Einstein Gravity from Matrix Integral

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CERN

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Based on works 2410.18173 and 2411.18678 with

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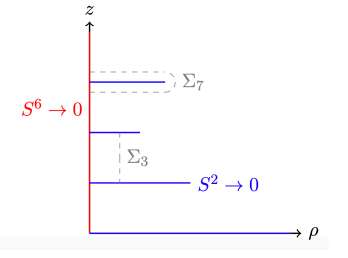
See also 2409.18706 by Sean Hartnoll & Jun Liu

Punchlines

Proposal:

- Duality between mass-deformed IKKT matrix integral and IIB backgrounds with exceptional F_4 SUSY.

$$Z = \int d^{10} X_I d^{16} \psi_\alpha \, e^{-S} \quad \longleftrightarrow$$



$$\times S^2 \times S^6$$

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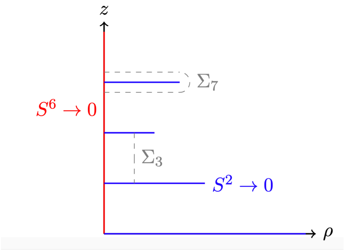

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Methods:

- SUSY localization of matrix integral
- Construction of BPS SUGRA solution.

Each saddle point of integral corresponds to a different SUGRA solution

cf. For mass-deformed BFSS (BMN): [Asano, Ishiki, Okada, Shimasaki] [Lin, Maldacena]

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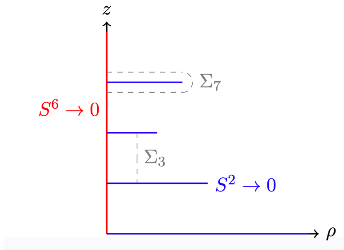

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Evidence:
- Saddle point equations = Data charactering SUGRA solutions

Other check: Probe brane analysis cf. [Hartnoll, Liu]

Plan

1. Review of IKKT & mass deformation
2. Analysis of matrix model
3. Analysis of gravity dual
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(Euclidean) IKKT matrix model

[Ishibashi, Kawai, Kitazawa, Tsuchiya]

$$S_{\rm IKKT} = {\rm Tr}\left(-\frac{1}{4}[X_I, X_J]^2 - \frac{i}{2}\bar{\Psi}\Gamma^I[X_I, \psi]\right)$$

- Theories of $N \times N$ matrices with 16 SUSY + SO(10) symmetry
- Low energy dynamics of D(-1) branes
- Proposed originally as non-perturbative def of IIB string (cf. BFSS conjecture)

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- Low energy dynamics of D(-1) branes
- Proposed originally as non-perturbative def of IIB string (cf. BFSS conjecture)
- The simplest top-down example of holography (with Einstein gravity)??
- Theories without time; emergence of time???

Holography for IKKT?

- Not many works on holography for IKKT in the past.

Matrix model side:

- Exact result for partition function (from SUSY localization):

[Green-Gutperle], [Moore-Nekrasov-Shatashvili], [Krauth-Nicolai-Staudacher]

$$Z_{\mathrm{IKKT}} = \frac{(2\pi)^{(10N+11)(N-2)/2}}{N^{3/2}\prod_{k=1}^{N-1}(k!)}\sigma_2(N)$$

