JT gravity in deSitter Space and Its Extensions

Sandip Trivedi

Tata Institute of Fundamental Research

Based On:

1) Kanhu K. Nanda, Sunil Sake, ST

``*JT Gravity in dS Space and The Problem of Time*'', *JHEP* 02 (2024) 145

2) Indranil Dey, Kanhu K. Nanda, Akashdeep Roy, Sunil Sake, ST,

``*JT Gravity in dS Space and Its Extensions*'', *hep-th/2501.03148*.

Several Additional References

In particular:

- M. Henneaux, Phys. Rev. Lett. 54 (Mar, 1985) 959–962.
- J. Maldacena, G. J. Turiaci and Z. Yang, *JHEP* 01 (2021) 139
- L. V. Iliesiu, J. Kruthoff, G. J. Turiaci and H. Verlinde, SciPost Phys. 9 (2020) 023, [2004.07242].

- J.Cotler, K. Jensen, JHEP12 (2021),089; 2401.01925

- E. Alonso-Monsalve, D. Harlow and P. Jefferson, hep-th/2409.12943.
- J. Held and H. Maxfield, hep-th/2410.14824.

Outline :

- Introduction and Motivation
- Classical Solutions
- Canonical Quantisation
- Comments:
  i) Extensions of JT Theory
  ii) Path Integral Understanding
  iii) Matrix Theory as a Hologram
- Conclusions

Motivation and Introduction

Jackiw-Teitleboim Theory is a very simple theory of 2 dimensional gravity.

With No Propagating degrees of Freedom.

Why Study It?

In the deSitter Context…

Motivation:

Many Conceptual Questions in Quantum Cosmology:

- Meaning and interpretation of Wave function

- Observers and Observables ?

- Problem of Time:

Physical Clock Must Arise Internal to the System. Hilbert Space Must be constructed at instances of physical time.

Motivation

These conceptual questions are present in 2 dimensions as well and not directly connected with UV divergence in higher dimensions

Hopefully what is learnt can also be of relevance in more realistic higher dimensional theories of Gravity.

Motivation

Holography:

What we learn could also shed some light how dS/CFT works in general.

JT deSitter gravity:

$$S = \frac{1}{16\pi G_N}\left[\int d^2x\sqrt{-g}\phi(R-\Lambda)\right] + S_{\rm bndry}$$

ϕ : Dilaton

G_N : Dimensionless

Λ : Only Scale, >0.
Set = 2

$S_{\rm bdry}$: GH Boundary term

G_N can be set to 1 by rescaling ϕ

Note: The classical limit $G_N \to 0$ corresponds to $\phi \to \infty$

While $G_N \to \infty$ corresponds to $\phi \to 0$

In addition a topological term:

$$S_T = S_0 \frac{1}{4\pi} \left(\int \sqrt{-g} R - 2 \int \sqrt{-\gamma} K \right)$$

$S_0 \propto \frac{1}{G_N}$

$\left(\int \sqrt{-g} R - 2 \int \sqrt{-\gamma} K \right) = \chi$

JT Gravity Can be obtained by dimensional reduction of a near extremal black hole in de Sitter space.

$S_0 \longleftrightarrow$ Higher dim. entropy

(Dilaton: Radius of internal space)

Dilaton: Physical Clock

The idea will be explore is that the dilaton plays the role of the physical clock.

This is similar to what happens in inflation (where the inflaton, also a scalar, plays this role).

Range of Dilaton

In dimensionally reduced case there is a minimum value for ϕ

We analyse the 2 dim quantum theory without imposing any restriction, and find that it allows for the range $-\infty < \phi < \infty$

Classical Solutions:

Global deSitter

$$ds^2 = -\frac{dr^2}{r^2+1} + (r^2+1)d\theta^2$$

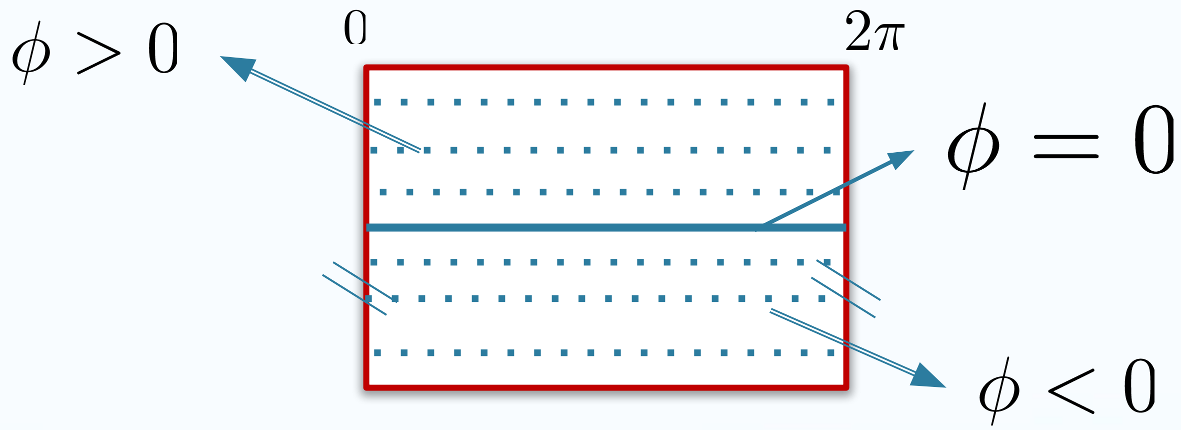
Spatial slices are compact

$$0 \le \theta \le 2\pi$$

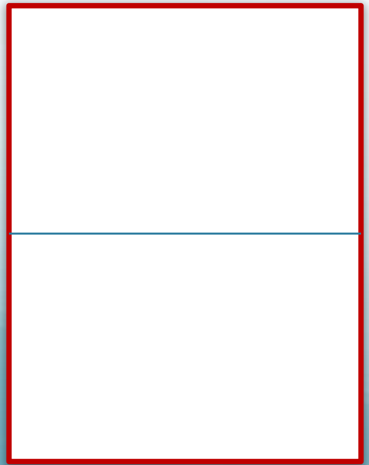
Dilaton $\phi = Ar$

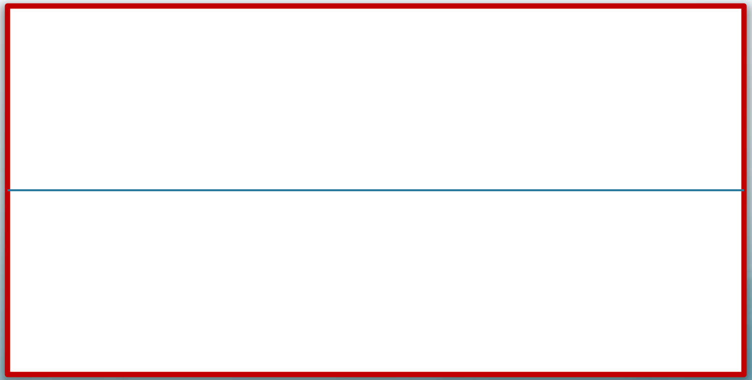
A: constant

Global deSitter:



Other Solutions :





