Exploring Confinement in Anti-de Sitter Space

[2407.06268] with R Ciccone, F De Cesare, M Serone

Strings 2025, NYU Abu Dhabi

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1924 2024
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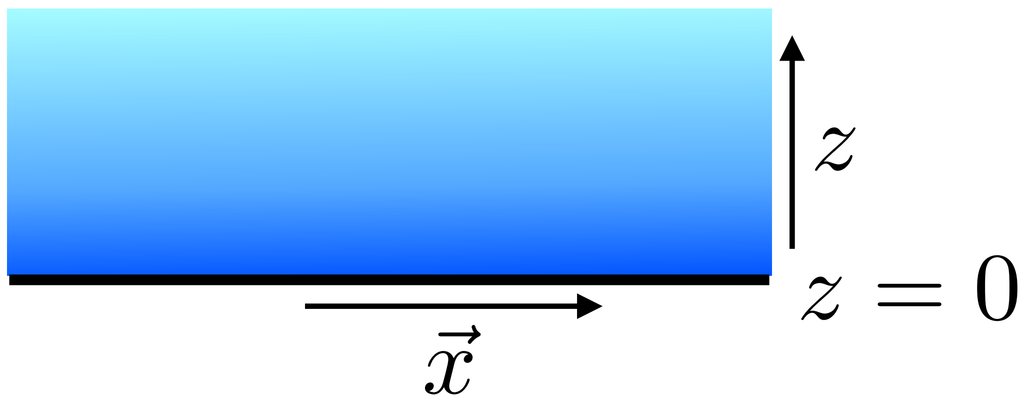
Why QFT in AdS ?

[Callan, Wilczek (1990)], ...

$$ds^2_{\rm AdS} = L^2 \frac{dz^2 + d\vec{x}^2}{z^2}$$



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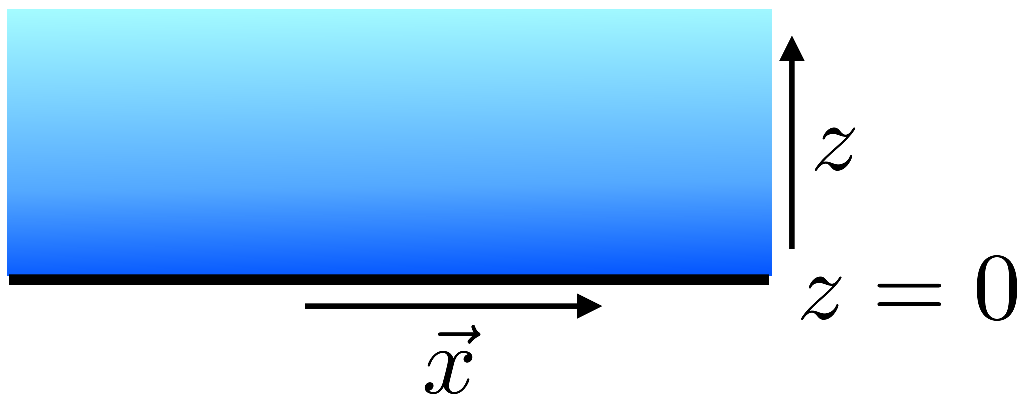

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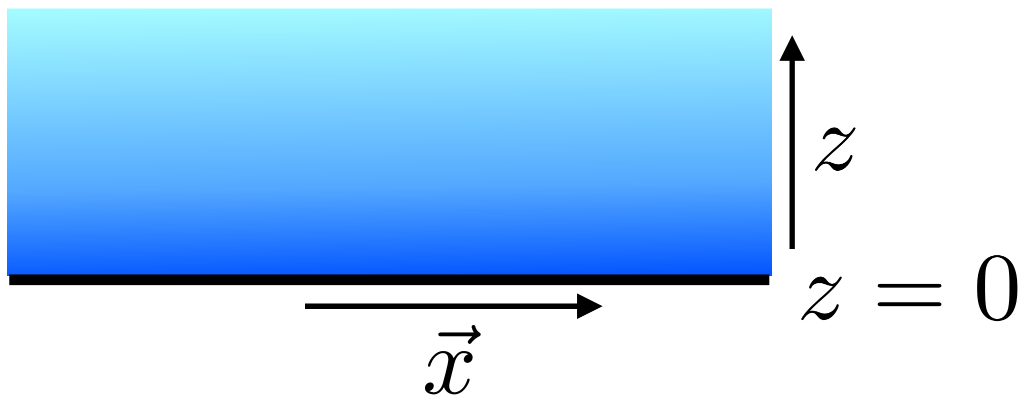

