Geometric Transitions in N=1 Theories

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Based on work with James Gray and Callum Brodie
arXiv:2211.05804
And with James Gray, Sunit Patil, and Caoimhin Scanlon,
25xx.xxxx
and James Gray and Xingyang Yu, 25xx.xxxxx

Motivating Questions

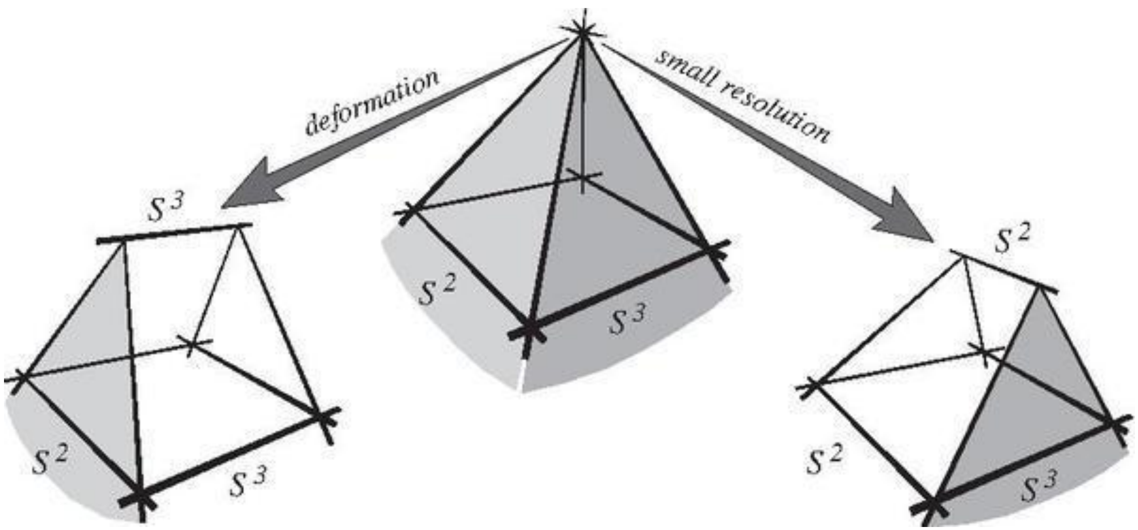
- 4D N=1 theories arising from string compactifications come from many topologically distinct geometries
- In string compactifications:

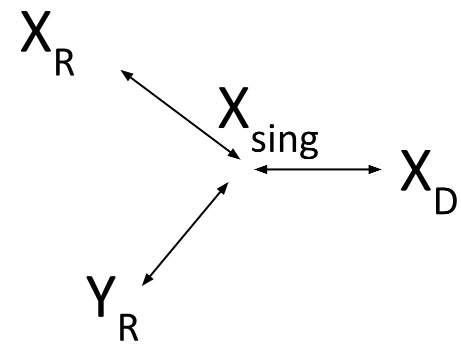
  which field theories <-> which geometries?

- Are these bounded? How do we classify what geometry (pieces/combinations of manifolds, branes, bundles, fluxes, etc) "matters" to the 4D EFT?
- Is it possibly to dynamically change topology?
- Can we understand geometric transitions in the N=1 context?

