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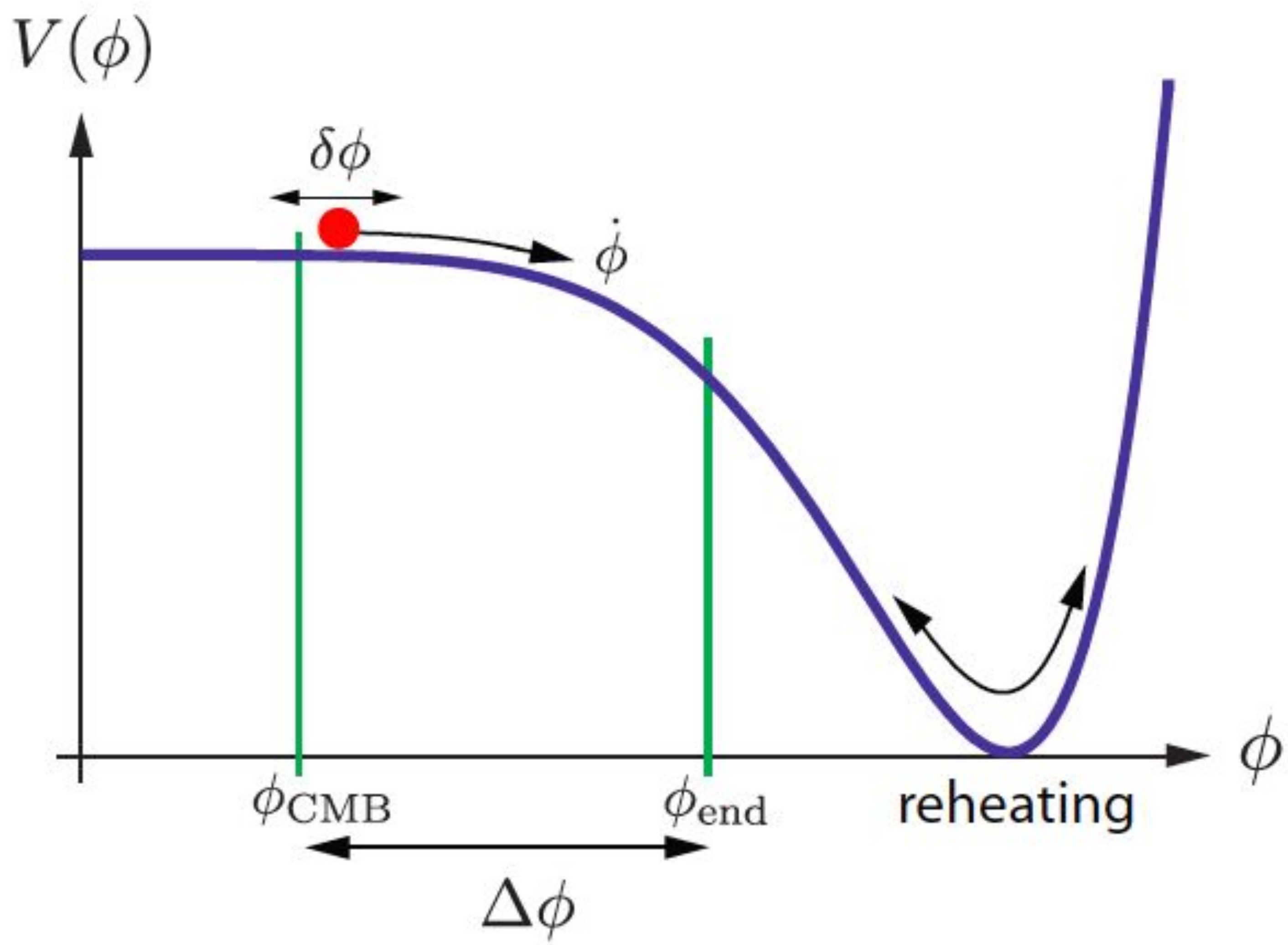
The Beginning of Inflation