Classical Solutions

Bounce Solutions $M < 0$

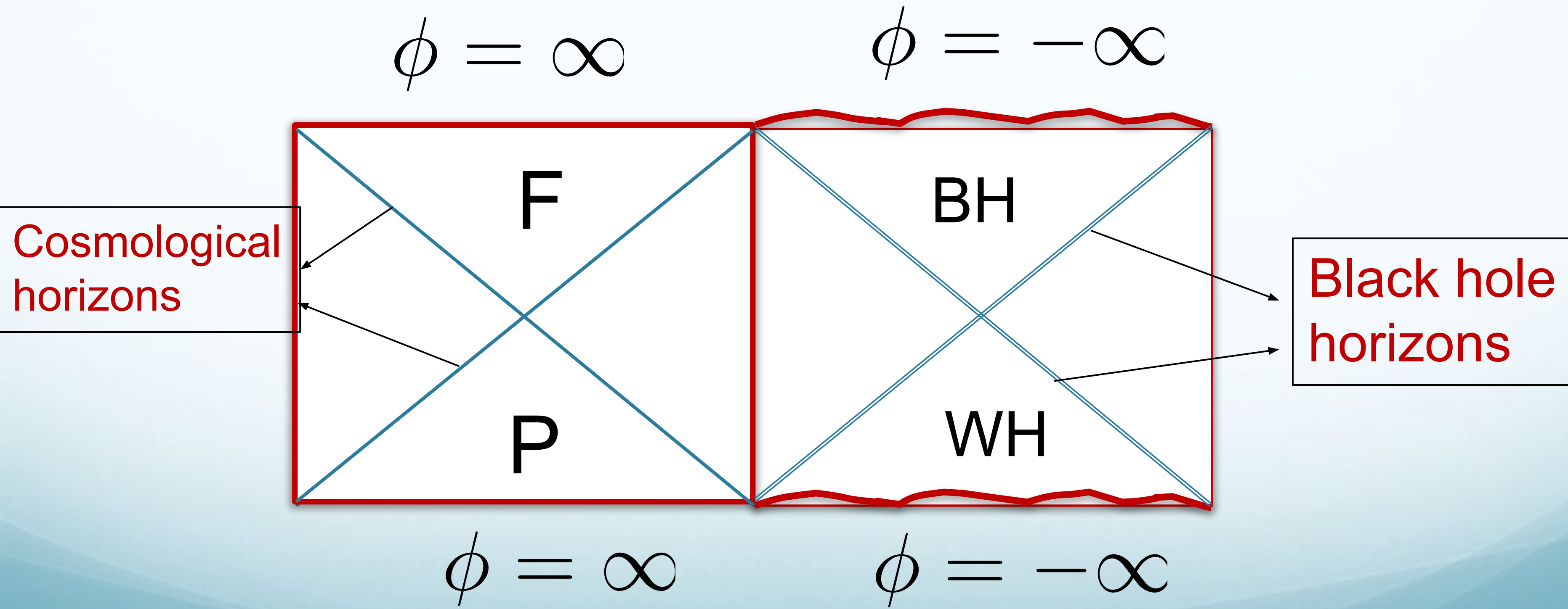
$$ds^2 = -\frac{dr^2}{r^2 - M} + (r^2 - M)\frac{dx^2}{A^2}$$

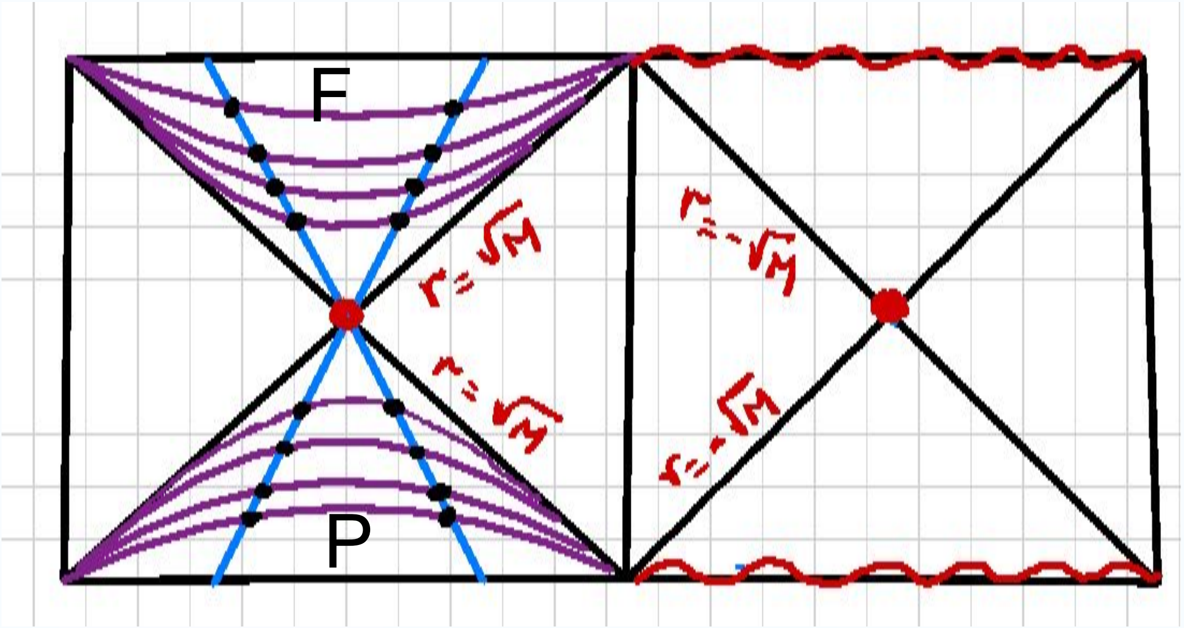
Spatial sections are compact $0 \leq x \leq 1$

Dilaton $\phi = r$

Two parameters worth of solutions (two dimensional phase space).

$M > 0$: Orbifolds of Black Hole Solutions



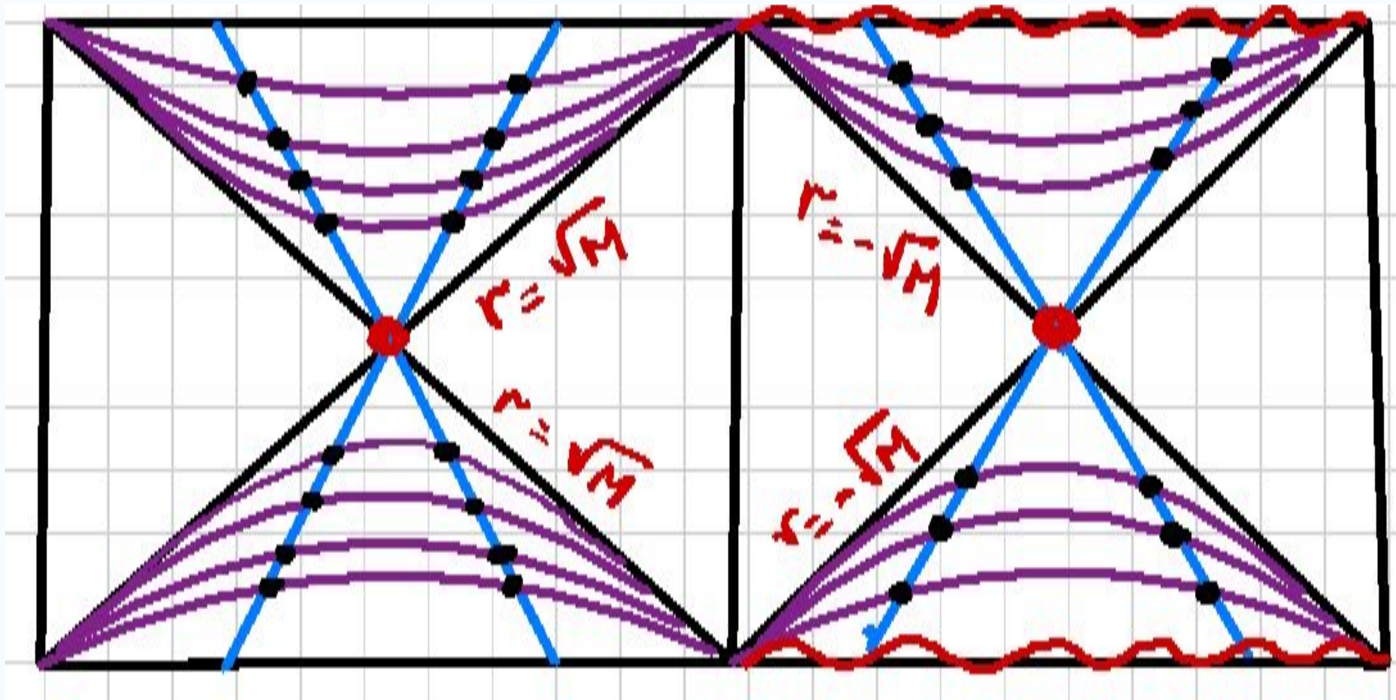


Orbifold Singularity, analogue of big bang/big crunch.

Contracting universe encounters the singularity.

Expanding Universe emerges from singularity.