Geometric Transitions

- Topology changing transitions in CY 3-folds: Conifolds and Flops

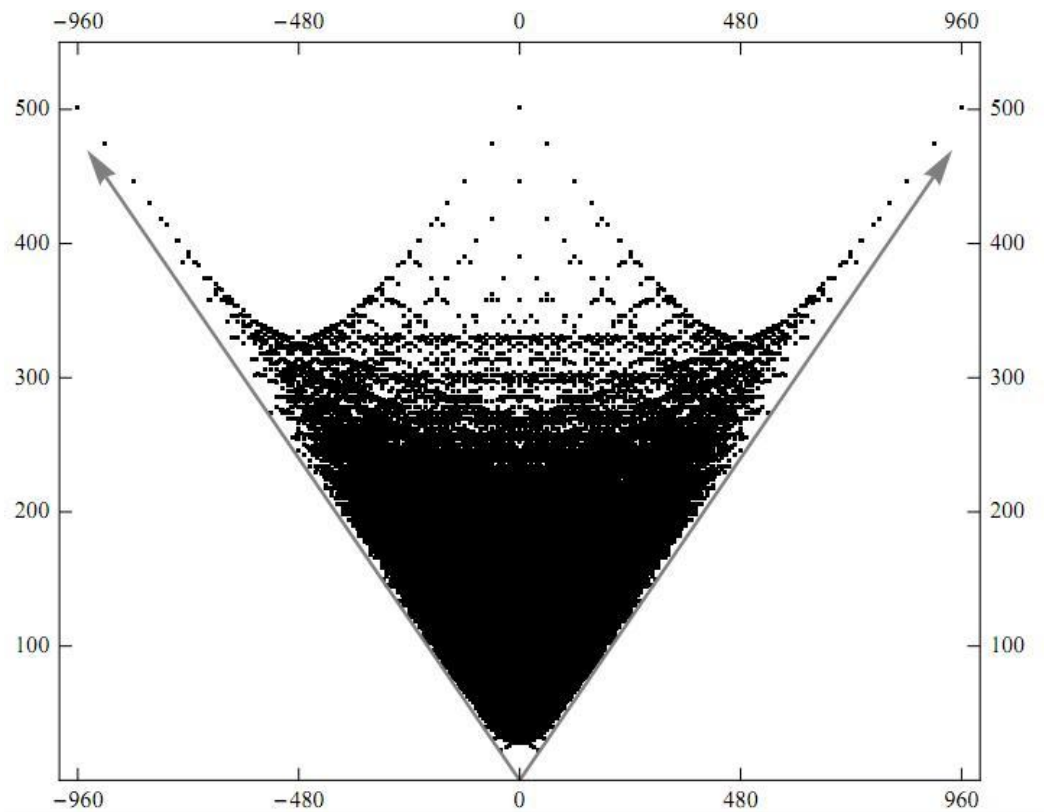




- At the level of CY geometry, these transitions are well understood.
- They are also well understood field-theoretically in some contexts (i.e. Type IIA/B, N=2 4D) (Strominger, Greene+Morrison, etc)

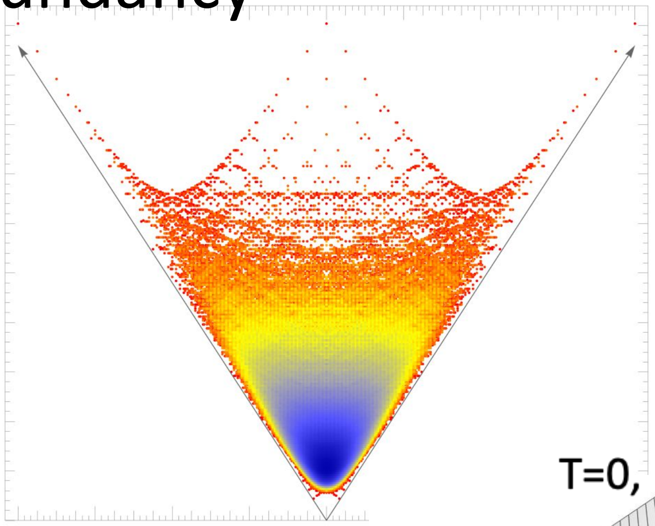
Geometric Transitions

- Connect (most) known CY manifolds
- All CY manifolds? (Reid)
- All SU(3) Structure manifolds??
- Key for attempts at bounding all CY or SU(3) structure manifolds
- More generally: Topology changing transitions touch a wide array of questions



Geometric Transitions

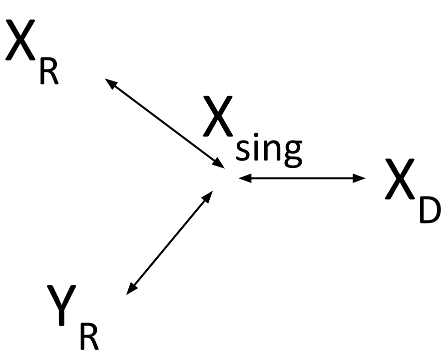
String Duality <-> Geometric “Redundancy”

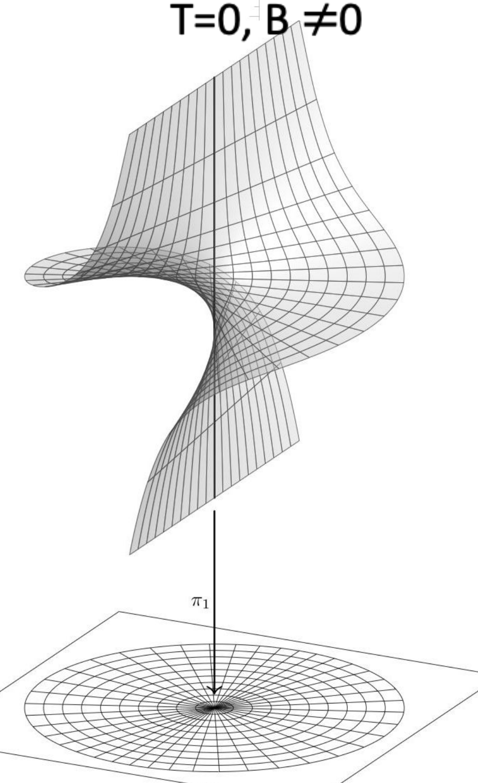


- We know in many contexts that more than one geometry can lead to the same EFT in string theory
- E.g. N=2 Theories
- Mirror Symmetry (Here Type IIB <-> IIA)
- Type IIB Flops (B-field

allows for field space to stay smooth, despite the CY singularity)

- E.g.s of dualities, branch changes…





Key questions for this talk

- Are there phenomena like mirror symmetry or "smooth" geometric transitions (i.e. flops) in the context of N=1 compactifications?
- Can we fully characterize what "geometry" matters for the N=1 compactifications (i.e. manifolds+bundles, branes, fluxes)? Redundancy?
- Can we understand/characterize/bound topology changing transitions in the N=1 context?
- Difficulties:

1. Notions of "moduli space" (For N=1: non-trivial superpotentials, etc).
2. Intrinsically coupled problem between manifolds and other background geometry (bundles, branes,etc)

- Conifolds in the N=1 context?
- In heterotic string theory cannot ignore the gauge fields/bundle. The theory has a Bianchi Identity:

$$dH = -\alpha' \mathrm{tr} F \wedge F + \alpha' \mathrm{tr} R \wedge R$$

Three-form

Negative sources from gauge and positive sources from gravitational sectors

- Chern class condition:

$$[\mathrm{tr} R \wedge R] = [\mathrm{tr} F \wedge F] \Rightarrow c_2(\Omega_X) = c_2(V)$$

- With 5-branes: $c_2(\Omega_X) = c_2(V) + [C]$

- This means that we cannot set the gauge fields to zero.

  Need to understand the change in the bundle during a geometric transition in addition to the CY geometry itself. This has been a stumbling block for decades.

  Motivation:

  Try and understand what happens to the gauge fields in heterotic string theory during a certain type of geometric transition (a conifold).

See also Collins, Gukov, Picard, Yau math.DG/2102.11170 and Candelas, de la Ossa, He, Szendroi hep-th/0706.3134

Where to start?…

Examples observed of N=1 “redundancy” (not descended from N=2)

- Known examples of heterotic compactifications with distinct geometry (i.e. pairs $(X, \pi: X \to V)$) that lead to the same massless spectrum
- E.g. (0,2) GLSM realizations (0,2) Target Space Duality (Distler/Kachru, Blumenhagen). (0,2) GLSMs that share a non-geometric (LG or hybrid) phase

Calabi-Yau Defining Relations $\longleftrightarrow$ Monad Maps

- Observation: base CY manifolds (X, X’) in these examples generically related by conifold transitions.

Distler and Kachru hep-th/9707198, Blumenhagen hep-th/9707198 and hep-th/9710021, Blumenhagen and Rahn 1106.4998, Anderson and Feng 1607.04628

Smooth quintic

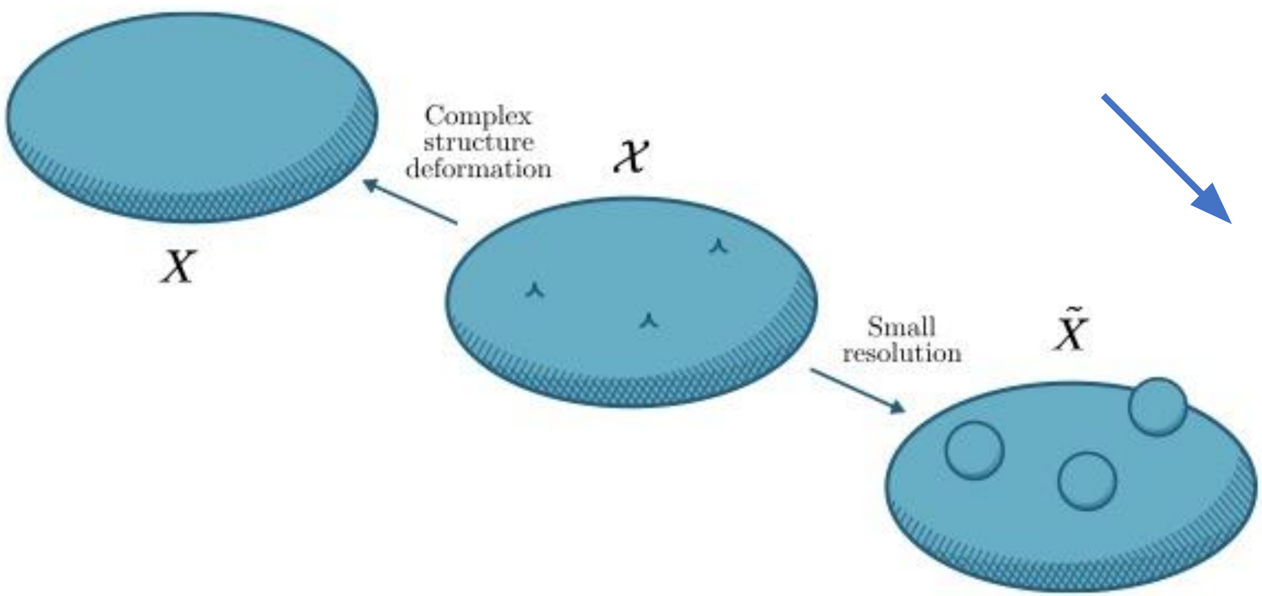
$$X_D = \left[\ \mathbb{P}^4 \ \middle|\ 5\ \right]$$

Nodal quintic

Deform complex structure

$$X_0 :\ l_1 q_2 - l_2 q_1 = 0$$

Blow up singular points of nodal quintic with (16) $\mathbb{P}^1 s$



$$X_R = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^4 & 1 & 4 \end{array}\right]$$