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  …like a sphere, but also

- Infinite volume: symmetry breaking and phase transitions

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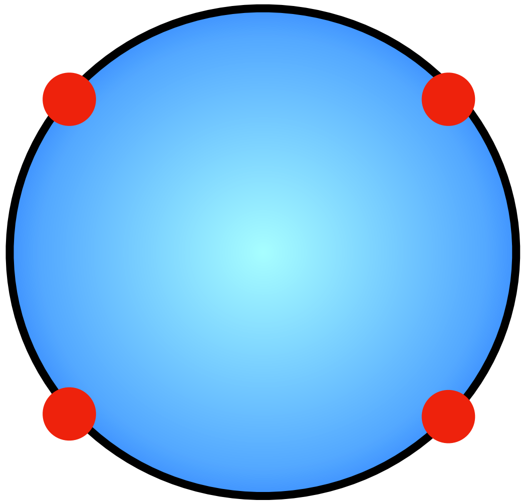

- Analogue of scattering amplitudes: local operators at boundary points

- Boundary correlators covariant under AdS isometries

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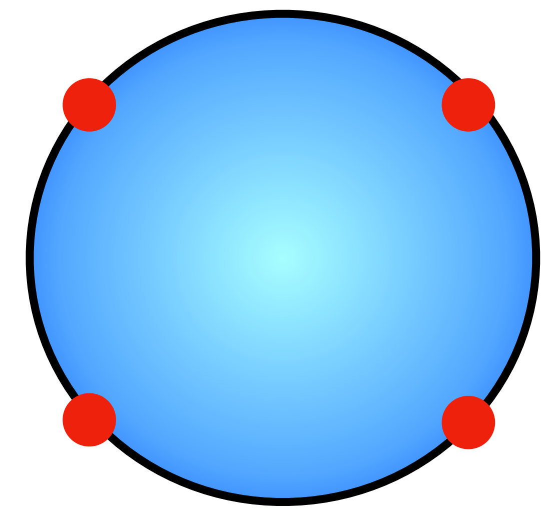

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→ **boundary CFT (bCFT)** in any QFT!

- bCFT data depend on: bulk couplings, boundary conditions

- Unlike AdS/CFT: no dynamical gravity, no boundary stress tensor, bCFT correlators are only a subset of all the observables

Setup: **Asymptotically free theories in AdS**

- UV theory: massless d.o.f, weakly interacting $g \ll 1$;
- IR theory: mass gap $\Lambda \sim \mu e^{-\frac{1}{\beta_0 g^2(\mu)}}$;

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- UV theory: massless d.o.f, weakly interacting $g \ll 1$;
- IR theory: mass gap $\Lambda \sim \mu e^{-\frac{1}{\beta_0 g^2(\mu)}}$;
- Varying the AdS radius L we can interpolate;
- In some cases, a **phase transition** between the two regimes must occur, detectable from the bCFT.

Opportunity: studying the bCFT as a way to **detect/explain the mass gap** in the bulk

Outline:

- Deconfinement transition for Yang-Mills in AdS_4 [Aharony, Berkooz, Tong, Yankielowicz (2012)]

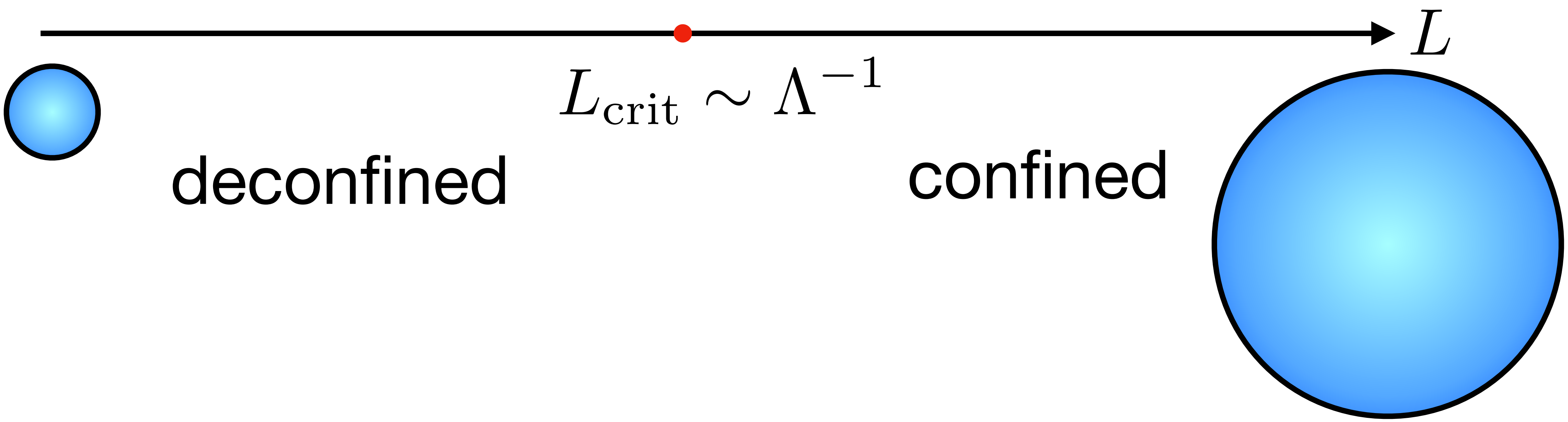
- [Copetti, DP, Ji, Komatsu (2023)]: transition in 2d examples, e.g. O(N) sigma model