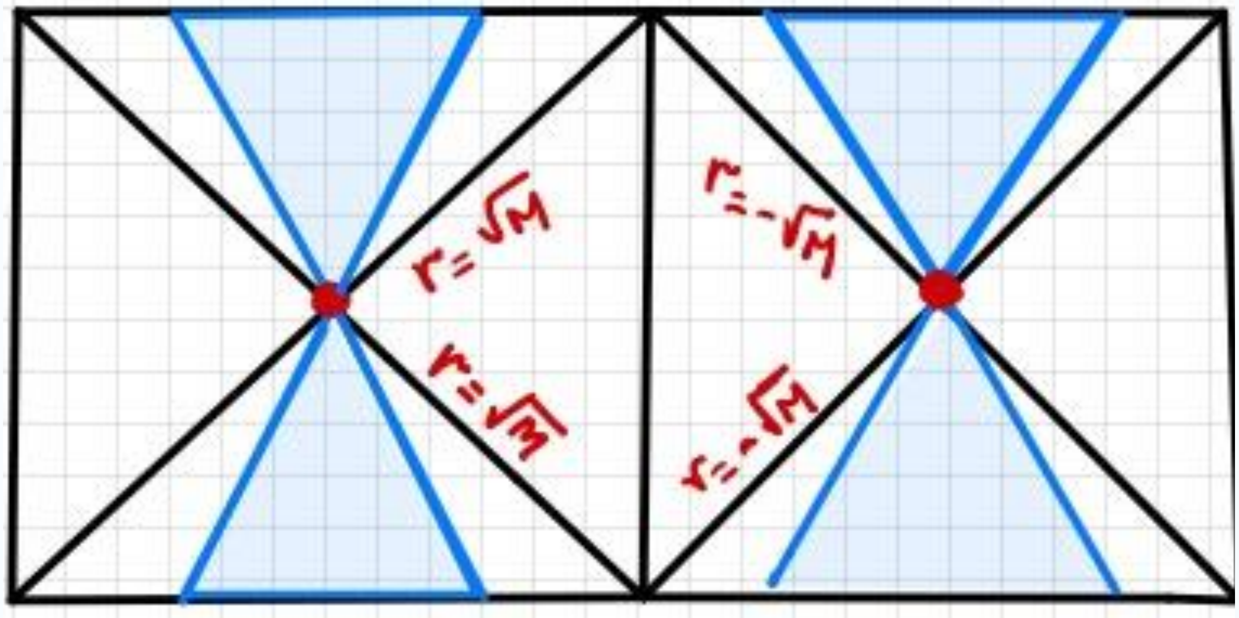
$\phi = \infty$ $\phi = -\infty$



$\phi = \infty$ $\phi = -\infty$

Expanding, Contracting Branches and Black Hole White Hole Branches.

$\phi = \infty$ $\phi = -\infty$



$\phi = \infty$ $\phi = -\infty$

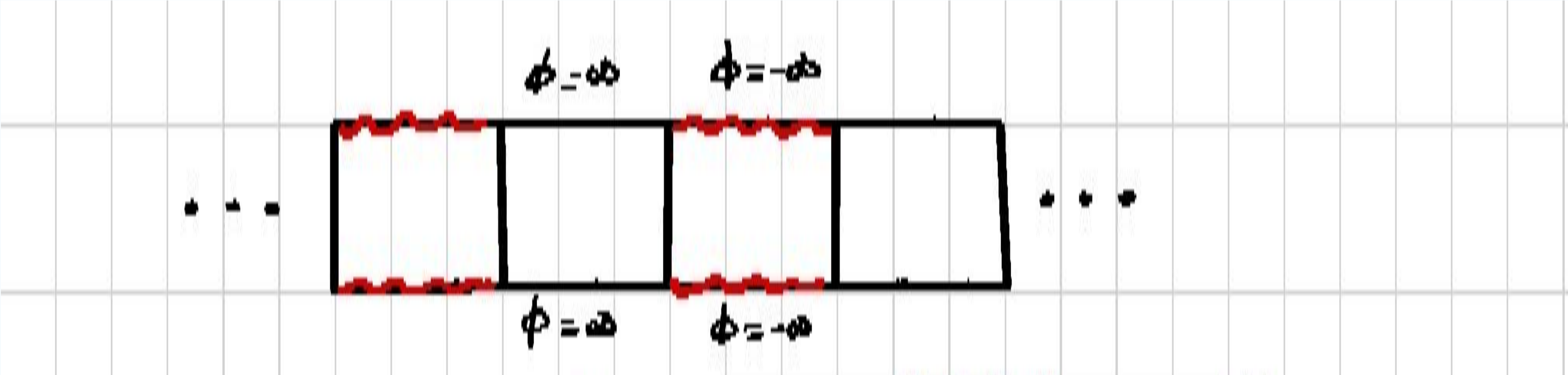
$M < 0$: Bounce Branch

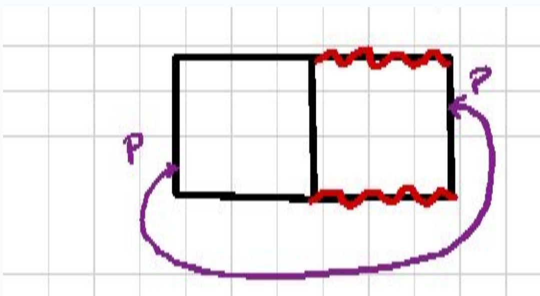
$M > 0$: Big Bang/Big Crunch Branch

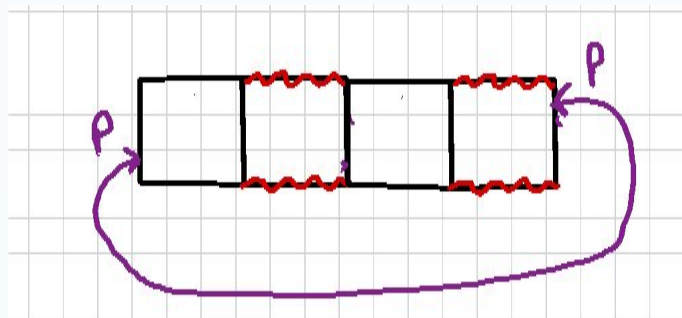
Two parameters worth of solutions, M, A. Two dimensional phase space.

However there is one more set of solutions

- E. Alonso-Monsalve, D. Harlow and P. Jefferson, hep-th/2409.12943.
- J. Held and H. Maxfield, hep-th/2410.14824.







Resulting spacetimes cannot be foliated by space-like hypersurfaces with constant dilaton.

Our discussion will only apply to the bounce and big bang/big crunch branches.

The parameter M corresponds to a conserved quantity

$$M = (\nabla\phi)^2 + \phi^2$$

$$\partial_r M = \partial_x M = 0$$

We will loosely refer to this parameter as the ``Mass".

It will play an important role in the following analysis

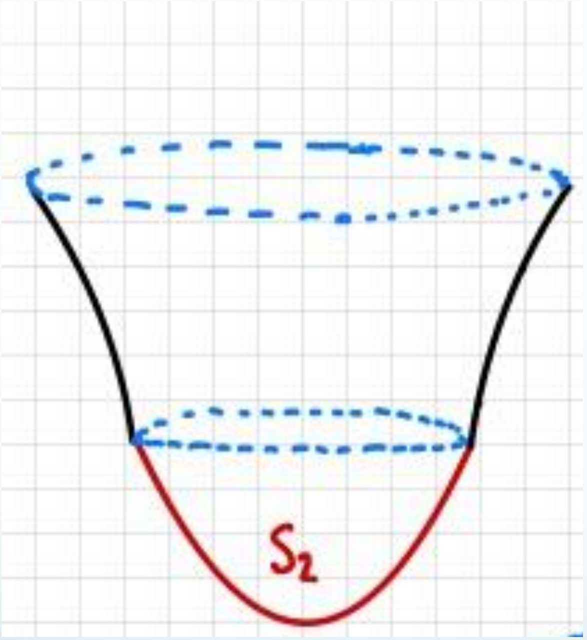
Path Integral

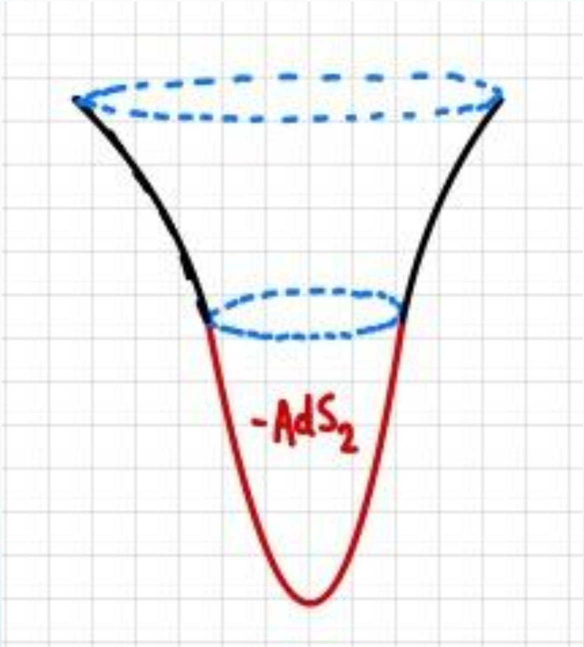
The wave function of the Hartle Hawking State can be calculated using the path integral Exactly.

After analytic continuation from $-AdS_2$, i.e. anti-deSitter space with two time like directions.

(Maldacena, Turiaci, Yang; Cotler, Jensen, Moitra, Sake, SPT;…
Stanford, Witten,)

This exact result will guide us in carrying out the canonical quantisation.