Strings 2025 – NYU Abu Dhabi



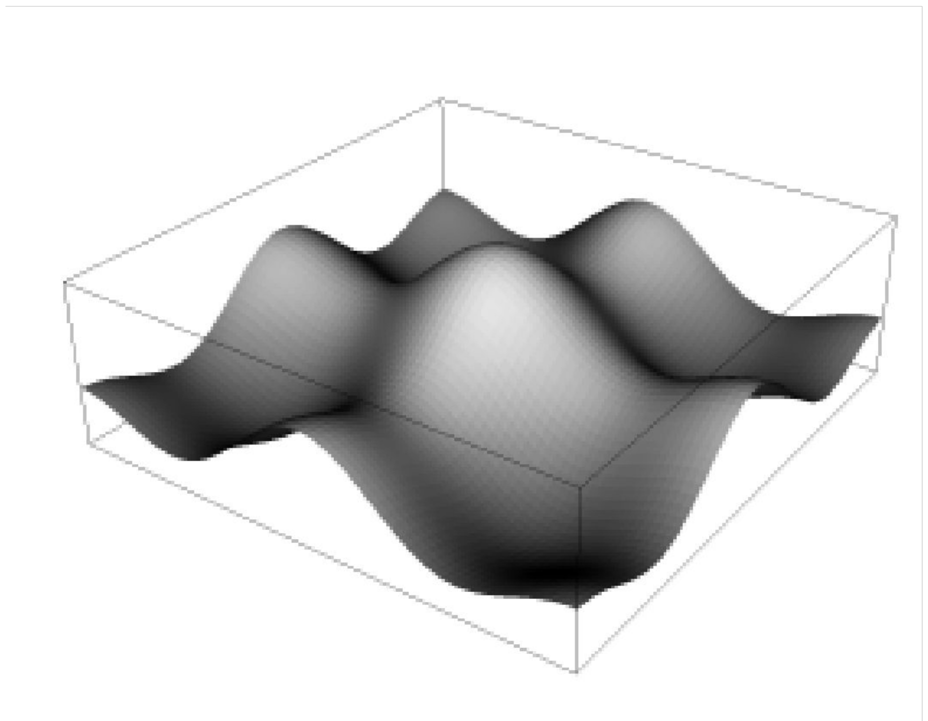
credit D. Baumann

Given an inflationary potential, does inflation "naturally" occur?

What are the initial conditions?

1. Attractor (cosmologists') approach

Show that slow-roll inflation occurs starting
from "generic" (classical) initial conditions