Additional computational handle: **Large N**

- [Ciccone, De Cesare, DP, Serone (2024)]: hints about the transition in Yang-Mills in AdS_4 using perturbation theory

Yang-Mills in AdS$_4$

[Aharony, Berkooz, Tong, Yankielowicz (2012)]



- $L \ll \Lambda^{-1}$: with **Dirichlet** boundary condition

$A^a_\mu \underset{z\to 0}{\sim} z J^a_\mu + \ldots$ ➡ G_{YM} global symmetry in bCFT

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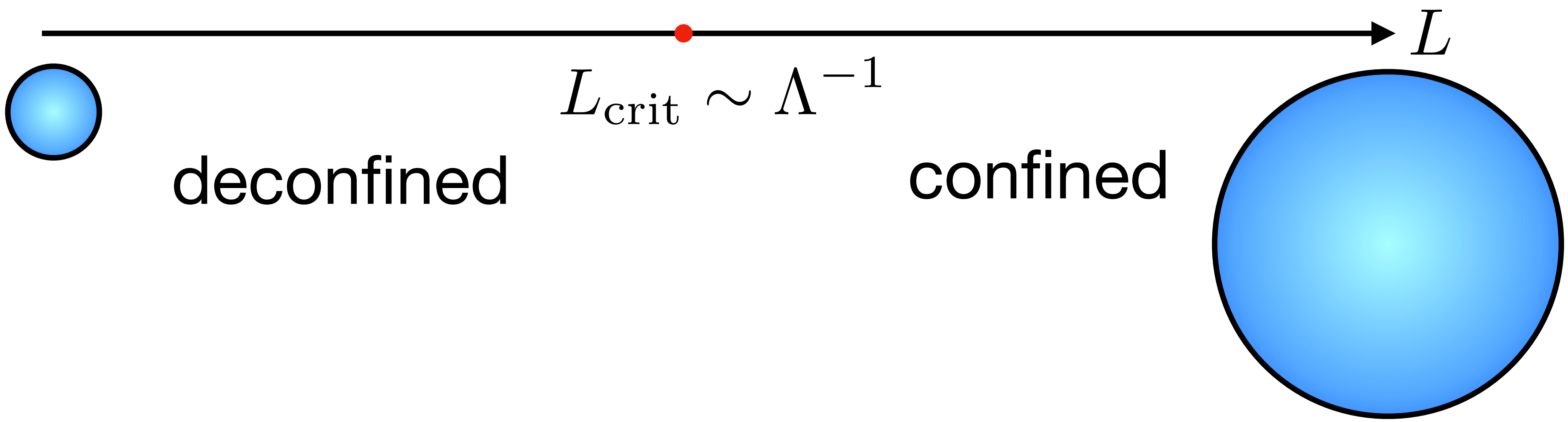

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Proving **disappearance of Dirichlet** is key to understand confinement and mass gap from AdS viewpoint

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bCFT: $\langle J^a_\mu(\vec{x}) J^b_\nu(0)\rangle = C_J\, \delta^{ab}\, \frac{\delta_{\mu\nu} - 2\frac{x_\mu x_\nu}{\vec{x}^{\,2}}}{(\vec{x}^{\,2})^2}$, $C_J \underset{L \to L_{\rm crit}}{\longrightarrow} 0$

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(3) **Tachyon:** (weakly-coupled) scalar ϕ with $M^2 < M^2_{\text{BF}}$

bCFT: $\Delta(\mathcal{O}_\phi) = \frac{3}{2}$, at weak coupling $\Delta(\mathcal{O}^2_\phi) = 3$
singlet marginal operator

We propose a generalization of (3), without need of weak coupling, which is realized in 2d toy models:

(4) **Marginality:**

bCFT: a singlet scalar operator $\mathcal{O}$ becomes marginal ($\Delta = 3$) and its boundary coupling runs

$$\int_{\mathrm{AdS}_4} \frac{1}{2g^2}\,\mathrm{tr}[F^2] + \int_\partial y\,\mathcal{O} \quad , \quad \beta_y \sim A\left(\frac{1}{g^2_{\mathrm{crit}}} - \frac{1}{g^2}\right) + B\,y^2$$

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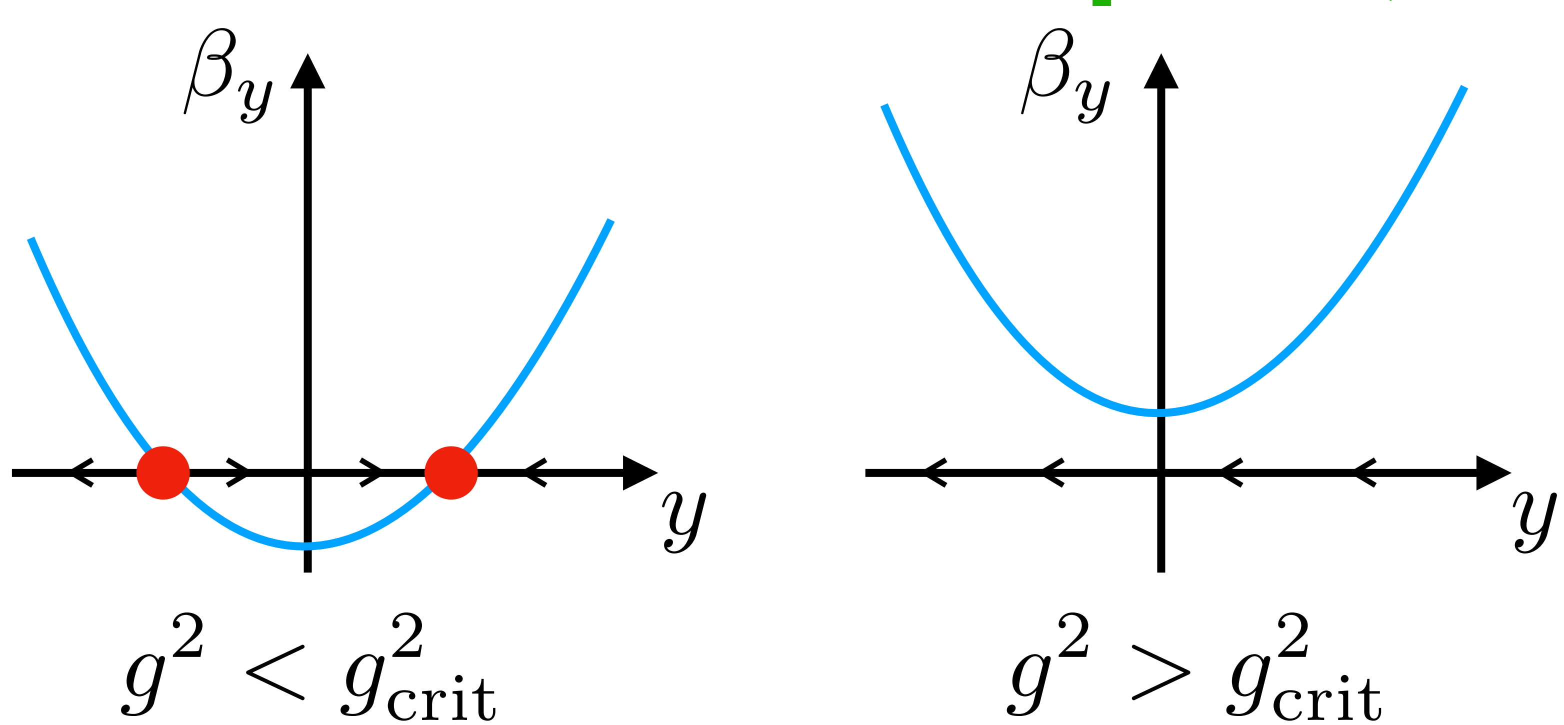

Merger and annihilation at the transition

2d Toy Models [Copetti, DP, Ji, Komatsu (2023)]