- These manifolds come equipped with bundles:

Deformation side bundle:

$$0 \to \mathcal{O}(-5) \to \mathcal{O}(-1)^{\oplus 5} \to V_D \to 0$$

Resolution side bundle:

$$0 \to \begin{array}{c} \mathcal{O}(-1,-5) \\ \oplus \\ \mathcal{O}(0,-4) \end{array} \to \begin{array}{c} \mathcal{O}(-1,0) \oplus \mathcal{O}(0,-5) \\ \oplus \\ \mathcal{O}(0,-1)^{\oplus 4} \end{array} \to V_R \to 0$$

What happens to the degrees of freedom of the theory?
E.g. Singlets:

Deformation side:

Kahler moduli: $h^{1,1} = 1$

Complex Structure: $h^{2,1} = 101$

Bundle Moduli: $h^1(\mathrm{End}_0(V)) = 324$

Total = 426

Resolution side:

Kahler moduli: $h^{1,1} = 2$

Complex Structure: $h^{2,1} = 86$

Bundle Moduli: $h^1(\mathrm{End}_0(V)) = 338$

Total = 426

(SO(10) Gauge symmetry and charged matter also agrees)

Heterotic Conifold Transitions

- With Gray + Brodie, we pulled apart the geometry of examples like these and developed more general rules for how a bundle must adjust across a conifold to maintain a consistent heterotic theory.
- Contains (0,2) TSD, but more general.
- Can be applied to other classes of N=1 compactifications (i.e. Type I, Type IIB orientifolds, F-theory, etc).
- Will give a heterotic overview and then talk about the more general effect.

How do the tangent bundles change?

- Topology change:

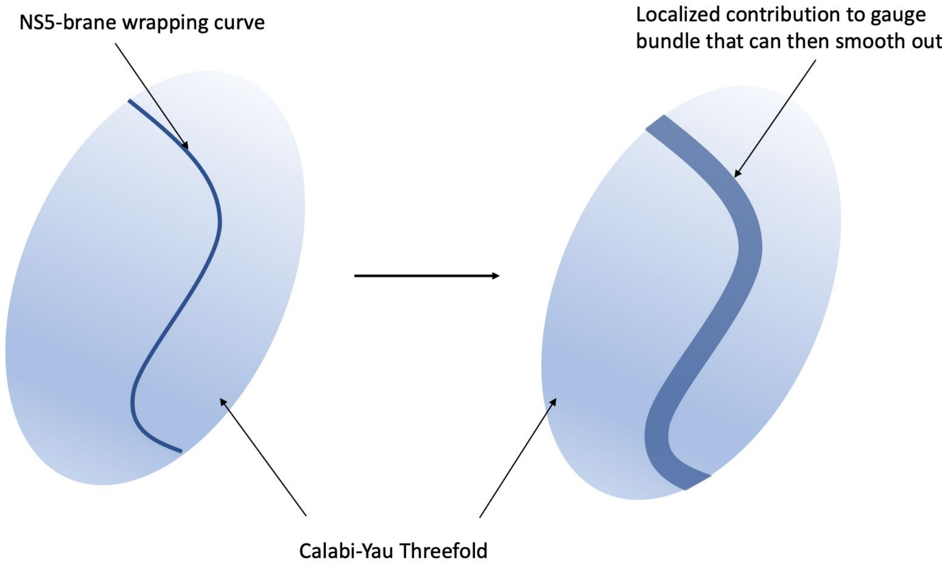
$$ch_2(X_R) = ch_2(X_D) + [\mathbb{P}^1]$$
$$h^{1,1}(X_R) = h^{1,1}(X_D) + 1$$
$$h^{2,1}(X_R) = h^{2,1}(X_D) - \Delta$$

- As bundles on the resolution manifold

$$0 \to f^*\Omega_{X_0} \to \Omega_{X_R} \to \mathcal{O}_{\mathbb{P}^1 s}(-2) \to 0$$

Small contraction: $f : X_R \to X_0$

- This looks familiar: Small Instanton Transition (Hecke Transform)



Including a gauge bundle in the transition

- Recall the anomaly cancelation condition in heterotic string-theory:

$$c_2(\Omega_{X_R}) = c_2(V_R)$$

- How does this change during the transition?

$$\underbrace{c_2(\Omega_{X_R}) + \left[\mathbb{P}^1 s\right]}_{\text{This is how the gravitational sector changes given the transition we have seen in the cotangent bundle.}} = \underbrace{c_2(V_R) + \left[\mathbb{P}^1 s\right]}_{\text{The gauge sector must change in the same way.}}$$

The full structure of the transition

- On the next slide is the map of the transition, presented at the level of classes for clarity.

- All of the sequences of sheaves required for this process to occur exist and can be written down explicitly.