$$\sigma_2(N) = \sum_{d|N} d^2$$

divisor function

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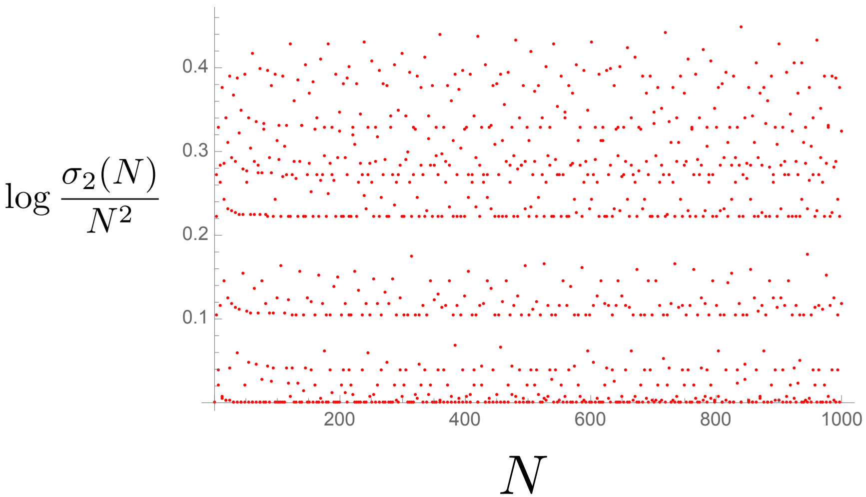

- Used for computing instanton correction to 4 graviton amplitudes in [Sen]

cf. [Green, Gutperle], [Green, Vanhove]

- No smooth large N limit ("erratic" in N)

Holography for IKKT?

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Gravity side:

- D(-1) brane geometry and its "decoupling limit"

[Gibbons-Green-Perry], [Ooguri-Skenderis]

$$ds^2_{\text{Einstein}} = (dx_I)^2$$

$$\tau = \chi + ie^{-\phi} = \frac{2i}{g_s\left(1 + \frac{g_s N \ell_s^8}{r^8}\right)}$$

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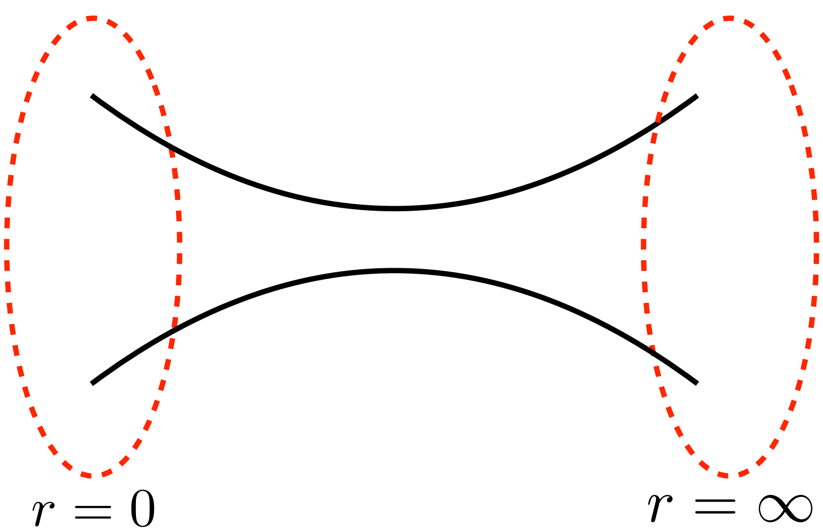


- String frame metric has $r \leftrightarrow 1/r$ symmetry

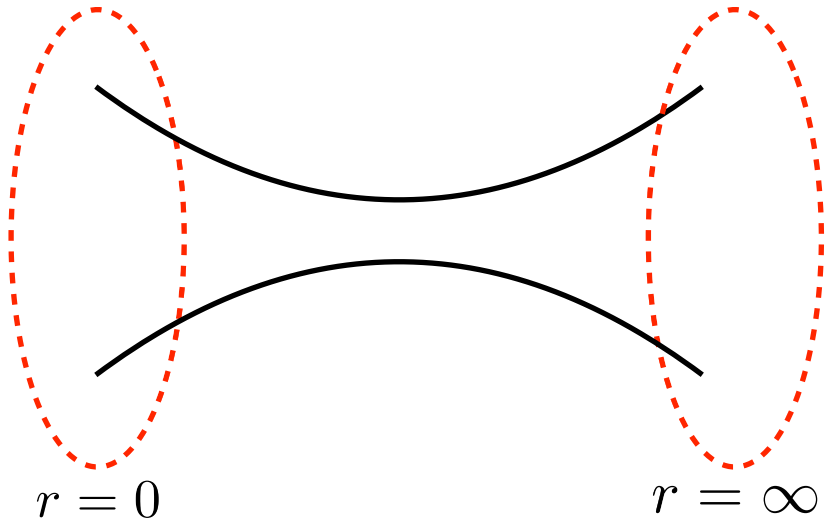
“wormhole” [Gibbons-Green-Perry]