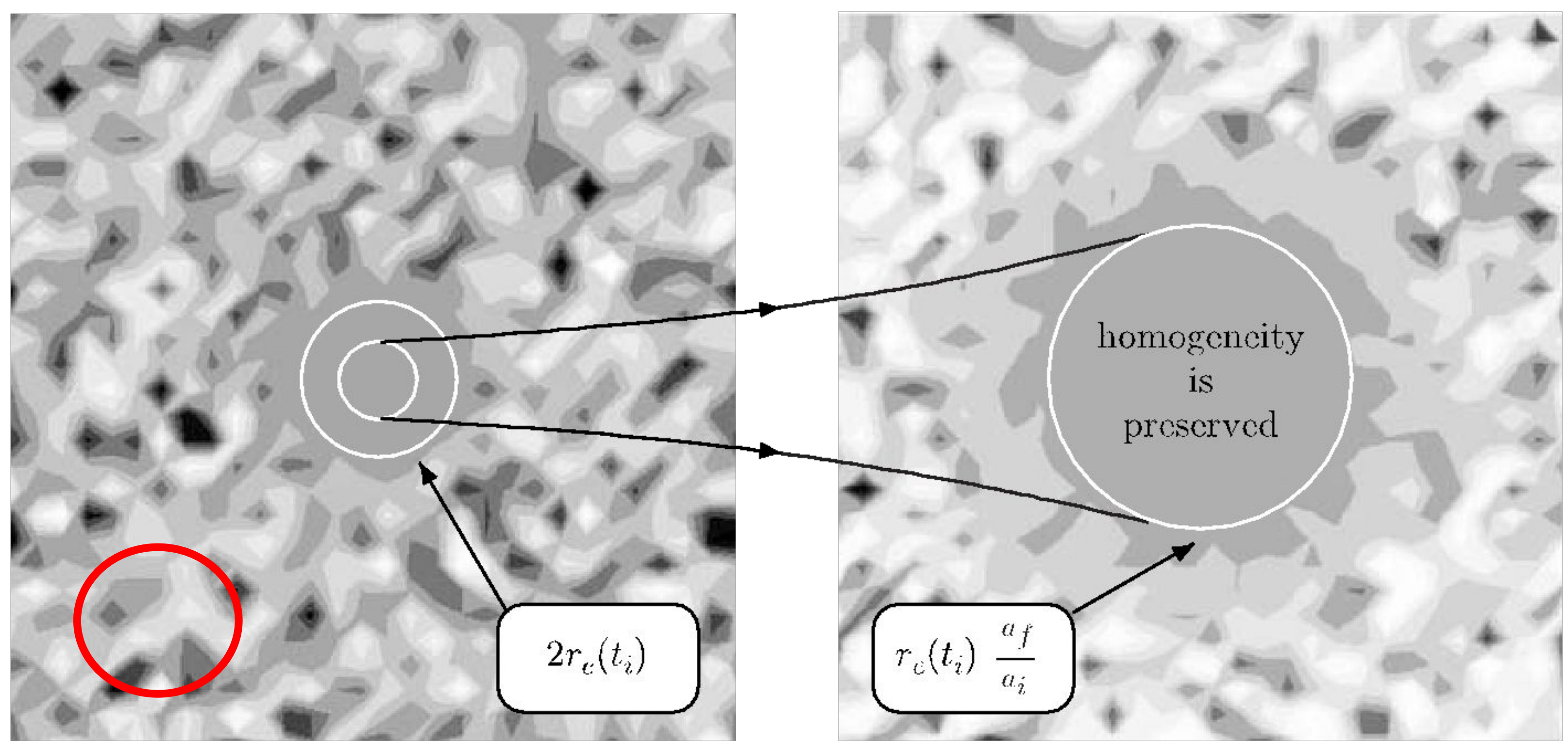


Numerical GR

East, Kleban, Linde, Senatore 15
Clough, Lim, DiNunno, Fischler, Flauger, Paban 16
Clough, Flauger, Lim 17
Aurrekoetxea, Clough, Flauger, Lim 19

Many regions collapse to BHs,
but inflation starts somewhere

No initial-patch problem



credit V. Mukhanov

Analytic results

In the presence of a positive c.c. one converges to dS: “no hair”

- Homogeneous, anisotropic universe. Go to de Sitter unless Bianchi IX.

  Wald 83

- No–recollapse theorem. For “most” topologies of 3-slices, the Universe cannot reach a maximal size