- In what follows V_s is a "spectator bundle" which goes through the transition in a trivial manner.

$$\mathrm{c}_2(\Omega_{X_R}) = \mathrm{c}_2(V_R)$$

“Pair create” curve supported sheaves $\Big\downarrow$ “Pair create” curve supported sheaves

$$\mathrm{c}_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = \mathrm{c}_2(V_R) + [\mathbb{P}^1 s]$$

SIT in cotangent bundle $\Big\downarrow$ SIT in gauge bundle

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_R] + [\mathbb{P}^1 s]$$

$\Big\downarrow$ “Brane” recombination

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_D]$$

$\Big\downarrow$ SIT in gauge bundle

$$\mathrm{c}_2(f^*\Omega_{X_0}) = \mathrm{c}_2(V_D)$$

$$c_2(\Omega_{X_R}) = c_2(V_R)$$

“Pair create” curve supported sheaves $\Big\downarrow$ “Pair create” curve supported sheaves

$$c_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = c_2(V_R) + [\mathbb{P}^1 s]$$

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$\Big\downarrow$ “Brane” recombination

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_D]$$

$\Big\downarrow$ SIT in gauge bundle

$$c_2(f^*\Omega_{X_0}) = c_2(V_D)$$

Bridging Branes

- The spectra matching we find in the heterotic case is based on the properties of special (spacetime filling) NS 5-branes (C_D, C_R) that are linked to the conifold pair. We call these Bridging Branes.
- Geometric property: Curves in X_R which do not collapse in conifold, but intersect collapsing cycles at a point.
- Moduli of pure 5-brane theory: $h^{1,1}(X) + h^{1,1}(X) + h^0(C, N_C)$

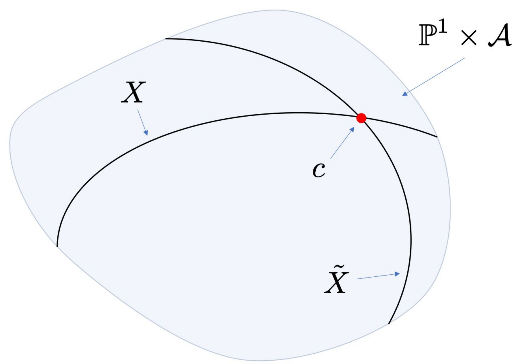
For Bridging Curves:

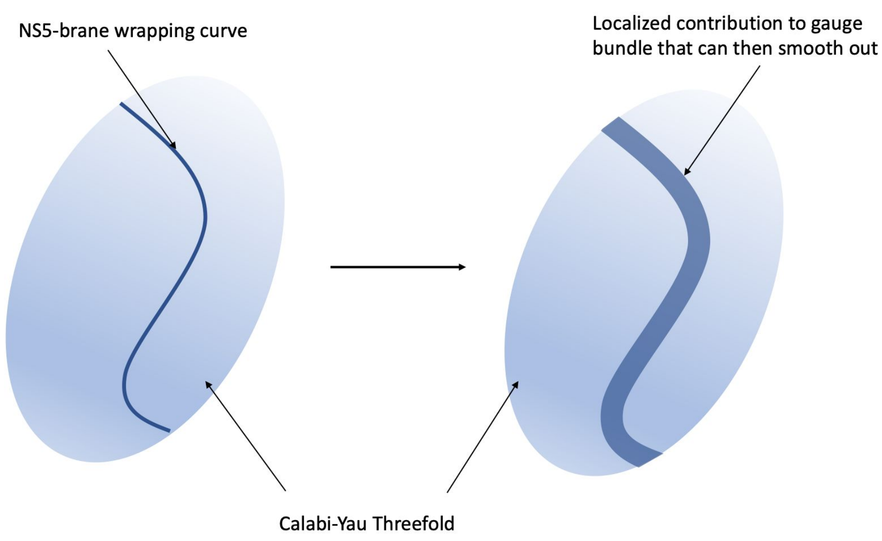
$h^0(C, N_C)$ adjusts across the conifold to exactly compensate for the change in hodge numbers! Every CY conifold determines a pair (C_D, C_R)

5-brane duality

$$\left[\ \mathcal{A}\ \middle|\ \mathbf{v}_0+\mathbf{v}_1 \quad \mathbf{R}\ \right] \longleftrightarrow \left[\begin{array}{c|ccc} \mathbb{P}^1 & 1 & 1 & 0 \\ \mathcal{A} & \mathbf{v}_0 & \mathbf{v}_1 & \mathbf{R} \end{array}\right]$$

- Pure 5-brane heterotic theories identical
- For CICY 3-folds bridging curves for 5-brane correspond to the intersection of the two CY 3-folds in an ambient space.
- General heterotic conifold duality induced from this phenomenon by small instanton transitions





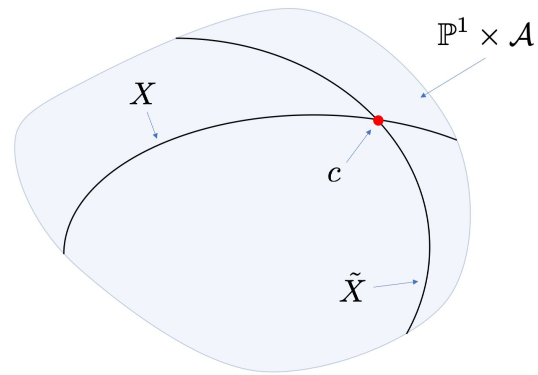
Questions:

- We observe correlated geometry and matching massless spectra.
- Is this a true duality? Or distinct theories with intersecting vacuum spaces?
- Can we use the geometry of these bridging branes in other string compactifications?

