Non-perturbative de Sitter JT gravity

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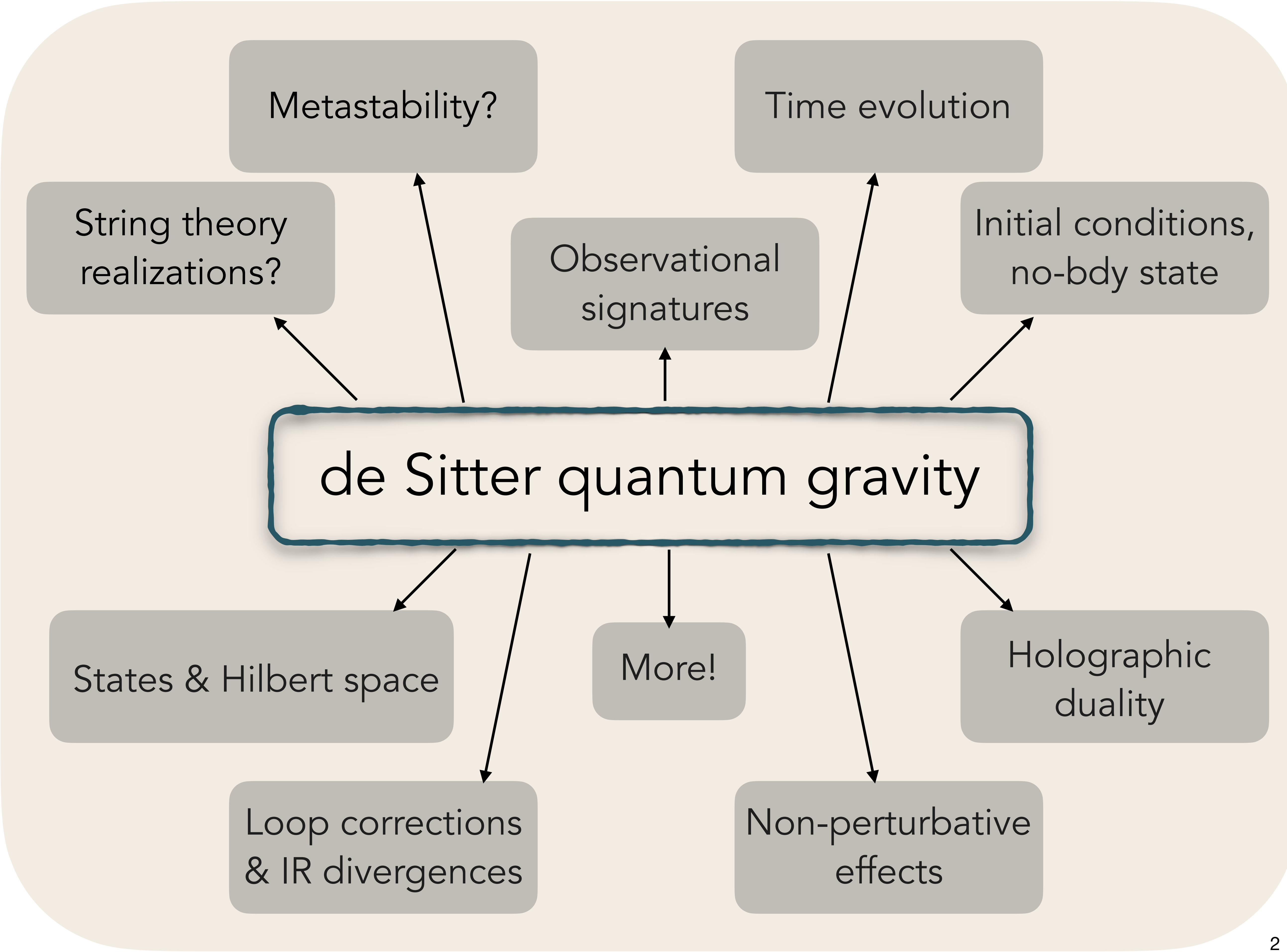
based on:
arXiv:2401.01925 [Cotler, KJ]

(see also arXiv:2302.06603, 1911.12358, 1905.03780)

Strings 2025
Abu Dhabi, January 6 2025

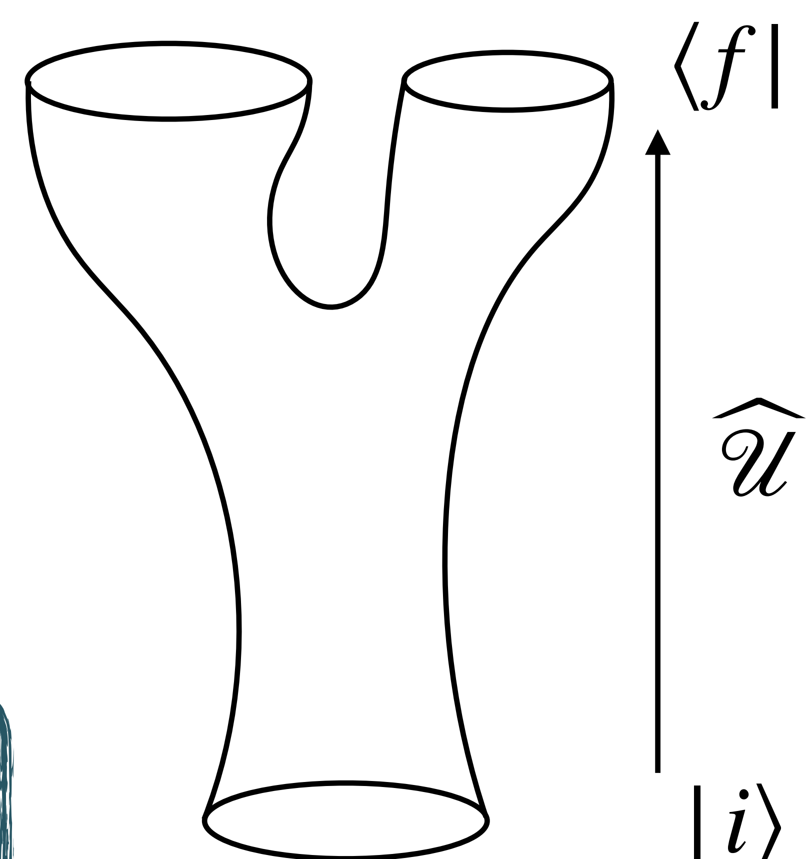

de Sitter quantum gravity
Metastability?
Time evolution
String theory realizations?
Observational signatures
Initial conditions, no-bdy state
States & Hilbert space
More!
Holographic duality
Loop corrections & IR divergences
Non-perturbative effects

This talk: a 2d toy model, Jackiw-Teitelboim gravity

AdS version has been en vogue since ~2016
as a stripped down model of AdS QG.
[Almheiri, Polchinski], [KJ], [Maldacena, Stanford, Yang],
[Engelsöy, Mertens, Verlinde], [Fu, Gaiotto, Maldacena, Sachdev],
[Saad, Shenker, Stanford], [Stanford, Witten], many more.

Today we are interested in the dS version ($\Lambda = 1$).
[Maldacena, Turiaci, Yang], [Cotler, KJ, Maloney], more!

Basic objective is to study transition amplitudes.



$$S_{\rm JT} = \frac{S_0}{4\pi}\int d^2x\sqrt{-g}\,R + \int d^2x\sqrt{-g}\,\varphi\,(R - 2\Lambda) + (\text{bdy})$$

$$g_s = e^{-S_0} \ll 1$$

Summary of major results:

1. Make sense of the gravitational path integral over $R = 2$ metrics.
2. States and inner products.
3. A topological expansion for transition amplitudes. "S-matrix."
4. Aspects of the no-boundary state, horizon entropy.
5. Quantum mechanical interpretation.
6. "Holographic dual."

[Cotler, KJ] '24

building on
[Maldacena, Turiaci, Yang] '19,
[Cotler, KJ, Maloney] '19
[Cotler, KJ] '19, '23

(cf also Trivedi's talk)

The plan:

1. Setting up

2. Results

3. Prospects

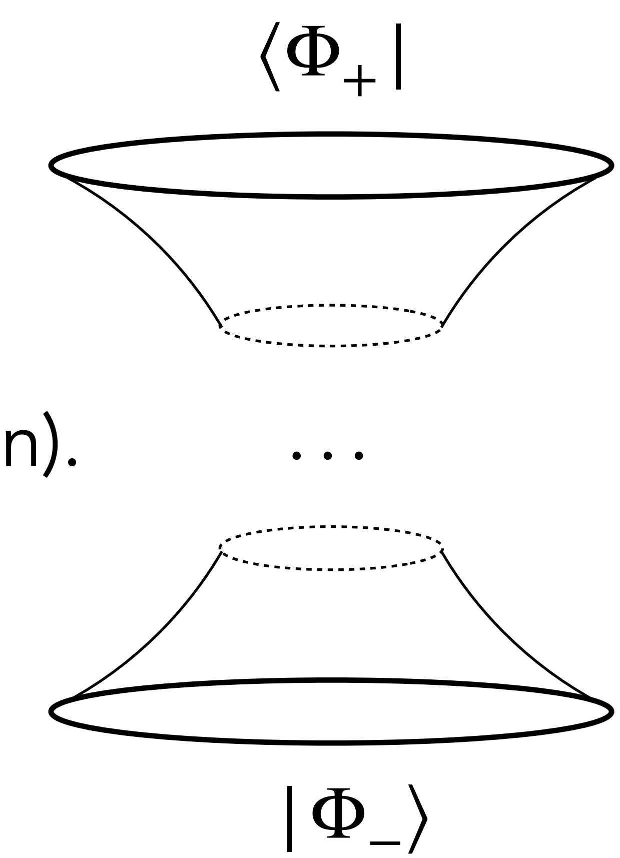
Asymptotic states:

$$S_{\rm JT} = \frac{S_0}{4\pi}\int d^2x\sqrt{-g}\,R + \int d^2x\sqrt{-g}\,\varphi\,(R-2) + (\text{bdy})$$

dS JT has boundary conditions analogous to nearly AdS bc.*

After a suitable coordinate transformation the metric/dilaton can be put into the form:

$$t \to \pm\infty : \begin{cases} ds^2 = -\,dt^2 + (e^{\pm 2t} + O(1))d\theta^2\,, \\ \varphi = \pm\frac{\Phi_\pm}{2\pi}e^{\pm t} + O(1)\,. \end{cases}$$



Future/past circles are labeled by $\Phi_\pm$ (with $i\epsilon$ prescription).