- “Decoupling limit”

$$\tau \propto \frac{1}{\cancel{1}+\frac{\#}{r^8}}$$

[Ooguri-Skenderis]

“Decoupling limit”?



- Normally, excitations near the throat $\leftrightarrow$ low energy modes on D-branes
- Gravity side: Flat metric. No redshift.
- Matrix model side: No time / energy, no clear notion of energy scale.
- No smooth large N limit of partition function.
- Better setup for studying holography?

Mass deformed IKKT ("Polarised IKKT")

[Bonelli]

$$S_\Omega = S_{\mathrm{IKKT}} + \mathrm{Tr}\left(\frac{3\Omega^2}{4^3} \sum_{i=1}^{3} X_i^2 + \frac{\Omega^2}{4^3} \sum_{p=4}^{10} X_p^2 + i\frac{\Omega}{3}\epsilon^{ijk} X_i X_j X_k - \frac{\Omega}{8}\bar{\psi}\Gamma^{123}\psi \right)$$

- SO(10) → SO(3) × SO(7)
- 16 SUSY (same as IKKT): exceptional F_4 Lie superalgebra

cf. 5d superconformal group

- Standard 't Hooft expansion.

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  cf. 5d superconformal group
- Standard ’t Hooft expansion.
- Classical vacua

$$[X_i, X_j] = \frac{3i\Omega}{8}\epsilon^{ijk}X_k$$

Fuzzy sphere

$$X_{4,\dots,10} = 0$$

$$X^i = \frac{3\Omega}{8}\begin{pmatrix} L^i_{N_1}\otimes 1_{n_1} & & & \\ & L^i_{N_2}\otimes 1_{n_2} & & \\ & & \ddots & \\ & & & L^i_{N_q}\otimes 1_{n_q} \end{pmatrix}$$

$L^i_{N_k}$: N_k-dimensional rep of $SU(2)$. n_k: multiplicity of each rep

# of vacua = # of partitions of N

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SUSY localization

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- Following [Moore-Nekrasov-Shatashvilli] [Asano-Ishiki-Okada-Shimasaki], one can compute the partition function using SUSY localization.
- Deform the action by $+tQ\Psi$ and send $t \to \infty$. Saddle + 1-loop = exact
- Localization saddle:

$$X^i = \frac{3\Omega}{8}\begin{pmatrix} L^i_{N_1}\otimes 1_{n_1} & & & \\ & L^i_{N_2}\otimes 1_{n_2} & & \\ & & \ddots & \\ & & & L^i_{N_q}\otimes 1_{n_q} \end{pmatrix}$$

$$X^{4,\ldots,9} = 0$$

$$X^{10} = \begin{pmatrix} 1_{N_1}\otimes M^{(1)} & & & \\ & 1_{N_2}\otimes M^{(2)} & & \\ & & \ddots & \\ & & & 1_{N_q}\otimes M^{(q)} \end{pmatrix}$$

$M^{(k)}$: diagonal matrix of size n_k

SUSY localization

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- Partition function:

$$Z = \sum_P Z_P \qquad\qquad P: \ \text{partition of } N$$

$$\boxed{Z_P \propto \int \prod_{s=1}^{q} \prod_{i=1}^{n_s} dm_i^{(s)} \ Z_{\text{1-loop}} \, e^{-\frac{3\Omega^4}{2^7} \sum_{s,i} N_s (m_i^{(s)})^2}}$$

$$Z_{\text{1-loop}} = \prod_{(si,tj)} f_{st}(m_i^{(s)} - m_j^{(t)})$$

"generalized van-der-monde factor"

$$f_{st}(x) = \prod_{J=\frac{|N_s-N_t|}{2}}^{\frac{N_s+N_t}{2}-1} \frac{((3J+2)^2+(8x)^2)^3((3J)^2+(8x)^2)}{((3J+1)^2+(8x)^2)^3((3J+3)^2+(8x)^2)}$$

