UV-IR matching of symmetries and anomalies in condensed matter

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The UV problem

The UV problem

Many systems of interest are modeled by fermions/spins on a lattice

U(1) charge conservation

SO(3) spin rotations

Time-reversal symmetry

Lattice symmetries

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Lattice symmetries

Generically, no exact higher-form or non-invertible symmetries in UV
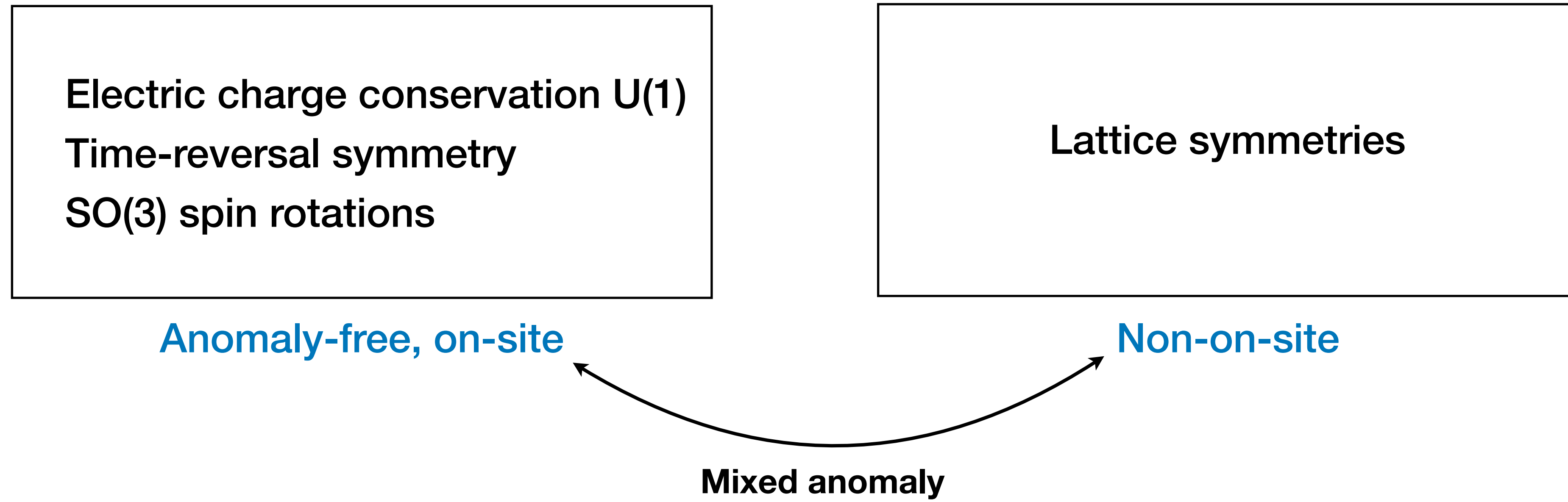
Special points in the parameter space with exact generalized symmetry