Canonical Quantisation:

(Standard ADM Quantisation)

$$ds^2 = -N^2 dt^2 + g_1 (dx + N^1 dt)^2$$

Work in ADM gauge

Set $N = 1, N^1 = 0$

In this gauge:

$$\pi_\phi = -\frac{\dot{g}_1}{2\sqrt{g_1}}$$

$$\pi_{g_1} = -\frac{\dot{\phi}}{2\sqrt{g_1}}$$

Hamiltonian and Momentum Constraints

$$\mathcal{H} = 2\pi_\phi\sqrt{g_1}\pi_{g_1} - \left(\frac{\phi'}{\sqrt{g_1}}\right)' - \sqrt{g_1}\phi$$

$$\mathcal{P} = 2g_1\pi'_{g_1} + \pi_{g_1}g'_1 - \pi_\phi\phi'$$

Equations of Motion of N and N_1

$\mathcal{H} = 0$ $\mathcal{P} = 0$

Classically the Constraints Close

Quantum Theory:

$$\pi_\phi = -i\frac{\delta}{\delta\phi}$$

$$\pi_{g_1} = -i\frac{\delta}{\delta g_1}$$

Physical states satisfy the conditions

$$\mathcal{H}|\Psi\rangle = 0$$

$$\mathcal{P}|\Psi\rangle = 0$$

$\Psi[\phi(x), g_1(x)]$: Functional of $\phi(x)$, $g_1(x)$

Quantum Theory:

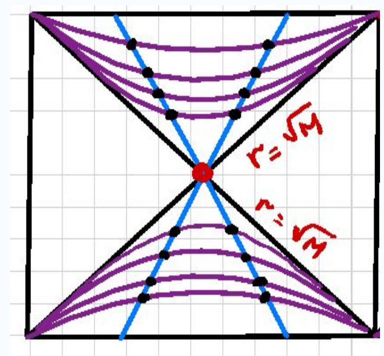
However the constraints have ordering ambiguities in the Quantum Theory.

Also UV divergences
($\mathcal{H}$ involves two functional derivatives, which acting on $\Psi[\phi, g_1]$ give $\delta(0)$ type singularities.)

We will choose hypersurfaces along which the dilaton is constant.

E.g.,

$\phi = \infty$



$\phi = \infty$

Using the spatial reparametisation invariance only other variable the wave function can depend on is, l the length of the spatial slice.

The wave function then becomes a simple function of : ϕ, l

$$\Psi(\phi, l)$$

The Hamiltonian constraint $\mathcal{H}$ leads to the Wheeler De Witt (WdW) equation.

However the WdW equation suffers from ambiguities (due to $\mathcal{H}$ being ill-defined).

E.g. in Mini-superspace approximation we get

$$\partial_l \partial_\phi \Psi + l\phi\Psi = 0$$

Instead the Hartle-Hawking wave function satisfies the equation

$$\partial_l \partial_\phi \Psi - \frac{1}{l}\partial_\phi \Psi + l\phi\Psi = 0$$

Subdominant in the WKB limit

The extra term can be understood as arising from an appropriate ordering prescription.

Taking this guidance from the Path Integral we will take the Wheeler-DeWitt equation to be:

$$\partial_l \partial_\phi \Psi - \frac{1}{l}\partial_\phi \Psi + l\phi\Psi = 0$$

Solutions of Wheeler DeWitt Equation:

$$\Psi(\phi, l) = e^{-il\sqrt{\phi^2 - M}}, e^{+il\sqrt{\phi^2 - M}}$$

Expanding Branch

Contracting Branch

$$\Psi = \int dM \rho(M) e^{-il\sqrt{\phi^2 - M}} + \int dM \tilde{\rho}(M) e^{il\sqrt{\phi^2 - M}}$$

Coefficient Functions

There are an infinite number of solutions.

One for each value of M in the expanding and contracting branches.

The parameter M corresponds to the Mass parameter we saw in the classical solutions

In Quantum Theory eigenvalue of the operator

$$\hat{M} = \partial_l^2 + \phi^2$$

But we are not done yet.

To obtain a Hilbert Space we need to define a norm (more generally inner product).

We will construct a Hilbert Space at constant values of ϕ.

So the Dilaton will play the role of the physical clock.

Since the WdW equation is akin to a Klein Gordon equation the natural norm:

$$\langle \hat{\Psi}, \hat{\Psi} \rangle = \int_0^\infty dl \; i(\hat{\Psi}^* \partial_l \hat{\Psi} - \hat{\Psi} \partial_l \hat{\Psi}^*),$$

$$\hat{\Psi} = \frac{\Psi}{l}$$

Wheeler deWitt equation

$$\partial_l \partial_\phi \hat{\Psi} + l \phi \hat{\Psi} = 0$$

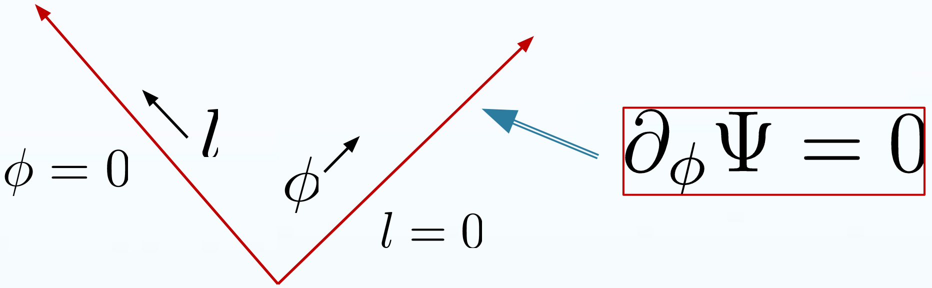
Norm Conservation:

Imposes condition

$$\mathcal{C}_N \equiv i(\hat{\Psi}\partial_\phi \hat{\Psi}^* - \hat{\Psi}^*\partial_\phi \hat{\Psi}) = 0 \quad \text{at } l = 0, \infty$$

The condition at $l = 0$ is in particular quite restrictive.

Norm Conservation Condition can be met by imposing suitable boundary conditions at $l = 0$.



(Analysis is best caried out in another basis, called the ``Rindler Basis")

Bottom line:

Boundary condition cuts down the number of states by half.

Number of states which remain are still infinite.

``Norm”

$$= 2\int_{k>0} dk \frac{\sinh(k\pi)}{\pi}\left(|a(k)|^2 - |b(k)|^2\right)$$

Not positive definite.

It is convenient to expand the wave functions in the ``Rindler Basis"

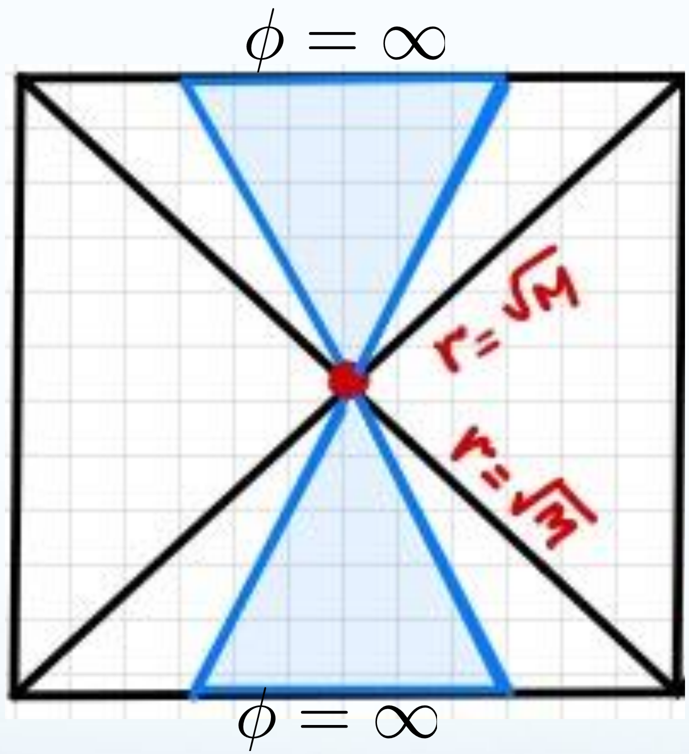
Defining : $\xi = l\phi \qquad e^{\theta} = \frac{\phi}{l}$