Path integral with these bc prepares asymptotic states.

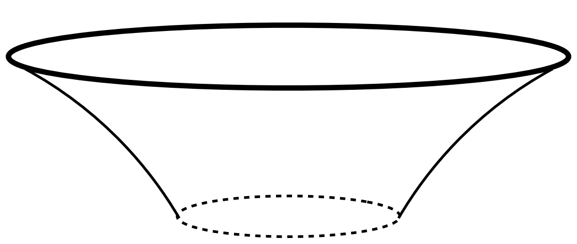
*More general bc are specified by $\varphi = \pm\frac{\Phi_\pm(\theta)}{2\pi}e^{\pm t} + O(1)$.

In this talk I focus on states where $\Phi(\theta)$ has no roots. Recently [Alonso-Monsalve, Harlow, Jefferson] and [Held, Maxfield] have observed that the case with roots is acceptable and physically interesting.

Setting up the gravitational path integral (GPI):

$$S_{\rm JT} = \frac{S_0}{4\pi}\int d^2x\sqrt{-g}\,R + \int d^2x\sqrt{-g}\,\varphi\,(R-2) + (\text{bdy})$$

dS JT gravity is a theory of $R = 2$ metrics.



Problem: What do we sum over?

On the Lorentzian cylinder we have a moduli space of such metrics.

More generally we must consider a complex time contour.

On a general surface $\Sigma_{g,n}$ we have hyperbolic metrics in (-,-) signature, i.e. $ds^2 = -ds^2_{\text{locally } \mathbb{H}^2}$. Akin to [Maldacena, '03].

Integration domain is symplectic with $\Omega_{\rm WP}$. We use measure Pf$(-\Omega_{\rm WP})$.*

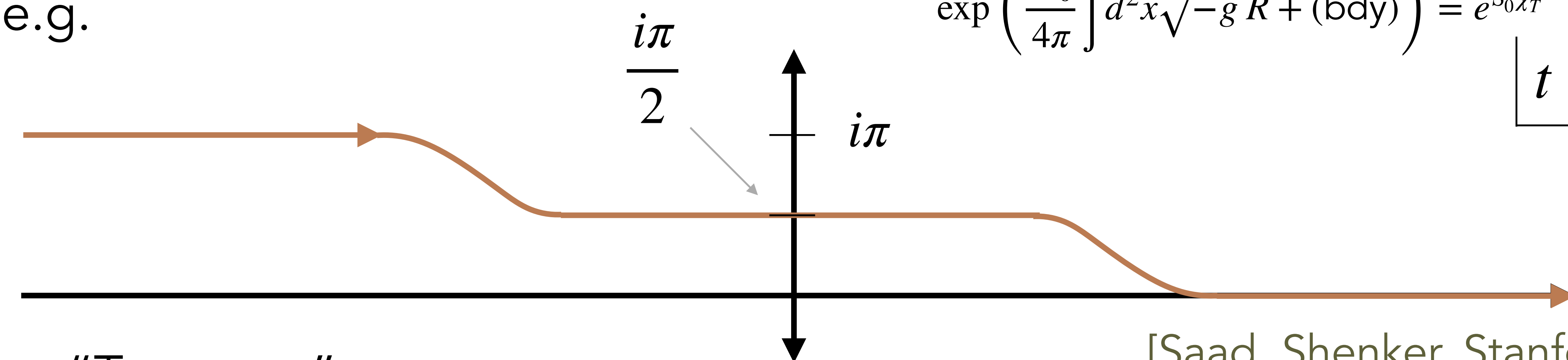
*This follows from using (-,-) metric to form inner product of fluctuations.

Mapping boundary conditions:

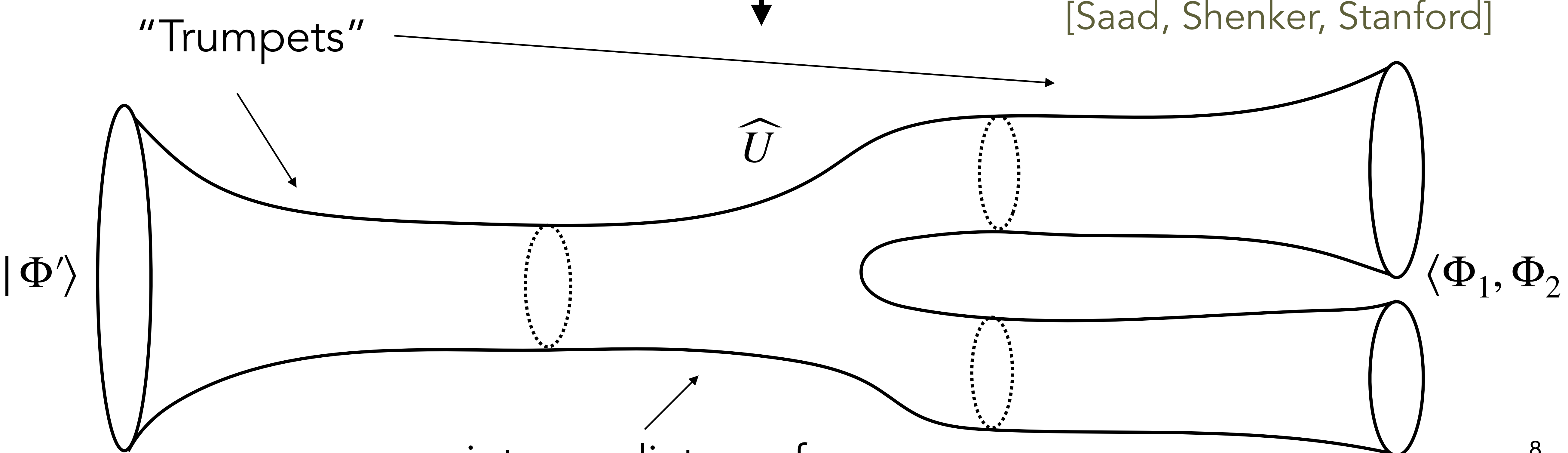
More precisely we consider complex time contours that connect asymptotically Lorentzian regions through an intermediate (-,-) surface.

e.g.

$$\exp\left(\frac{iS_0}{4\pi}\int d^2x\sqrt{-g}\,R + (\text{bdy})\right) = e^{S_0\chi_T}$$



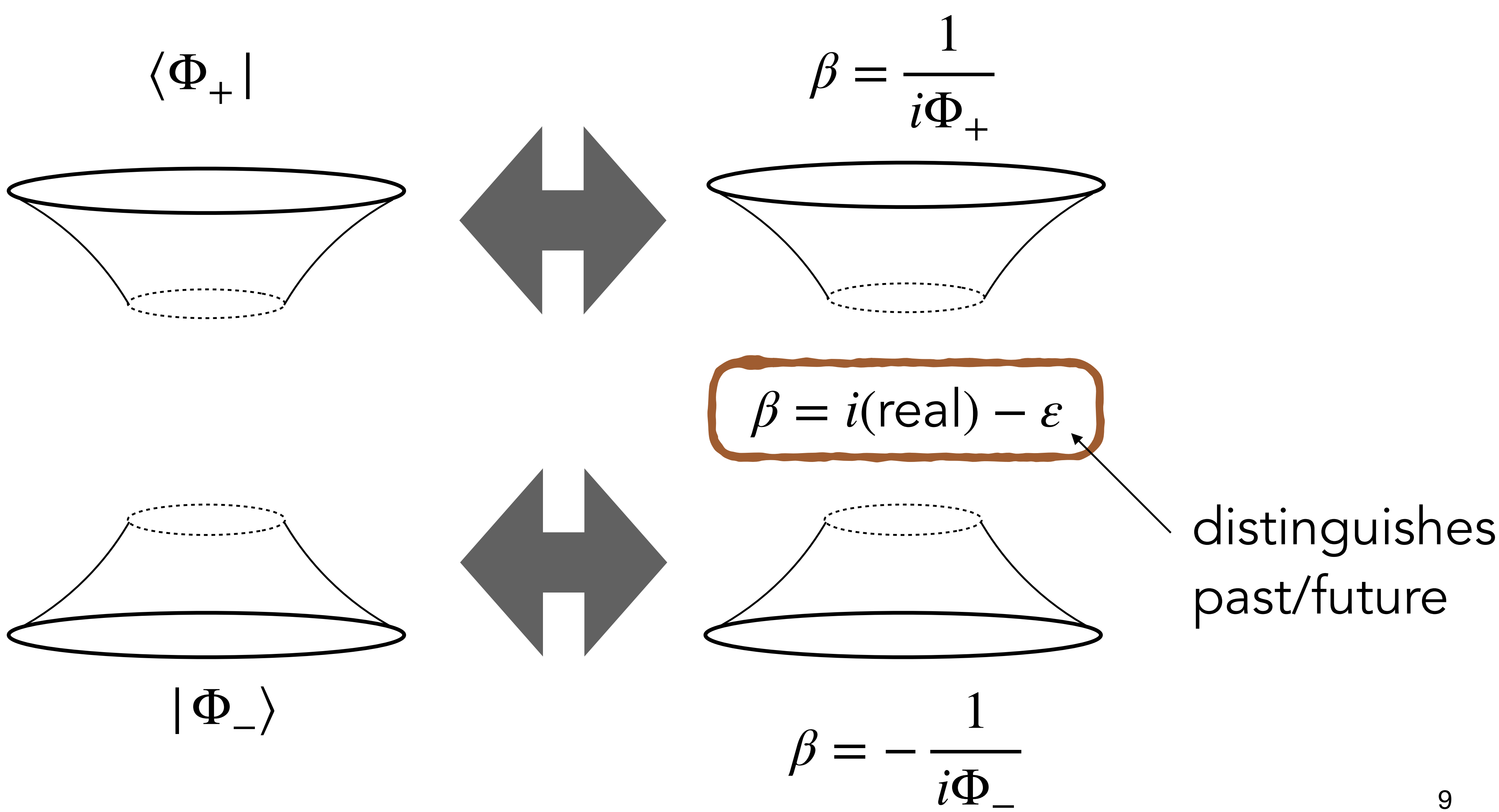
[Saad, Shenker, Stanford]