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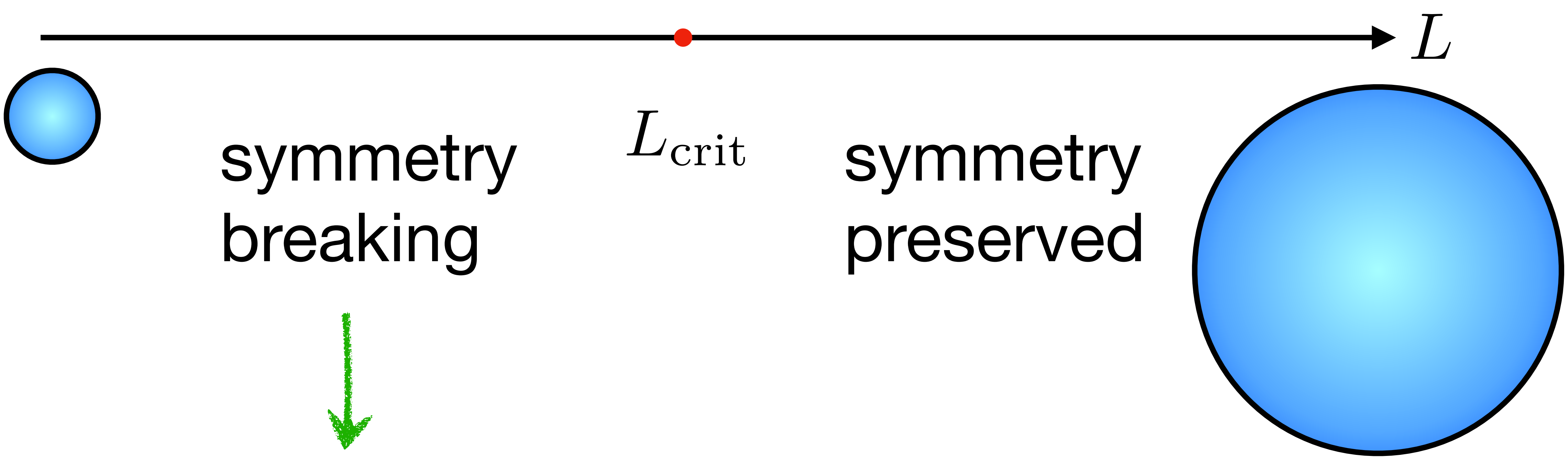
In theories with continuous global symmetry:



Massless scalars.
Forbidden for $L \to \infty$ by Coleman-Mermin-Wagner

2d Toy Models Example: O(N) sigma model in AdS_2

$$S = \int \tfrac{1}{2}(\partial\phi^i)^2 \quad , \quad i = 1, \ldots, N \quad , \quad (\phi^i)^2 = \frac{N}{g^2}$$

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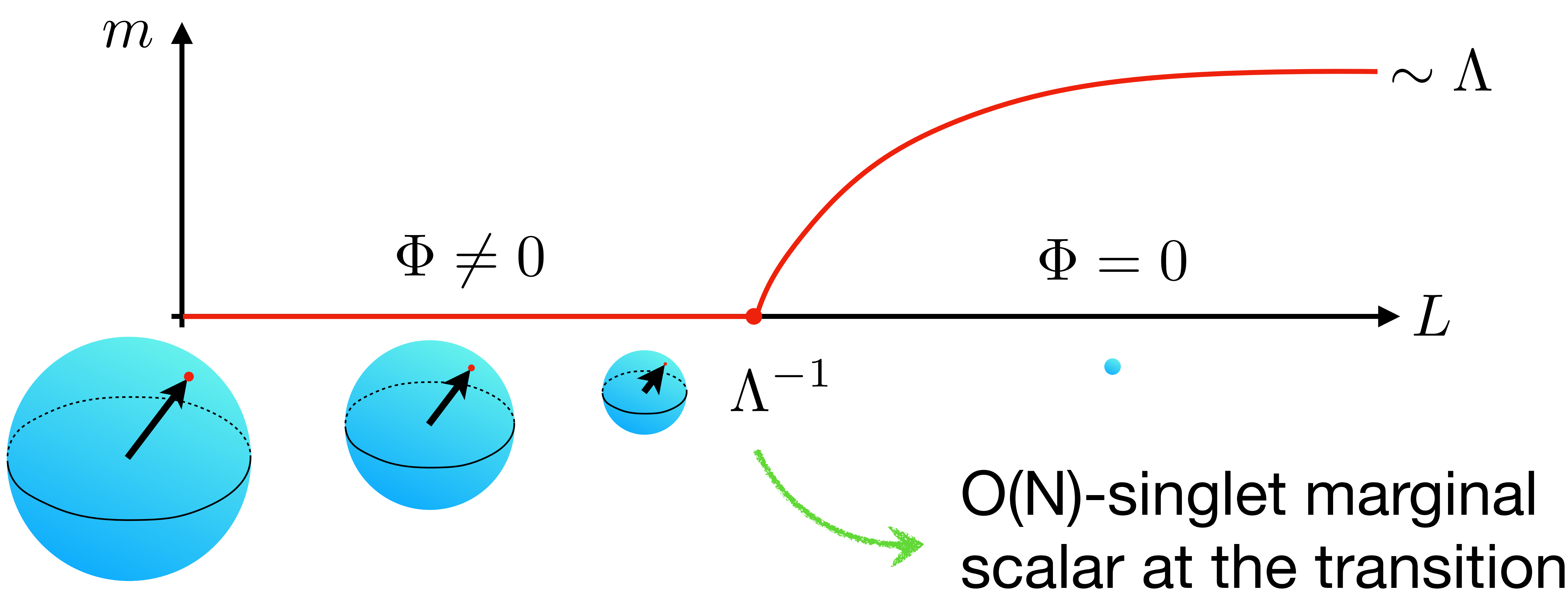
Large N exact solution:



Back to Yang-Mills:

[Ciccone, De Cesare, DP, Serone (2024)]

Goal: use perturbation theory to test which of the scenarios is more plausible.

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Leading candidates for (a)-(b): “double-trace” operators

$J^a_\mu J^{a\mu}$ singlet, $d^{abc} J^b_\mu J^{c\mu}$ adjoint

$$\Delta_{[JJ]} = 4 + \gamma^{(1)}_{[JJ]} g^2 + \mathcal{O}(g^4)$$

Test for (c): $C_J = \frac{2}{\pi^2 g^2}(1 + c^{(1)}_J g^2 + \mathcal{O}(g^4))$

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Results:

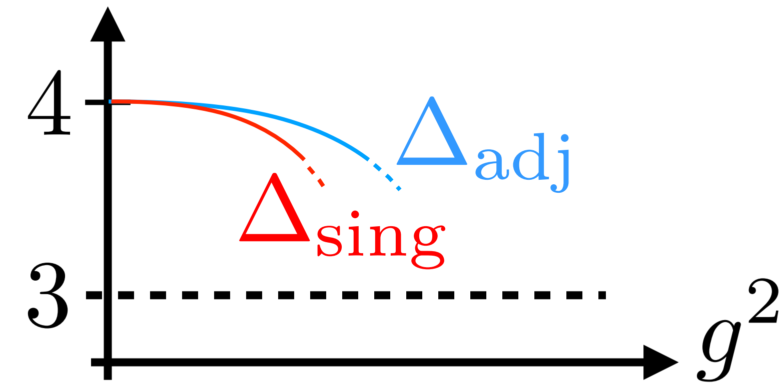
$SU(n_c)$

$$\gamma^{(1)}_{[JJ],\mathrm{singlet}} = -\frac{22 n_c}{3}\frac{1}{16\pi^2}$$

$$\gamma^{(1)}_{[JJ],\mathrm{adjoint}} = -\frac{11 n_c}{3}\frac{1}{16\pi^2}$$

$$c^{(1)}_J = -\frac{(10 + 3\gamma_E) n_c}{324\pi^2}$$

$\gamma^{(1)}_{[JJ],\mathrm{sing}}$ largest coefficient



Estimates from truncating at NLO:

$$\Delta_{\mathrm{sing}} = 3 \quad \rightarrow \quad n_c g^2_{\mathrm{crit}}|_{\mathrm{NLO}} \approx 21.5$$

$$\Delta_{\mathrm{adj}} = 3 \quad \rightarrow \quad n_c g^2_{\mathrm{crit}}|_{\mathrm{NLO}} \approx 43.1$$

$$C_J = 0 \quad \rightarrow \quad n_c g^2_{\mathrm{crit}}|_{\mathrm{NLO}} \approx 272$$