  Barrow, Tipler 85
  Kleban, Senatore 16

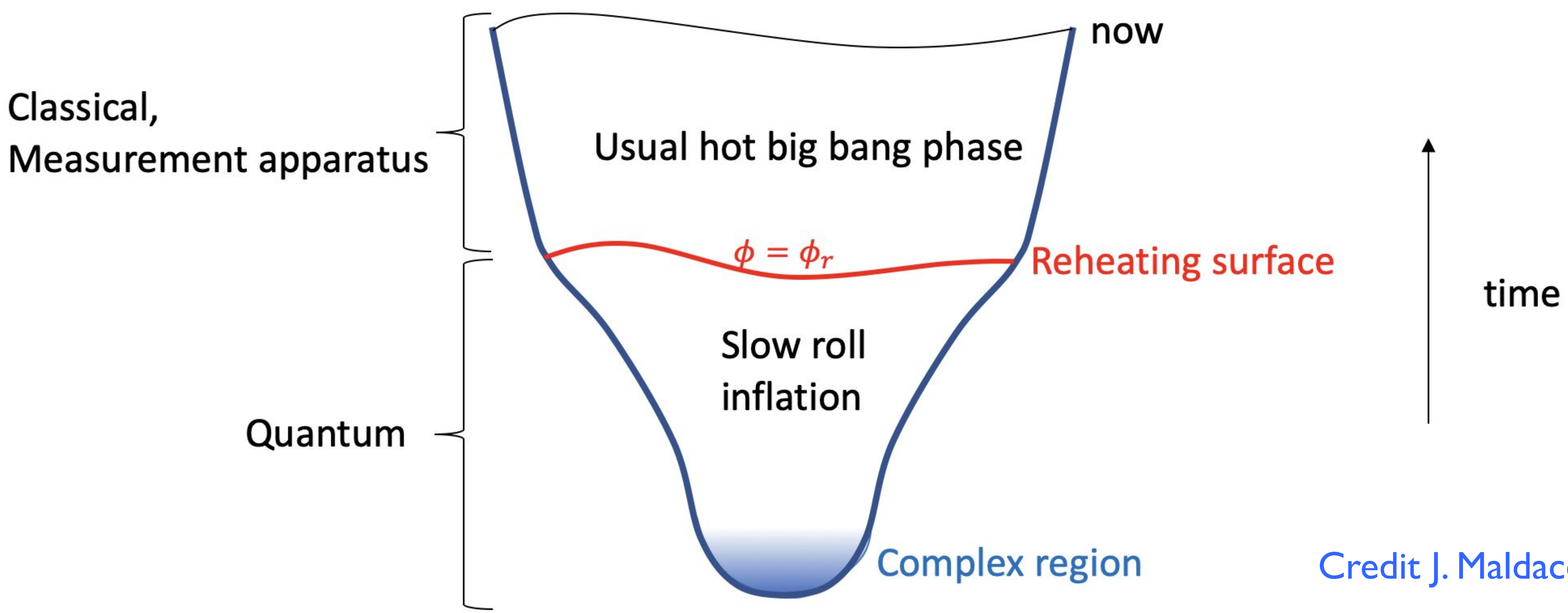
- Convergence to dS in 2+1 PC, Senatore, Vasy 19