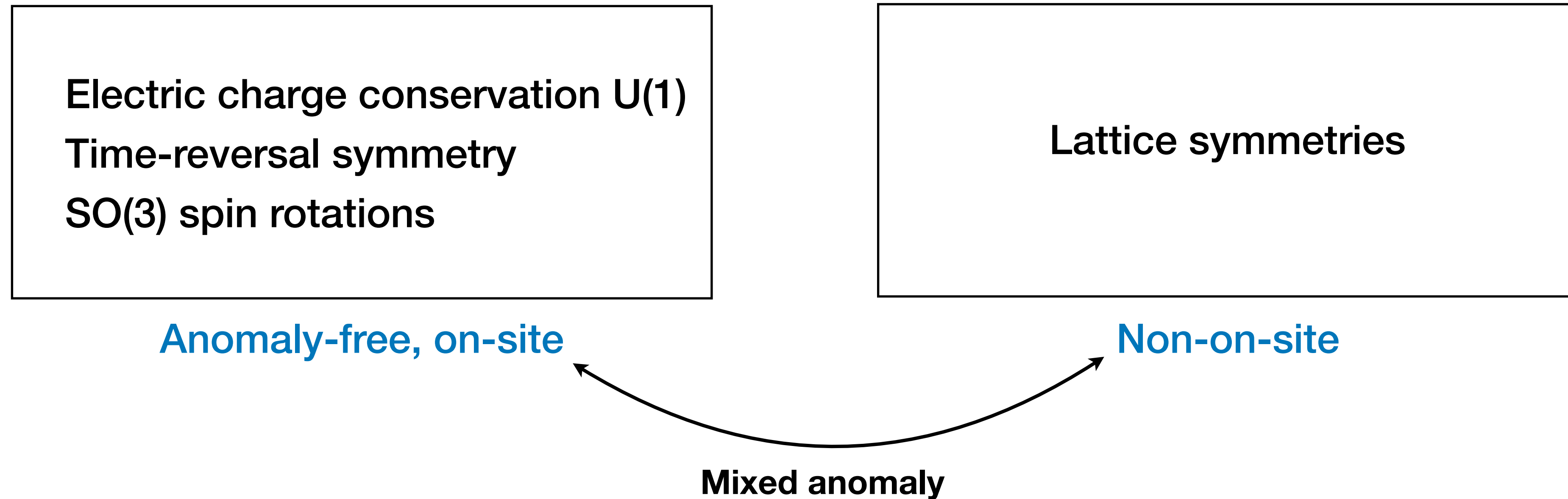Many examples in 1+1, Ising chain, XY model, anyon chain, etc
2+1: Kitaev's honeycomb model, fusion surface models etc

UV symmetries and anomalies

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Electric charge conservation U(1)

Time-reversal symmetry

SO(3) spin rotations

Lattice symmetries

Anomaly-free, on-site

Non-on-site

Mixed anomaly

UV symmetries and anomalies

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Lattice symmetries

Anomaly-free, on-site   Non-on-site

Mixed anomaly

Filling anomaly: a translation-invariant system with U(1) symmetry
and **fractional** filling (charge per unit cell)

LSM anomaly: a translation-invariant lattice with SO(3) symmetry
and a **half-integer** spin per unit cell

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Given a UV system, what are the possible IR phases?

"UV system" = Hilbert space + global symmetry

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Assumption: the IR phase is described by a relativistic QFT

Notable exceptions: Fermi liquid, fracton models, …

Classifying symmetry-enriched QFTs

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A QFT is characterized kinematically by its "generalized symmetries" G_{IR}

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A QFT is characterized kinematically by its "generalized symmetries" G_{IR}

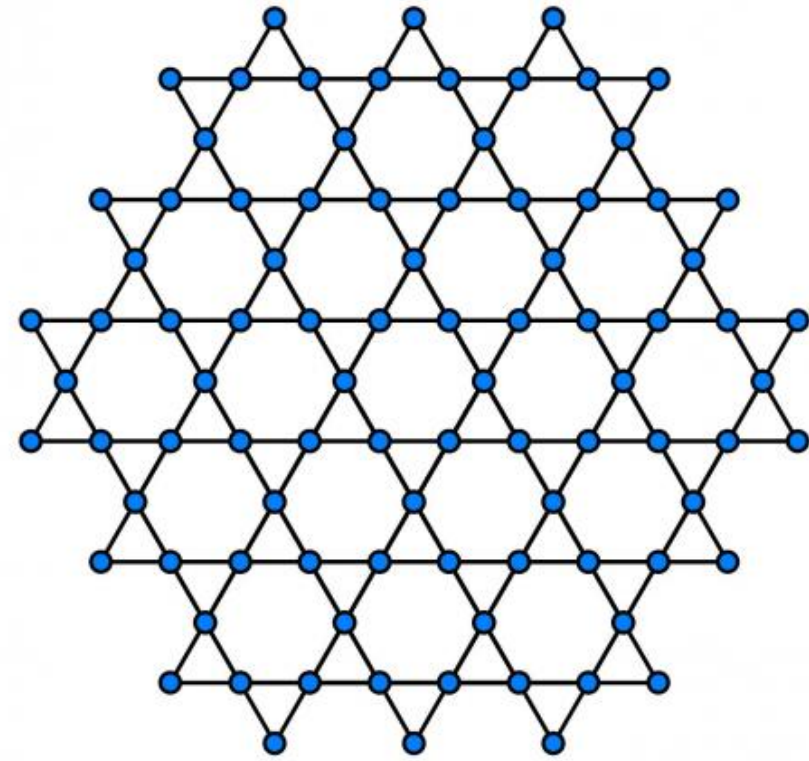For **UV** (0-form) symmetry G_{UV}, a formal solution to the "enrichment" problem:

A "homomorphism" $\rho : G_{\mathrm{UV}} \to G_{\mathrm{IR}}$

Anomaly matching: $\omega_{\mathrm{UV}} = \rho(\omega_{\mathrm{IR}})$

More in Maissam's presentation

Chen et al 2015; Barkeshli et al 2019; Benini et al 2019; Else et al 2021; …
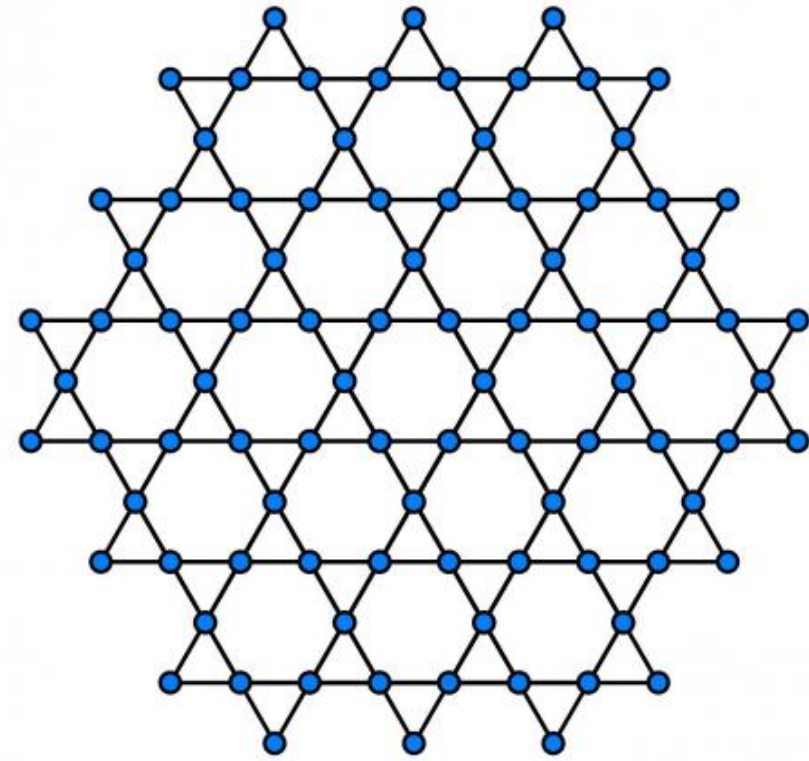
Topological spin liquid

