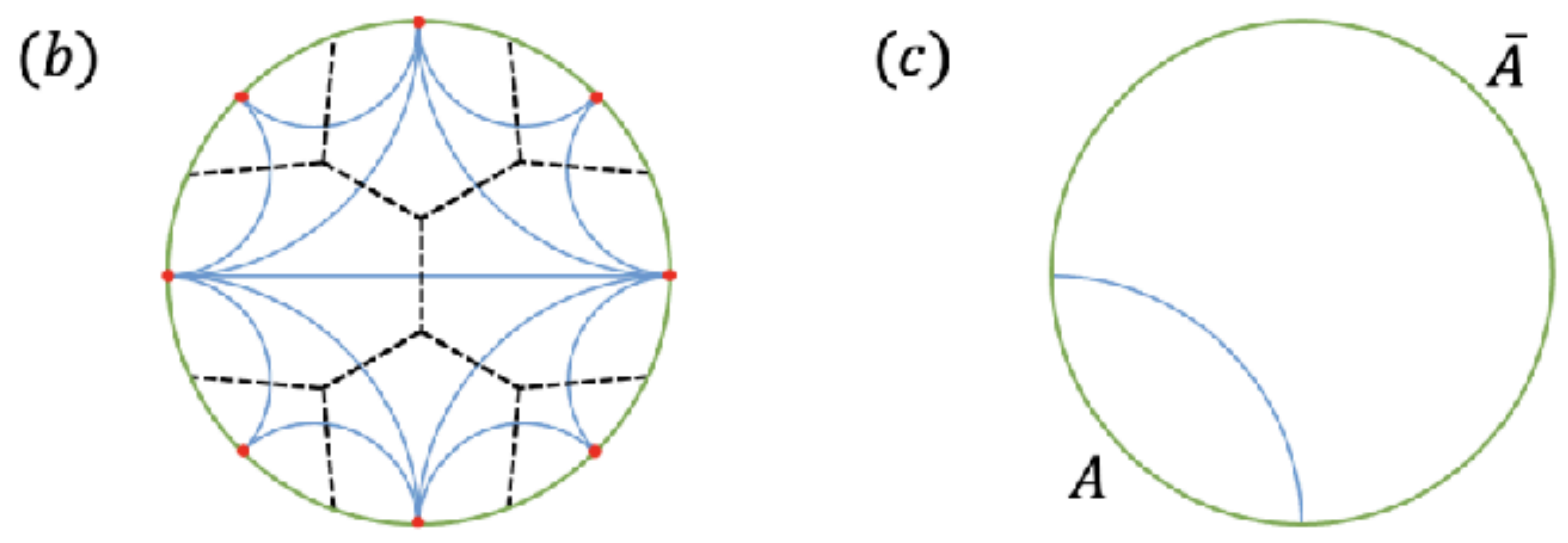


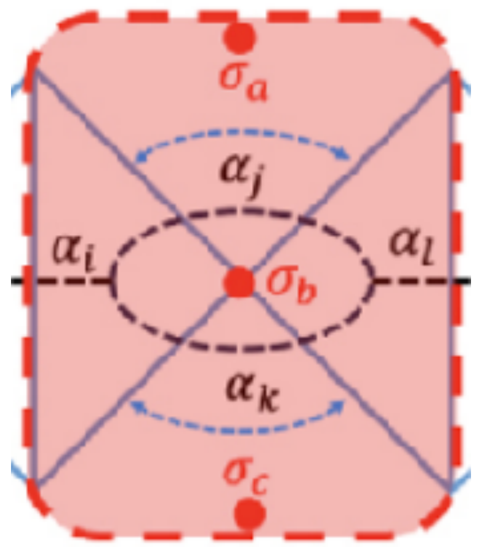
Thank you!