Mapping boundary conditions:

In JT gravity we can push the contour so as to always have (-,-) signature.

Nearly dS bc are mapped to nearly (Euclidean) AdS bc:



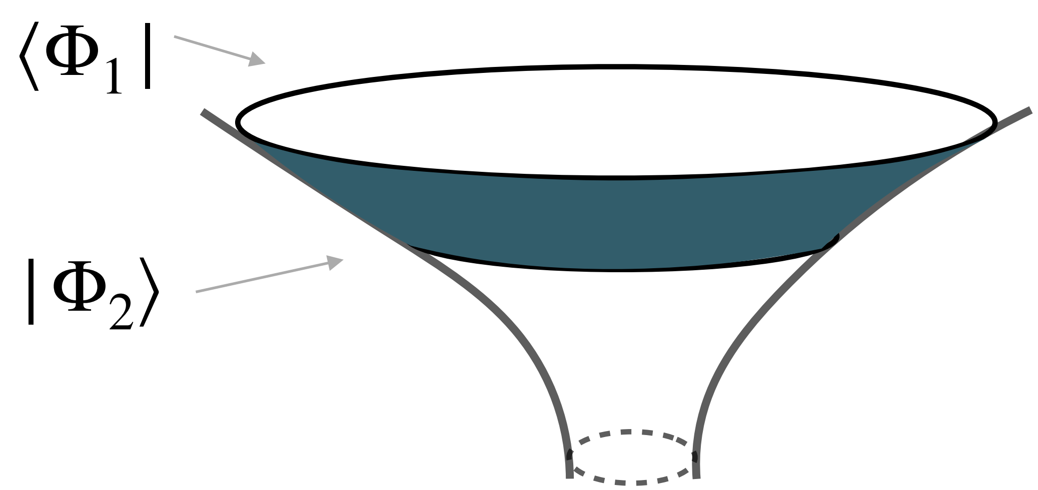
Inner product:

With the definitions in place we can compute transition amplitudes between unnormalized versions of $|\Phi\rangle$.

We define a GPI that computes an inner product of asymptotic states in order to obtain the *normalized* version of transition amplitudes.

$$\lim_{t\to 0} e^{-i\hat{H}t} = 1$$

Basic idea in [Cotler, KJ] '19.



Extends to multi-bdy states, where *n*-universe states form a Fock space isomorphic to that of identical bosons.

No genus corrections.

δ-normalized

$$\langle\Phi_1|\Phi_2\rangle = \sqrt{\Phi_1\Phi_2}\,\delta(\Phi_1 - \Phi_2)$$

Sign depends on measure!

Non-trivial.

Transition amplitudes:

Extract normalized amplitudes by rescaling $|\Phi\rangle \to \dfrac{|\Phi\rangle}{\sqrt{i\Phi}}$.

Also dS JT version of no-boundary state $|\varnothing\rangle$ prepared by capped-off geometry. GPI with no past asymptotic region computes its wavefunction.

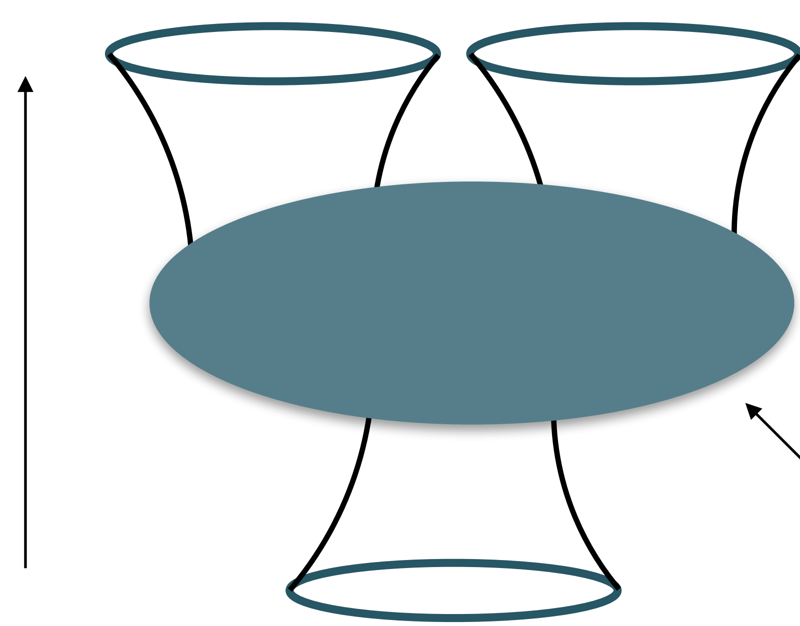
However, both $|\varnothing\rangle$ and its evolution to the far future, Hartle-Hawking, $|\mathrm{HH}\rangle$, are non-normalizable in dS JT, and so in studying infinite-time transition amplitudes we project it out.

$\langle \Phi_1, \Phi_2 |$

cap off with no past or no future

$$\widehat{\mathscr{U}} = \widehat{U} - |\mathrm{HH}\rangle\langle \mathrm{HH}|$$

fill in all possible ways



$= \langle \Phi_1, \Phi_2 | \, \widehat{\mathscr{U}} \, | \Phi' \rangle$

smoothly connects past to future

$|\Phi'\rangle$

Takeaways:

1. Integrate over metrics with a complex time contour.
   Path integral measure for dS JT different than for AdS JT.

2. GPI defines an inner product of asymptotic states as well as infinite-time evolution $\widehat{\mathscr{U}}$.

3. $i\epsilon$ prescription distinguishes kets/bras.

4. In dS JT, no-boundary state is non-normalizable ($\langle \varnothing | \varnothing \rangle \approx Z_{\mathbb{S}^2} \to \infty$)
   Consequently divergent horizon entropy.

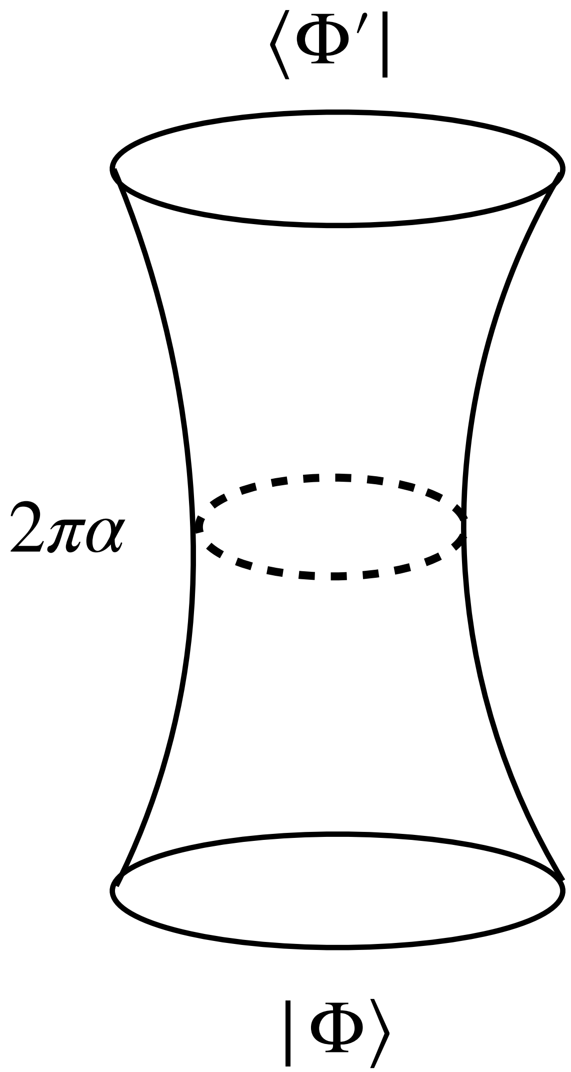
   (Unlike other minimal strings [Anninos, Bautista, Muehlmann], [Mahajan, Stanford, Yan] and higher-dim Einstein gravity at one-loop. [Anninos, et al].)

The plan:

1. Setting up
2. Results
3. Prospects

Cylinder:

Begin with the cylinder (global dS_2). We can compute it directly in Lorentzian signature, or as a sum over (-,-) double trumpets.



$$ds^2 = -\,dt^2 + \alpha^2 \cosh^2(t) d\theta^2$$

Moduli (α, τ), bdy Schwarzian modes.