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- When $n_s \gg 1$, standard large N analysis gives

$$\boxed{\frac{3\Omega^4}{2^7} N_s x^2 - \mu_s + \frac{2}{3} \sum_t \int dy \frac{((\frac{3}{2}|N_s - N_t|)^2 - (8(x-y))^2}{\left((\frac{3}{2}|N_s-N_t|)^2 + (8(x-y))^2\right)^2} \rho^{(t)}(y) = 0}$$

$\rho^{(s)}(x):$ eigenvalue density

Surprise

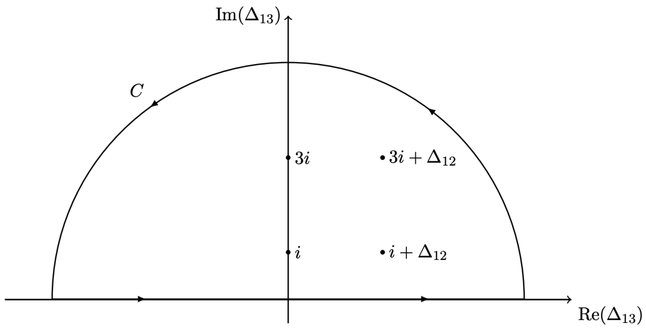
$$\lim_{\Omega \to 0} Z^{(\Omega)}_{\text{polarized}} \neq Z_{\text{IKKT}}$$

- Partition function

$$Z^{(\Omega)}_{\text{polarized}} \overset{\Omega \to 0}{\sim} \Omega^{-2N} \to \infty$$

$$Z_{\text{IKKT}} \propto \sigma_2(N) : \ \text{finite}$$

- Instead of computing the localization integral on the real axis, evaluating the integral by residues on UHP gives Z_{IKKT}



$$\overset{\Omega \to 0}{\sim} Z_{\text{IKKT}}$$

- Physical implication? (More later)

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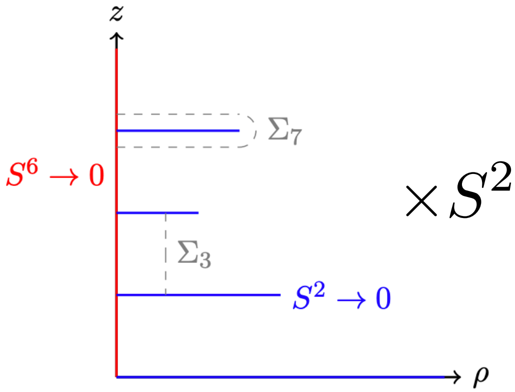
IIB background with exceptional F_4

- BPS solutions dual to 5d SCFTs preserving exceptional F_4 SUSY have been constructed in [d'Hoker, Gutperle, Karch, Uhlemann] [Legramandi, Nunez]
- Appropriate modifications of their analysis give

$$ds^2 = H(\rho,z)^2(d\rho^2 + dz^2) + R_2(\rho,z)d\Omega_2^2 + R_6(\rho,z)^2 d\Omega_6^2$$

cf. [Lin-Lunin-Maldacena], [Lin-Maldacena]



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with $D1$, $NS5$, $D5$-charges

(no $F1$ charges)