Topological spin liquid

$$H = \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \cdots$$

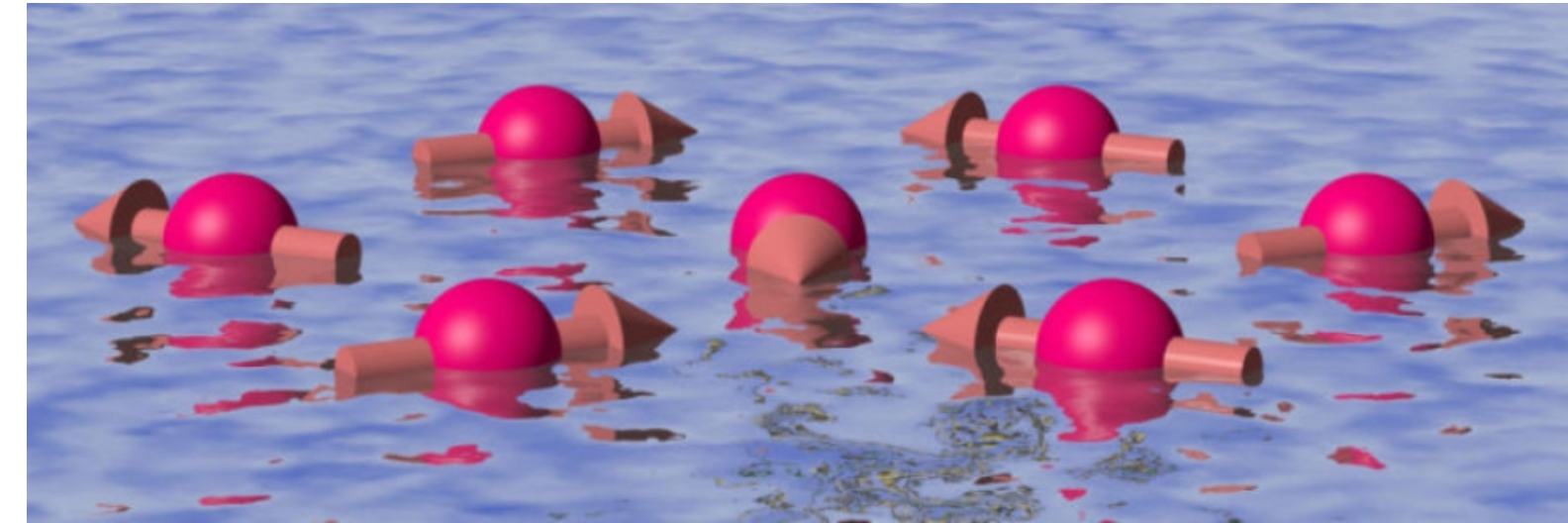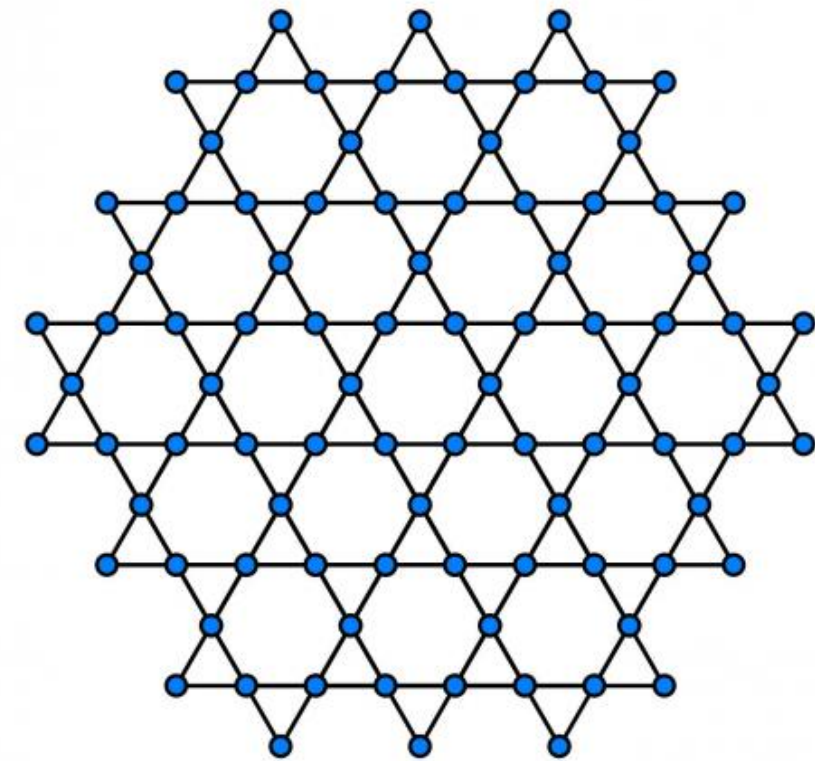$$G_{\mathrm{UV}}^{(0)} = \mathrm{SO}(3) \times \mathbb{Z}^2$$