Solutions: $J_{\pm ik}(\xi)e^{\pm ik\theta}$

No leakage of probability:

$$\hat{\Psi} = \int_{k>0} [a(k)J_{-ik}e^{ik\theta} + b(k)J_{ik}e^{-ik\theta}$$

$$\Psi = \mathcal{N} e^{-il\phi} e^{-\sigma(\frac{l}{\phi} - \frac{1}{A})^2}$$



Running time back, the expanding universe goes through the singularity and emerges in the far past as a contracting one

$$\Psi = \tilde{N} e^{il\phi} e^{-\sigma(\frac{l}{\phi} - \frac{1}{A})^2}$$

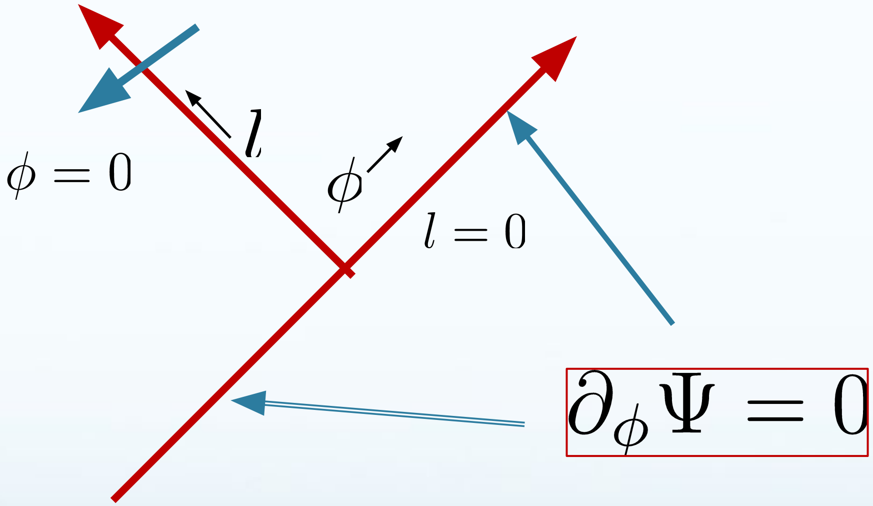
Range of Dilaton

In dimensionally reduced case there is a minimum value for ϕ

So far we have not considered what the range of ϕ is in the 2 dim quantum theory.

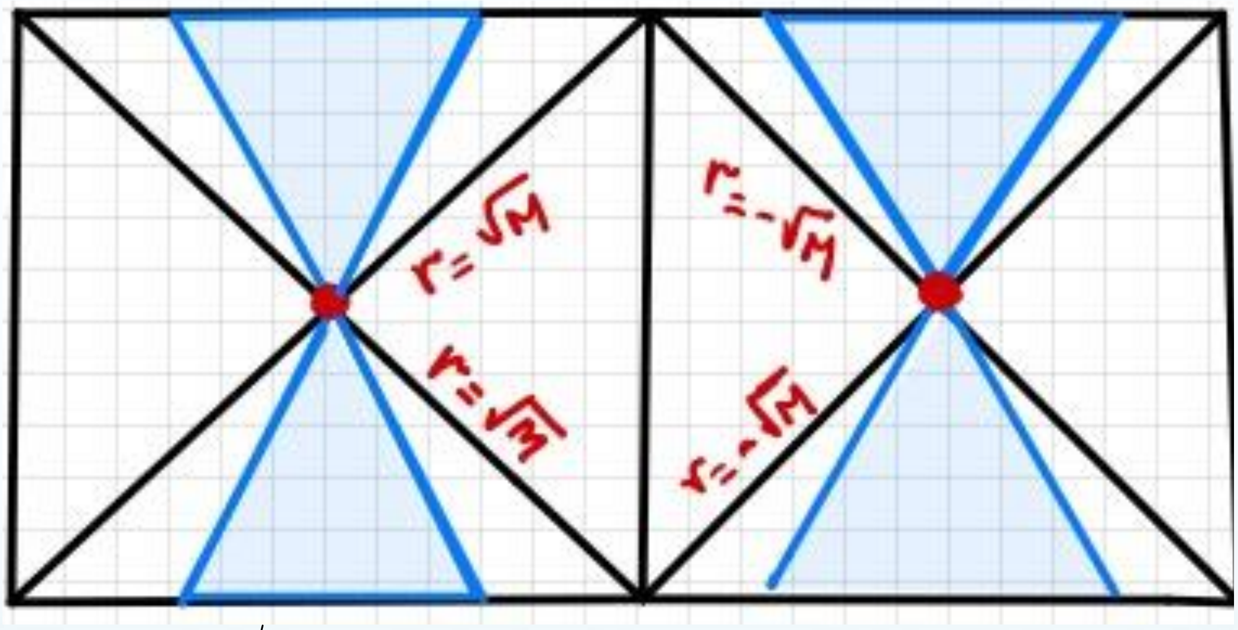
Making Ψ vanish at $\phi = 0$ leaves no non-trivial solutions.

Wave function can be analytically continued at $\phi = 0$



Now the wave function have non-zero components in all four regions

$\phi = \infty$ $\phi = -\infty$



$\phi = \infty$ $\phi = -\infty$

Other norms can also be defined.

Using the idea of group invariants.
(can be adapted to the discussion here as well).

One can also use a different clock to define physical time.

For example the extrinsic curvature of a hypersurface,

J. Held and H. Maxfield, hep-th/2410.14824.

Path Integral

There is a limited understanding based on the path integral for some of the other states.

Basic idea: Deform the JT theory, so that the corrections become unimportant when $\phi \to \infty$

HH state in the deformed theory gives rise to other states in the JT theory asymptotically.

Can be checked in some examples.

This is an example of mapping different states in dS to different theories.

- Chakraborty, Chakravarty, Godet, Paul, Raju, *JHEP* 12 (2023) 120, *JHEP* 01 (2024) 132

E.g.:

$$U'(\phi) = 2\phi + 2\sum_{i=1}^{n} \epsilon_i e^{-\alpha_i \phi}$$

$$\pi < \alpha_i < 2\pi \qquad \sum_i \epsilon_i = 0$$

The Path Integral can be done exactly.

Witten, Proc. Royal Soc. A 476 (2020) 20200582.
L. Eberhardt and G. J. Turiaci, arXiv:2304.14948 (2023)

$$\rho(M) = e^{S_0}\left[\frac{\sinh(2\pi\sqrt{M})}{4\pi^2} + \sum_i \frac{\epsilon_i}{2\pi\sqrt{M}} \cosh((2\pi - \alpha_i)\sqrt{M})\right]$$

HH State

Correction

Extensions of JT Theory

This idea of deforming the JT theory can be explored in canonical quantisation as well.

And in fact taken quite far.

Interestingly, the canonical quantisation can be carried out for a large class of potentials $U(\phi)$, by extending the procedure followed for JT theory.

(For branches which allow for a foliation along constant ϕ slices.)

Extensions of the JT Model

$$S_{\mathrm{JT}} = \frac{-i}{2}\left(\int d^2x\sqrt{-g}\,(\phi R - 2\phi) - 2\int_{bdy}\sqrt{-\gamma}\phi K\right)$$

$$S_{\mathrm{U}} = \frac{-i}{2}\left(\int d^2x\sqrt{-g}\,(\phi R - U'(\phi)) - 2\int_{bdy}\sqrt{-\gamma}\phi K\right)$$

$$U = \int U'(\phi)d\phi$$

$$U_{JT} = \phi^2$$

$$U(\phi) \to \phi^2$$

Solutions:

$$\Psi(\phi, l) = e^{-il\sqrt{U(\phi)-M}}; \qquad e^{il\sqrt{U(\phi)-M}}$$

Expanding branch — Contracting Branch

$$\Psi = \int dM [\rho(M) e^{-il\sqrt{U(\phi)-M}} + \tilde{\rho}(M) e^{il\sqrt{U(\phi)-M}}$$

Can construct a Hilbert space on constant ϕ slice. Etc.

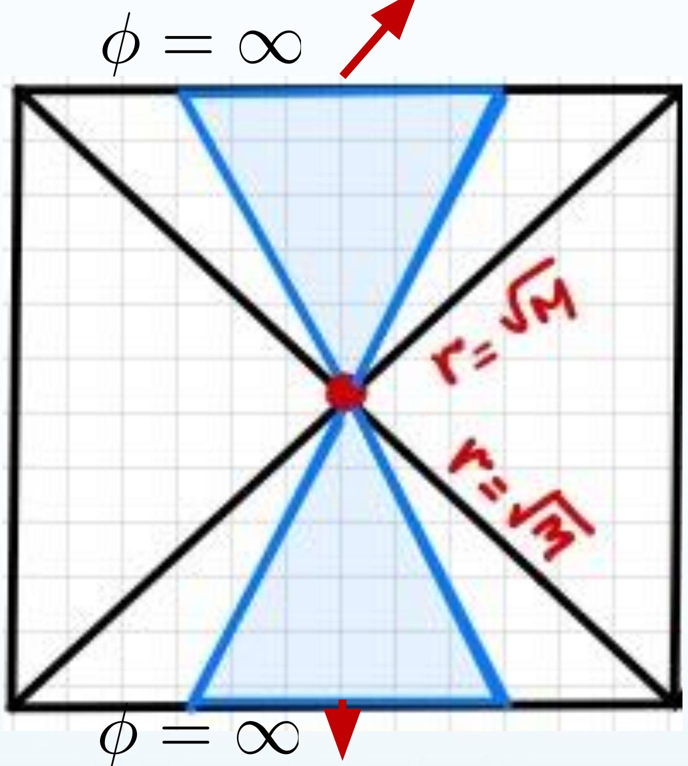
States in the deformed theory can be also interpreted as states in JT theory, asymptotically.