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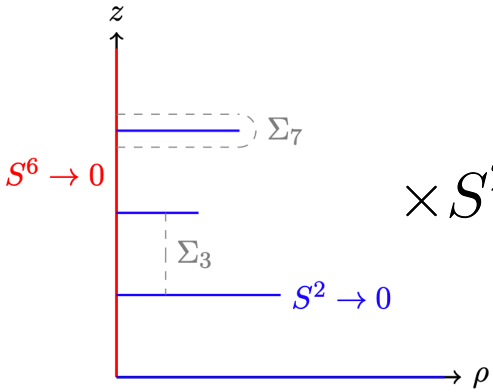

$\times S^2 \times S^6$ with $D1$, $NS5$, $D5$-charges (no $F1$ charges)

- At $z, \rho \gg 1$, it approaches flat metric of [Ooguri-Skenderis] with const C_2 flux.

$$dC_2 = -i\mu dx^1 \wedge dx^2 \wedge dx^3 \qquad \mu \sim \Omega$$

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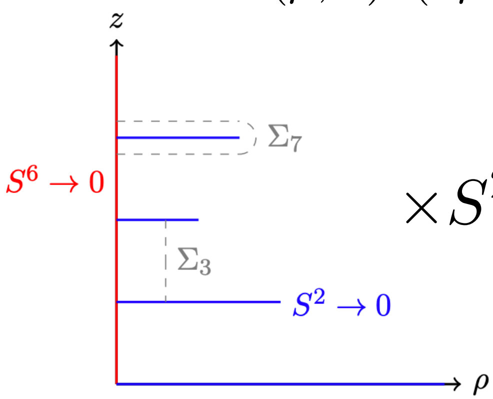

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$$dC_2 = -i\mu dx^1 \wedge dx^2 \wedge dx^3 \qquad \mu \sim \Omega$$

- H and $R_{2,6}$ are given by axi-sym solution to 4d electrostatic potential V

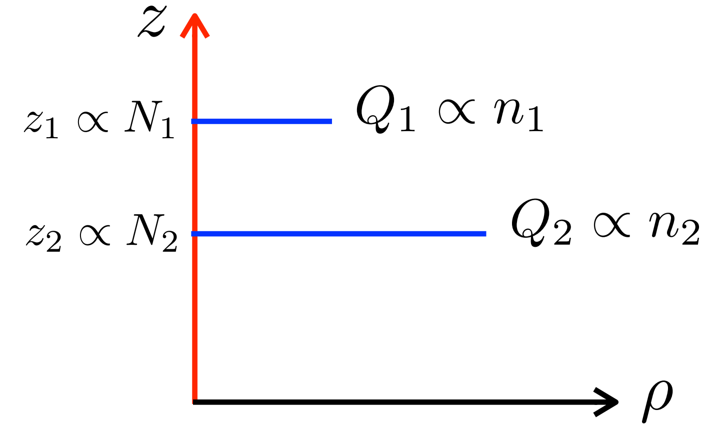
$$\partial_z^2 V + \frac{2}{\rho}\partial_\rho V + \partial_\rho^2 V = 0$$

Boundary condition: Horizontal lines (blue) = conducting balls ($V =$ const)

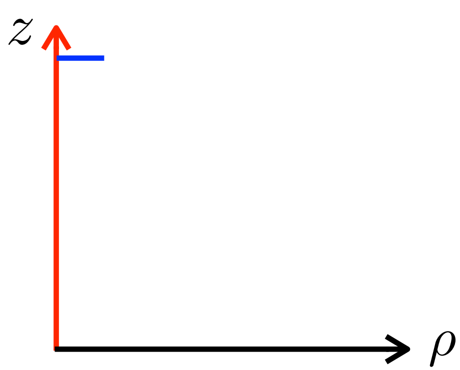
+ background potential $\mu^5(z\rho^2 - z^3)$ (for positivity of metric)

