Symmetry - enriched phases of matter and higher categorical structures

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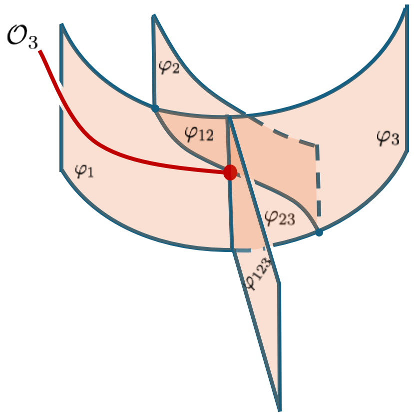
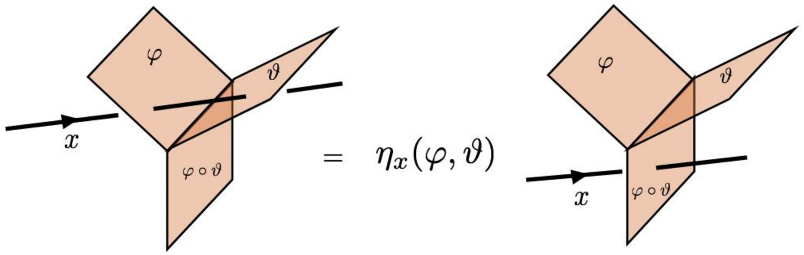




UV - IR group homomorphism

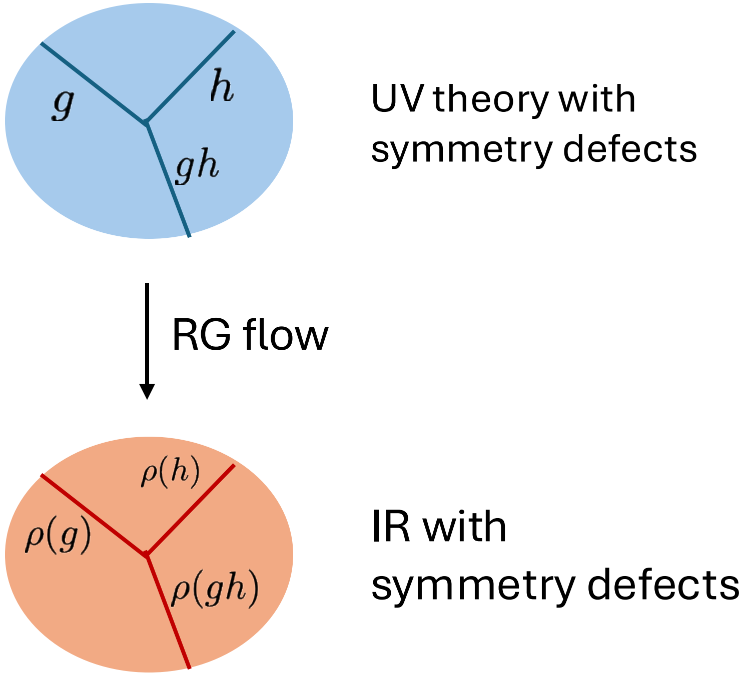
- In condensed matter physics, we are usually interested in understanding the possible phases of matter when the UV description has a global symmetry group G = G_{UV}
- In general there is an IR fixed point that governs the universal behavior (TQFT, CFT, etc).

  The IR fixed point has a symmetry group G_{IR} containing internal and space-time symmetries

- There is a UV-IR group homomorphism $\rho : G_{UV} \to G_{IR}$

  ρ can be viewed as a topological invariant of the UV system

  This means we have a **collection** of RG flows from the UV to IR, one for each symmetry defect configuration



- We can have different UV theories described by same IR fixed point, but different ρ → symmetry-enriched phase of matter

IR symmetries and higher categories

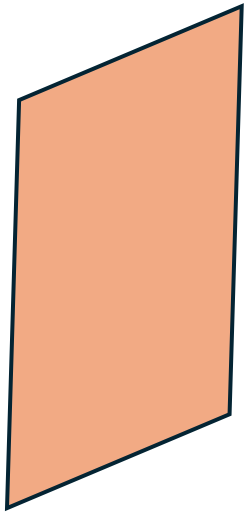
- In general, the IR symmetry is not a group
- For a (d+1)D TQFT, it is expected to be a fusion d-category $\mathcal{B}^{(d+1)}$

This is understood using the correspondence

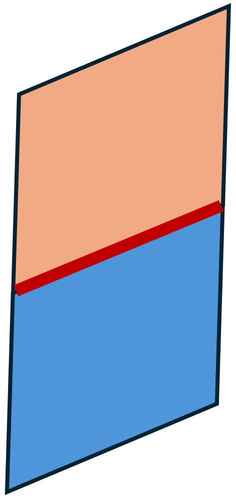
Symmetry ⟷ Topological defects

The higher category is characterizing topological defects of varying codimension and their mutual interactions