→ **Marginality scenario favoured**

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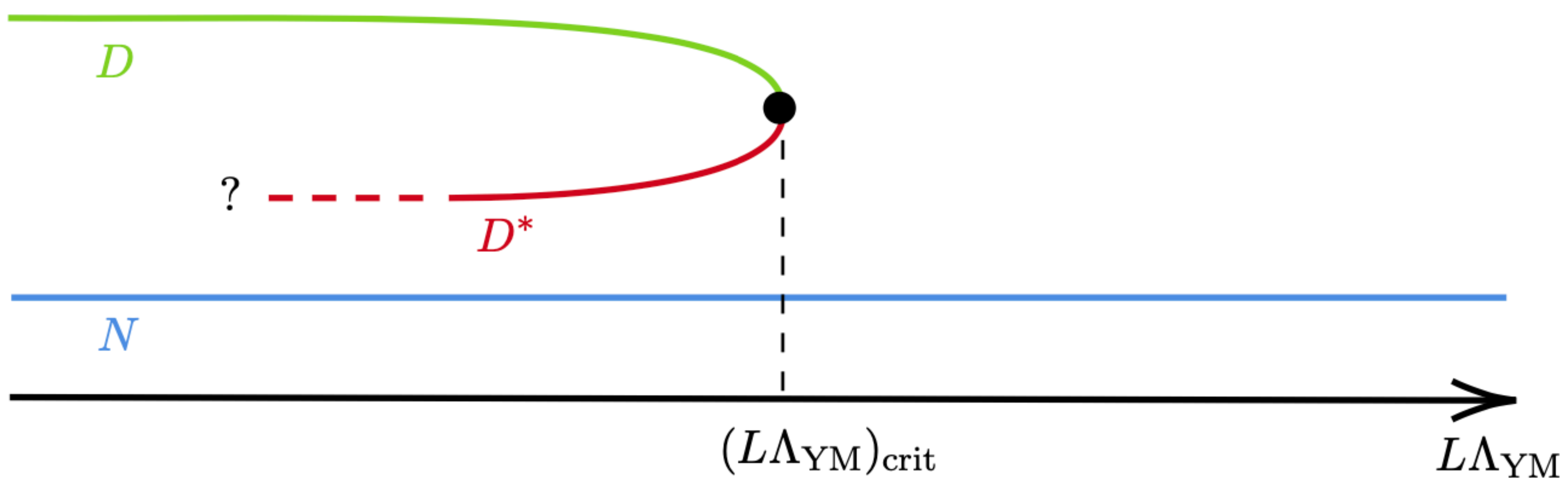
Lightest scalar for Neumann boundary condition:

$$\mathrm{Tr} f^a_{\mu\nu} f^{a\mu\nu} \; , \quad \Delta = 4 + \frac{22 n_c}{3} \frac{g^2}{16\pi^2} + \mathcal{O}(g^4)$$

➡ away from marginality



D
?
D^*
N
$(L\Lambda_{\mathrm{YM}})_{\mathrm{crit}}$
$L\Lambda_{\mathrm{YM}}$

Summary

- Transitions from gapless to gapped phases in AdS
- Solvable examples in 2d: merger and annihilation as mechanism for disappearance of gapless b.c.
- Perturbative results for 4d YM point in the same direction

Future directions

- Numerical conformal bootstrap for YM in AdS$_4$: rigorous bounds on C_J , Δ_{sing} , Δ_{adj} from $\langle J^a J^b J^c J^d \rangle$ [wip w/ Kousvos, Meineri, Piazza, Serone, Vichi]
- Localization for $\mathcal{N}=2$ SYM in AdS$_4$: transition from $SU(2)$ (deconfined) to $U(1)$ (Coulomb) [wip w/ Bason, Copetti, Komatsu, Ji]

Thank You

Gluon propagator in AdS$_{d+1}$

$$\Pi_{\mu\nu}(x,y) = -g_0(u)\nabla_\mu\nabla_\nu u + g_1(u)\nabla_\mu u \nabla_\nu u$$

$x = (z, \vec{x})$, $y = (w, \vec{y})$ $\qquad u = \dfrac{(\vec{x}-\vec{y})^2 + (z-w)^2}{2zw}$

Generic $\frac{1}{2\xi}(\partial_\mu A^{a\mu})^2$ gauge-fixing: $g_0,\, g_1$ are derivatives of hypergeometric functions

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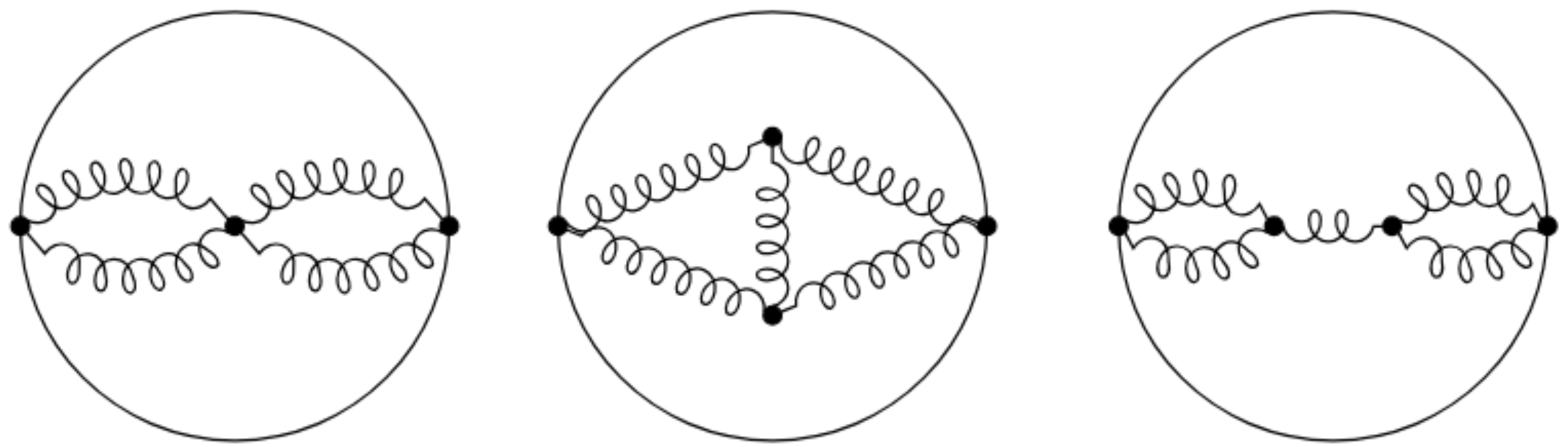
Simplification in **Fried-Yennie gauge** $\quad \xi = \dfrac{d}{d-2}$