Away from asymptotic limit the behaviour can be different.

A large class of two dimensional cosmologies which can be canonically quantized are obtained in this way.

Holography

Boundary Hologram



Boundary Hologram

Forward/Past Regions of Big Bang/Big Crunch branch

Hartle Hawking State, in the expanding branch, can be expressed as a Trace

$$\Psi_{HH}(\phi, l) = e^{-il\phi} < Z(\beta = -\frac{il}{2\phi}) >$$

Matrix

$$\Psi_{HH}(\phi, l) = e^{-il\phi} < Tr(e^{\frac{ilM}{2\phi}}) >$$

In the Double Scaled Matrix Model, studied by Saad, Shenker, Stanford

(In asymptotic region $l, \phi \to \infty, \frac{l}{\phi} : \text{constant}$)

Other States in The Hologram

Other states can be expressed as a Generalized Trace in the Matrix Model.

Expanding Branch, asymptotically

$$\Psi(\phi, l) = e^{-il\phi} < Tr(f(M)e^{\frac{ilM}{2\phi}}) >$$

Different functions $f(M)$ correspond to different states.

Similarly for the contracting branch.

$$\Psi(\phi, l) = e^{il\phi} < Tr(\tilde{f}(M)e^{\frac{-ilM}{2\phi}}) >$$

Trace in the Double Scaled Matrix Theory

One-to-one map between solutions in bulk, in expanding branch, and states in the Matrix Theory.

$$\Psi_M(\phi, l) = e^{-il\phi} e^{\frac{ilM}{2\phi}} \leftrightarrow |E = M >$$

Leads to a norm and Hilbert space for the bulk states, namely the norm and Hilbert space in the Matrix theory.

$$(\Psi_f, \Psi_f) = \int dM \rho_{MM}(M) |f(M)|^2$$

Matrix Model density of states

$$\rho_{MM}(M) = \frac{e^{S_0}}{4\pi^2} \sinh(2\pi\sqrt{M})$$

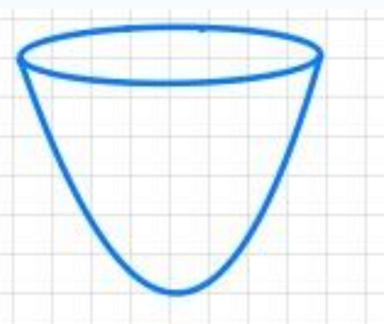
Manifestly positive definite.

(Hilbert Space is different from what we got by canonical quantisation on constant dilaton slices.

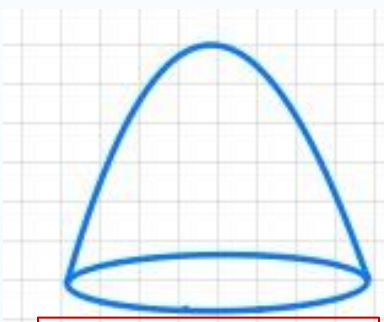
States which were thrown out due to norm conservation are included here.)

Matrix Theory computes Topology changing processes for the Hartle Hawking state.

These allow a single of multiple universes to be ``born” from nothing.



Nothing to One



One to Nothing

Nothing to Two

Two to Nothing

Matrix Theory results agree with the Path Integral (Maldacena, Turiaci, Yang)

Additional checks are needed, beyond single universe sector.

Better Path Integral understanding for general states will help.

Finite Rank and deSitter Entropy

JT theory is dual to the double scaled limit of Matrix theory, with rank $L \to \infty$

Now suppose JT theory can be modified so that the bulk theory is dual to a matrix theory at large but finite rank.

(Or dual to SYK model with finite but large number of flavours N)

Finite Rank and deSitter Entropy

At finite L

$$e^{S_0} = L$$

Topology counting parameter in JT/Bulk
=
Higher dimensional Entropy

Topology counting Parameter in Matrix Theory
=
Number of States

Suggesting that entropy of deSitter space can be accounted for by a hologram at $\mathcal{I}^{\pm}$

Finite Rank and deSitter Entropy

If after the modification the identification between bulk states and in the matrix theory continues to be valid

$$e^{S_0} = L$$

Corresponds to higher dim entropy.

Number of states

Suggesting that entropy of deSitter space can be accounted for in a hologram at $\mathcal{I}^{\pm}$

In the finite rank case corrections would set in when

$$\frac{l}{\phi} \sim O(1)$$

These will need to be understood as corrections in JT theory.

Conclusions

- Canonical quantisation revealed an infinite number of physical states.
- Suggests that states other than the Hartle Hawking state should be given further attention in Cosmology.
- Better Path Integral understanding of these states would be helpful

Conclusions

Holographic correspondence needs to be better understood.

Including the possibility of accounting for deSitter entropy.

Need to go beyond 2 dimensions

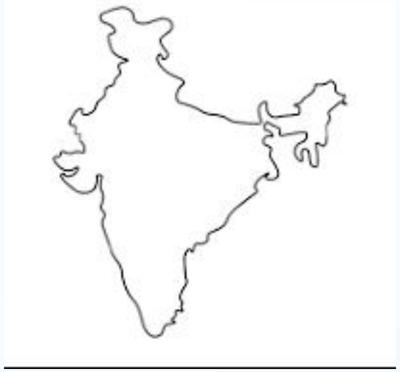
Shukran شكرا

Shukriya शुक्रिया

Thank You!

Several Directions For the Future

Perhaps Time Will Tell!



780 living languages

Holography

Hartle Hawking State (Expanding Branch)

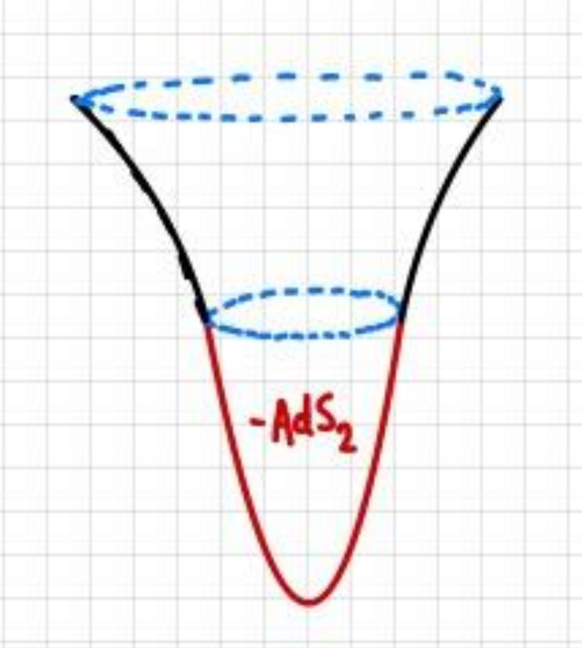
$$\Psi_{HH}(\phi, l) = e^{-il\phi}\frac{e^{S_0}}{4\pi^2}\int_0^\infty dM \sinh(2\pi\sqrt{M})e^{i\frac{lM}{2\phi}}$$

Density of states of Double Scaled Matrix Model with Energy E=M.

$$e^{-\beta M}$$

$$\beta = -\frac{il}{2\phi}$$

Perhaps not surprising since the HH state wave function can be obtained by analytic continuation from $-AdS_2$



What about other states?

$$\Psi(\phi, l) = \quad e^{-il\phi} \int_{M>0} dM \rho(M) e^{\frac{ilM}{2\phi}}$$

$$\rho(M) = f(M) \rho_{MM}(M)$$

$$\Psi(\phi, l) = e^{-il\phi} < Tr(f(M) e^{\frac{ilM}{2\phi}}) >$$

$$\rho_{MM}(M) = \frac{e^{S_0}}{4\pi^2} \sinh(2\pi\sqrt{M})$$

(Similarly for contracting branch)

Finite Rank and deSitter Entropy

Canonically quantising JT theory in the single universe sector suppresses topology change, $e^{S_0} \to \infty$

Topology counting parameter in JT, which corresponds to higher dim entropy.

Finite Rank and deSitter Entropy

In JT theory there is a topology counting Parameter S_0, which corresponds to higher dimensional entropy.

Working in the Single Universe sector corresponds to taking $S_0 \to \infty$

Perhaps not surprisingly then we found an infinite number of states.