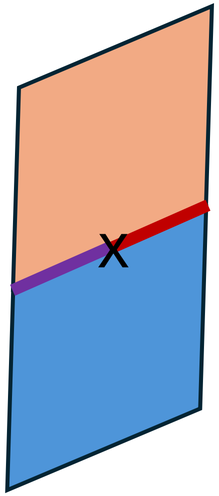
Fusion d-category of topological defects in (d+1)D TQFTs



0–morphisms:
Codimension-1 defects
gapped domain walls



1–morphisms:
Codimension-2 defects
Domain walls between
domain walls



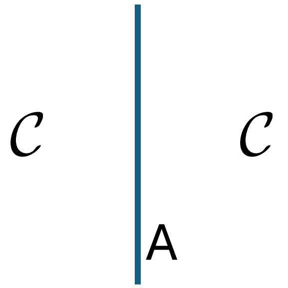
2–morphisms:
Codimension-3 defects
Domain walls between
codimension-2 defects

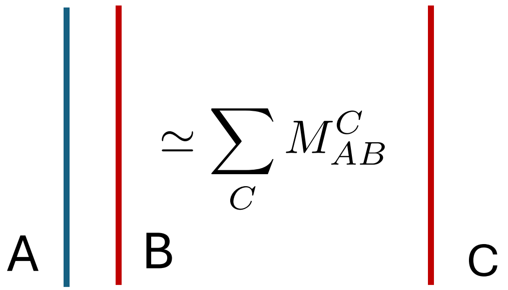
- Continue until we get to d-morphisms (codimension-(d+1) defects → maps between codimension-d defects)

Fusion d-category of topological defects in (d+1)D TQFTs

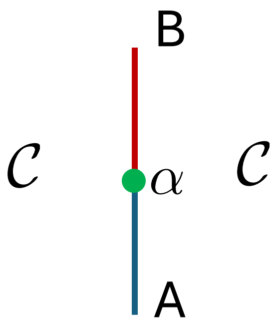
Tensor structure: Can fuse defects

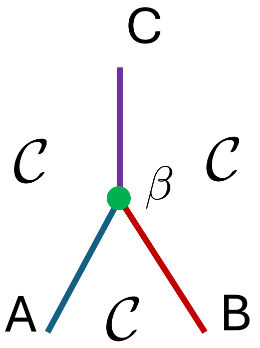
0–morphisms: Codimension-1 defects; gapped domain walls



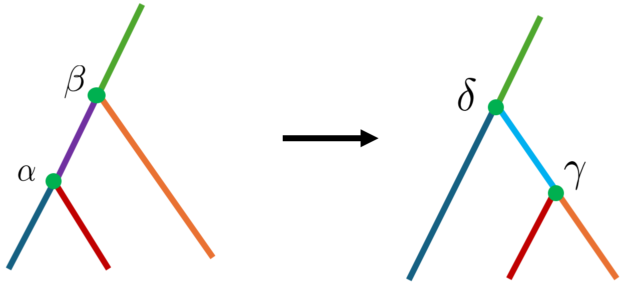


1-morphisms: Codimension-2 defects;





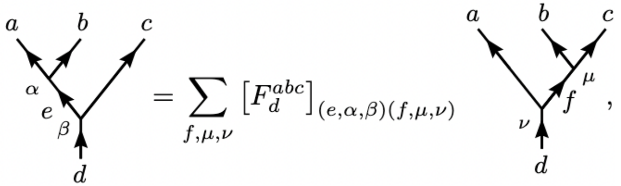
2-morphisms: maps between codimension-2 defects



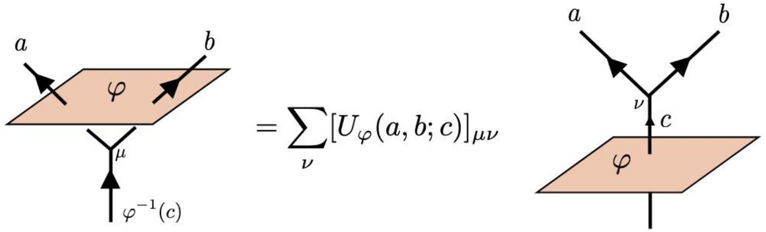
Example: Symmetry of (2+1)D topological phases of matter

IR categorical symmetry

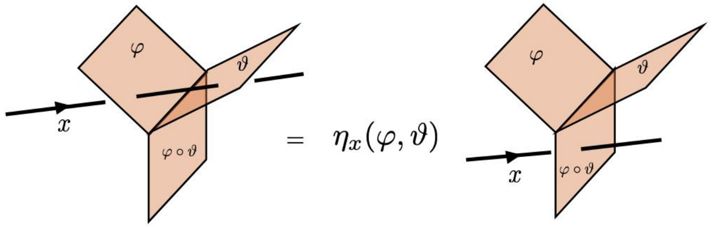
- Unitary modular tensor category of anyons $\mathcal{C}$