Moduli space integral converges with our $i\varepsilon$ prescription.

Normalized result: $\langle \Phi' | \,\widehat{\mathscr{U}}\, | \Phi \rangle \approx \dfrac{i}{2\pi} \dfrac{1}{\Phi' - \Phi}$

Global dS_2 saddle when $\Phi = \Phi'$

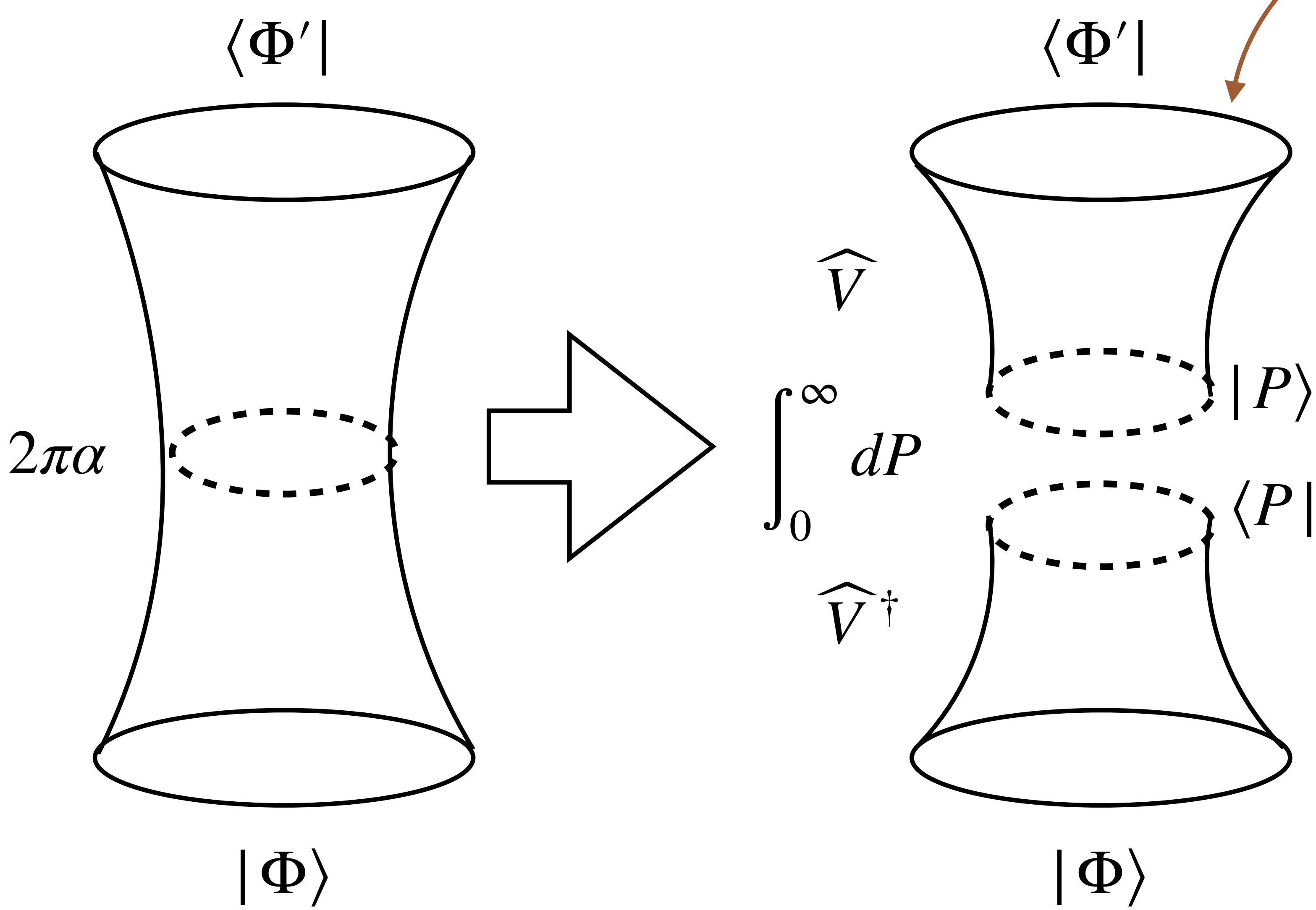
Cutting:

One more Lorentzian definition that is useful to dissect amplitudes.

We can cut the cylinder amplitude in half to obtain bulk states $|P=\alpha^2\rangle$, with $P>0$:

[Cotler, KJ] '23

dS JT "trumpet"



Sign depends on $\mathrm{Pf}(-\Omega_{\mathrm{WP}})$!

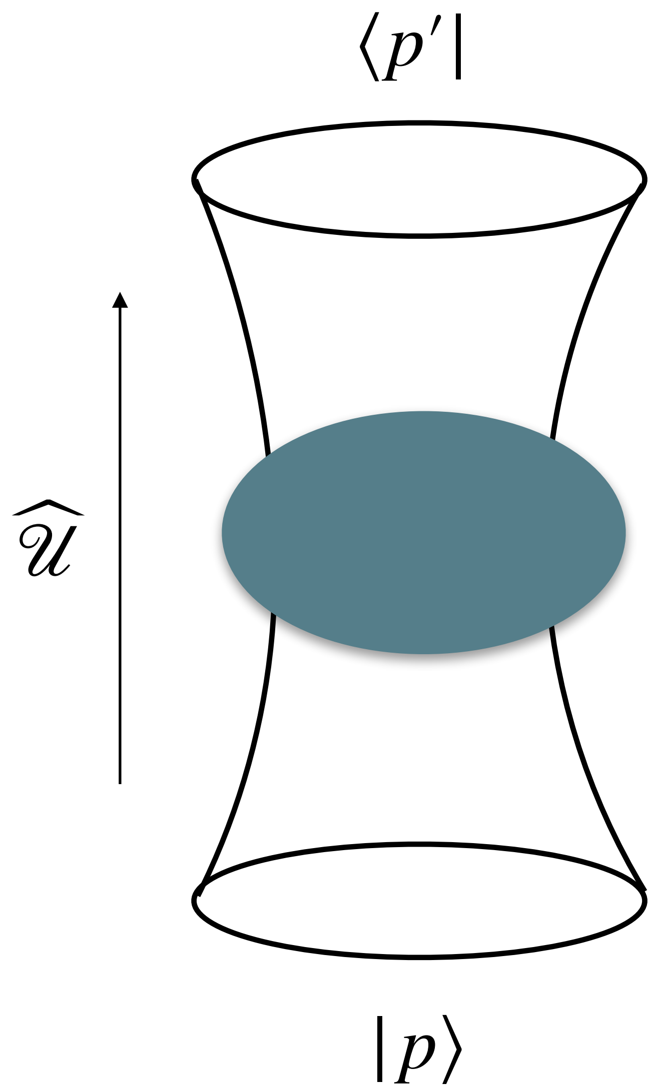
$$\langle P | P' \rangle = \delta(P-P') + O(e^{-2S_0})$$

$$\langle \Phi | \, \widehat{V} \, | P \rangle = \frac{1}{\sqrt{2\pi}} e^{i\Phi P}$$

$$\langle P | \varnothing \rangle = e^{S_0}\delta'(P-1) + O(e^{-S_0})$$

Leading order unitarity:

Useful: go to canonical conjugate variable: $[\hat{\Phi}, \hat{p}] = i$.



Then to leading order in topological expansion and between states with any number of circles,

$$\widehat{\mathscr{U}} = \Theta(\hat{p}) + O(e^{-S_0})$$

Evolution is approximately *unitary* for $p > 0$ states.

$p < 0$ states prepare crunching spacetimes. Projected out under evolution thanks to sum over non-singular metrics.

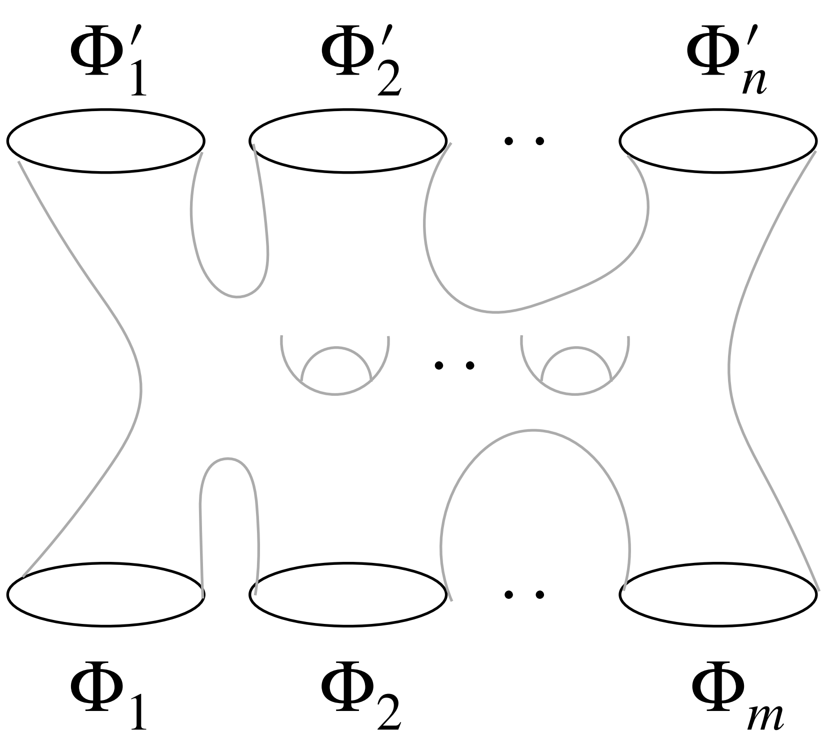
Transition amplitudes:

On a general surface of genus g with
n future circles, m past circles:

Analytically continued WP volumes

$$\frac{(-1)^{3-3g-n-m}}{\sqrt{n!m!}}\int_0^\infty dP_1'..dP_n'dP_1..dP_m V_{g,n+m}(2\pi i\sqrt{P_1'},..,2\pi i\sqrt{P_n'},2\pi i\sqrt{P_1},..,2\pi i\sqrt{P_m})$$

$$\times \widetilde{Z}_T(\Phi_1',P_1')..\widetilde{Z}_T(\Phi_n',P_n')\widetilde{Z}_T^*(\Phi_1,P_1)..\widetilde{Z}_T^*(\Phi_m,P_m)$$



"dS trumpet"

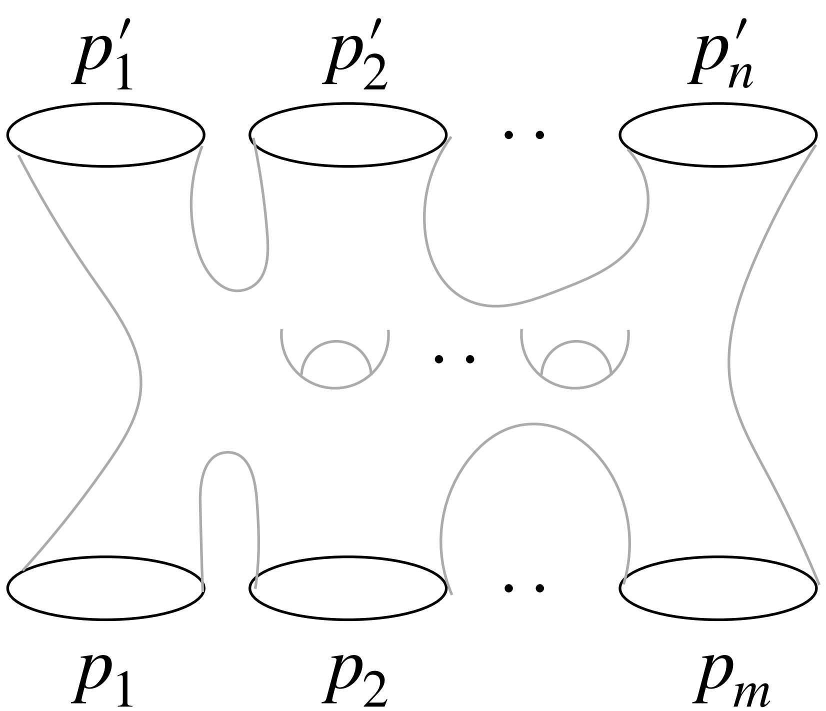
$$\widetilde{Z}_T(\Phi,P)=\langle\Phi|\,\widehat{V}\,|P\rangle=\frac{1}{\sqrt{2\pi}}e^{i\Phi P}$$

Transition amplitudes:

In the conjugate $\hat{p}$ basis things are even simpler:

$$\langle p_1', .., p_n' | \, \widehat{\mathcal{U}} \, | p_1, .., p_m \rangle_{g,\text{conn}} = \frac{(-1)^{3-3g-n-m}}{\sqrt{n!m!}} \Theta(p_i', p_j)$$

$$\times V_{g,n+m}(2\pi i\sqrt{p_1'}, .., 2\pi i\sqrt{p_n'}, 2\pi i\sqrt{p_1}, .., 2\pi i\sqrt{p_m})$$



Decomposition:

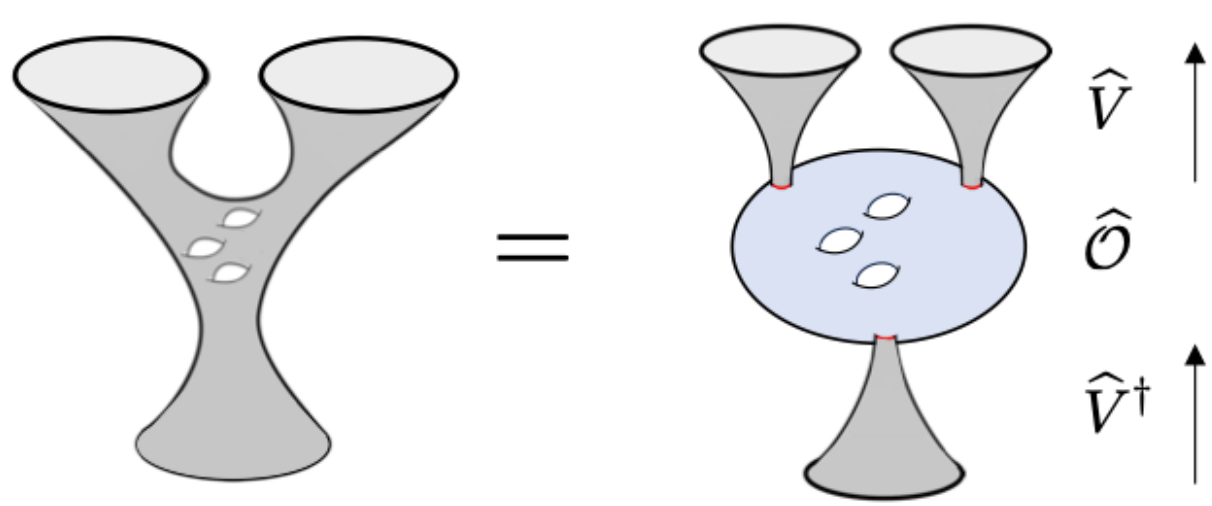
This allows us to decompose these amplitudes in an interesting way!

$$\widehat{\mathscr{U}} = \widehat{V}\,\widehat{O}\,\widehat{V}^{\dagger}$$

$\widehat{V}$: dS trumpet, no genus corrections.

$\widehat{O}$: Bulk inner product, receives genus corrections, described by continued WP volumes.

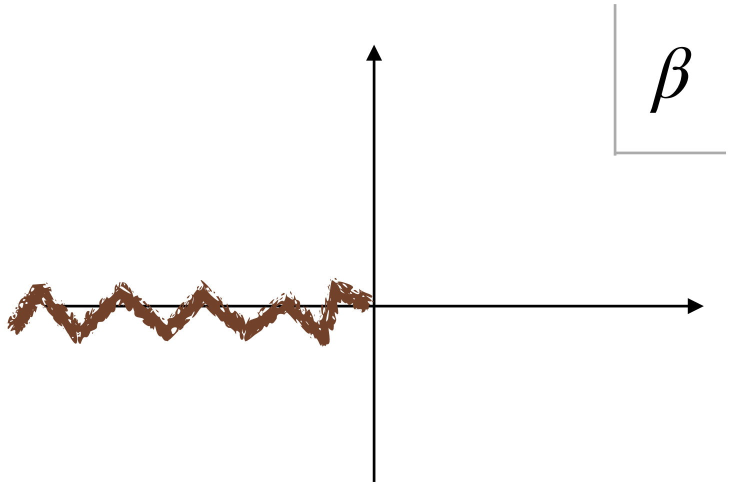
$$\widehat{O} = \Theta(\hat{P}) + O(e^{-S_0})$$



Holographic dual?:

I have described a direct computation of dS JT amplitudes as a sum over (-,-) metrics.

Ex post facto, the de Sitter amplitudes are suitable continuations of (Euclidean) AdS JT amplitudes, in both the string coupling and β's.



To go from AdS amplitudes to dS ones,

1. $S_0 \to S_0 + \frac{3i\pi}{2}$, i.e. $e^{-S_0} \to ie^{-S_0}$,
2. Continue from Re$(\beta) > 0$ to Re$(\beta) < 0$ by going CCW around 0.

(1) Turns the genus expansion into an alternating series.
(2) Means that, after inverse Laplace transform, amplitudes are supported along $E < 0$.

Holographic dual?:

After a suitable integral transform (analogous to the AdS one) the dS JT amplitudes obey *topological recursion*, with a spectral curve and an effectively negative number of dof:

$$\tilde{y}(\tilde{z}) = -\frac{1}{\sqrt{2}}\sinh(2\tilde{z})\,, \qquad \tilde{z}^2 = E \qquad N_{\text{eff}} = -\,e^{2S_0}$$

This is an analytic continuation of the AdS JT spectral curve.

If this is itself a formal matrix integral*, then it has a leading, *oscillatory* density of states:

$$\tilde{\rho}_0(E) = \frac{1}{\sqrt{2}\pi}\sin(2\sqrt{-E})\Theta(-E)$$

*"Formal matrix integral" is a standby for the formal power series (with poly V):

$$\sum_{n=0}^{\infty}\frac{1}{n!}\int dH\, e^{-V_{\text{gaussian}}(H)}(-V_{\text{non-gaussian}}(H))^n$$

This may yet come from a formal matrix integral with a complex potential.