A moduli map for bridging branes

- We'll explore now the pure NS 5-brane heterotic theories and attempt to determine a mapping of fields.
- Recall mirror symmetry (map: Comp. Struc. $\leftrightarrow$Kahler). What happens here with manifold + branes?
- This can be generalized to bundles directly via SIT (won't go through this)
- This is hard to do directly since some parts of the EFT unknown (e.g. matter field Kahler potential):
- Two primary tools:
  - Full geometries (manifold+brane) must "meet in the middle" at the conifold locus
  - Geometric d.o.f. live in intricate equivalence classes. Must map physical to physical d.o.f.

The rough idea



- Example Brane/manifold system of eqns:

y_0	y_1	y_2	y_3	y_4	x_0	x_1
0	0	0	0	0	1	1
1	1	1	1	1	3	0

- Deformation:

$$G_{1,3}{}^{(D)} = G_{0,5}{}^{(D)} = 0 \text{ (manifold) } \mathrm{P}_{1,4}{}^{(\mathrm{D})} = \mathrm{P}_{1,4}^{(\mathrm{D})} = 0 \text{ (brane)}$$

$$h^{1,1} + h^{2,1} + h^0(\mathcal{N}_{C_D}) = 1 + 101 + 38 = 140$$

- Resolution:

$$G_{1,4}{}^{(R)} = G_{1,4}{}^{(R)} = 0 \text{ (manifold) } \mathrm{P}_{1,3}{}^{(\mathrm{R})} = \mathrm{P}_{0,5}^{(\mathrm{R})} = 0 \text{ (brane)}$$

$$h^{1,1} + h^{2,1} + h^0(\mathcal{N}_{C_R}) = 2 + 86 + 52 = 140$$

- Tempting guess: Interchange brane <-> manifold defining equations…

Problem with interchanging polynomials

- What goes wrong with naive interchange of polynomials?
- Deformation side: Consider fluctuating the brane defining equation:

$$P_{1,4} \to P_{1,4} + L_{0,1} G_{1,3}$$

This change is unphysical

$$G_{1,4} \to G_{1,4} + L_{0,1} P_{1,3}$$

- Resolution side: Same fluctuation of the manifold

	Deformation	Resolution
Manifold	$G_{1,3}^{(D)}, G_{0,5}{}^{(D)}$	$G^{\alpha}_{1,4}{}^{(R)}$
Brane	$P^{\alpha}_{1,4}{}^{(D)}$	$P_{1,3}{}^{(R)}, P_{0,5}{}^{(R)}$

Is physical.

More generally, fluctuations defined by geometric equivalence classes

Def:

$$H^1(TX_D) \quad : \begin{cases} \delta P_{0,5} \sim \delta P_{0,5} + hP^{(D)}_{0,5} + l^i_{0,1}\partial_{y^i}P^{(D)}_{0,5} \\ \delta P_{1,3} \sim \delta P_{1,3} + mP^{(D)}_{1,3} + l^i_{0,1}\partial_{y^i}P^{(D)}_{1,3} \\ \qquad +l_{1,3}\partial_{x^0}P^{(D)}_{1,3} + l_{1,0}\partial_{x^1}P^{(D)}_{1,3} \end{cases}$$

$$\begin{aligned} H^0(\mathcal{N}_{\mathcal{C}_D}|_{\mathcal{C}_D}) : \delta P^\alpha_{1,4} \quad &\sim \quad \delta P^\alpha_{1,4} + A^\alpha_\beta P^{(D)\beta}_{1,4} + L^\alpha_{0,1}P^{(D)}_{1,3} \\ &\quad +l^i_{0,1}\partial_{y^i}P^{(D)\alpha}_{1,4} + l_{1,3}\partial_{x^0}P^{(D)\alpha}_{1,4} + l_{1,0}\partial_{x^1}P^{(D)\alpha}_{1,4} \end{aligned}$$

Res:

$$\begin{aligned} H^1(TX_R) \quad &: \delta P^\alpha_{1,4} \sim \delta P^\alpha_{1,4} + A^\alpha_\beta P^{(R)\beta}_{1,4} \\ &+l^i_{0,1}\partial_{y^i}P^{(R)\alpha}_{1,4} + l_{1,3}\partial_{x^0}P^{(R)\alpha}_{1,4} + l_{1,0}\partial_{x^1}P^{(R)\alpha}_{1,4} \end{aligned}$$

$$H^0(\mathcal{N}_{\mathcal{C}_R}|_{\mathcal{C}_R}) : \begin{cases} \delta P_{0,5} \sim \delta P_{0,5} + hP^{(R)}_{0,5} + BP^{(R)}_{\text{nodal}} \\ \quad +L^\alpha_{0,1}\left(q^{(R)\alpha}\right) + l^i_{0,1}\partial_{y^i}P^{(R)}_{0,5} \\ \delta P_{1,3} \sim \delta P_{1,3} + mP^{(R)}_{1,3} + l^i_{0,1}\partial_{y^i}P^{(R)}_{1,3} + \\ \quad l_{1,3}\partial_{x^0}P^{(R)}_{1,3} + l_{1,0}\partial_{x^1}P^{(R)}_{1,3} \end{cases}$$