Correspondence with matrix side

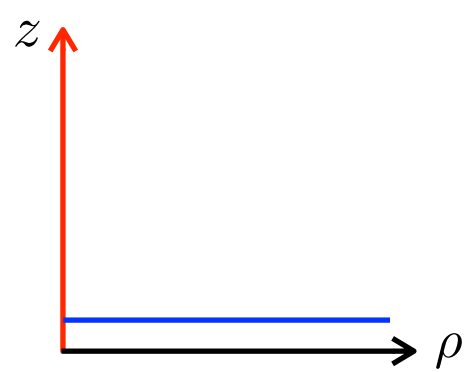
$$X^i = \frac{3\Omega}{8}\begin{pmatrix} L^i_{N_1}\otimes 1_{n_1} & & & \\ & L^i_{N_2}\otimes 1_{n_2} & & \\ & & \ddots & \\ & & & L^i_{N_q}\otimes 1_{n_q} \end{pmatrix}$$



- Size of irrep $N_k \longleftrightarrow$ Height of conducting ball $z_k = NS5$ charges
- Multiplicity $n_k \longleftrightarrow$ Charge of conducting ball $Q_k = D1$ charges



spherical (S^2) probe $D1$-brane



spherical (S^6) probe $NS5$-brane

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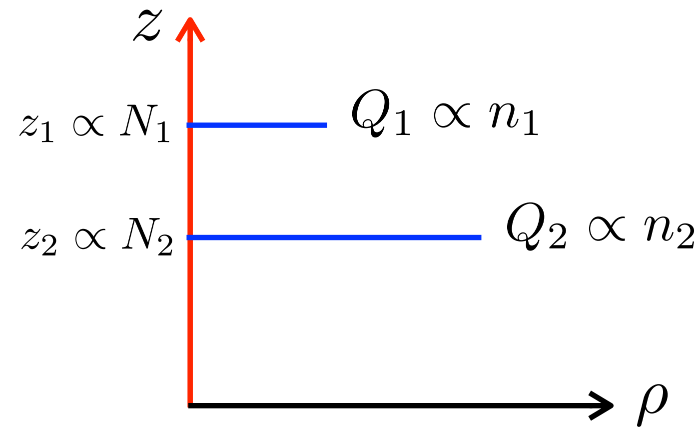

- Size of irrep N_k $\longleftrightarrow$ Height of conducting ball z_k $= NS5$ charges
- Multiplicity n_k $\longleftrightarrow$ Charge of conducting ball Q_k $= D1$ charges
- Requiring the potential V to be constant on each ball gives

$$\underbrace{\frac{3\Omega^4}{2^7}N_s x^2 - \mu_s}_{\text{Constant potential + bkd}} + \underbrace{\frac{2}{3}\sum_t \int dy \frac{((\frac{3}{2}|N_s - N_t|)^2 - (8(x-y))^2}{\left((\frac{3}{2}|N_s - N_t|)^2 + (8(x-y))^2\right)^2}\rho^{(t)}(y)}_{\text{Potential created by each ball}} = 0$$

$\rho^{(s)}(x)$: charge density

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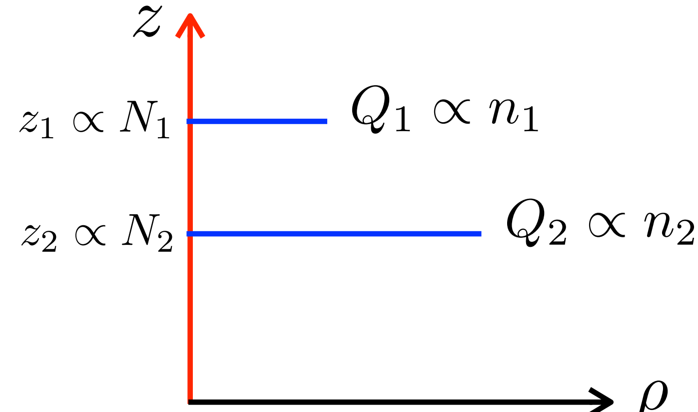

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$\rho^{(s)}(x)$: charge density

- Precise agreement with the saddle-point eq of SUSY localization

Implication on undeformed IKKT?

- In many ways, polarized IKKT is better behaved than the original IKKT.