Topological spin liquid

Figure from web

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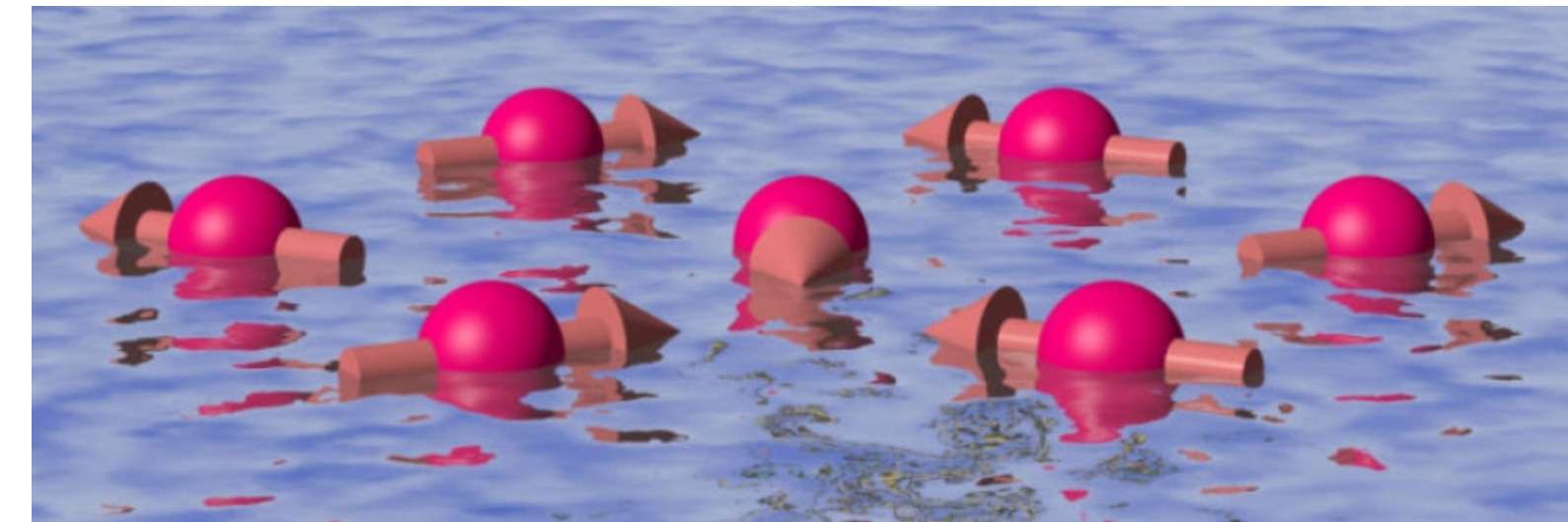$$G_{\mathrm{UV}}^{(0)} = \mathrm{SO}(3) \times \mathbb{Z}^2$$

2+1 TQFT, e.g. $\mathrm{U}(1)_2$

1-form symmetry

Topological spin liquid

Figure from web

$$H = \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \cdots$$

$$G^{(0)}_{\mathrm{UV}} = \mathrm{SO}(3) \times \mathbb{Z}^2$$

$\longrightarrow$

2+1 TQFT, e.g. $\mathrm{U}(1)_2$

1-form symmetry

Coupling to $G^{(0)}_{\mathrm{UV}}$ background by activating 1-form background

"Symmetry fractionalization"