Proposed mapping

- Field mapping almost interchanging, but with important correlation:

$$
\begin{aligned}
G_{1,4}^{(R)\alpha} &= P_{1,4}^{(D)\alpha} = P_{1,4}^{\alpha} = l^{\alpha} x_0 + q^{\alpha} x_1 \\
P_{1,3}^{(R)} &= G_{1,3}^{(D)} = P_{1,3} = x_0 \\
P_{0,5}^{(R)} &= G_{0,5}^{(D)} = \alpha \epsilon_{\alpha\beta} l^{\alpha} q^{\beta} + P'_{0,5}
\end{aligned}
$$

(Trivial in X_R but not X_D)

- This decomposition of the quintic correlates the deformation and resolution complex structure (as expected by conifold geometry).
- Maps physical <-> physical degrees of freedom
- Decomposition of $G_{0,5}$ non-unique (polynomial long division only defined up to multiples of q^{α}). As we'll see this ambiguity drops out of the N=1 theory's superpotential.

The new Kahler modulus

- There is exactly one physical fluctuation of the complex structure on the deformation side which cannot map to physical comp. struc. or bundle mod. on the resolution side —> New Kahler modulus?
- $\delta G_{0,5} \sim \alpha \epsilon_{\alpha\beta} l^{\alpha} q^{\beta}$ (scaling of the nodal piece)
- In the limit $\alpha \to \infty$, approach the nodal limit. Corresponds to $T_{new} \to 0$
- Schematically (actually linear combinations):

Deformation		Resolution
86 C.S.	$\rightarrow$	C.S.
14 C.S.	$\rightarrow$	Brane mod.
1 C.S.	$\rightarrow$	Kahler

Testing a duality: Heterotic Superpotenials

- Explore whether the theories (not just their spectra) are identical? N=1 theories -> superpotential.
- Need a full mapping of fields to do this. Test of our proposal
- Perturbative Yukawa couplings
  - E.g. E6 theory, **27**^{3} coupling: $H^1(V) \times H^1(V) \times H^1(V) \to H^3(\wedge^3 V)$
  - Complex functions of bundle/complex structure moduli
- Non-perturbative terms $W \sim \sum Pfaff(c,b)_i e^{-T}$
  - Could obstruct the transition? Agreement?

An E6 example:

Deformation side:

$$\left[\mathbb{P}^4 \middle| 5\right]$$

$$0 \to V_D \to \mathcal{O}(1)^3 \oplus \mathcal{O}(2) \to \mathcal{O}(5) \to 0$$

Resolution side:

$$\left[\begin{array}{ccccccc|cc} y_0 & y_1 & y_2 & y_3 & y_4 & x_0 & x_1 & p_1 & p_2 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 3 & 0 & 4 & 4 \end{array}\right]$$

$$0 \to V_R \to \mathcal{O}(0,2)^2 \oplus \mathcal{O}(0,1) \oplus \mathcal{O}(1,0) \to \mathcal{O}(1,5) \to 0$$

Matching superpotentials

In this example we find that all 147,440 independent, non-vanishing, Yukawa couplings correctly match as holomorphic functions of the moduli on either side of the duality (95 families in this case).

- More generally, for instanton contributions, we can prove
- Potentially obstructing terms

$$W \sim \sum \psi_i e^{-T_{new}}$$

vanish

- Remaining non-perturbative terms have non-trivial matching of Pfaffians, moduli dependence (work in progress)

Other N=1 theories?

- Most of the above story is a feature of CY conifold geometries, not heterotic compactifications
- Generalize bridging branes to other N=1 vacua?
- Easy to see that SO(32) heterotic and Type I behave very similarly.
- What about Type IIB orientifolds? Try vacua with D5/O5 system.

Type IIB orientifolds (D5/O5)

- CY3 Conifold pair

$$\left[\begin{array}{c|ccccc} \mathbb{P}^4_y & 1 & 1 & 1 & 1 & 1 \\ \mathbb{P}^4_z & 1 & 1 & 1 & 1 & 1 \end{array}\right]_{2,52} \leftrightarrow \left[\begin{array}{c|cccccc} \mathbb{P}^1_x & 1 & 1 & 0 & 0 & 0 & 0 \\ \mathbb{P}^4_z & 1 & 0 & 1 & 1 & 1 & 1 \\ \mathbb{P}^4_z & 0 & 1 & 1 & 1 & 1 & 1 \end{array}\right]_{3,47}$$