The plan:

1. Basic features
2. Some physics
3. Prospects

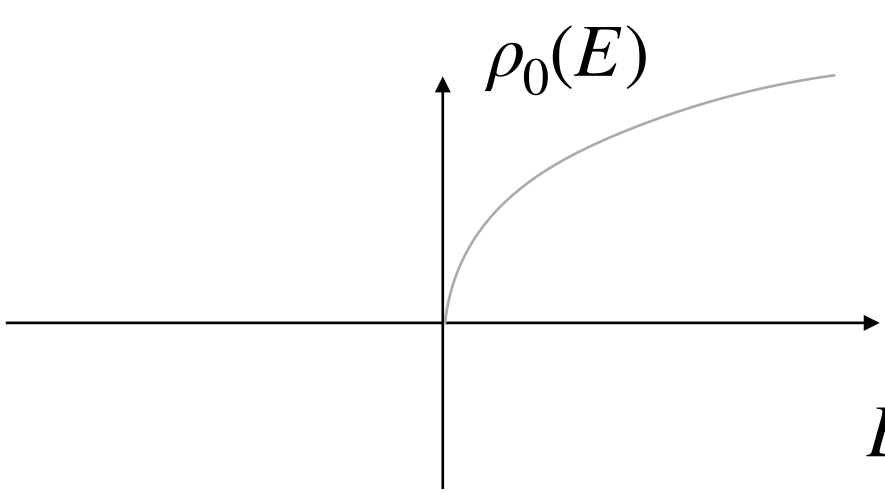
Holographic duality?

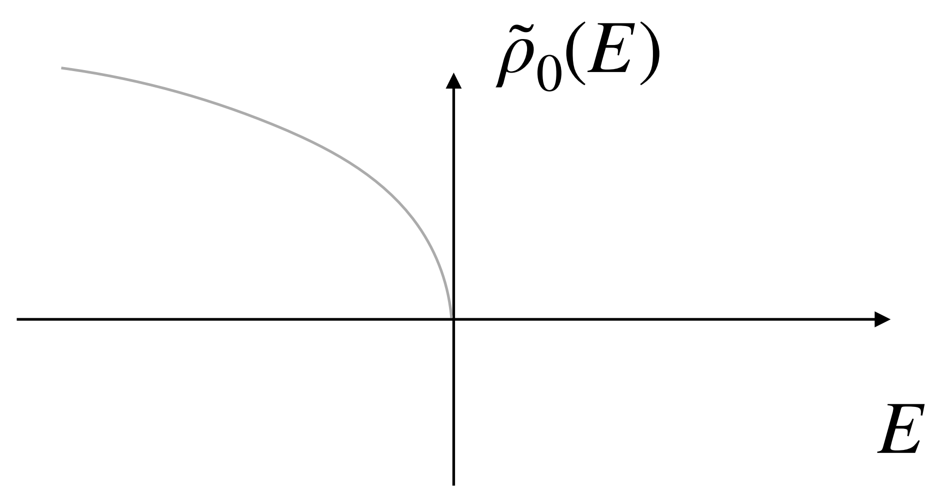
We can obtain dS JT amplitudes from a continuation of AdS JT.

Q: Does dS JT admit a bona fide matrix integral completion?

Easier: Consider topological gravity (i.e. the Airy model) under the same continuation.

Under $e^{-S_0} \to ie^{-S_0}$ the Airy model is sent to another double-scaled matrix integral, namely itself but with $E \to -E$.
Resonant with Re$(\beta) < 0$. What about in dS JT?!





Relation to complex Liouville string?

Recently [Collier, et al] have studied a variant of the minimal string with two copies of Liouville at $c = 13 \pm i\nu$. "Complex Liouville string."

They provide a 2d dilaton gravity target space interpretation with Euclidean AdS and dS (really Euclidean AdS in (-,-)) saddles.

$$S_{\text{dil}}[\Phi, g] = \frac{i}{2b^2} \int_{\Sigma_{g,n}} \mathrm{d}^2 x \sqrt{g} \left(\Phi \mathcal{R} - \pi^{-1} \sin(2\pi\Phi) \right) ,$$

Propose a two-matrix model dual, although also oscillatory $\langle \rho \rangle$.

Q: Are these two constructions related? If so, how?

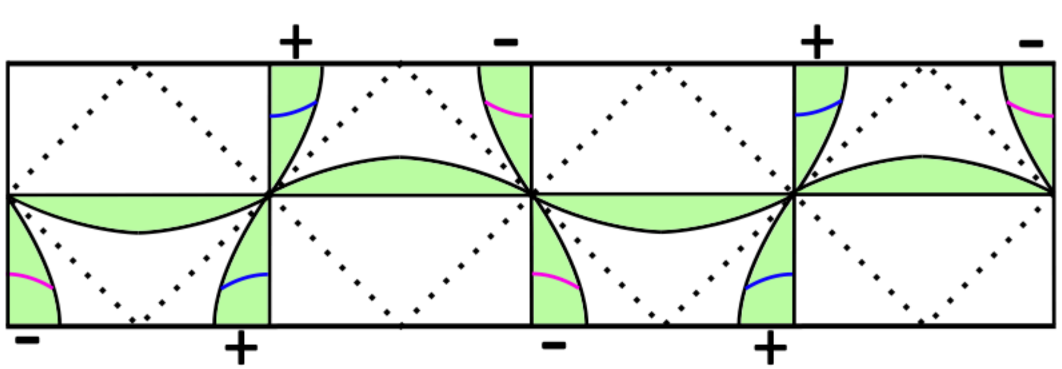
Other asymptotic states?

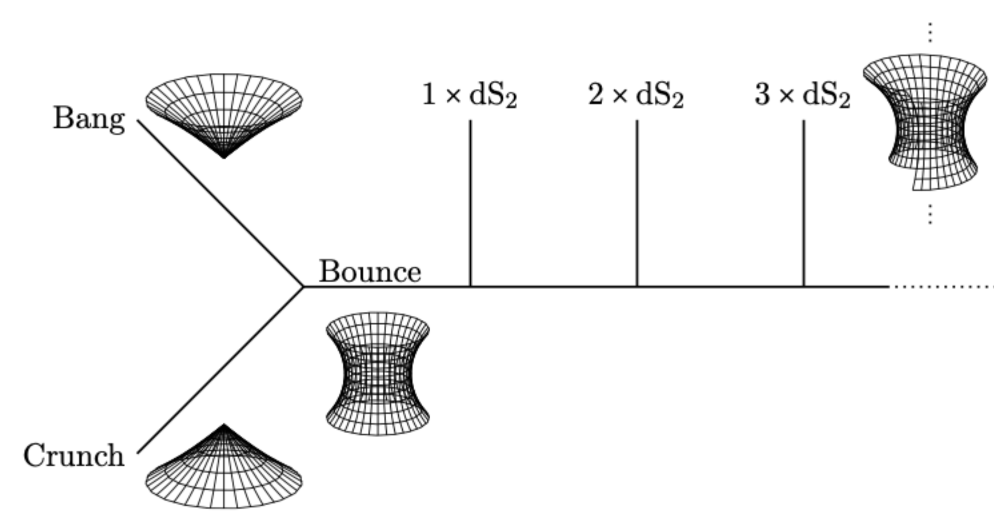
In this talk I focused on states prepared by bc where the asymptotic dilaton has no roots. Recently [Alonso-Monsalve, Harlow, Jefferson] and [Held, Maxfield] have found classical saddles where the asy dilaton has roots.

The resulting states appear to decouple from those with no roots. [WIP]

Q: What is the physics of these states? Dual description?

Q: Amplitudes between these states?



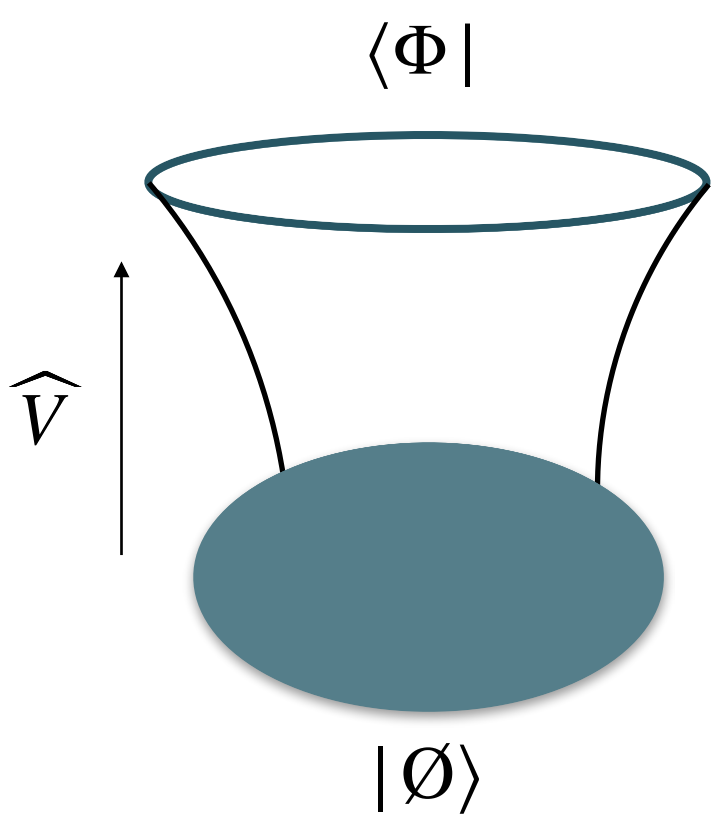


Thank you!