Barkeshli et al 2019; Benini et al 2019; Hsin et al 2020; Delmastro et al 2022; …

Quantum many-body kinematics

Anomaly as UV kinematic constraints
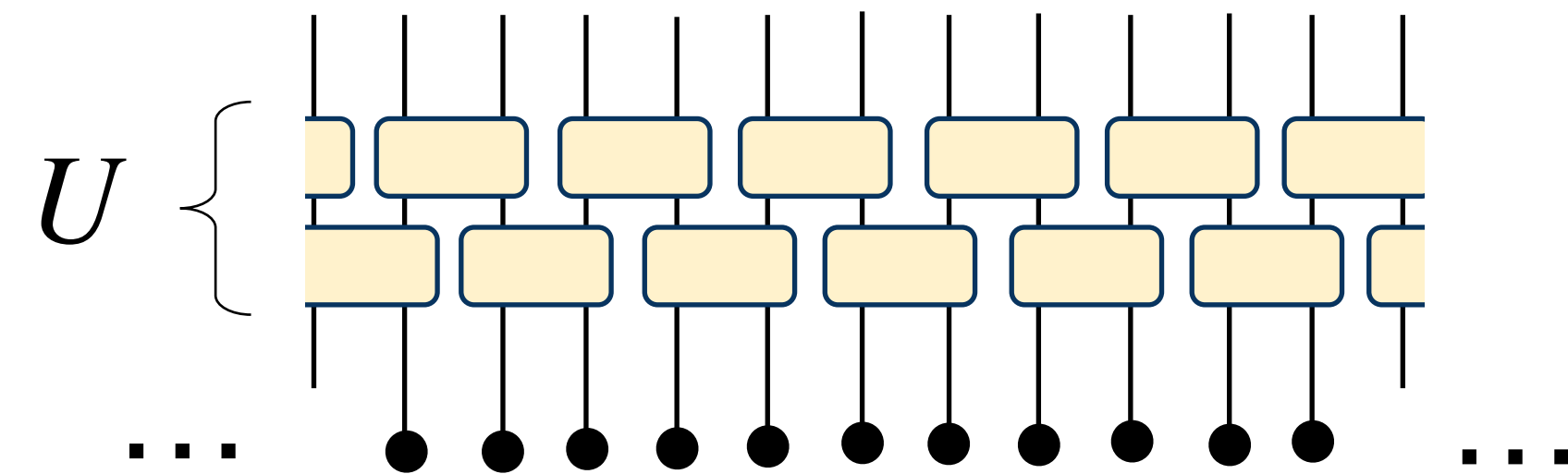
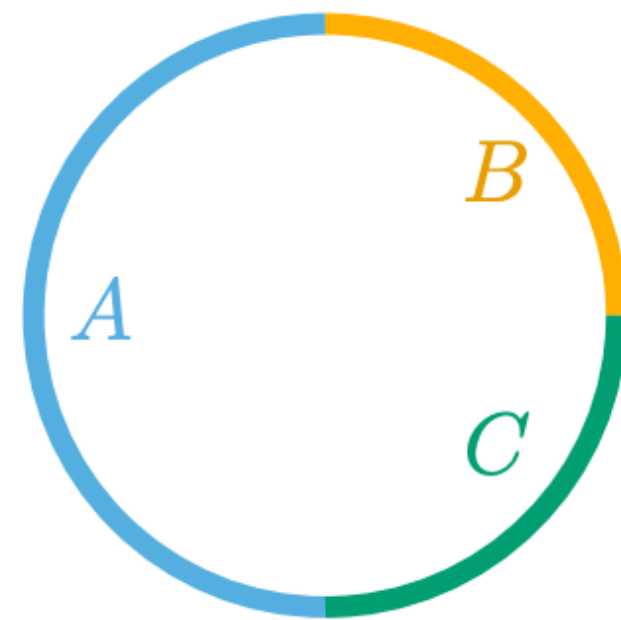Forbid "short-range-entangled" states