- Orientifold actions

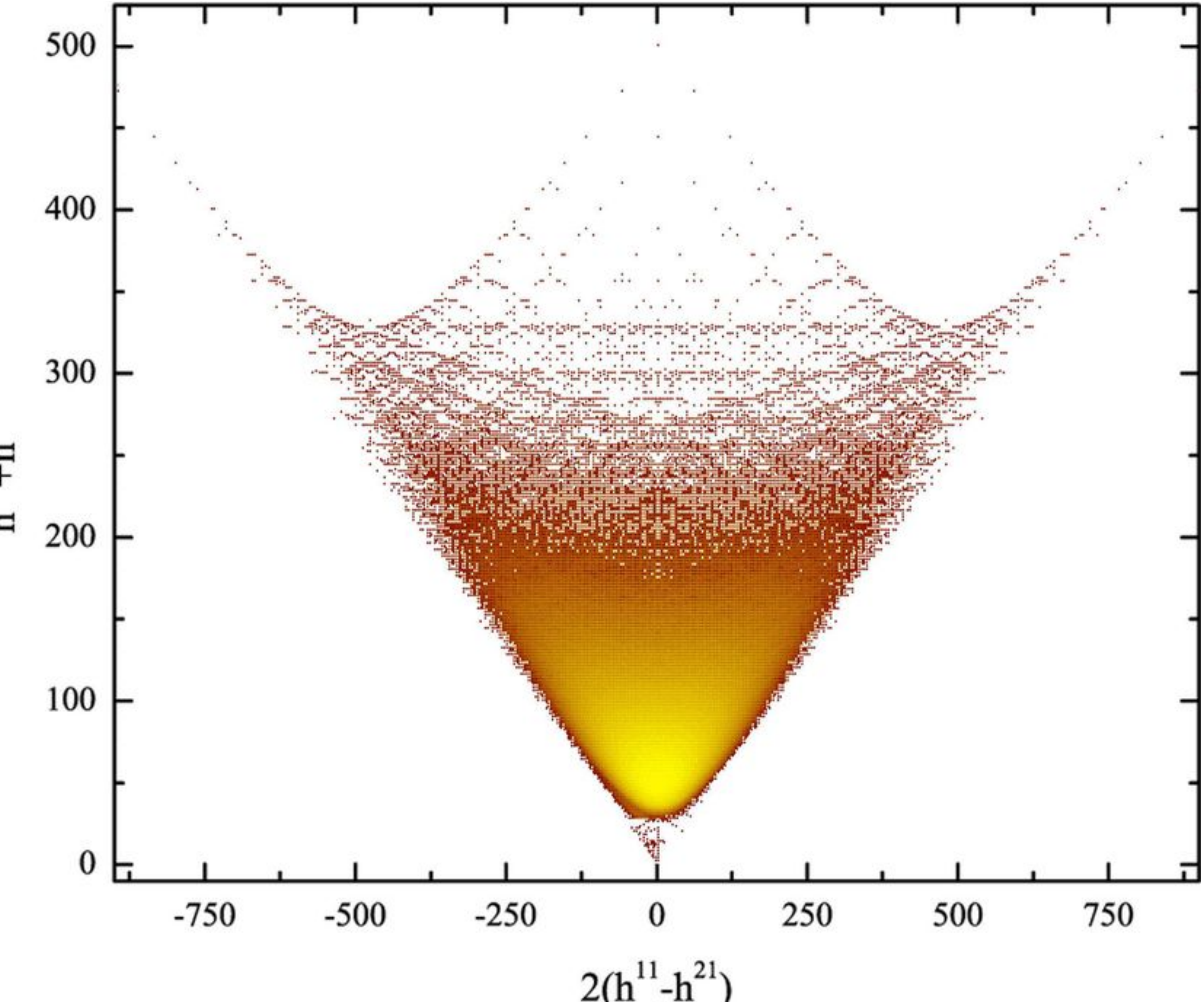
$$(y_0 \to -y_0 \quad , \quad z_1 \to -z_1) \leftrightarrow (x_0 \to -x_0 \, , \; y_0 \to -y_0 \, , \; z_1 \to -z_1)$$

- D5s (bridging curves) and O5s (by action above) satisfy tadpoles
- Spectrum agrees

closed string d.o.f.	
chiral multiplets	$h^{1,1}_+ + h^{1,1}_- + h^{2,1}_+ + 1$
vector multiplets	$h^{2,1}_-$
open string d.o.f.	
chiral multiplets	$h^0_+(C, N_C) + h^{1,0}(C)$
vector multiplets	1

Further questions

- We are developing new and intrinsically N=1 dualities(?)
- Not dual "pairs" of theories, but long chains.
- Of clear relevance for model building/scans. Computation of EFT. Large redundancy of theories
- Embedding of theories (low $h^{1,1}$ —> high $h^{1,1}$) is potentially powerful.
- What geometry matters? N=1 "Wall's Data"?
- Work to appear (ask me if curious):
- What about N=1 F-theory/CY 4-folds?
- Is the physical transition smooth across the geometric conifold? Could presence of bridging branes lift extra light states from branes wrapping collapsing cycles?


500
400
300
200
100
0
-750
-500
-250
0
250
500
750
$2(h^{11}-h^{21})$