$$g_0(u) = \frac{\Gamma\left(\frac{d+1}{2}\right)}{2\pi^{\frac{d+1}{2}}(u(u+2))^{\frac{d-1}{2}}(d-2)}$$

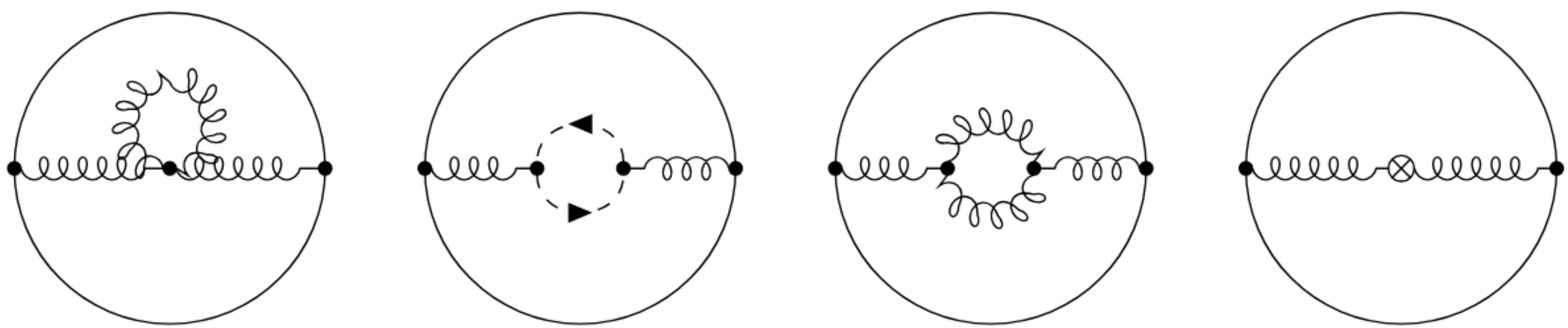
$$g_1(u) = \frac{u+1}{u(u+2)}g_0(u)$$

Diagrams for $\Delta_{[JJ]}$:



$$\langle [JJ](\vec{x})[JJ](0)\rangle \supset \#\, g^2 \log|\vec{x}|$$

Diagrams for C_J :



finite term in $\langle J^a_\mu(\vec{x})J^b_\nu(0)\rangle$ after renormalization

Alternative approach for Δ_{sing} : **multiplet recombination**

- $g^2 = 0$ is a (free) CFT ➡ general CPT argument
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$$T_{zz}|_{z=0} = \mathcal{D} \ , \qquad \Delta_{\mathcal{D}} = d+1$$

- Bulk relevant operator: $\delta S_{\text{bulk}} = \int_{\text{AdS}_{d+1}} \lambda\, O$

$T^{\mu}_{\mu} = \beta_{\lambda} O \neq 0$ ➡ $\mathcal{D}$ not protected any more

$$\gamma_{\mathcal{D}} = c \times \beta_0 \lambda + \mathcal{O}(\lambda^2)$$

In the CFT at $g^2 = 0$:

$$\mathcal{D} = J^a_\mu J^{a\mu}$$

Using $\gamma_{\mathcal{D}} = c \times \beta_0 \lambda + \mathcal{O}(\lambda^2)$, with $\lambda \equiv g^2$, we reproduce:

$$\gamma^{(1)}_{[JJ],\mathrm{singlet}} = -\frac{22 n_c}{3} \frac{1}{16\pi^2}$$

For Neumann boundary condition at $g^2 = 0$:

$$\mathcal{D} = \mathrm{Tr} f^a_{\mu\nu} f^{a\mu\nu}$$

➡ we can apply the same method and get:

$$\gamma^{(1)}_{\mathrm{tr}[ff]} = +\frac{22 n_c}{3} \frac{1}{16\pi^2}$$