Quantum many-body kinematics

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U
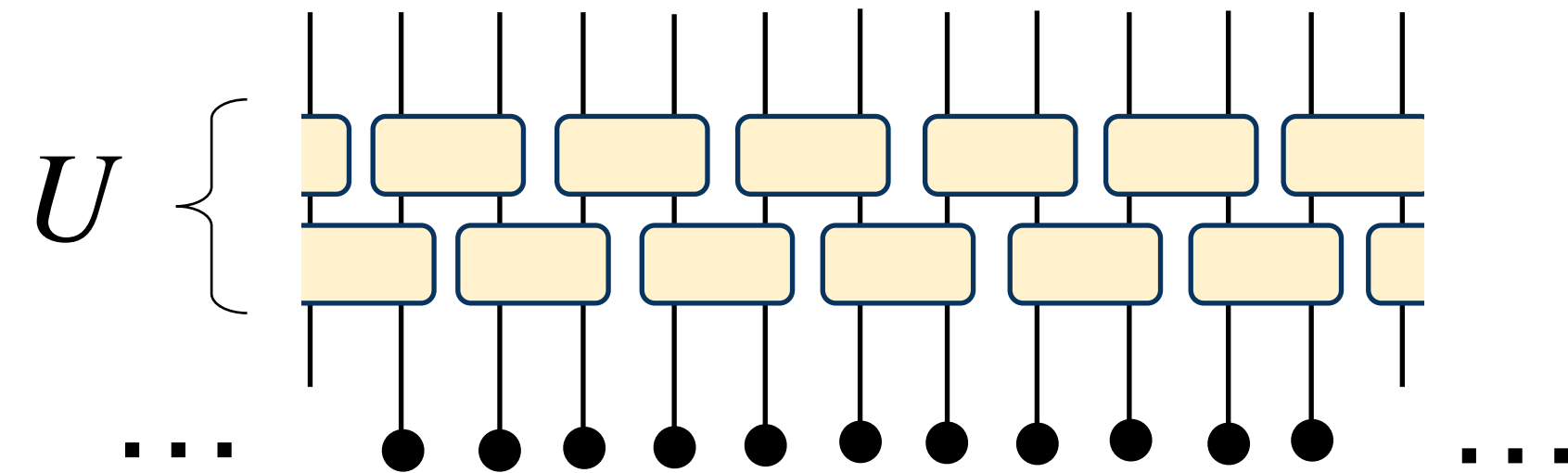
Refined constraints on entanglement structure: multipartite separability