有り難うございます。
धन्यवाद
ধন্যবাদ
ధన్యవాదాలు
ਧੰਨਵਾਦ
Npeże
நன்றி
Thank You
Khublai
Kadinche
ধন্যবাদ
આભાર

Example: Gaussian Wave packet

$$\hat{\Psi} = \frac{1}{l}\int_{-\infty}^{\phi^2} dM\,\rho(M)\; e^{-il\sqrt{\phi^2-M}} + \int_{\phi^2}^{\infty} dM\rho_1(M)e^{-l\sqrt{M-\phi^2}}$$

$$\rho(M) = 2\sin(Mx_0)e^{-\frac{1}{2\sigma}(M)^2},\; M \in [-\infty,\infty]$$
$$\rho_1(M) = 2\sin(Mx_0)e^{-\frac{1}{2\sigma}(M)^2}\; M \in [0,\infty]$$

Two parameters, σ, x_0

Take $x_0 > 0,$ $x_0 \sim O(1)$

Gaussian Wave Packet

For sufficiently late time, $\phi \gg \sigma^{\frac{1}{4}}$

Wave function takes form

$$\hat{\Psi} = \frac{\sqrt{2\pi\sigma}}{2il} e^{-il\phi} \left(e^{-\frac{\sigma}{2}(x_0 + \frac{l}{2\phi})^2} - e^{-\frac{\sigma}{2}(-x_0 + \frac{l}{2\phi})^2} \right)$$

$$\hat{\Psi} \simeq -\frac{\sqrt{2\pi\sigma}}{2il} e^{-il\phi} e^{-\frac{\sigma}{2}(-x_0 + \frac{l}{2\phi})^2}$$

If $\sigma \gg 1$ we have a ``sharp peak”

Purely expanding universe.

(momentum π_l has the correct sign).

$$\frac{l}{\phi} = 2x_0$$

$$\frac{\langle l^2\rangle - \langle l\rangle^2}{\langle l\rangle^2} \sim \mathcal{O}\left(\frac{1}{\sigma}\right).$$

Similarly $\langle \pi_l \rangle = -\phi$

Agrees with the classical solutions (at late time) we had found earlier.

(More general wave packets can also be constructed which depend on two parameters, corresponding to A, m, in the classical solutions).

Interestingly, Hartle Hawking state is not well behaved.

Norm diverges.

$$\Psi = \int_0^{\phi^2} dM \sinh(2\pi\sqrt{M}) e^{-il\sqrt{\phi^2 - M}} + \int_{\phi^2}^{\infty} dM \underbrace{\sinh(2\pi\sqrt{M})}_{} e^{-l\sqrt{M-\phi^2}}$$

Diverges at $l = 2\pi$

Note:

It might be premature to drop states which are not normalisable.

Once matter is added some observables could be well defined. e.g., correlations of the matter fields at fixed values of l, ϕ

$$\hat{\Psi}(l, \phi, \zeta)$$

↘ Matter

Concluding Comments And Summary

- The number of allowed physical states is infinite. Even after imposing various conditions: a finite norm, finite expectation values, good classical limit.

- The Hartle Hawking state (and related Vilenkin state) have received a lot of attention, but there are many other states, in some ways better behaved.

Concluding Comments And Summary

- Understanding these states from the path integral perspective will be worthwhile.

- Could potentially be of interest for the real world.

(U. Moitra, S. Sake, S.P.T JHEP 06 (2022) 138)

Concluding Comments And Summary:

The coefficient function ρ (and $\tilde{\rho}$) give the amplitude for various values of the momentum P.

But unlike the AdS case ρ can be complex.

This suggests the hologram of dS has some key new features.

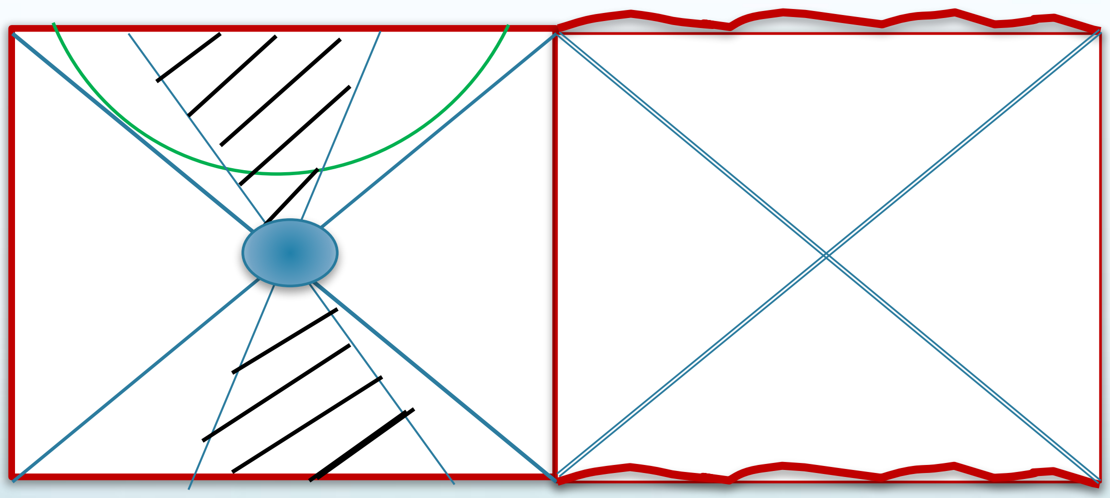
Concluding Comments And Summary:

In some cases a conserved norm only arises at late times, $\phi > \phi_c$

(determined by ρ)

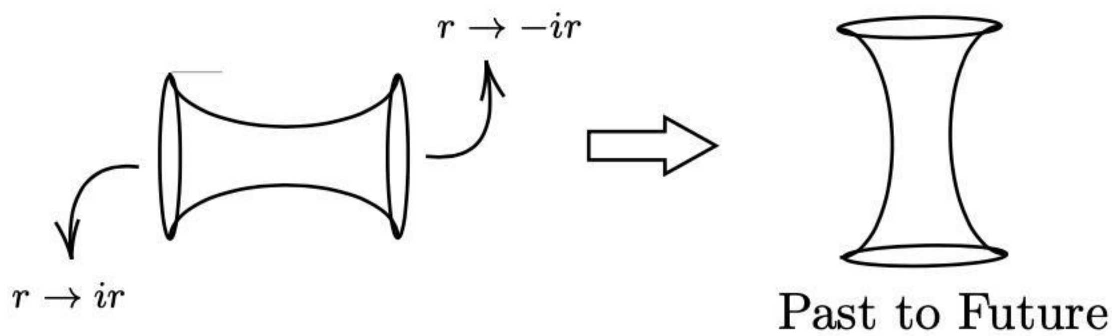
For earlier times, $\phi < \phi_c$ one does not have a conserved norm and conventional (Hamiltonian) quantum mechanics

Occurs for solutions where we sum over Orbifolds of Black Hole Geometries. Suggests some leakage of probability for $\phi < \phi_1$ near the orbifold region.



$\phi > 0$

Amplitude for tunneling from a contracting orbifold geometry in far past to far future is non-vanishing



(Moitra, Sake, SPT)

Resulting matter power spectrum has deviations, from the Bunch Davies vacuum, for correlations at large wave lengths.

In other cases the norm is not conserved even at late times, to a small extent.

Suggests a breakdown of conventional quantum mechanics even at late times.

Concluding Remarks And Summary

In yet other cases the behaviour is perfectly well defined (with conserved norm etc) all the way till $\phi = 0$ and even beyond for $\phi < 0$

(for geometries of non-orbifold type)

Behaviour in the presence of matter needs to be studied (in semiclassical approximation matter back reaction is finite).

Quantisation with matter may not be too hard.

Interestingly, Hartle Hawking state is not well behaved.

Norm diverges.

$$\hat{\Psi}(l,\phi) = \left|\hat{\mathcal{N}}\right| e^{S_0} \frac{\phi^2}{(l^2 - 4\pi^2)} \left(e^{i\alpha} H_2^{(2)}(\phi\sqrt{l^2 - 4\pi^2}) + e^{-i\alpha} H_2^{(1)}(\phi\sqrt{l^2 - 4\pi^2})\right)$$

↓ Contracting Branch ($e^{i\alpha} H_2^{(2)}$ term)

↓ Expanding Branch ($e^{-i\alpha} H_2^{(1)}$ term)

Path Integral gives (Moitra, Sake, SPT):

$$e^{i\alpha} = \pm i$$

Near turning point gives rise to divergence

$$\hat{\Psi} \sim \frac{1}{(l-2\pi)^2}$$

(unless $\alpha = 0, \pi$)

(See also: G. Fanaras and A. Vilenkin)

Summary:

- We canonically quantized JT gravity in dS space and found all the gauge invariant states. (Henneaux)
- There are an infinite number of such states
- With dilaton as the physical clock, we constructed a space of physical states, defined their norm and expectation values for operators on these states.