IKKT

- No smooth large N limit
- Gravity dual is not fully understood

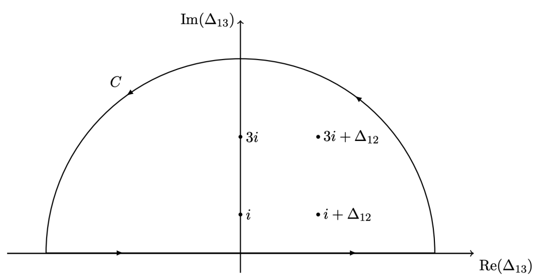
Polarized IKKT

- Standard large N expansion
- Some quantitative match with gravity dual

- Partition function $$\lim_{\Omega\to 0} Z^{(\Omega)}_{\text{polarized}} \neq Z_{\text{IKKT}}$$

$$Z^{(\Omega)}_{\text{polarized}} \overset{\Omega\to 0}{\sim} \Omega^{-2N} \to \infty$$

- By evaluating localization integral by residues on UHP gives Z_{IKKT}



$$\overset{\Omega\to 0}{\sim} Z_{\text{IKKT}}$$

- The asymptotic metric $(\rho, z \gg 1)$ becomes [Ooguri-Skenderis] when $\Omega \to 0$.

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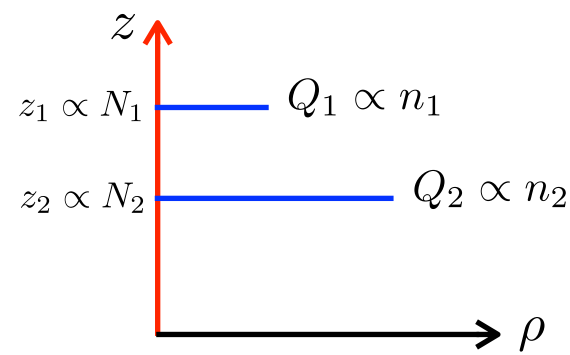

- Proposed duality between polarized IKKT & IIB SUGRA backgrounds
- SUSY localization

$$Z = \sum_P Z_P \qquad\qquad Z_P \propto \int \prod_{s=1}^{q}\prod_{i=1}^{n_s} dm_i^{(s)}\, Z_{\text{1-loop}}\, e^{-\frac{3\Omega^4}{2^7}\sum_{s,i} N_s (m_i^{(s)})^2}$$

- Saddle point equations = Data charactering SUGRA solutions

Some loose ends

- Evaluating on-shell SUGRA action (determining the boundary term)
- Prove $\lim_{\Omega\to 0} Z^{(\Omega)}_{\text{residue}} = Z_{\text{IKKT}}$ for general N
- SUGRA solutions with D7 branes?

Conclusion

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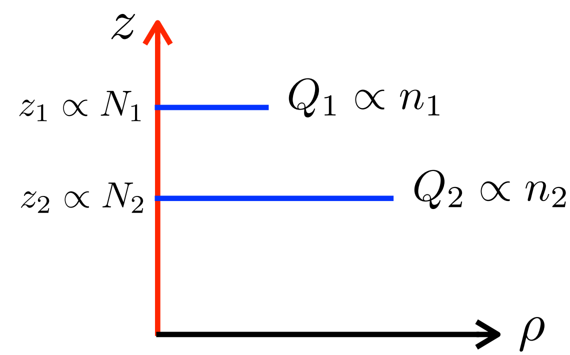

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- Holographic dictionary? (Operator insertions ↔ supergravity modes?)
- Localization of correlation functions ($\mathrm{Tr}\left((X_3 + iX_{10})^m\right)$ etc.)
- Matrix bootstrap? (Measure is not positive) [Lin], [Kazakov, Zheng],….
- Implication to original "IKKT conjecture" ? (Non-perturbative def of IIB string in flat space)

$\mathrm{Tr}(e^{ikX}) \leftrightarrow$ graviton vertex op? Lorentzian IKKT? (Divergent partition fn)