Computing Entanglement Entropy

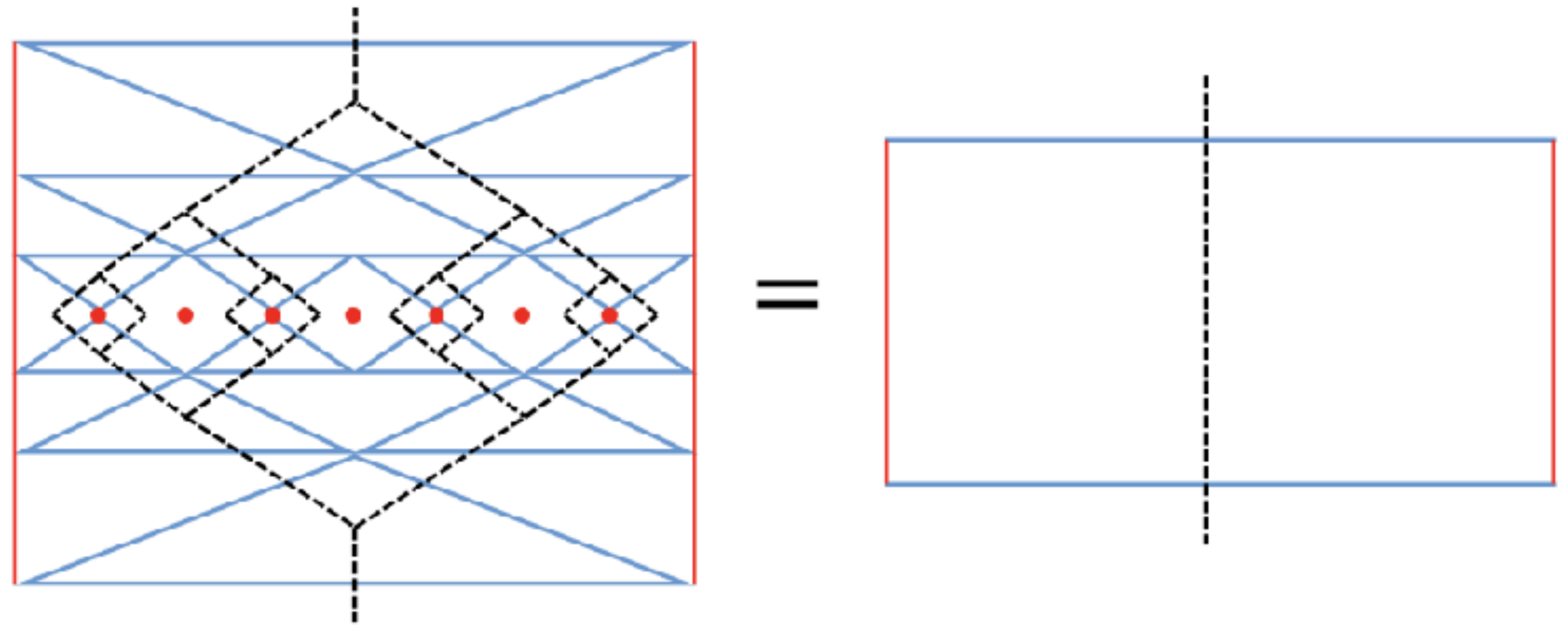
Wave function can be computed with a path-integral on a disk .







This is almost like a perfect tensor condition



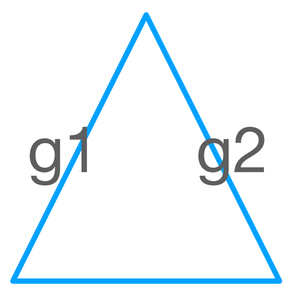
Lattice gauge theory in 2D and JT gravity

Lin Chen, Haochen Zhang, Kaixin Ji, Ce Shen, Ruoshui Wang [2210.12127]