No-boundary state:

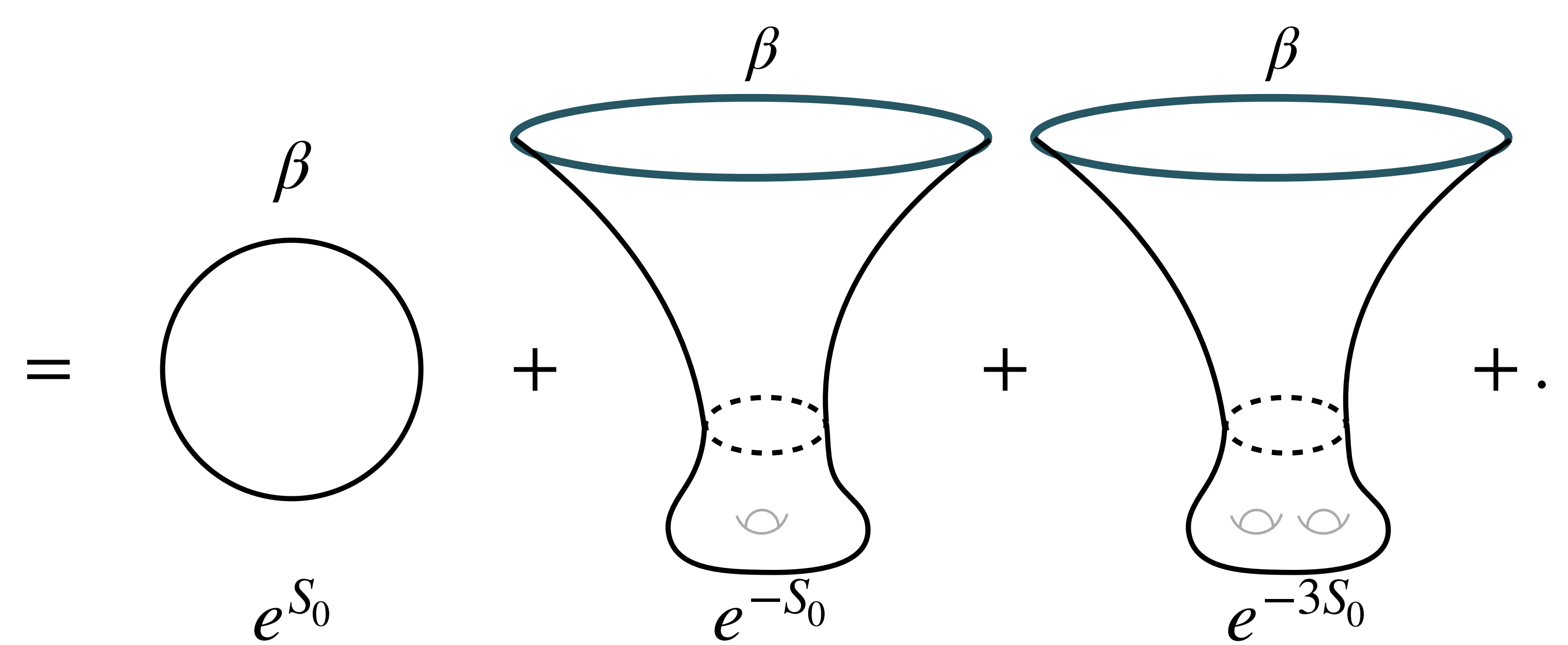
Besides asymptotic states corresponding to asymptotically dS regions, we have the dS JT version of the no-boundary state $|\emptyset\rangle$.

We interpret GPI with only future asymptotic regions as computing the wavefunction of this state in the basis of asymptotic states.
(Only past computes the conjugate wavefunction.)



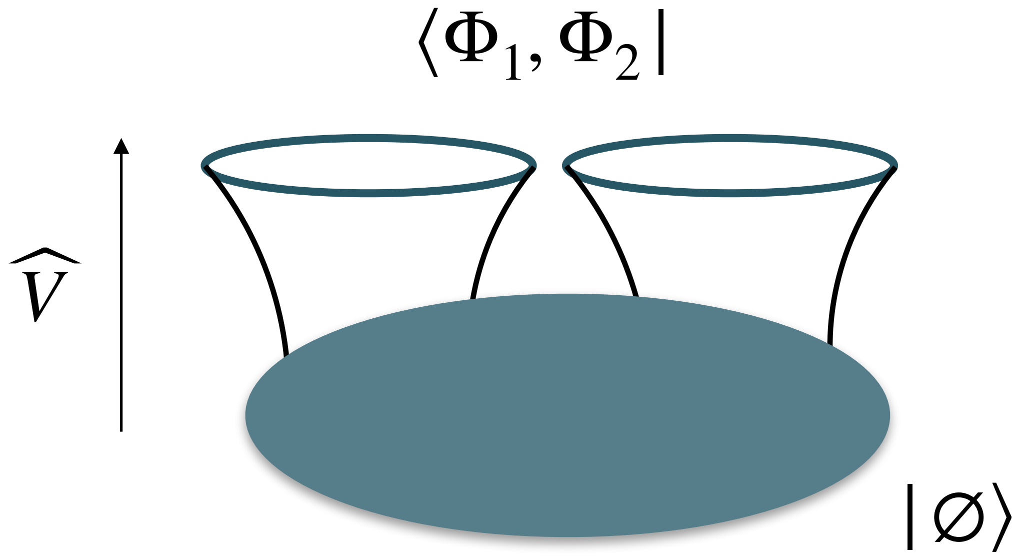
Smooth cap

$$= \langle\Phi|\,\widehat{V}\,|\emptyset\rangle = \langle\Phi|\mathrm{HH}\rangle = \Psi_{\mathrm{HH}}(\Phi)$$



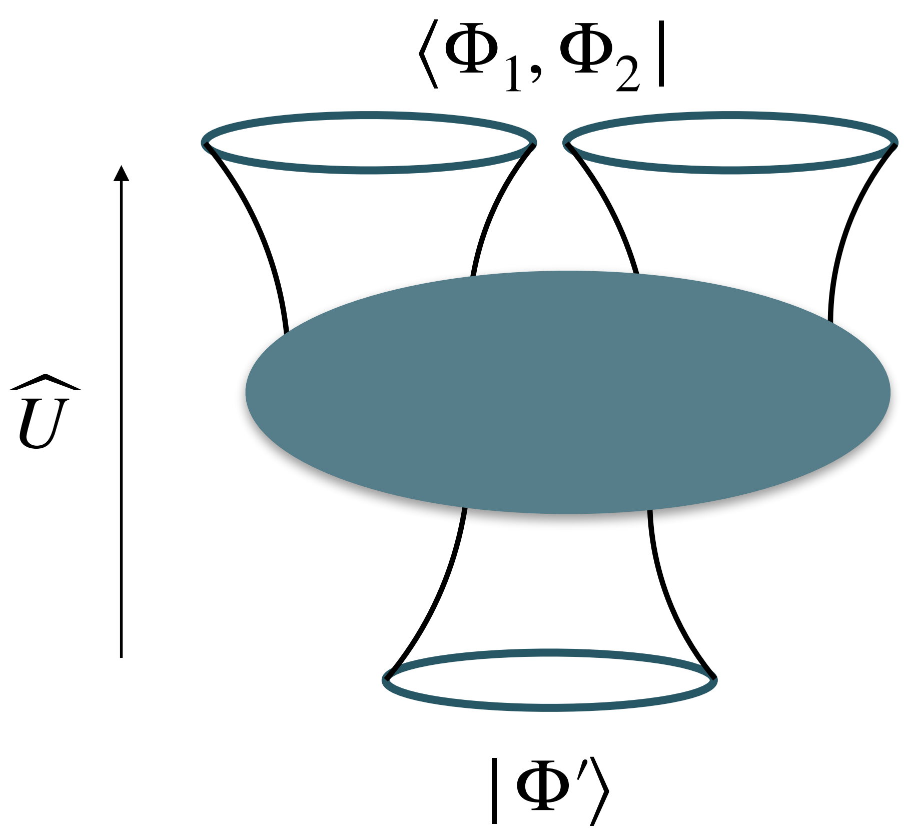
Amplitudes:

We propose the interpretation e.g.



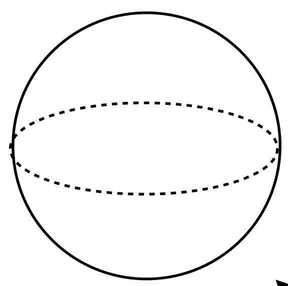
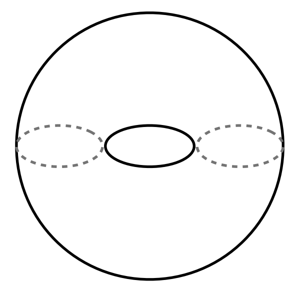
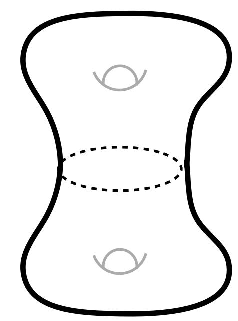
$$= \langle \Phi_1, \Phi_2 |\, \widehat{V}\, | \varnothing \rangle = \langle \Phi_1, \Phi_2 | \mathrm{HH} \rangle$$