If $|\psi\rangle$ is invariant under anomalous 0-form internal symmetry

$$|\psi\rangle \neq U|A\rangle|B\rangle|C\rangle$$

However, $|\psi\rangle$ can be bipartite-separable

Lessa, MC, Wang 24

Lattice ↔ Continuum

Rigorous classification of gapped phases on lattice

Kitaev, Kapustin, Sopenko, Ogata, Bachmann, …

Exact lattice regularization of QFTs, e.g. Chern-Simons theories

Seiberg, Shao, Gorantla, Jacobson, Sulejmanpasic, Chen, …
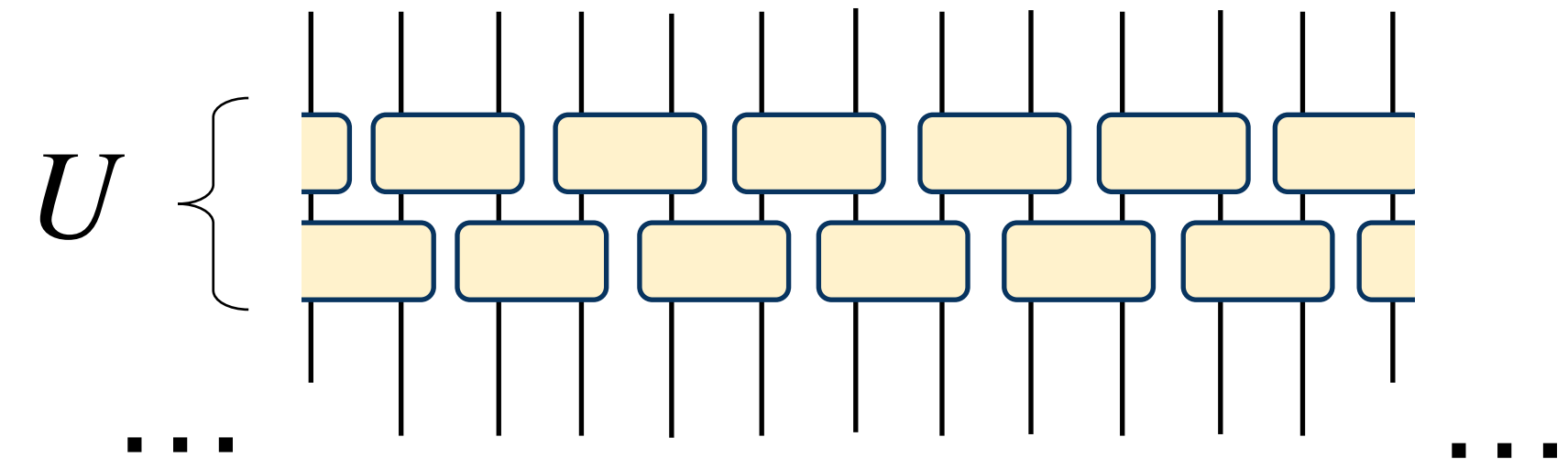
General framework for symmetry on lattice?

Lootens, Verstraete, Perez-Garcia, Jones, …

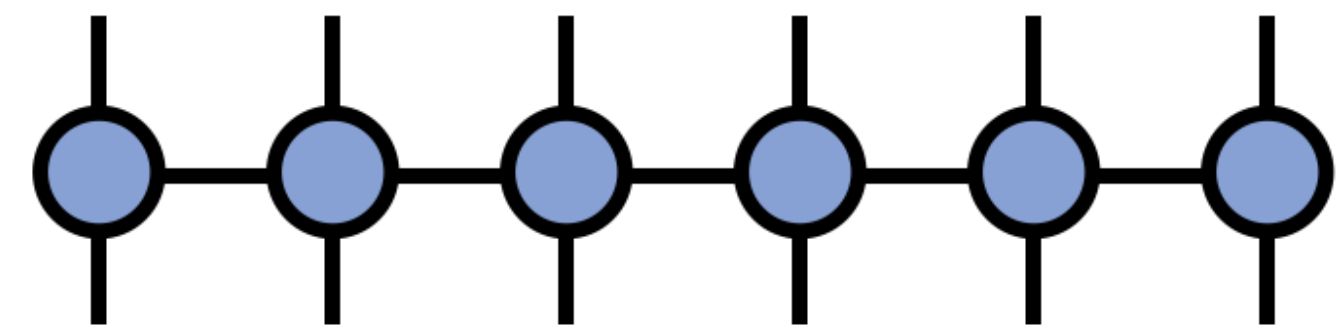Beyond traditional QFT phases: additional structures?

Backup slides

Symmetry on lattice

As finite-depth unitary circuits U $\left\{\vphantom{\rule{0pt}{3em}}\right.$

...   ...

As locality-preserving automorphisms of (sub)algebra of local operators
include translations (but not rotations!), KW duality, $\mathrm{Rep}(D_8)$, etc

Tensor-network operators
including all known symmetries in 1+1

The most general?