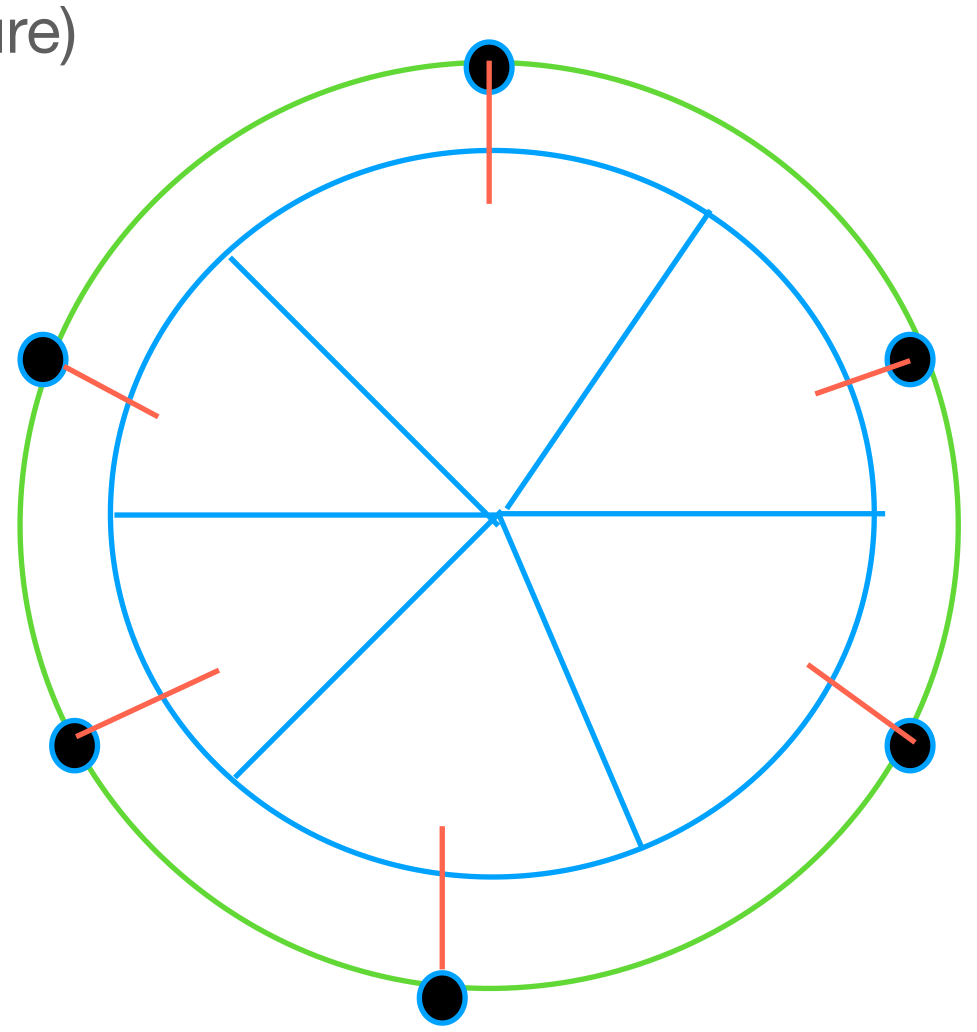
$$\langle \Omega | = \sum_{\{g\}} \mathrm{tr}(\cdots \rho(g_i)\rho(g_{i+1})\rho(g_{i+2})\cdots)\langle \cdots g_i, g_{i+1}, g_{i+2}\cdots |,$$

(Replace by Haar Measure)