$$= \Psi_{\mathrm{HH}}(\Phi_1, \Phi_2)$$



$$= \langle \Phi_1, \Phi_2 |\, \widehat{U}\, | \Phi' \rangle$$

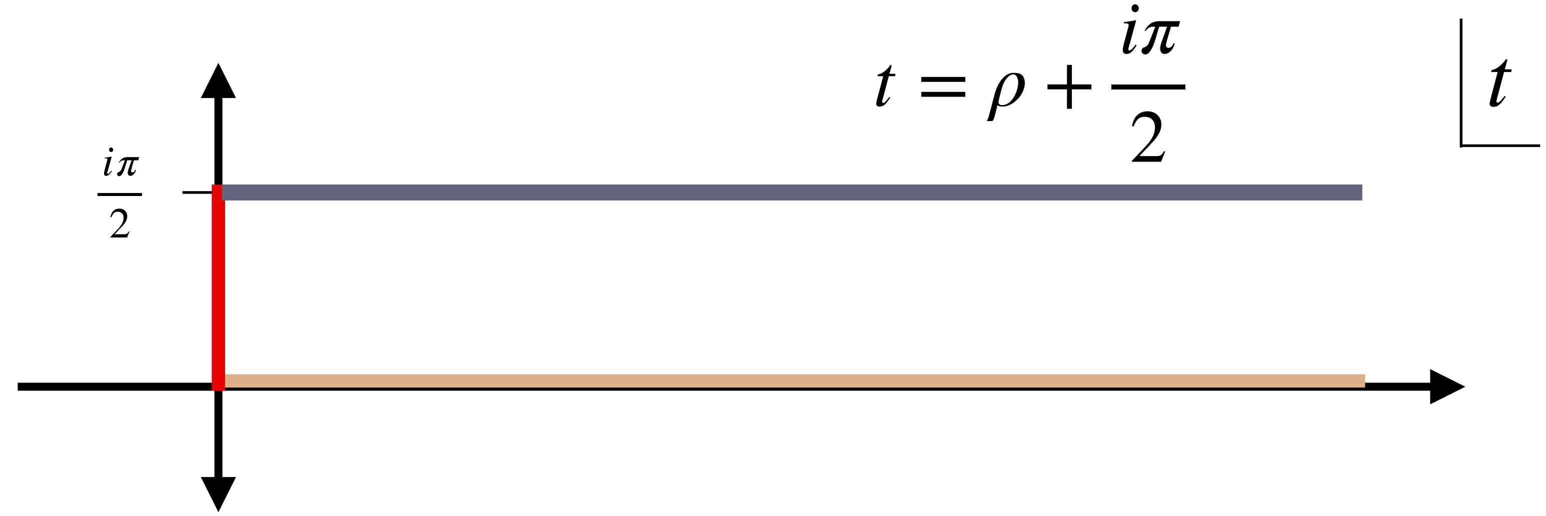
Non-normalizability of the no-boundary state:

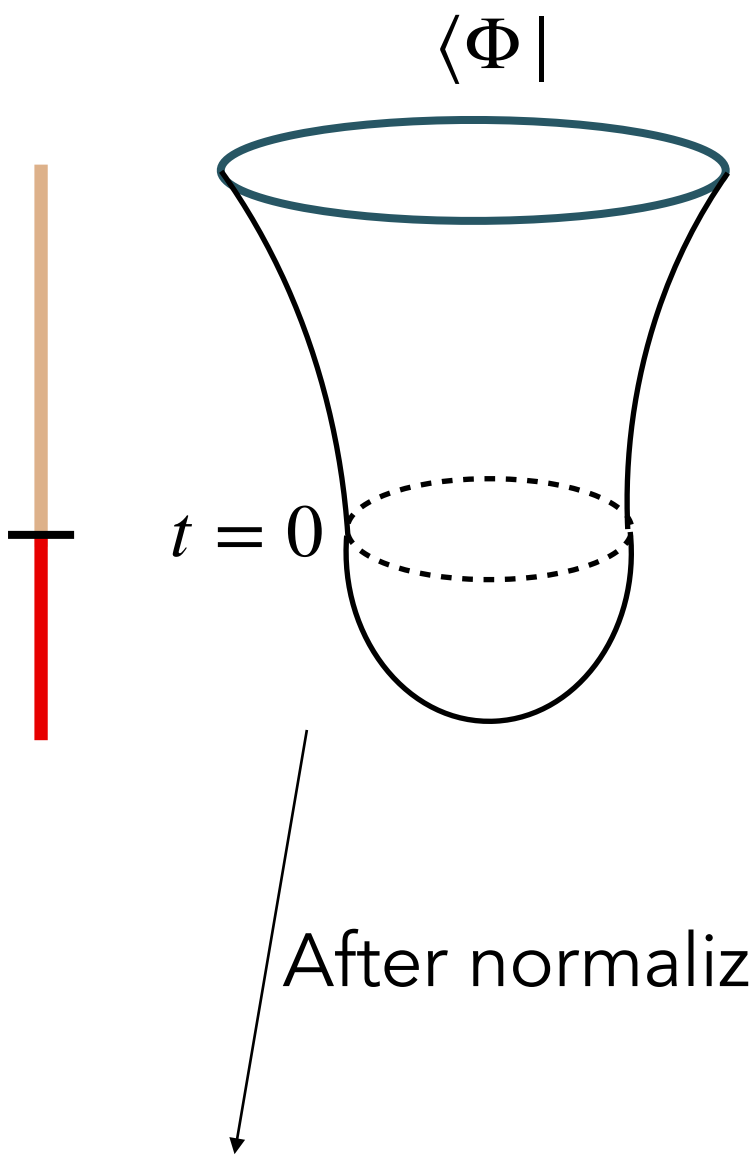
$\langle \varnothing | \varnothing \rangle =$  $+$  $+$  $+ \ldots +$ (disconnected)

$= \infty$

[Mahajan, Stanford, Yan] (noncompact $\ell = 1$ zero mode of dilaton)

Disk/HH:





$$\begin{cases} ds^2 = -dt^2 + \cosh^2(t) d\theta^2 \,, \\ \varphi = \frac{\Phi}{2\pi} \sinh(t) \,. \end{cases} \Rightarrow \begin{cases} ds^2 = -(d\rho^2 + \sinh^2(\rho) d\theta^2) \,, \\ \varphi = -\frac{i\Phi}{2\pi} \cosh(\rho) \,. \end{cases}$$

[Maldacena, Turiaci, Yang], [Cotler, KJ, Maloney] '19

After normalizing $\langle \Phi |$, we get: $\Psi_{\rm HH}(\Phi) \approx \frac{-i\Phi}{\sqrt{2\pi}} e^{S_0 + i\Phi} \left(1 + O(e^{-2S_0})\right)$

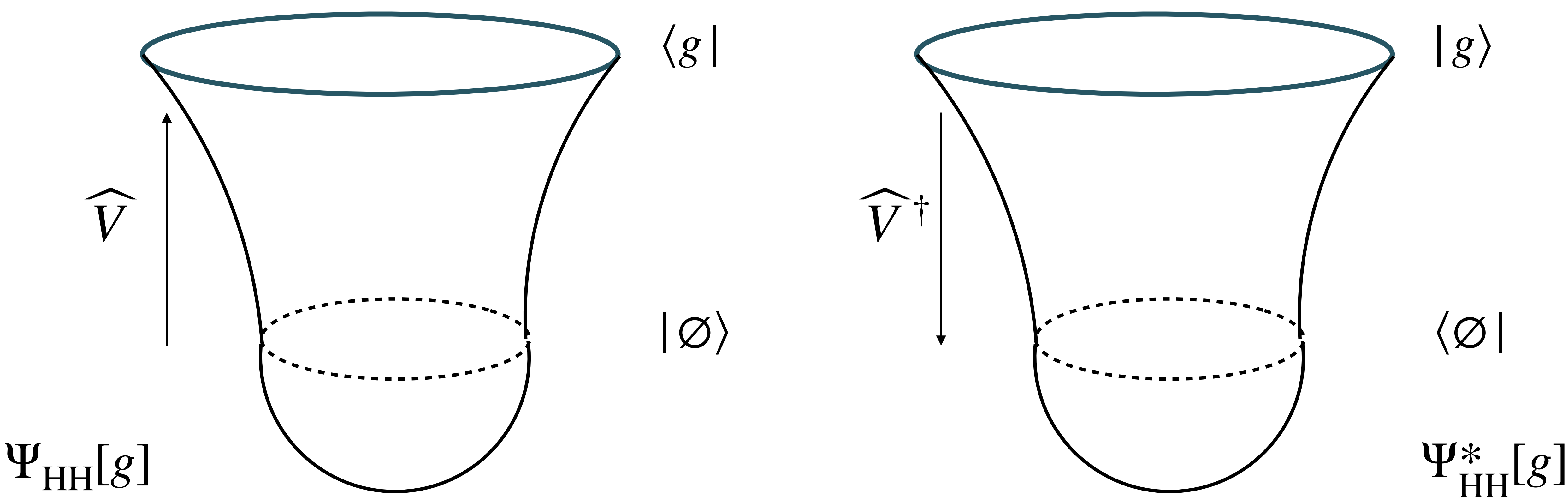
No moduli, bdy Schwarzian mode $\quad (\langle \mathrm{HH} | \mathrm{HH} \rangle \approx Z_{\mathbb{S}^2})$

HH is nonnormalizable! $\langle \mathrm{HH} | \mathrm{HH} \rangle \approx \int_{-\infty}^{\infty} d\Phi \, |\Psi_{\rm HH}(\Phi)|^2 = \infty$

Another prospect: $|Z_{\mathbb{S}^{d+1}}| \approx_? \langle \text{HH} | \text{HH} \rangle$

There is an expectation in higher-dim gravity that, at least semiclassically, and at the level of one-universe states,

$$|Z_{\mathbb{S}^{d+1}}| = \langle \text{HH} | \text{HH} \rangle$$



Q: Is it?

dS$_3$: [WIP, Cotler, KJ]
dS$_4$: [WIP, Cotler, Harvey, KJ]

Holographic dual?:

The more conservative statement is that the genus expansion of dS JT is a suitable continuation of the AdS JT "formal matrix integral."

Even so, this is the most complete realization of dS holography to date.

In a slogan, bulk QM emerges from (a continuation) of $\int dH$.

Because we do not know if dS JT comes from a bona fide matrix integral the status of doubly non-perturbative effects is unclear.