- 0-form invertible symmetry: braided tensor autoequivalences $\mathrm{Aut}(\mathcal{C})$



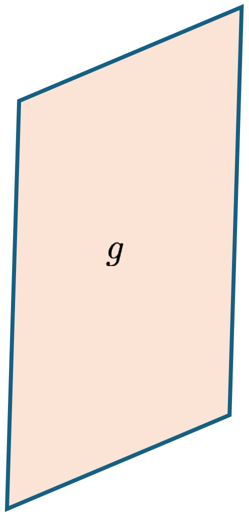
- Symmetry fractionalization data:



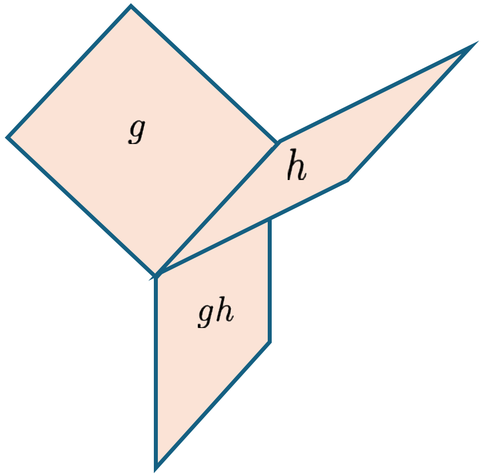
- And more .... (F and R symbols of endpoints of codimension-1 defects, non-invertible domain walls, ...)

Symmetry-enriched phases of matter

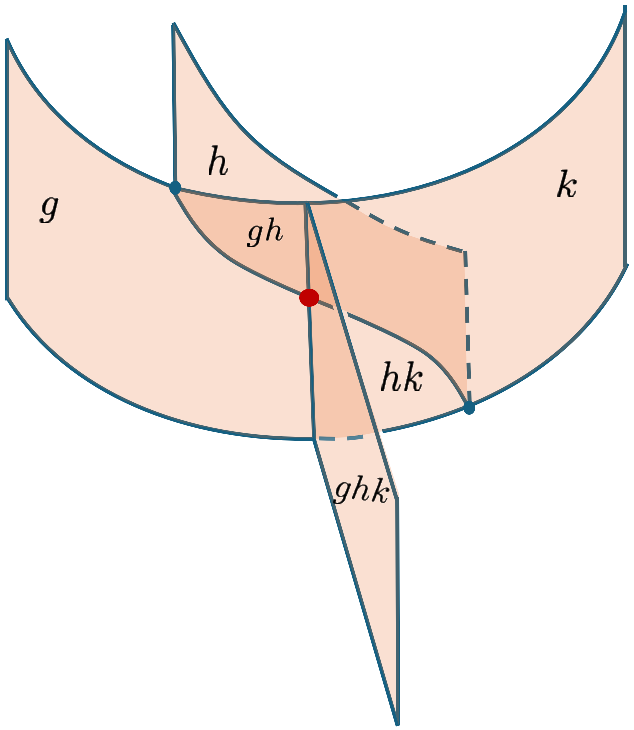
- Symmetry-enriched phases of matter are properly described by viewing the UV symmetry group G_{UV} as a higher category $\mathcal{G}_{UV}^{d+1}$



0-morphisms



1-morphisms: junctions between 0-morphisms



2-morphisms: maps between 1-morphisms

Symmetry-enriched phases of matter and higher homomorphisms

- And now we have a higher (d+1)-homomorphism

$$\rho^{d+1} : \mathcal{G}_{UV}^{d+1} \to \mathrm{Inv}(\mathcal{B}^{d+1})$$

- Specializing to (2+1)D symmetry-enriched gapped phases of matter gives the framework of G-crossed braided tensor categories

MB, Bonderson, Cheng, Wang arXiv:1410.4540

See also: Etingof, Nikshych, Ostrik 2010; Jones, Penneys, Reuter 2023

- All topological phases of matter can be understood this way
  (SPTs, topological insulators, FQH states, quantum spin liquids, anomalies, etc)

$\mathcal{B}^{d+1}, \rho^{d+1}$ characterize symmetry-enriched topological phases

Outlook

- It is crucial to unpack these abstract mathematical structures (abstract nonsense) into concrete algebraic data, consistency equations, and equivalence relations (skeletonization)

  This is a job for physicists

  e.g. MB, Bonderson, Cheng, Wang 2014

- It is important to relate these mathematical structures to the operator algebra that describes the UV lattice model or quantum field theory

- What is the full symmetry structure of a gapless QFT in (d+1)D? $\mathcal{B}^{d+2}$?

- We should develop efficient computational algorithms that can:

  - Give the complete classification / characterization of phases of matter given the symmetries of interest

  - Determine the full set of topological data characterizing a phase of matter given only access to ground state correlators.

Relevant for quantum state tomography in quantum computing