Dijkgraaf- Witten lattice gauge theory



g3 = g1.g2



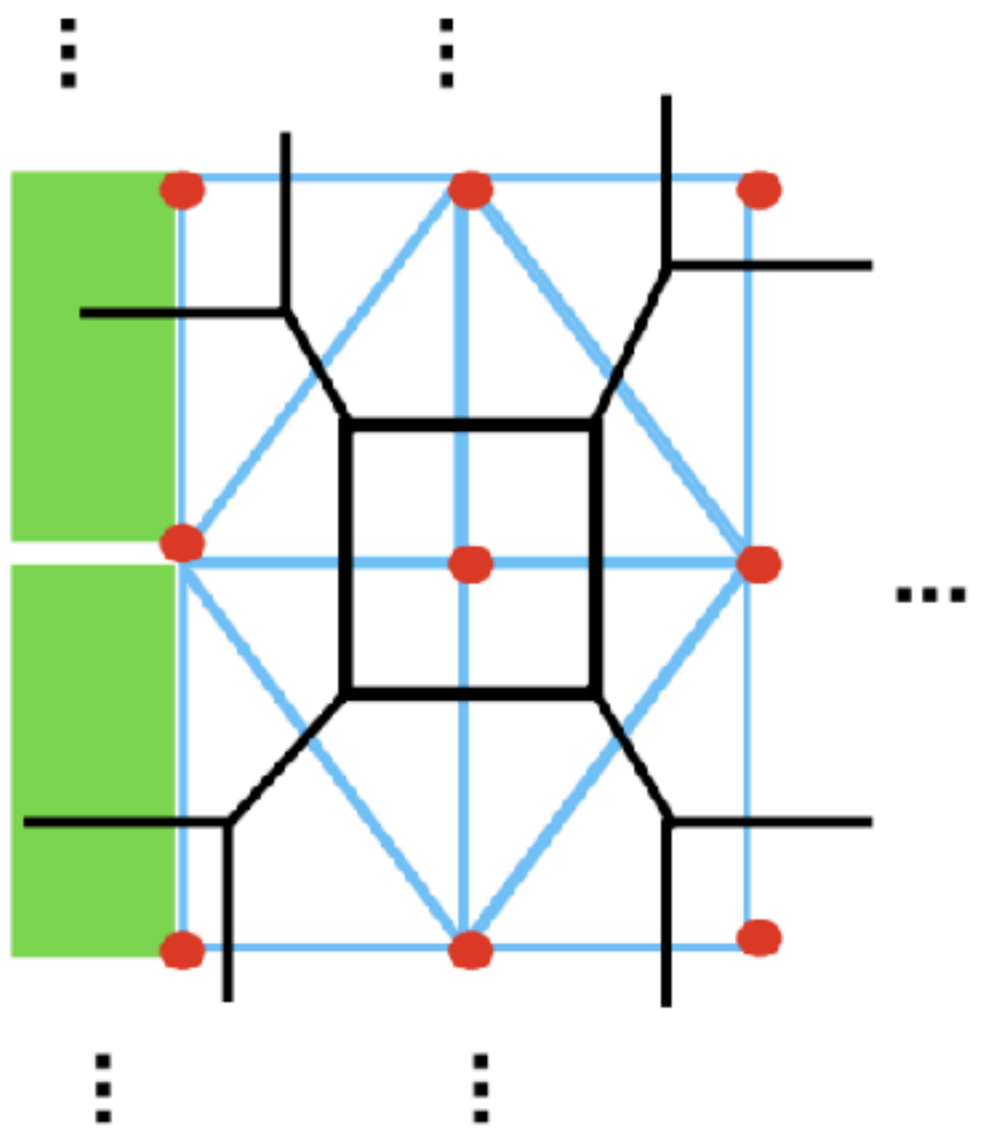
$$g \;\; || \;\; \rho(g)_{I_1 I_2}$$

$$\exp(-\tau C_\rho)$$

Comparing with Blommaert, Mertens and Verschelde 2018 - seems to reproduce their results of the Schwarzian.
Hao Geng, Yikun Jiang, Lin Chen, 2501.xxxxx

We are also upgrading this story to finite GN.

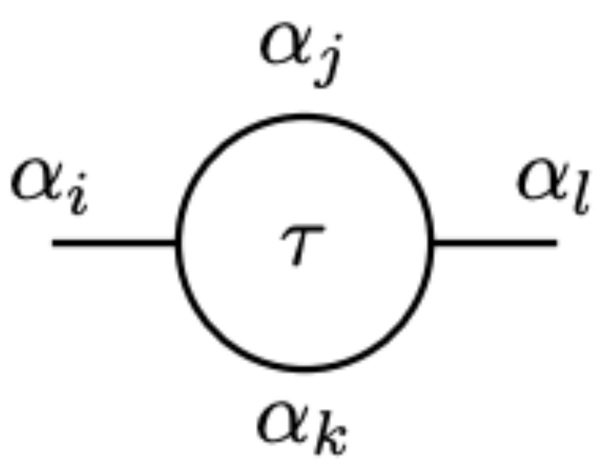
BCFT and bulk brane



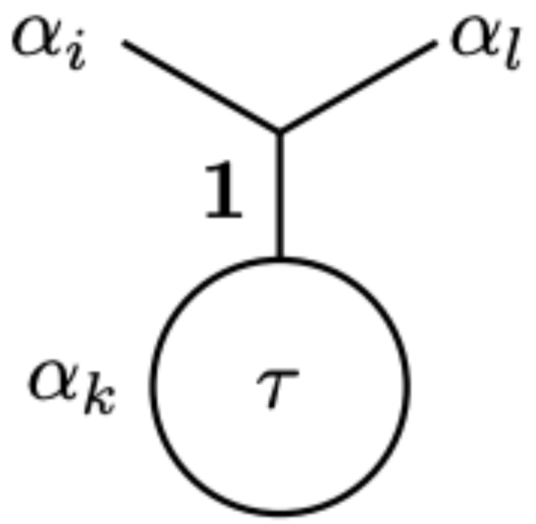
One can obtain an analytic expression for the boundary states!
We can now try to test the bulk dual of this brane that indeed extend into the bulk.

TTbar deformation

$$\int_0^\infty dP_{\alpha_j} dP_{\alpha_k} dP_{\sigma_b}\mu(P_{\sigma_b}) C^{\sigma_b,\sigma_c,\sigma_a}_{\alpha_i,\alpha_j,\alpha_k} C^{\sigma_c,\sigma_b,\sigma_a}_{\alpha_j,\alpha_l,\alpha_k}$$



$$= \delta(P_{\alpha_i}-P_{\alpha_l})\frac{2^{\frac{1}{4}}}{c_b}\left(\frac{1}{\sqrt{\gamma_0}\Gamma_b(Q)}\right)^2\int_0^\infty dP_{\alpha_k}\mu(P_{\alpha_k})$$



The saddle point is located at :

$$P_k^* = \frac{1}{2b\epsilon}.$$

Taking the conformal block to be approximated by

$$\chi_{P_k}(\tau) = \frac{e^{2\pi i \tau P_k^2}}{\eta(\tau)}.$$

$\int_0^\infty dP_k \mu(P_k)\frac{e^{2\pi i\tau P_k^2}}{\eta(\tau)}$ in the limit $b \to 0$.