Summary

- Imposing various conditions, like a finite norm, finite expectation values, good classical limit, cut down the number of states, but still left many allowed ones.

- The Hartle Hawking state (and related Vilenkin state) have received a lot of attention, but there are many other states, in some ways much better behaved.

Some, but not all, states have a norm which is conserved and finite.

Some, but not all, have a good classical limit.

Even after imposing all these requirements we found there were an infinite number of solutions.

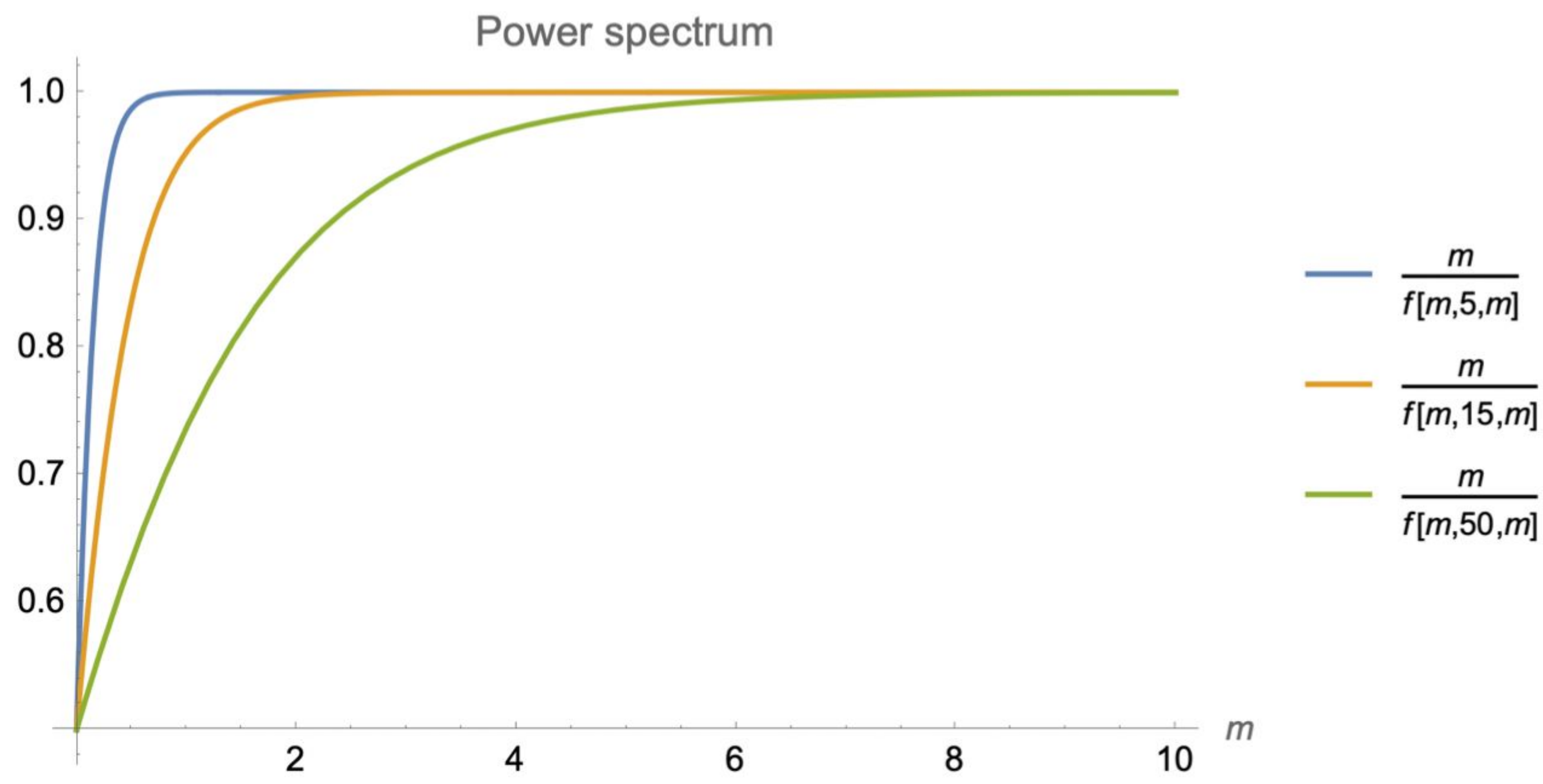
The Hartle Hawking (and related Vilenkin) states have received a lot of attention. But there are many others, in some ways even better defined,

Going forward:

- Understand the extra states we have found using path integral method. What replaces the no-boundary proposal?
- Add matter
- Consider inflation.

Additional states could have interesting departures in CMB spectrum.

One example (shown in Moitra, Sake SPT), using path integral methods, gives spectrum with deviations at longer wave lengths (small l)


Power spectrum
1.0
0.9
0.8
0.7
0.6
2
4
6
8
10
m
m / f[m,5,m]
m / f[m,15,m]
m / f[m,50,m]

Note that the solutions we have obtained turn out to be closely related to those of a Klein Gordon equation for a massive field:

$$(\partial_l \partial_\phi + \phi l)\hat{\Psi}[\phi, l] = 0.$$

$$\hat{\Psi} = \frac{1}{l}\Psi$$

(Equation is First order in ∂_ϕ)

Klein Gordon Equation:

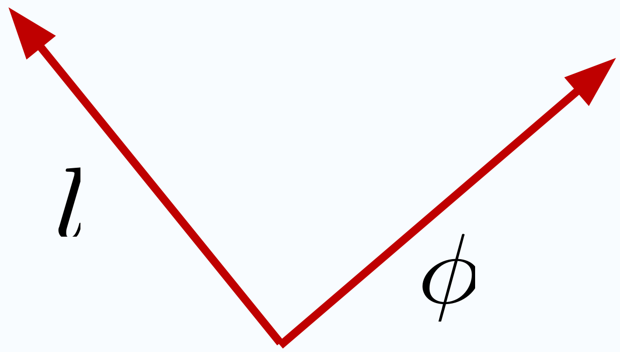
$$(\partial_l \partial_\phi + \phi l)\hat{\Psi}[\phi, l] = 0.$$

$$\partial_u \partial_v \hat{\Psi} + \frac{1}{4}\hat{\Psi} = 0$$

$$u = l^2, \quad , v = \phi^2$$

Klein Gordon equation for field of mass ¼.

Hyperbolic equation. Infinite number of solutions



ϕ, l are null directions.

Minisuperspace approximation ϕ, g_1

Independent of x and only dependent on t.

$$ds^2 = -dt^2 + g_1 dx^2$$

$$I = \int d^2x[-\dot{l}\dot{\phi} - l\phi]$$

$$H = -\pi_l \pi_\phi + l\phi$$

Minisuperspace approximation ϕ, g_1

Independent of x and only dependent on t.

$$ds^2 = -dt^2 + g_1 dx^2$$

$$I = \int d^2x[-\dot{l}\dot{\phi} - l\phi]$$

$$H = -\pi_l \pi_\phi + l\phi$$

Hamiltonian constraint (also referred to as Wheeler Dewitt constraint) leads to :

$$(\partial_l \partial_\phi + \phi l)\hat{\Psi}[\phi, l] = 0.$$

$$\hat{\Psi} = \frac{1}{l} e^{\pm i l \sqrt{\phi^2 - M}}$$

(Note, $\hat{\Psi}$, not, Ψ)

(Illiesiu, Kruthoff, Turiaci, Verlinde; Maldacena)

Similar to what we got above.

Not identical due to the ordering ambiguities.

Covariant Phase Space:

The phase space corresponding to the classical solutions

$$ds^2 = -\frac{dr^2}{r^2 - m} + (r^2 - m)dx^2$$

$$\phi = Ar$$

Is two dimensional.

The symplectic form in phase space can be worked out. (Crnkovic, Witten)

Setting, $M = mA^2$

We get:

$$[M, \frac{1}{A}] = \frac{i}{\pi}$$

An eigenstate of M, with eigenvalue, -M, is then

$$\Psi = e^{i\,\frac{\pi M}{A}}$$

In the asymptotic dS limit

$$l, \phi \to \infty, \frac{l}{\phi} \text{ fixed}$$

They come close.

$$\Psi_{cov} = e^{+i\frac{Ml}{2\phi}}$$

$$\Psi = e^{-il\phi} e^{+i\frac{Ml}{2\phi}}$$

Away from this limit it would be nice to understand the connection between the two ways of quantisation better.

Using the relation (from the classical solutions) that

$$\frac{1}{A} = \frac{l}{2\pi\sqrt{\phi^2 - M}}$$

We find that $\Psi_{cov} = e^{i\frac{Ml}{2\sqrt{\phi^2 - M}}}$

Which is quite different from what we got earlier. $\Psi = e^{-il\sqrt{\phi^2 - M}}$