  In 3+1 with homogeneity in two directions PC, Herschkovits, Senatore, Vasy 20

Tool: Mean Curvature Flow

2. No boundary WFU

I "know" the state of the universe and I work out its consequences



Credit J. Maldacena

$$\Psi[h_{ij}, \phi] \propto \exp\left(iI[g_{\mu\nu}, \phi]\right)$$

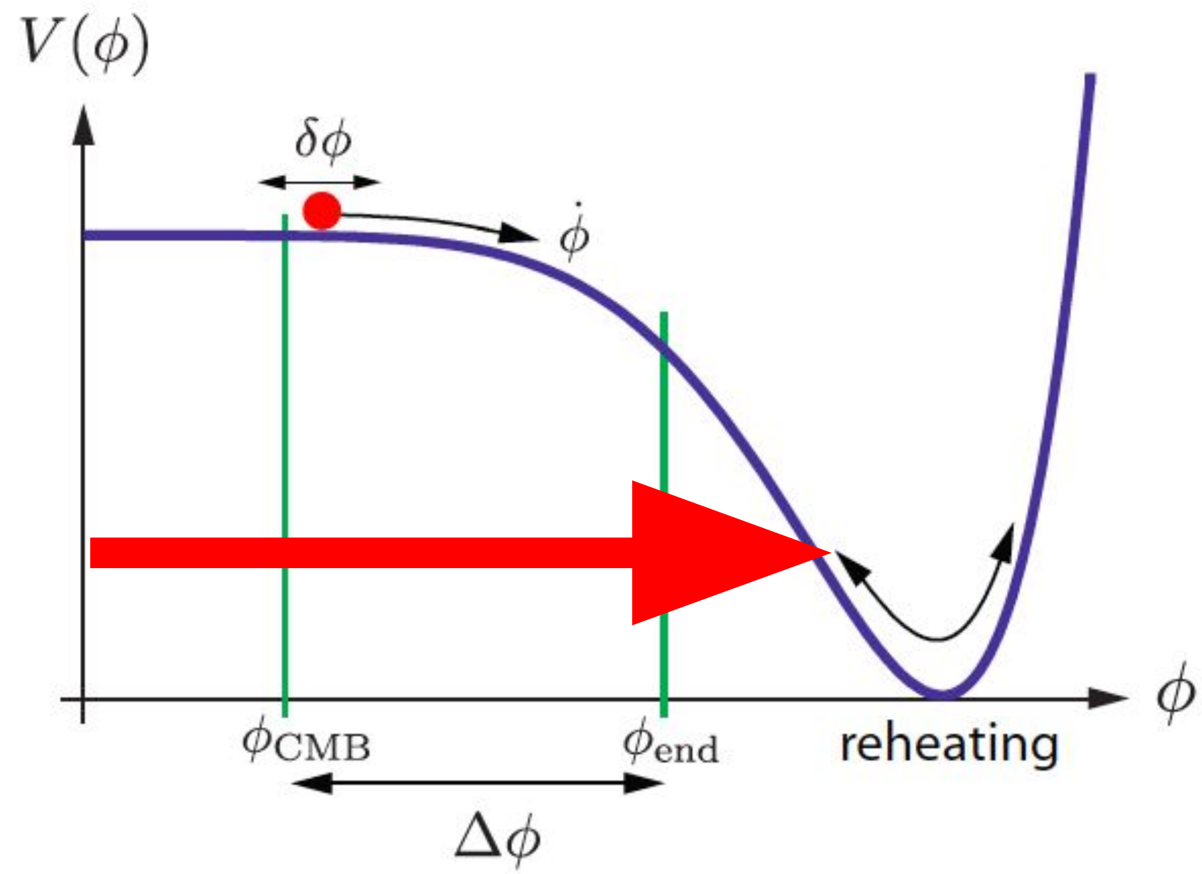
This prescription works well for the fluctuations we observe in CMB and LSS

2. No boundary WFU

$$|\Psi|^2 \propto \exp\left(24\pi^2 \frac{M_p^4}{V(\phi_*)}\right)$$

……
Janssen 20
Maldacena 24

φ_* is the inflaton value when inflating solution starts



One predicts a very positively curved universe. Observations $|\Omega_k| < 0.003$

Can eternal inflation change the picture?

Wrong prescription for the WFU?

Backup slides

No recollapse theorem

Barrow, Tipler 85
Kleban, Senatore 16

Does inflation start somewhere?

If we assume some topology for spatial slices, one cannot have global recollapse

MCF cannot stop and reach a maximal slice, with K = 0

$$G_{\mu\nu}n^\mu n^\nu = 8\pi G T_{\mu\nu}n^\mu n^\nu \longrightarrow {}^{(3)}R + \frac{2}{3}K^2 - \sigma^2_{\mu\nu} = \frac{2}{3}K^2_\Lambda + 16\pi G T_{\mu\nu}n^\mu n^\nu$$

$$= 0 \quad \leq 0 \qquad > 0 \quad \geq 0 \quad \text{(WEC)}$$

${}^{(3)}R > 0$ everywhere. But this cannot happen for some topology, for example 3-torus

Topology for dummies (me)

In 2D Gauss-Bonnet

$$\int d^2x\sqrt{h}\ {}^{(2)}R = 4\pi\chi$$

χ=2 for sphere, χ=0 for torus,
χ< 0 for higher genus

3D topology is very rich (e.g. Poincaré conjecture)

Topology can be of 3 kinds:

- "Closed", e.g. S^3, RP^3, S^2 x S^1. ${}^{(3)}R$ completely unconstrained

- "Flat", e.g. 3-tori and other quotients of R^3. ${}^{(3)}R$ < 0 somewhere or vanishes identically

- "Open", quotients of H^3, H^2 x S^1, Nil, Solv... ${}^{(3)}R$ < 0 somewhere

Connected sums that contain an open factor are open. "Most" topologies are open